Stability and Efficiency in Coalition Formation*

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December 20, 2002

* I wish to thank M. Jackson for a comment on a related paper that prompted this work. This paper benefited also from conversations with Shlomo Weber and, of course, from the help and guidance of Salvador Barberà. All errors are mine.

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Abstract

This paper considers a situation in which a society of heterogeneous individuals partitions into several groups, each choosing its own public good. A strong Tiebout equilibrium is a coalition structure in which there does not exist a group of agents who unanimously decide to form a new coalition and in which no agent migrates to another existing coalition. We characterize strong Tiebout equilibria and show that there always exists one that fulfills a strong efficiency requirement. Refinements of this concept based on rules for country formation are also analyzed.
JEL Classification Numbers: C71, H73.
Keywords: local public good economies, coalition formation, (strong) Tiebout equilibrium.

"... to test the validity of one's conclusions, the model user should engage wherever possible in what ... is called sensitivity analysis. ... That is, the modeler should make slight variations in the parameters or the other assumptions of the model and observe their effect on the solution. ... Examples ... are ... (4) the choice of a solution concept.´´ (Shubik [12], page 14)

1 Introduction

This paper analyzes a society of heterogeneous agents with different preferences over a given set of feasible policies. The agents may group together in one large group or form several smaller groups.[1] Since each policy is costly, there are economies of scale when large groups form. However, agents may feel better-off in small groups because the policy chosen will be more in line with their most preferred choice.

This conflict between increasing returns to scale and heterogeneity of agents' preferences arises in many situations. One example is the formation of political parties. Larger parties offer e.g. a higher level of material, political or career benefits for its members while the political platform chosen may be less satisfactory for members with rather extreme political views. Another example is the formation of jurisdictions. Here the tax burden may be lower in larger communities while again the need to choose one policy may create incentives for smaller units. An important example of the trade-off concerns the formation of countries. Here the heterogeneity of agent's preferences may origin in their cultural, ethnic or religious background and motivate the se-

[1] Throughout this paper we use the words "groups", "country", "jurisdiction" and "coalition" interchangeably.

cession of sub-groups of agents.[2]

The main purpose of this work is to analyze the efficiency properties of the partition of a society in coalitions driven by this trade-off. In carrying out this analysis we try to fill a small gab between two previous studies.

Alesina and Spolaore [1] – AS henceforth – provide a simple positive and normative framework for the analysis of country formation. AS use a spatial model where the world's population is a continuum of agents distributed uniformly on the line segment $[0, 1]$. Each individual consumes a (local) public good and incurs a transportation cost proportional to the distance between her location and the location of the public good in her country. In each country the location of the public good is decided by majority rule and the costs to produce the public good are covered using a proportional tax scheme. Thus, driving force of the process of country formation is the trade-off between the benefits of large countries and the costs of heterogeneity in large populations. *The main implication of this model is that in equilibrium one generally observes an inefficiently large number of countries.*

Haimanko, Le Breton and Weber [9] – HLBW henceforth – consider also a spatial model driven by the trade-off between heterogeneity of preferences and increasing returns to scale. HLBW assume also a one-dimensional policy space and single-peaked transportation costs in the policy space while in many other aspects their model is more general than the AS-model (e.g. the distribution of individual ideal points). HLBW analyze "sustainable" partitions, in which the policy related costs can be distributed in a way that no subgroup has an incentive to break away from the rest and to set up its own policy. *The main result of HLBW is that efficiency and sustainability*

[2] See Haimanko, Le Breton and Weber [9] for a more detailed exposition. Other examples can be found in Demange [4].

are equivalent[3].

In our view the main differences between AS and HLBW are two. Firstly, there is the assumption whether utility is transferable or not. While HLBW allow for transfer schemes in their analysis, AS assume a proportional taxation scheme and impose that income is the same for all agents.[4] The second mayor difference concerns the stability notions employed. As already said, HLBW investigate "sustainable" partitions, in which the policy related costs can be distributed in a way that no sub-group has an incentive to break away from the rest and to set up its own policy. AS employ a "mix" of stability notions relying on the free mobility of agents, concepts related to "sustainability" and specific rules for country formation.[5]

It is well known that in models driven by the trade-off between heterogeneity of preferences and increasing returns to scale appropriate transfer schemes may reconcile efficiency and stability (see Haimanko, Le Breton and Weber [8]). The starting point for our analysis is the question whether this aim can be reached when transfers are not available. Although the stability concepts of AS are reasonable in the context of country formation, the

[3] HLBW offer an extensive review of the related literature. For other papers offering an economic analysis of the formation of countries see Le Breton and Weber [11] Haimanko, Le Breton and Weber [8], Bolton and Roland [2] or the survey of the literature in Bolton, Roland and Spolaore [3].

[4] A consequence of this is that the partition that AS consider to be stable is also efficient in the sense that it would be chosen chosen by a Rawlsian social planner. Both papers consider (broadly speaking) the same partition to be efficient. While in HLBW this efficiency notion corresponds to Pareto-efficiency, in AS the efficiency requirement is "stronger".

