Social Preferences and Skill Segregation[*]

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Abstract

This paper shows that models where preferences of individuals depend not only on their allocations, but also on the well-being of other persons, can produce both large and testable effects. We study the allocation of workers with heterogeneous productivities to firms. We show that even small deviations from purely "selfish" preferences leads to widespread workplace skill segregation. That is, workers of different abilities tend to work in different firms, as long as they care somewhat more about the utilities of workers who are "close".

Keywords: contract theory, mechanism design, envy, social preferences, skill segregation.

JEL Classification: A13, C72, D64, D80, J41.

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1 Introduction

We have by now ample evidence that preferences of individuals between allocations do not depend only on their own material well-being. Rather, the actions and material allocations of other individuals impact directly a person's utility, and are thus taken into account when making a decision. But the research in models of "social preferences," as they are sometimes called, has not delivered empirical implications which change qualitatively our view of economic behavior. We show, however, that these models produce both large and testable e¤ects. We study worker allocation to …rms in a contract-theoretic framework, where agents di¤er in their productivity. We show that even small deviations from purely "sel…sh" preferences leads to widespread workplace skill segregation.

The current interest in social preferences' models arises in a large part to explain "anomalous" results from experimental economics. The papers in the area typically devote entire sections to show that their models can robustly account for the data generated by many di¤erent experiments. In doing so, they often estimate coe¢cients for the models. The coe¢cients estimated are, however, typically small, even for the relatively small stakes games played in the laboratory. The approach is, then, subject to the criticism that social preferences will lead only to small scale e¤ects in the real world. Therefore, it could be argued that it is not useful to incorporate them into mainstream models of labor markets, consumer behavior, and so on. Our aim is to show that this view is incorrect.

We study a labor market in which …rms compete for workers of heterogeneous (and unobservable) quality by o¤ering (menus of) contracts. Social preferences' models involve interpersonal comparisons of utility across agents. It is natural to assume that these comparisons do not necessarily span the whole population, but only individuals who are "close." This is implicitly acknowledged by current research on social preferences, as, in the typical application, the comparisons are only among agents playing a particular game. However, the range of interpersonal comparisons has been a generally neglected issue. To make the notion of closeness precise, we introduce a spatial structure in the model. Firms choose locations in a ring, and workers compare their material payo¤s to those of workers in their same …rm and in other …rms located within a certain distance in the ring.[1]

The e¢ciency units of workers' labor are perfect substitutes but the individual endowments of e¢ciency units are the private information of each worker. That is, some workers are more productive/skilled than others, but workers of di¤erent skills are perfectly substitutable in some …xed proportions. With this structure, and the traditional "sel…sh" preferences, the equilibria would not make a prediction on the distribution of skill levels by …rm or location. Any distribution would be consistent with equilibrium. With the introduction of social preferences, of however small strength, the equilibrium becomes both skill and spatially segregated, that is, …rms hire only from one skill pool and …rms employing workers of a given skill level form spatial

[1]Fehr et al. (2000) and Fershtman et al. (2001) are other attempts to embed social preferences in standard economic models. Those papers consider a moral hazard contracting setting with fairness-minded agents, and analyze the optimal incentive structure and workforce …rm composition. As in our paper, social preferences, however small, generate interesting twists with respect to the traditional approach with self-interested individuals.

clusters.[2]

The segregation and clustering results would also hold in a model with complete information. We introduce incomplete information for two reasons. First of all, the incomplete information makes it more evident that the externality driving segregation is di¤erent than the one in models of say, racial segregation. We deal here with a pecuniarity externality, that is, high-skilled types do not separate from low-skill types because they intrinsically dislike them. They do it, rather, because the market tends to produce di¤erent material payo¤s for both. Second, having a model that is robust to incomplete information is an obvious strength that is introduced at a relatively low complexity cost.

2 Background and related work

We bring together several strands of the economics literature.

Research on social preferences originated in large measure to give account of the growing empirical and experimental evidence that human behavior could not be explained only by the hypothesis of self-interested material payo¤ maximization. For instance, contribution to public goods is higher than would be expected under purely sel…sh maximization.[3] More importantly from our point of view, there is vast amounts of evidence that people reject lopsided o¤ers in ultimatum bargaining games (Güth, Schmittberger and Schwarze 1982).[4] Several models have been proposed to account for these observations. Bolton (1991), Rabin (1993), Levine (1998), Bolton and Ockenfels (1999), Fehr and Schmidt (2000a), Charness and Rabin (2000). It would be too di¢cult to discuss all those models in detail, so we refer to the excellent surveys of Sobel (2000) and Fehr and Schmidt (2000b). A feature that many of the models share is that individuals dislike payo¤ inequality. Our innovation with respect to this literature is that we think explicitly about the set of individuals to which the utility comparisons apply. We also provide further testable implications for the model (and implicitly relevant economic applications).

There is also evidence that …rms workforces are more homogenous than simple "random matching" would suggest. People of di¤erent skill levels sort themselves into di¤erent …rms. For instance, Kramarz et al. (1996) compute a measure a specialization for di¤erent professional categories proposed by Kremer and Maskin (1996). They …nd that specialization increased massively in France between 1986 and 1992.[5] Davis and Haltinwanger (1991) note the continuous rise in wage inequality in the U.S. which is imputable in part to ability sorting of workers across …rms. Brown and Medo¤ (1991) investigate explanations for wage-size di¤erentials. They …nd evidence in support only for explanations based on sorting by worker skill. Theoretical explanations for this evidence usually resort to the introduction of some form of complementarities

[2] In a sense we can argue that social preferences operate here as a kind of "equilibrium-re…nement." The advantage of this way of re…ning equilibria is that the payo¤ perturbation is economically and empirically well-motivated.

[3] See Ledyard's (1995) survey on public goods in the Handbook of Experimental Economics.

[4] See also Roth's (1995) survey on bargaining in the Handbook of Experimental Economics.

[5] "Blue collar unskilled workers are more and more separated from other types of workers, and therefore, tend to work together in the same …rms. This is true for each of the six categories of skills. The number even doubled for clerks." Kramarz et al. (1996), p. 375.

between individuals of the same skill levels. Good examples of these explanations are de Bartolomé (1990), Bénabou (1993), Kremer and Maskin (1996) and Saint-Paul (2001). We depart from this by not postulating any form of production complementarities between worker's types. The externality that arises between workers is of a pecuniary nature. It arises because market outcomes favor more productive workers, and individuals are averse to inequalities in their own neighborhood.[6]

3 The model

There are N workers, with two types, L and H, which are their private information. The productivity of a worker of type $t \in \{L; H\}$ is μ_t. We assume that $\mu_H > \mu_L$. The prior probability of an H type is $1 > p > 0$. The material payoff function of a worker i who receives a wage w, and exerts effort e, is:

$$u_i(w; e|t) = w - c_t(e)$$

The function $c_t(e)$ represents the disutility experienced by a worker of type t when exerting effort e. For a given effort level, $e \neq 0$, the cost of effort of an L type is higher than that of an H type, that is, $c_L(e) > c_H(e)$. We also assume that $c_{t;e}(e; \mu) > 0$ and $c_{t;ee}(e; \mu) > 0$, for all $t \in \{L; H\}$.[7] Effort levels are verifiable.

Individuals are embedded in a network of social relationships. In addition to the utility they obtain from their own wage and effort, which we call their material payoffs, they also experience utility (or disutility) from the material payoffs of close neighbors in their network. Denote by N_i the set of neighbors of i (excluding himself) and by n_i its size. Individuals dislike inequality, so their extended "social payoffs" are of the form

$$U_i = u_i - \frac{1}{n_i} \sum_{j \in N_i} V(u_j - u_i)$$

where $V(0) = 0$, and $V(x) > 0$, when $x \neq 0$. We assume that $|V'(x)| < 1$: That is, the marginal impact of inequality (even considering the whole group) is not larger than the impact of a marginal increase in material payoff of the same size. Our results are robust to heterogeneity in fairness concern between individuals, and we may allow for a player specific inequality aversion term $V_i(\cdot)$, $i \in N$.[8]

[6] There are other models of segregation which rely on group externalities. Seminal works in this area are Becker (1957) and Schelling (1971). Contrary to our paper, in that literature the individuals have an intrinsic like or dislike of workers in their or other groups. In our case, the spillover is related only to the market outcome. High and low types would live happily together if wages were equal.

[7] In fact, we need to ensure that indifference curves are non-thick and generate strictly convex upper contour sets.

[8] Given that the type of a player is private information, in the expression for worker i's social payoffs, the u_j in $V(u_j - u_i)$ should be understood as the expected value of u_j given i's information. However, the equilibrium contracts are separating. So, in equilibrium, worker i will, in fact, know worker j's type just by observing either her wage or her effort. We assume that one of these variables is, indeed, public knowledge.

There are $F > N$ identical firms.[9] They locate in at most ¸ ¸ $3F + 1$ different nodes of a ring. In particular, we allow for more than one firm to occupy the same location. Each firm can employ any number of workers, and technology is constant returns to scale. Net profit for each worker is equal to his productivity μ, minus the wage w he receives. Firms' profits are determined by the sum of profits per worker. If the firm does not employ any worker, it makes zero profits.

The game proceeds in three stages. First, each firm chooses a location in the ring. Second, each firm offers a menu of contracts to some workers which specifies the wage and effort required of different worker types. Recall that types are private information of the workers, but effort levels are verifiable, thus contractible. Third, each worker i specifies the menus acceptable to him, and the contracts within this menu that he would take. A worker who does not accept any contract obtains a reservation payoff of zero.