[5] Because of the uniform cost sharing scheme in AS, subgroups that break away from a larger group have to use this scheme also. Hence, even when the concepts are close in spirit there are important differences.

question remains whether in other applications where these concepts are not appropriate a similar inefficiency result can be expected to hold. The approach we take to answer this question is to apply other stability concepts to the AS framework. In this sense we carry out a test of robustness of the AS-model to *ceteris paribus* changes in the equilibrium concept.

The first ingredient of the AS-equilibrium concept is the idea that agents have, to some extend, the possibility to migrate between existing countries and to create new coalitions. These are frequent requirements for an equilibrium in the literature on coalition formation or on local public goods economies. A concept which strengthens considerably both requirements and assures therefore additional desirable properties has been called by Greenberg and Weber [5] strong Tiebout equilibrium.[6] We characterize strong Tiebout equilibria and show that there always exists an efficient one. However, the inefficiency result of AS is robust to the use of this concept in the sense that one can understand the unique inefficient equilibrium in AS as the outcome of a selection among strong Tiebout equilibria by means of specific rules of country formation. This is nice because it implies that the fact that all coalitions consist exactly of one interval on the line segment is an implication and not an assumption of the model.

These specific rules of country formation are the second ingredient of the AS-equilibrium concept. First, there is the requirement that after a perturbation at borders agents should move as to restore the initial situation. We show that this condition is very important because it rules out many reasonable strong Tiebout equilibria. Second, there is a rule that models international agreements over modifications of borders which must be ratified

[6] For papers that use one or both requirements see also e.g. Greenberg and Weber [6], Demange [4], Jehiel and Scotchmer [10] or Haeringer [7] among others.

by simple majority rule. We follow a suggestion of AS and analyze "which supermajority rules for changing borders might enforce the efficient number of countries as a stable equilibrium".[7] Unfortunately, under qualified majority rule an equilibrium does not exist.

The remainder of this paper is organized as follows. The next Section reviews the AS country formation model. Section 3 characterizes efficient coalition structures. The following Section deals with stable structures. It is divided in three parts which analyze the implications of free mobility, the consequences of the possibility to form new coalitions and the application of country formation. We conclude in the last Section. All proofs are relegated to the Appendix.

2 The Model

AS consider an economy with a population $\mathcal{P}$ consisting of a continuum of agents with mass one, uniformly distributed over the line segment $[0, 1]$. We denote a generic member of $\mathcal{P}$ and its location by i.

A coalition S is any subset of $\mathcal{P}$ with $|S| > 0$, where $|S|$ denotes its size.[8] A coalition structure $\mathcal{W} = \{S_1, ..., S_N\}$ is any partition of $\mathcal{P}$ in coalitions, fulfilling that $|S_n \cap S_{n'}| = 0, \forall n \neq n'$ and $\cup_{n=1}^{N} S_n = \mathcal{P}$. This definition allows to think of coalitions as unions of closed intervals where boundary points

[7] Because of the fact that in reality different countries specify very different rules that govern the ratification of these international agreements it is reasonable to ask for some robustness of the AS-inefficiency result to changes in the majority requirements.

[8] The assumption that $|S| > 0$ assures a finite number of coalitions.

of these intervals are intersections of coalitions.[9] We assign coalitions their subindex by the following procedure. Coalition S_1 has its left border at point 0. Starting at 0 and going to the right, the first agent belonging to another coalition than S_1 belongs to S_2. The next agent belonging to another coalition than S_2 forms part of either S_1 or S_3 and so on.

A ***connected coalition*** consists exactly of one interval and we say that a ***coalition structure is connected*** if all its coalitions are connected. If a coalition S_n consists of several closed intervals, we denote them by $S_{n,1}, ..., S_{n,K}$.[10]

We say that an agent i belongs to the border of two coalitions S_n and S_m if she belongs to both S_n and S_m. We denote these agents by $b(S_{n,k}, S_{m,l})$ (where $n < m$). Neighboring coalitions are those which share a border. The set $B(\mathcal{W})$ is the set of all border agents given $\mathcal{W}$.

For a given coalition structure $\mathcal{W}$, $S_i(\mathcal{W})$ is the coalition i belongs to. To save on notation, we often simply write S_i or even S for $S_i(\mathcal{W})$, b instead of $b(S_n, S_m)$ and B for $B(\mathcal{W})$. Also, given $\mathcal{W}$, we denote by $|S_{max}(\mathcal{W})|$ and $|S_{min}(\mathcal{W})|$ the largest and smallest coalition size in $\mathcal{W}$. For simplicity we will write $|S_{max}|$ and $|S_{min}|$ whenever it is clear which coalition structure $\mathcal{W}$ is meant.