An employed worker gets the material payoffs derived by the implemented contract in the firm for which he works. The neighborhood of some employed worker i, N_i, is composed by those workers (if any) employed by firms located in i's employer node, and in the two adjacent nodes. This neighborhood is the one that enters in the determination of the final social payoffs.

4 Results

In this section we show that, for the game we just described, in all the subgame perfect equilibria where agents do not use dominated strategies, different types of workers earn a wage equal to their productivity, but they work in different locations. Workers earn their productivity for the usual reasons in a model with competitive wage-setters. The intuition for the spatial segregation result is simple. Since wages equal productivities, and those differ across workers, a low type working in an environment with high types suffers because of his aversion to inequality. A competitor firm which is making zero profits in that environment can profitably deviate. He can do so by moving to an empty location and offering a wage slightly below his productivity to the low type that works around high types. Provided this wage is close enough to the productivity, the worker will accept and the firm makes strictly positive profits.

Given the simplicity of the intuitions involved, it may come as a bit of a surprise that we need to resort to undominated subgame perfect equilibrium as a solution concept. The reason becomes more apparent once we look at the following example, which we have stripped down to the essentials to be easier to follow. In particular we have even dispensed with the incomplete information and the cost of effort.

Example 1 Let two workers, L and H, whose respective productivities, μ_L and μ_H, are common knowledge. They have no cost of effort. There are 4 firms and 13 nodes in a ring.[10] The following actions form part of a subgame perfect equilibrium outcome. Firm 1 locates on node 1 and offers worker L a wage equal to μ_L and worker H a wage equal to μ_H, firm 2 locates on node 1 and offers worker L a wage equal to μ_L, firm 3 locates on node 6 and offers worker L a

[9] Alternatively, we could assume that the number of firms is endogenously determined, and our results would not change.

[10] In fact, 8 locations are enough for our purpose.

wage $w_L^3 = \theta_L - V(\theta_H - \theta_L)$, and worker H a wage equal to θ_H, firm 4 locates on node 6 and offers worker H a wage equal to θ_H: Worker H accepts the offer of firm 1 and worker L accepts the offer of firm 3.

The use of dominated strategies by both the firms and the workers is crucial in the construction of the example. In the example, firms make many offers of wages equal to productivity that are not used in the equilibrium path. Those unused offers, which are weakly dominated, are what (out of equilibrium) supports the equilibrium outcome we postulate. Even more importantly, the responses of the players are also (almost) dominated. Take, for example, a deviation by firm 2 to location 3 that offers the L worker a salary w_L^2 higher than the one he obtains in equilibrium. If L accepts this offer, he is sure to obtain a utility equal to w_L^2, as he is sure not to experience disutility from inequality. On the other hand, if he accepts (as we postulate in the proof) the standing offer of firm 1, his utility depends on whether worker H indeed decides to accept the standing offer of 4. In fact, for w_L^2 arbitrarily close to θ_L, he has to be arbitrarily sure that H will indeed move. We find this rather unsatisfactory because of its probable unrealism.

There is one problem that arises if we choose to eliminate dominated strategies. When wages can be chosen from the real numbers, the set of undominated strategies is open. Any wage that is strictly smaller than the productivity of a worker is undominated, but a wage equal to productivity is weakly dominated. So we cannot construct Nash equilibria in undominated strategies, as any wage offer different from the productivity can always be defeated by a nearby proposal. To get rid of this difficulty, we discretize the wage space. We consider a family of discrete wage spaces with increasingly fine grids that approaches the continuum when the grids becomes infinitely fine.

More precisely, let $n^0; n^1; n^2; \ldots$ be an increasing sequence of integers such that $n^k \to +\infty$. For each $k \in \mathbb{N}$, let

$$\Theta^k = \left\{ \frac{a}{n^k} \mid a \in \mathbb{N} \right\}:$$

We assume that $\theta_t \notin \Theta^k$, for all $k \in \mathbb{N}$ and $t \in \{L; H\}$.[11] For all $k \in \mathbb{N}$, let $\varepsilon^k = 1/n^k$, and for all $t \in \{L; H\}$, let $\theta_t^k = \arg\max \left\{ x \leq \theta_t \mid x \in \Theta^k \right\}$. By definition, θ_t^k is the highest element in the discrete wage space Θ^k smaller than type t's productivity. We have, $\varepsilon^k > \theta_t - \theta_t^k > 0$, for all $t \in \{L; H\}$.

The location and contracting game where firms chose wages in Θ^k is denoted by G^k.

Proposition 1 There exists an integer K such that, for all $k \geq K$, $k \in \mathbb{N}$, at every subgame perfect Nash equilibrium of G^k, contracts accepted with positive probability are different across types, and pay t type employees a wage θ_t^k, $t \in \{L; H\}$.