Coalitions have to provide a local public good bundle $l(S) \in [0, 1]$. All agents have the same utility function, which is decreasing in the distance to $l(S)$ and increasing in $|S_i|$. This function can be represented by the individual cost function

$$c_i(S) = cd_i(S) + \frac{1}{|S_i|}, \tag{1}$$

[9] This implies that every agent i belongs to at least one coalition but some agents (those located at boundary points) belong to more than one coalition. However, the measure of these agents is zero.

[10] Note that $S_{n,k}$ may be a singleton.

where $d_i(S) = |i - l(S)|$ and the nonnegative parameter c measures the relative importance of the costs of being in a heterogeneous coalition with respect to the 'public good provision' costs.[11] For convenience we will also use the simplifying notation $c_i(\mathcal{W})$.

The decision over the location of the public good $l(S)$ is taken by majority voting. This implies, since individual utilities are single-peaked with respect to $l(S)$, that the median voter determines $l(S)$. In case of ties in unconnected coalitions we suppose that $l(S)$ coincides with the left median position.[12]

3 Efficient Coalition Structures

A coalition structure is efficient if it minimizes the sum of the individual cost functions subject to the constraint that the sum of individual contributions to public good provision must equal its total costs. This implies the following important result.

PROPOSITION **1** *If $\mathcal{W}$ is an efficient coalition structure, then it is connected.*

The other properties of efficient coalition structures are characterized by proposition 1 in AS.

PROPOSITION **2** *[Alesina and Spolaore 1997] The social planner (i) locates*

[11] AS postulate $U_i(S) = \alpha(1 - \beta d_i(S)) + y - \frac{\gamma}{|S_i|}$, where y represents income and α, β and γ are positive parameters. Hence $c = \frac{\alpha\beta}{\gamma}$. We could have defined $g = \frac{\gamma}{\alpha\beta}$ as in HLBW.

[12] This assures that each location $l(S)$ of a local public good is an element of the union of intervals which constitute the coalition S.

the local public good in the middle of each coalition, and (ii) chooses N^* *coalitions of equal size, such that*

$$N^* = \frac{\sqrt{c}}{2}, \tag{2}$$

provided that $\frac{\sqrt{c}}{2}$ *is an integer. Otherwise the efficient number of coalitions* N^* *is given by either the largest integer smaller than* $\frac{\sqrt{c}}{2}$*, or the smallest integer larger than* $\frac{\sqrt{c}}{2}$*.*

4 Stable Coalition Structures

4.1 Tiebout equilibria

The purpose of this Section is to analyze the consequences of free mobility of agents for coalition structures. A partition into coalitions is a Tiebout equilibrium if no individual wants to migrate to any other existing coalition.[13] More formally:

DEFINITION 1 *A coalition structure* $\mathcal{W} = \{S_1, ..., S_N\}$ *is a* ***Tiebout equilibrium [TE]*** *if for all* $i \in [0,1]$*, we have,*

$$c_i(S_i) \leq c_i(S'), \forall S' \in \mathcal{W}. \tag{3}$$

Since we work with individual cost instead of utility, condition (3) states that each individual prefers his own coalition to any existing coalition. An important consequence of this requirement is the following result.

[13] Different from AS or Jehiel and Scotchmer [10] we do not require a stability condition. But we will investigate the implications of such a requirement in a later Section.

LEMMA **1** *If a coalition structure $\mathcal{W}$ is a Tiebout equilibrium, then $\mathcal{W}$ is connected.*

REMARK: An important consequence of this result and proposition 1 is that the implicit assumption of AS that coalition structures must be connected can in fact be derived as resulting from the requirements of efficiency and free mobility of agents.

The next proposition characterizes TE.

PROPOSITION **3** *A coalition structure $\mathcal{W} = \{S_1, ..., S_N\}$ is a Tiebout equilibrium if and only if $\mathcal{W}$ is connected and*

- *either all coalitions have the same size*
- *or there exist exactly two different sizes of coalitions with $|S_{min}||S_{max}| = \frac{2}{c}$.*

This result says that the requirement of free mobility of agents implies a very specific structure for $\mathcal{W}$. Coalitions are intervals and there can be at most two different coalition sizes.

REMARK: The definition of a Tiebout equilibrium is stronger than the definition of an A-equilibrium in AS because it allows any agent (and not only individuals located at borders) to move to any coalition (and not only to neighboring ones). However, the reader familiar with the work of AS will have noticed that the set of Tiebout equilibria coincides with the set of A-equilibria. Therefore, in the AS-model the fulfillment of a very weak free mobility condition (A-equilibrium) assures the desirable properties of a free mobility (Tiebout) equilibrium. This is true because in a given coalition S_n everyone is at least as well off as the agents at the border. Moreover, if an agent joins another coalition S_m she will be worse-off than the border agents in S_m. Since in an A-equilibrium all border agents get the same utility, no agent has an incentive to move.

4.2 Strong Tiebout Equilibria

The purpose of this Section is to impose on Tiebout equilibria the additional requirement that there should be no group of agents that can all become better off by creating a new coalition.

One reason for the creation of a new coalition may be that, given c, coalitions in the initial structure may be so small that agents would like to form larger coalitions. Note that proposition 3 does not imply any minimum size for coalitions in a Tiebout equilibrium. A first step is to look if there exist two coalitions whose agents would unanimously agree to merge to one large coalition S^M. A coalition structure in which such a consent cannot be reached is pairwise-merger-proof.

DEFINITION **2** *A coalition structure $\mathcal{W} = \{S_1, ..., S_N\}$ is* ***pairwise-merger-proof [PMP]*** *if in any two coalitions S_n and S_m of $\mathcal{W}$ there is no unanimous consent to merge and form S^M, that is, there exists $i \in S_n \cup S_m$ with,*

$$c_i(S^M) \geq c_i(\mathcal{W}). \tag{4}$$

This requirement just says that in any merger of coalitions that can be proposed, there should be at least one agent who cannot (strictly) increase her utility. Note that in principle proposed mergers do not necessarily involve neighboring coalitions. But proving the following proposition, which characterizes pairwise-merger-proof Tiebout equilibria, we show that if no neighbors merge, non-neighbors do not merge either.

PROPOSITION 4 *A Tiebout equilibrium $\mathcal{W}$ is pairwise-merger-proof if and only if one of the following is true*
(i) $\mathcal{W}$ contains two coalitions of different sizes and there are no neighboring coalitions both of size $|S_{min}|$ or
(ii)

$$|S_{min}| \geq \sqrt{\frac{1}{c}}. \tag{5}$$

The requirement of pairwise-merger-proofness puts a lower bound on coalition sizes. There exist some Tiebout equilibria in which, given c, coalitions may be too small and it pays to merge. However, in some Tiebout equilibria the lower bound on coalition sizes is automatically fulfilled.[14]

A second reason for the creation of new coalitions may be that, given c, coalitions in the initial structure are too large and it is advantageous to create smaller ones. Again, proposition 3 puts no upper bound on the size of coalitions. We investigate now when agents from two neighboring coalitions do not want to secede and create a smaller (connected) coalition. This is the case which AS analyze. They call a coalition structure in which those secessions do not take place C'-stable.[15]

[14] Note that the proof of proposition 4 focuses on the new border agents of the merged coalition S^M and that exactly one-half of the population in both coalitions experience the same change in utility as their border agent. This implies that the unanimous approval can be weakened to simple majority without affecting the result.

[15] AS also investigate the possibility of secessions involving only agents from one coalition, which they call C-stability. C-stability implies C'-stability.

Definition 3 *A coalition structure* $\mathcal{W} = \{S_1, ..., S_N\}$ *is* ***C'-stable*** *if in any two neighboring coalitions* S_n *and* S_{n+1} *of* $\mathcal{W}$ *there is no connected set of individuals* z *with* $|z| \leq |S_{min}|$ *that unanimously agrees to form* z*, that is, there exists* $i \in z$ *with,*

$$c_i(z) \geq c_i(\mathcal{W}). \tag{6}$$

C'stability requires that in any proposed secession of a connected set of agents with cardinality smaller than $|S_{min}|$ there is at least one agent who puts her veto on the proposal because her utility will not (strictly) increase. We have the following result.[16]

Proposition 5 *A Tiebout equilibrium* $\mathcal{W}$ *is C'-stable if and only if one of the following is true*
(i) $\mathcal{W}$ *contains two coalitions of different sizes or*
(ii)

$$|S| \leq \frac{\sqrt{2}+2}{\sqrt{c}}. \tag{7}$$

This result is very surprising. On one hand it says in part (ii), as we expected, that, depending on c, coalitions should not be too large. But on the other hand it says that Tiebout equilibria containing coalitions of different sizes do have a structure assuring that it is never beneficial to create smaller coalitions.[17]

We incorporate now the last two concepts into a stronger one. A coalition structure is a strong Tiebout equilibrium if it is a Tiebout equilibrium and no

[16] If $\mathcal{W}$ is the grand coalition condition (7) becomes $|S| \leq \frac{\sqrt{6}+2}{\sqrt{c}}$.

[17] The crucial information in proposition 3 is that $|S_{min}||S_{max}| = \frac{2}{c}$. For Tiebout equilibria containing coalitions of the same size, $|S| \leq \sqrt{\frac{2}{c}}$ (which assures $|S|^2 \leq \frac{2}{c}$) is a sufficient condition for C'-stability.

group of individuals can form a new coalition that makes all of them better off.

DEFINITION 4 *A coalition structure $\mathcal{W} = \{S_1, ..., S_N\}$ is a* ***strong Tiebout equilibrium [STE]*** *if it is a Tiebout equilibrium and for any coalition $z \notin \mathcal{W}$ there exists $i \in z$ such that*

$$c_i(z) \geq c_i(\mathcal{W}). \tag{8}$$

REMARK: The definition of a strong Tiebout equilibrium is stronger than both pairwise-merger-proofness and C'-stability. On one hand it allows for secession proposals of any size and on the other these proposals may be unconnected.