Corollary 2 When $k \to +\infty$, contracts accepted with positive probability pay employees exactly their productivity.

The presence of social preferences does not change the contracts observed in equilibrium, with respect to the equilibrium contracts when agents do not have extended preferences. The proof is very similar as the one for the standard model. One needs to be a bit careful with the

[11] Precisely, to avoid including a weakly dominated strategy in the wage space.

deviations that defeat non-equilibrum outcomes. The problem is that those deviations could increase inequality, so either they would not be followed, or they would be too expensive to be profitable. However, we have assumed that a marginal increase in inequality (even considering the whole group) is not more valuable than an increase in material payoff of the same size. We have also assumed that the number of locations is high enough for any firm to be able to relocate at an empty location with no firms close by. This allows to construct deviations that are just like the ones in the standard proofs, adjusted for the potential increase in the inequality. Example 2 at the end of this section shows that without this second assumption, our segregation result would not hold.

The main difference between the equilibria in our model and the ones in the standard model is that firms, here, do not employ workers of different types. Otherwise some firm would have a deviation that would allow it to earn strictly positive profits by attracting workers of just one type with a lower salary. Their decrease in material payoffs is compensated by a decrease in disutility due to a more egalitarian work environment. So in any equilibrium, types are geographically separated. One consequence of this segregation is that, at equilibrium, contracts accepted with positive probability are identical within types, irrespective of employee's location.

Proposition 3 *There exists an integer K such that, for all $k \geq K$, $k \in \mathbb{N}$, at every subgame perfect Nash equilibrium of G^k, firms are spatially segregated by types separated by empty locations.*

Social preferences thus predict both skill and spatial workplace segregation as, at equilibrium, firms hire only from one skill pool and firms employing workers of a given skill level form spatial clusters.

Remark 1 All previous results hold when individuals are averse to wage inequality, rather than inequality in material payoffs (that is, wages minus cost of contracted effort), and extended social payoffs are of the form

$$U_i = u_i - \frac{1}{n_i} \sum_{j \in N_i} V_i(w_j - w_i);$$

where, for all $i \in N$, $V_i(0) = 0$, $V_i(x) > 0$, when $x \neq 0$, and $|V_i'(x)| < 1$.[12]

Remark 2 All previous results hold with arbitrary neighborhood structures, as long as the number of available locations λ and the number of firms F are such that $\lambda \geq (\max_{i \in N} \{N_i\} + 1)F + 1$.

We have assumed that the number of possible locations, λ, is such that $\lambda \geq 3F + 1$, where F is the number of firms. The following example shows that firms may not be spatially segregated by types (separated by empty locations) when this assumption does not hold.

Example 2 There are $F = 4$ firms locating on at most $\lambda = 4$ different nodes, 2 workers of type L and 2 workers of type H. Individual productivities are common knowledge and workers have

[12] See Bramoullé (2001) for a critical account of different structures of social preferences: (i) concern for others' allocations, (ii) concern for others' material payoffs, and (iii) concern for others' extended social payoffs.

no cost of effort. Extended preferences are of the form

$$U_i = u_i - \frac{1}{n_i} \sum_{j \in N_i} \alpha |u_j - u_i|, \; 0 \leq \alpha < 1.$$

There exists a non-segregated equilibrium with one H type worker at nodes 1 and 2, and one L type worker at nodes 3 and 4. Each worker is employed by one firm and wages are equal to productivities.

5 Conclusion

This paper shows that small deviations from "selfish" preferences leads to a very stark sorting of workers into firms by abilities. This coincides with empirically observed sorting patterns. A natural question is whether our explanation is more important than others for explaining the observation. One competing hypothesis, which would lead to similar results in our context, is that workers of the same type have complementary sets of skills. The two hypothesis are observationally distinguishable in other environments, however.

In our model, the pecuniary externality is driven by the fact that firms compete between themselves. In the absence of that externality there would be no reason for separation. So if a firm had market power in the labor market, and the outside option of workers was not related to their type (say, the skills were highly job-specific), all workers would be paid the same. Thus, our model would not predict sorting, whereas the model with complementarities would still predict them. While it is not easy to think of markets that precisely fit those conditions, there are many markets for qualified workers in Europe, like those of physicians and teachers, where the public sector has strong market power. If the amount of sorting in those markets were somewhat smaller than in others for workers of similar characteristics, our hypothesis would clearly have explanatory power. More empirical field work seems like a good avenue for further research.

On the other hand, experimental work appears to be more challenging for this topic than for others that have to do with social preferences. It will be difficult to control in the lab the network structure of preferences. Perhaps by choosing subjects from physically distant places, and running the experiment on the Internet, one could emulate the social structure of the model. In any case, we believe that a contribution of this paper is that it confronts the field with the important issue of who is included in the interpersonal comparisons and how much. Perhaps a better understanding of this issue would also contribute to clarify the other important (at least from an evolutionary point of view) question of why agents care about payoff differences.

One other observation on empirical testing arises from the fact that individuals may not be averse to inequality when the output measure of others is very objective. It may be debatable who is the best economist in a certain department (the current fashion for ranking individuals notwithstanding), but is is less controversial who is the top scorer in a soccer team. If indeed aversion to inequality depends on the objectivity of the output measure, then one would expect less sorting by skill-type (thus more within-firm inequality) in soccer teams that in universities.

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Let $k \in \mathbb{N}$ and G^k the corresponding game. We denote by $m^k_{f,i} = \left\langle w^k_{f,i,L}, e^k_{f,i,L}, w^k_{f,i,H}, e^k_{f,i,H} \right\rangle$ the menu of contracts offered by firm f to player i. For all $i \in N$, let $M_i^k = \{m^k_{f,i}\}_{f \in F}$ denote the set of contracts offered to player i by all firms. A pure strategy Nash equilibrium of G^k's second stage (acceptance) game is a profile of accepted menus $\left(s^k_1, \ldots, s^k_n\right) \in \times_{i \in N} M_i^k$.

Proof of Proposition 1. We decompose it into the following lemmata.

Lemma 4 For all $k \in \mathbb{N}$, at every subgame perfect Nash equilibrium of G^k, firms ex ante profits are nonnegative and strictly smaller than ε^k.

Proof. Suppose not. Let $k \in \mathbb{N}$ and G^k the corresponding game. Then there exists some subgame perfect Nash equilibrium (SPNE) of G^k where some firms ex ante profits are higher or equal than ε^k. Consider such a SPNE, denoted by *SPNE.

Let $\overline{m}^{k*}$ be the menu that makes the highest expected profit at *SPNE. This menu is offered by some firm f to some player i, that is, $\overline{m}^{k*} = m^{k*}_{f,i,L} = \left\langle w^{k*}_{f,i,L}, e^{k*}_{f,i,L}, w^{k*}_{f,i,H}, e^{k*}_{f,i,H} \right\rangle$, and player i accepts it. Let $t_i \in \{L, H\}$ denote player i's type. Given that f's ex ante profits are higher or equal than ε^k, necessarily $\theta_{t_i} - w^{k*}_{f,i,t_i} \geq \varepsilon^k$. We distinguish two cases.

Case 1: $\theta_L - w^{k*}_{f,i,L} \geq \varepsilon^k$. Consider some firm $g \neq f$ making zero profits at *SPNE. The condition $F > N$ guarantees that such a firm exists. Let g deviate by locating at an empty location surrounded by two empty adjacent locations. The condition $\geq 3F + 1$ guarantees that such a location exists. Let g offer player i the menu of contracts $m^{k\delta}_{g,i} = \left\langle \theta^k_L, e^{k*}_{f,i,L}, w^{k*}_{f,i,H}, e^{k*}_{f,i,H} \right\rangle$ at this location. We have $\theta_L - w^{k*}_{f,i,L} \geq \varepsilon^k > \theta_L - \theta^k_L$, implying in particular that $\theta^k_L > w^{k*}_{f,i,L}$. Player i may be simultaneously receiving offers from other firms (besides from g) which are equivalent, in terms of material payoffs, to $m^{k\delta}_{g,i}$. But, if player i didn't accept those offers at the *SPNE, it is because player i would have faced a strict disutility due to inequality in case of accepting them. At g's new location, there is certainly no inequality. At any other location, though, the extended utility accruing from any menu equivalent to $m^{k\delta}_{g,i}$ in terms of material payoffs depends, in general, on the reactions of other players. Therefore, it is a weakly dominant strategy for player i to accept $m^{k\delta}_{g,i}$, and g's deviation is profitable in expected terms.

Case 2: $\theta_L - w^{k*}_{f,i,L} < \varepsilon^k$. Then, necessarily, $\theta_H - w^{k*}_{f,i,H} \geq \varepsilon^k$. Let $g \neq f$ making zero profits at *SPNE, deviating by locating at an empty location surrounded by two empty adjacent locations, and offering player i the menu of contracts $m^{k\delta}_{g,i} = \left\langle w^{k*}_{f,i,L}, e^{k*}_{f,i,L}, \theta^k_H, e^{k*}_{f,i,H} \right\rangle$ at this location. It is a weakly dominant strategy for player i to accept g's offer given that it increases his material payoffs, and there is no disutility due to inequality at g's new location (and g's deviation is profitable). Indeed, switching contracts modifies both the material payoffs and the inequality payoffs accruing to some individual. Given that $|V'(x)| < 1$, variations in inequality induced by unilateral switching of contracts do never offset the corresponding variations in material payoffs, and unilateral decisions to pick up a contract out of an array of alternatives are governed solely by material payoff concerns. Therefore, no L type worker accepts $\left(\theta^k_H, e^{k*}_{f,i,H}\right)$ because the corresponding material payoffs are strictly lower than those obtained with some alternative offered contract. ■

Lemma 5 There exists an integer K such that, for all $k \geq K$, $k \in \mathbb{N}$, at every subgame perfect Nash equilibrium of G^k, contracts of different types accepted with positive probability are different.