PROPOSITION **6** *A Tiebout equilibrium $\mathcal{W}$ is a strong Tiebout equilibrium if and only if it is pairwise-merger-proof and it is C'stable.*

This result establishes that saying $\mathcal{W}$ is a strong Tiebout equilibrium is equivalent to saying that its coalition sizes lie in an interval depending on c. The *lower bound* comes from the possibility to create larger coalitions. If coalitions are too small it pays to form larger ones. It turns out that pairwise-merger-proofness is a sufficient condition to prevent such a creation. The *upper bound* comes from the possibility to create smaller coalitions. If coalitions are too large they will break up in smaller ones. Here C'-stability is sufficient.

REMARK: Note that Tiebout equilibria containing coalitions of different sizes are almost always automatically strong Tiebout equilibria.

The last result of this Section concerns the existence of efficient strong Tiebout equilibria.

PROPOSITION **7** *An efficient strong Tiebout equilibrium always exists.*

4.3 Stable Countries

Strong Tiebout equilibria have many nice properties. However, given the specific application one has in mind, other additional properties may be desirable. One such application is the formation of countries. AS model specific rules for country formation and establish the existence of a unique stable equilibrium $\mathcal{W}(\widetilde{N})$ in which all coalitions are equally sized and the number of coalitions $\widetilde{N}$ is too large. Since this inefficiency result holds (strictly) if and only if $c > 50$, we assume in what follows that this holds for c.

There are three additional requirements AS impose. The first one is called "stability under rule A". For our purpose the following definition is sufficient.[18]

DEFINITION **5** *A coalition structure* $\mathcal{W} = \{S_1, ..., S_N\}$ *is* ***stable under rule A*** *if after any small perturbation that shifts any border between two neighboring coalitions and moves a small amount of agents from one coalition to the other one, these agents will move in such a way as to restore the initial coalition structure.*

From proposition 2 in AS it follows directly the following corollary.

[18] Jehiel and Scotchmer [10] also impose such a condition.

COROLLARY 1 *A coalition structure $\mathcal{W} = \{S_1, ..., S_N\}$ is a strong Tiebout equilibrium which is stable under rule A if and only if all coalitions are of the same size and*

$$\frac{\sqrt{c}}{\sqrt{2}+2} \leq N < \sqrt{\frac{c}{2}}.^{19} \tag{9}$$

REMARK: Note that stability under rule A has two important consequences. On one hand it strengthens pairwise-merger-proofness by establishing a lower upper bound for the number of coalitions. On the other hand and most importantly it excludes all coalition structures that contain coalitions of different sizes.

The second requirement that AS impose is called B-stability. It is intended to capture border rearrangements obtained by international agreements among existing countries, ratified by majority rule votes within each country. We test the robustness of the AS-inefficiency result to changes in the votes necessary to ratify an agreement.

DEFINITION **6** *Consider strong Tiebout equilibria which are stable under rule A. A coalition structure $\mathcal{W} = \{S_1, ..., S_N\}$ changes to another structure with $N-1$ or $N+1$ coalitions (of the same size) by applying **rule B(Q)** if the modification is approved by a population share of Q in each coalition of $\mathcal{W}$.*

DEFINITION **7** *A coalition structure $\mathcal{W} = \{S_1, ..., S_N\}$ is **B(Q)-equilibrium** if rule B(Q) is not applied.*

DEFINITION **8** *A B(Q)-equilibrium $\mathcal{W} = \{S_1, ..., S_N\}$ is **B(Q)-stable** if after any perturbation in the number of coalitions the system returns to $\mathcal{W}$ with*

[19] Note that, since $c > 50$ implies $N \geq 2$, the lower bound is unambiguous.

repeated applications of rule B(Q).

We analyze two different majority requirements.

4.3.1 Simple Majority Rule

AS analyze the case in which for ratification a simple majority is necessary. The following result follows directly from the two facts that AS apply B($\frac{1}{2}$)-stability to a larger set than we do and that their unique B($\frac{1}{2}$)-equilibrium is a strong Tiebout equilibrium.[20]

COROLLARY 2 *Let $\widetilde{N}$ be the largest integer strictly smaller than $\sqrt{\frac{c}{2}}$. The unique $B(\frac{1}{2})$-stable coalition structure has $\widetilde{N}$ coalitions of the same size. For $c > 50$, $\widetilde{N} > N^*$.*

REMARK: This shows that the inefficiency result of AS is robust to strengthening the ideas of giving agents the possibility to migrate freely among existing coalitions and to create new coalitions. The selection of $\mathcal{W}(\widetilde{N})$ can be understood as the selection of one specific strong Tiebout equilibrium motivated by the rules of country formation.

[20] The set of A-equilibria that are stable under rule A is strictly larger than the set of strong Tiebout equilibria that are stable under rule A, since the former does not include the requirement of C'-stability.