Proof. Suppose not. Let $k \in \mathbb{N}$ and G^k the corresponding game. We distinguish two cases.

Case 1. There exists one firm $\bar f$ that offers a menu $\bar m^k = \left(\bar w^k; \bar e^k; \bar w^k; \bar e^k\right)$ with identical wage $\bar w^k$ and effort level $\bar e^k$ to both workers' types. In the effort-wage space, denote by $U^{\pm}_H$ the strict upper contour set corresponding to the material payoffs of an H type worker applying for firm $\bar f$ at its location. Similarly, denote by U_L the upper contour set of the material payoffs of an L type worker applying for firm $\bar f$ at its location. Consider some firm g making zero profits. Suppose that g deviates to an empty location and offers a menu $\left(\bar w^k; \bar e^k; \tilde w^k; \tilde e^k\right)$ to some of $\bar f$'s current workers, where $\left(\tilde w^k; \tilde e^k\right)$ is chosen in $\Omega^k = (U^{\pm}_H \setminus U_L) \cap \left\{w < \theta_H \mid w \in \Theta^k\right\}$. We show that for k high enough, $\Omega^k \neq \emptyset$. By assumption, for all $e \in \mathbb{R}_+$, $c'_L(e) > c'_H(e)$. Therefore, for k high enough, $U^{\pm}_H \setminus U_L \neq \emptyset$. We are left to prove that $(U^{\pm}_H \setminus U_L) \cap \left\{w < \theta_H \mid w \in \Theta^k\right\} \neq \emptyset$. It suffices to show that, for k high enough, $\bar w^k < \theta^k_H$. Suppose on the contrary that, for all $k \in \mathbb{N}$, $\bar w^k \geq \theta^k_H$. For k high enough, $\theta^k_H > \theta_L$. For such values of k, $\bar f$'s ex post profits made with H type workers are smaller or equal than ε^k, whereas $\bar f$'s ex post profits made with L type workers are strictly negative. There is a positive probability that L type workers accept menu $\bar m^k$. Therefore, given that $\varepsilon^k \downarrow 0$, when $k \to +\infty$, there exists an integer K such that, for all $k \geq K$, $\bar f$'s ex ante profits are negative, which violates Lemma 4. Therefore, for all $k \geq K$, we have $\bar w^k < \theta^k_H$. With such menu of contracts, it is a weakly dominant strategy for all H type workers in $\bar f$'s workforce to accept g's offer given that it increases their material payoffs, and there is no disutility due to inequality at g's new location. This deviation is profitable to g.

Case 2. There exists one firm $\bar f_1$ who offers a menu $\bar m^k_1$ including contract $\left(\bar w^k; \bar e^k\right)$ only accepted by L type workers and a firm $\bar f_2$ who offers a menu $\bar m^k_2$ including contract $\left(\bar w^k; \bar e^k\right)$ only accepted by H type workers. But then, by Lemma 4, all ex post profits of firm $\bar f_1$ with L type workers are nonnegative and smaller or equal than ε^k, implying that $\bar w^k = \theta^k_L$. Similarly, all ex post profits of firm $\bar f_2$ with H type workers are nonnegative and smaller or equal than ε^k, implying that $\bar w^k = \theta^k_H$, which is impossible as, for high enough values of k, we have $\theta^k_L \neq \theta^k_H$. ∎

Lemma 6 There exists an integer K such that, for all $k \geq K$, $k \in \mathbb{N}$, at every subgame perfect Nash equilibrium of G^k, contracts accepted with positive probability by L type workers (resp. H type workers) offer wage θ^k_L (resp. wage θ^k_H), that is, contracts accepted with positive probability make ex post profits which are nonnegative and strictly smaller than ε^k.

Proof. Let $k \in \mathbb{N}$ and G^k the corresponding game. We first show that for any firm f and independently of its location, the wage $w^k_{f;i;L}$ proposed by f to some player i, and accepted by i whenever $t_i = L$, is such that $w^k_{f;i;L} \geq \theta^k_L$. Suppose on the contrary that some firm f offers at some location a wage $w^k_{f;i;L} < \theta^k_L$ which is part of a contract accepted with positive probability. Consider some firm g making zero profits. Suppose that g deviates to an empty location and offers the contract $\left(\theta^k_L; e^k_{f;i;L}\right)$ to some of f's current workers. Then, g makes ex post profits which are higher or equal than ε^k with any worker eager to accept such wage offer, whatever his type. Therefore, g makes ex ante profits which are higher or equal than ε^k, which is impossible by Lemma 4.

We now show that the wage $w^k_{f;i;H}$ proposed by any firm f to some player i, and accepted by i whenever $t_i = H$, is such that $w^k_{f;i;H} \geq \theta^k_H$. Suppose not. Then, there exists some firm f offering a contract $\left(w^k_{f;i;H}; e^k_{f;i;H}\right)$ accepted with positive probability by some H type workers, where $w^k_{f,i,H} < \theta^k_H$. Lemma 5 implies that, for k high enough, no L type worker accepts this contract. In other words, for k high enough, the extended social payoffs of any L type worker accepting $\left(w^k_{f,i,H}, e^k_{f,i,H}\right)$ are strictly lower than the extended utility obtained with some alternative contract. Switching contracts modifies both the material payoffs and the inequality payoffs accruing to some individual. Given that $|V'(x)| < 1$, variations in inequality induced by unilateral switching of contracts do never offset the corresponding variations in material payoffs, and unilateral decisions to pick up a contract out of an array of alternatives are governed solely by material payoff concerns. Therefore, for k high enough, no L type worker accepts $\left(w^k_{f,i,H}, e^k_{f,i,H}\right)$ because the corresponding material payoffs are strictly lower than those obtained with some alternative offered contract. Consider some firm g making zero profits. Suppose that g deviates to an empty location and offers the contract $\left(\theta^k_H, e^k_{f,i,H}\right)$ to some of f's current workers. It is a weakly dominant strategy for all H type workers in f's workforce to accept g's offer given that it increases their material payoffs, and there is no disutility due to inequality at g's new location. The increase in material payoffs is $\theta^k_H - w^k_{f,i,H} = q\varepsilon^k$, for some $q \in \mathbb{N}$. We know that, for k high enough, no L type worker accepts f's original contract $\left(w^k_{f,i,H}, e^k_{f,i,H}\right)$, and this decision is taken by comparing only material payoffs from different contracts. Also, $\varepsilon^k \downarrow 0$, when $k \to +\infty$. Therefore, there exists an integer K such that, for all $k \geq K$, no L type worker accepts g's contract offer. When $k \geq K$, only H type workers accept firm g's offer, and g's ex post profits with all of them are strictly higher than ε^k, which is impossible by Lemma 4.

Therefore, for all $k \geq K$, $k \in \mathbb{N}$, $f \in F$ and $i \in N$, we have $w^k_{f,i,L} \geq \theta^k_L$ and $w^k_{F,i,H} \geq \theta^k_H$. By Lemma 4, firms make ex ante profits which are nonnegative and smaller or equal than ε^k. Therefore, $w^k_{f,i,L} = \theta^k_L$ and $w^k_{f,i,H} = \theta^k_H$. ∎

Proof of Proposition 3. Let $k \in \mathbb{N}$ and G^k the corresponding game. Consider a subgame perfect Nash equilibrium of G^k, denoted by *SPNE. Given a location ℓ, denote by n_ℓ the number of workers employed at ℓ and at its two adjacent nodes at *SPNE. We have $n_\ell = n_{\ell,L} + n_{\ell,H}$, where $n_{\ell,t}$ denotes the number of t type workers employed at ℓ and at its two adjacent nodes, $t \in \{L, H\}$. For all $t \in \{L, H\}$, let

$$q_{\ell,t} = \begin{cases} \frac{n_{\ell,t}}{n_\ell}, & \text{if } n_\ell \neq 0 \\ 0, & \text{otherwise} \end{cases}$$

We prove that $q_{\ell,t} \in \{0, 1\}$, for all $t \in \{L, H\}$. Suppose not. Let ℓ such that $0 < q_{\ell,L} < 1$.[13] Let ℓ' be an empty location surrounded by two empty locations. The assumption $\lambda \geq 3F + 1$ guarantees that such an ℓ' exists.

We now prove that workers employed at ℓ experience a nonzero disutility due to inequality at *SPNE. Suppose not. By assumption, $x \neq 0$ implies $V(x) > 0$. Denote by u^*_i the material payoffs of player i at *SPNE and by U^*_i its extended social payoffs. Then, for all i, j employed at ℓ and its two adjacent nodes, $U^*_i = u^*_i = u^*_j = U^*_j$. Given that $0 < q_{\ell,L} < 1$, there exists at least two workers of different types employed at ℓ or its vicinity which are in the direct neighborhood of each other. We denote those workers by i_L and i_H, where $t_{i_L} = L$ and $t_{i_H} = H$. In the effort-wage space, denote by U^δ_H the strict upper contour set corresponding to the material

[13] Note that $q_{\ell,L} = 1 - q_{\ell,H}$, and $0 < q_{\ell,H} < 1$ is equivalent to $0 < q_{\ell,L} < 1$.