4.3.2 Qualified Majority Rule

In many countries changes of the territory, of the borders or the independence of regions require a modification of the constitution. For constitutional changes normally more than a simple majority is required. For simplicity in what follows we require $Q = \frac{2}{3}$.[21] We have the following result.

PROPOSITION 8 *All strong Tiebout equilibria which are stable under rule A, including the efficient coalition structure, are $B(\frac{2}{3})$-equilibria. There exists no $B(\frac{2}{3})$-stable coalition structure.*

This result has a simple intuition. When a higher quorum is needed fewer people are enough to block a proposal. Thus, one should expect more B(Q)-equilibria. Once there are multiple B(Q)-equilibria any perturbation from one B(Q)-equilibrium to another persists *because* we reached a new B(Q)-equilibrium. Thus with multiple B(Q)-equilibria the requirement of B($\frac{2}{3}$)-stability is too strong.

REMARK: Note that proposition 8 is robust to the following two modifications of rule B(Q). First, enlargements of countries must be approved by simple majority while secessions need the approval of a qualified majority.

[21] In Spain the independence of any region needs a change of the constitution. The constitutional change requires *in this case* a majority of $\frac{2}{3}$ in both the Congress (Congreso de los diputados) and the Senate (Senado). Then there are general elections (to both the congress and the senate), both newly elected organs have to ratify the decision and approve the new text of the constitution with a majority of $\frac{2}{3}$. Afterwards a referendum takes place. In Germany any change of the constitution must be approved by a majority of $\frac{2}{3}$ in both the Lower House of Parliament (Bundestag) and Upper House of Parliament (Bundesrat).

Second, a rule B'(Q) in which the majority must be reached in each of the regions that would constitute new countries under the proposed modification.[22]

5 Conclusions

In this paper we have analyzed how a society of heterogeneous agents with different preferences over a given set of feasible policies partitions into coalitions. The main stability concept we have employed is the notion of a strong Tiebout equilibrium. This is a coalition structure in which there does not exist a group of agents who unanimously decide to form a new coalition and in which no agent migrates to another existing coalition. We have characterized strong Tiebout equilibria and shown that a strong Tiebout equilibrium always exists. The concepts of A-equilibria and C'-stability, that seem very weak on first sight, can be seen as a kind of "sufficient conditions" for more desirable properties. Together with another intuitive requirement that we called pairwise-merger-proofness they assure that an equilibrium has all the properties of a strong Tiebout equilibrium.

It was also shown that there always exists an equilibrium that is efficient. Efficiency is a strong concept because our setting does not allow for transfers between agents. Underlying the notion of a strong Tiebout equilibrium we have not identified a force inherent in this concept that goes in the direction

[22] This is true, since in the proof of proposition 8 we use the results of AS concerning the enlargements of countries. In secessions we show that some agents who in both coalition structures form part of the first country are enough to block the proposal. Since the new country is smaller than the old one, our result is robust. Moreover, since AS claim that their result holds through for rule B'(Q), ours does, too.

of the AS inefficiency result. Therefore, in other applications of local public goods economies than the formation of countries there is no reason to expect the inefficiency result to hold true.

We have also shown that the specific rules for country formation employed by Alesina and Spolaore [1] select one specific strong Tiebout equilibrium out of a multiplicity and that qualified majority rule for changing borders cannot enforce the efficient number of countries as a stable equilibrium in their model.

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Appendix

PROOF OF PROPOSITION **1**:

Let $\mathcal{W}$ be an unconnected coalition structure. We show that $\mathcal{W}$ cannot be efficient, since there exists another coalition structure $\mathcal{W}'$ with the same number of coalitions where the sum of distances between agents and its local public good is strictly lower. Let $S \in \mathcal{W}$ be the unconnected coalition with the lowest subindex in $\mathcal{W}$. Then there exists a line segment T such that for some agents $i \in T$ holds $i \notin S$ but their locations lie between the highest and the lowest agent in S. Note that T cannot be a singleton because then S were connected. Construct $\mathcal{W}'$ such that it is identical to $\mathcal{W}$ but all but the first part of S are moved to the left such that S is connected. Since $|T| > 0$, on one hand, the sum of distances between agents in coalition S and $l(S)$ must be strictly lower in $\mathcal{W}'$ than in $\mathcal{W}$. On the other, since all coalitions with a lower subindex than S are connected in $\mathcal{W}'$, for the remaining coalitions the sum of distances cannot have increased in comparison to $\mathcal{W}$. ■

PROOF OF LEMMA **1**:

Let $\mathcal{W}$ be an unconnected Tiebout equilibrium and let $S \in \mathcal{W}$ be the unconnected coalition with the lowest subindex in $\mathcal{W}$. Denote the first part of S by S_L. Since S has at least two parts choose S_R in the following way. If $l(S) \in S_L$, then choose the second part. Otherwise choose S_R such that $l(S) \in S_R$. Call $b2$ and $b3$ the right border agent of S_L and the left border agent in S_R, respectively. Denote the coalition immediately on the right of S_L by S'. It is sufficient to distinguish four cases.