payoffs of i_H, and by U_L the upper contour set corresponding to the material payoffs of i_L. Let $© ^k = (U_H^{\pm} \backslash U_L) \cap \{w < µ_H^k \mid w \in £^k\}$. For k high enough, $©^k \neq ;$. Indeed, denote by $(w^{¤}_{\ell;i_H}; e^{¤}_{\ell;i_H})$ the contract accepted by i_H at location ℓ at ¤SPNE, where $w^{¤}_{\ell;i_H} \in £^k$. Let $(w; e^{¤}_{\ell;i_H})$, $w \in £^k$, such that $u_{i_L}(w; e^{¤}_{\ell;i_H}) = u_{i_H}(w^{¤}_{\ell;i_H}; e^{¤}_{\ell;i_H})$. Given that, for all $e \in \mathbb{R}_+$, $c_L(e) > c_H(e)$, necessarily $w > w^{¤}_{\ell;i_H}$. For k high enough, there exists some $w' \in £^k$ such that $w > w' > w^{¤}_{\ell;i_H}$, implying that $U_H^{\pm} \backslash U_L \neq ;$. If k is high enough, we also have $©^k \neq ;$. Consider some firm g making zero profits at ¤SPNE. Suppose that g deviates to ℓ' and offers a contracts $(\mathbf{w}; \mathbf{e}) \in ©^k$. We know from Lemma 5 that, at equilibrium, when k is high enough, no L type worker accepts the contract with which i_H obtains $U^{¤}_{i_H} = u^{¤}_{i_H}$ at ℓ. Recall also from the proof of Lemma 6 that unilateral deviations to pick up a contract out of an array of alternatives are governed solely by material payoffs concerns. Therefore, for high enough values of k, $(\mathbf{w}; \mathbf{e}) \in ©^k$ can be chosen so as not to be accepted by any L type worker. Then, g only attracts H type workers to ℓ' (those initially employed at ℓ, and possibly some others). We deduce from Lemma 6 that H type workers are paid $µ_H^k$ at equilibrium. By construction of $©^k$, $\mathbf{w} < µ_H^k$. Therefore, g makes ex ante profits which are higher or equal than $"^k$, which is impossible by Lemma 4.

Therefore, at ℓ, employed workers face a strictly positive disutility due to inequality. Any L type worker employed at ℓ would be strictly better off at ℓ' with the same contract because he would face a smaller disutility due to inequality. Therefore, any firm making zero profits at the current equilibrium (the assumption $F > N$ guarantees that such a firm exists) moving to ℓ' and offering a contract $µ_L^k - "^k$, where k is high enough, could attract such L type workers (and possibly some H type workers too) and make ex ante profits strictly higher than $"^k$, thus violating Lemma 4. ∎

Proof of Example 1. To show that this is indeed part of a subgame perfect equilibrium, we need to specify the responses of the workers to deviations by the firms. In fact we do not need to specify responses to all possible deviations, but only to unilateral deviations of one firm. Worker H is already obtaining a salary equal to productivity, so no deviation that intends to attract H can ever be profitable. Thus, the only possibly profitable deviations are those that affect worker L. Clearly, firm 3 is already making the maximum possible profit in this environment, so only deviations by firms 1, 2 and 4 need to be considered:

(a) Suppose that firm 1 deviates by offering L, at some location, the wage w_L^1, with $µ_L > w_L^1 > w_L^3$. If worker H responds to this deviation by choosing to work for firm 4, and worker L responds by choosing to work for firm 2, then the deviation by 1 is not profitable.

(b) Suppose that firm 2 deviates by offering L, at some location, the wage w_L^2, with $µ_L > w_L^2 > w_L^3$. If worker H responds to this deviation by choosing to work for firm 4, and worker L responds by choosing to work for firm 1, then the deviation by 2 is not profitable.

(c) Suppose that firm 4 deviates by offering L, at some location, the wage w_L^4, with $µ_L > w_L^4 > w_L^3$. If worker H responds to this deviation by choosing to work for firm 3, and worker L responds by choosing to work for firm 2, then the deviation by 4 is not profitable. ∎

Proof of Example 2. It is readily checked that this game has two subgame perfect Nash equilibria (modulo a relabelling of nodes). In both cases, workers are paid exactly their productivity at equilibrium:

(a) a segregated equilibrium, where both H type workers are located at node 1, and both L type workers are located at node 2, and individual extended payoffs at equilibrium are $U_i = \theta_{t_i}$, $i \in \{1; 2; 3; 4\}$.

(b) a non-segregated equilibrium, where H type workers are located at nodes 1 and 2, and L type workers at nodes 3 and 4, and extended payoffs are $U_i = \theta_{t_i} - \alpha\Delta/2$, $i \in \{1; 2; 3; 4\}$. $\blacksquare$