Case 1: Suppose $l(S) \leq b2 < l(S') \leq b3$. Denote $i = l(S')$. Observe that because $\mathcal{W}$ is a Tiebout equilibrium, $b2$ is indifferent between the two coalitions that are closest to her. This implies $c_i(S) > c_{b2}(S) = c_{b2}(S') > c_i(S')$.

But $c_i(S) > c_i(S')$ if and only if $\frac{|S|-|S'|}{|S||S'|} < c(l(S') - l(S))$. Consider now $j = b3$. Here $c_j(S) \leq c_j(S')$ if and only if $\frac{|S|-|S'|}{|S||S'|} \geq c(l(S') - l(S))$, which is a contradiction.

Case 2: Suppose $l(S) \leq b2 < b3 < l(S')$. Denote $i = b2$. We have that $c_i(S) = c_i(S')$ if and only if $\frac{|S'|-|S|}{|S||S'|} = c(l(S') + l(S) - 2b2)$. Consider now $j = b3$. Here $c_j(S) \leq c_j(S')$ if and only if $\frac{|S'|-|S|}{|S||S'|} \leq c(l(S')+l(S)-2b3)$. Both conditions imply $b3 \leq b2$, which contradicts the definition of those agents.

Case 3: Suppose $l(S') \leq b3 < l(S)$. Denote $i = b2$. Now it holds that $c_i(S) = c_i(S')$ if and only if $\frac{|S|-|S'|}{|S||S'|} = c(l(S) - l(S'))$. Note that, since $l(S') < l(S)$, we have also that $|S| > |S'|$. Denote the agent with the highest position in S' whose position is on the left of $l(S)$ by j. Suppose $l(S') < j$. The fact that $c_j(S') \leq c_j(S)$ leads to $\frac{|S|-|S'|}{|S||S'|} \leq c(l(S) + l(S') - 2j)$. Both conditions imply $l(S') \geq j$, a contradiction. Suppose now $l(S') = j$. It follows that there must exist an agent $k \in S'$ with $k > l(S)$. We have that $c_k(S') \leq c_k(S)$ leads to $\frac{|S'|-|S|}{|S||S'|} \geq c(l(S) - l(S'))$. Here $l(S') < l(S)$, implies that $|S| < |S'|$, also a contradiction.

Case 4: The case $b3 < l(S) < l(S')$ is proven along the lines of the third case. One just has to interchange each S and S'. ■

PROOF OF PROPOSITION **3**:

Because of lemma 1 consider a TE $\mathcal{W}$ which is connected. Suppose there are at least two coalitions. Border agents are indifferent between the two coalitions they belong to if and only if [3.1] $c_b(S_n) = c_b(S_{n+1})$, $\forall b(S_n, S_{n+1}) \in B(\mathcal{W})$. This holds if either [3.2] $|S_n| = |S_{n+1}|$ or [3.3] $|S_n||S_{n+1}| = \frac{2}{c}$.

We show first that $\mathcal{W}$ contains at most two different coalition sizes. Suppose there are three or more different sizes $|S_l|$, $|S_m|$ and $|S_n|$. Then there exists at least two borders at which [3.3] must hold. That is, $|S_l||S_m| = \frac{2}{c}$ and $|S_m||S_n| = \frac{2}{c}$. This implies $|S_l| = |S_n|$, a contradiction.

We show now that if [3.1] $c_b(S_n) = c_b(S_{n+1})$, $\forall b \in B$, then $c_i(S_n) \leq c_i(S_m)$, $\forall i \in S_n$ and $\forall S_n, S_m \in \mathcal{W}$. Suppose [3.1], take (by symmetry) any $S_n \in \mathcal{W}$ with $n \leq N-1$, consider $b = b(S_n, S_{n+1})$ and choose S_m such that $m \geq n+1$. Observe that because of [3.1] and since all coalitions are symmetric, $c_b(\mathcal{W}) = \bar{c}$, $\forall b \in B(\mathcal{W})$. Define $b' = (S_{m-1}, S_m)$. Note that since $d_b(S_m) \geq d_{b'}(S_m)$, we have that $c_b(S_m) \leq \bar{c}$. hence $c_b(S_n) \leq c_b(S_m)$. On one hand, since $d_i(S_n) \leq d_b(S_n)$ for all $i \in S_n$, we have for all $i \in S_n$ that $c_i(S_n) \leq c_b(S_n)$. On the other hand it is true that $d_i(S_m) \geq d_b(S_m)$ for all $i \in S_n$. This implies $c_b(S_m) \leq c_i(S_m)$ for all $i \in S_n$. Thus the desired inequality $c_i(S_n) \leq c_i(S_m)$, for all $i \in S_n$ and for all $S_n, S_m \in \mathcal{W}$ follows. ■

PROOF OF PROPOSITION **4**:

Let $\mathcal{W}$ be a TE. *We show first that the proposition holds for neighboring pairs of coalitions.* Take any pair of neighboring coalitions S_n and S_{n+1} in $\mathcal{W}$ and focus first on S_n. If the merger occurs all agents in S_n have the same advantage of lower public good provision costs, but the left border agent $b = b(S_{n-1}, S_n)$ is among those agents who have to support the biggest increase of distance to the public good. Agent b refuses to form S^M if and only if $c_b(S_n) = \bar{c} = \frac{|S_n|}{2}c + \frac{1}{|S_n|} \leq \frac{|S_n|+|S_{n+1}|}{2}c + \frac{1}{|S_n|+|S_{n+1}|} = c_b(S^M)$. This gives [4.1] $|S_n|^2 + |S_n||S_{n+1}| \geq \frac{2}{c}$. Suppose $|S_n| \neq |S_{n+1}|$. By proposition 3 condition [4.1] is fulfilled. Suppose $|S_n| = |S_{n+1}| = |S|$. Condition [4.1] boils down to $|S| \geq \frac{1}{\sqrt{c}}$. Suppose $\mathcal{W}$ is a TE containing coalitions of different sizes and some coalitions of size $|S_{max}|$ are neighbors. Suppose $|S_{max}| < \frac{1}{\sqrt{c}}$. This contradicts $\sqrt{\frac{2}{c}} < |S_{max}|$, which must be true by proposition 3.

It remains to show that if neighbors do not merge, non-neighboring coalitions do not merge either. Take two non-neighboring coalitions S_n and S_m. Suppose $|S_n| \neq |S_m|$. Since now distances are higher than in the case of neighbors, we know that the merger does not take place. The same holds in the case that $|S_n| = |S_m| = |S_{max}|$. Suppose $|S_n| = |S_m| = |S_{min}|$. If there are only coalitions of size $|S_{min}|$ in between it is clear that the fact that neighboring coalitions do not merge implies that S_n and S_m do not merge either. Assume now that there is one coalition of size $|S_{max}|$ in between. Consider the agent with the highest position in the merger. Similar reasoning as above leads to $2|S_{max}||S_{min}| + |S_{min}|^2 \geq \frac{1}{c}$, which is fulfilled. ■

Proof of proposition **5**:

Since part (ii) follows from equation (11) on page 1036 in AS, we show (i) only. Suppose $\mathcal{W}$ is a TE containing two neighboring coalitions S_n and S_{n+1} of different sizes. Let $|S_n| > |S_{n+1}|$. We know from proposition 3 that [5.1] $|S_n||S_{n+1}| = \frac{2}{c}$ holds. We show that no $i \in \mathcal{W}$ wants to secede to any z with $|z| \leq |S_{n+1}|$ where i will be border agent. Consider $S \in \{S_n, S_{n+1}\}$. We have that $c_i(\mathcal{W}) \leq \frac{|S|}{2}c + \frac{1}{|S|} \leq \frac{|z|}{2}c + \frac{1}{|z|} \leq c_i(z)$ must hold (the last inequality is strict if z is unconnected). If $|S| = |z|$ this is trivially fulfilled, if not it implies [5.2] $|S||z| \leq \frac{2}{c}$. Since $|S| > |z|$ for all $S \in \{S_n, S_{n+1}\}$ and [5.1], [5.2] is true. For later reference note that the proof holds through for unconnected z. ■

Proof of Proposition **6**:

Step 1: We show first that C'stability implies that there is no z with $|z| \leq |S_{min}|$ that blocks $\mathcal{W}$. If $\mathcal{W}$ contains two coalitions of different sizes this follows from the proof of proposition 5. Suppose $\mathcal{W}$ contains only coalitions of the same size. It remains to show that for any unconnected z, denoted by z_u with $|z_u| \leq |S|$, which blocks $\mathcal{W}$ there exists a connected, denoted by z_c, which also blocks $\mathcal{W}$. It is immediate that equation (8) is fulfilled for individuals with $d_i(S) = 0$. Denote the individuals with the lowest and highest position in a secession proposal by $b_1(z_u)$ and $b_2(z_u)$, respectively.

If the interval between $b_1(z_u)$ and $b_2(z_u)$ contains at least one position of government, then there exists an individual in z_u with a higher or equal distance to government than in S. If the unconnected secession proposal z_u does not contain any government location, then it is possible to construct a connected proposal z_c with the government at the same position and $|z_u| = |z_c|$. Consider the border agent who is closest to $l(S)$. The fact that $c_{b(z_c)}(z) > c_{b(z_c)}(S)$ implies $c_{b(z_u)}(z) > c_{b(z_u)}(S)$.

Step 2: For the reminder of this proof suppose $|z| > |S_{min}|$. Fix a secession proposal z and consider its border agents $b(z)$. Since individuals on the border of S have the minimum utility in $\mathcal{W}$, equation (8) is fulfilled for $b(z)$ if [6.1] $c_{b(z)}(z) \geq \frac{|z|}{2}c + \frac{1}{|z|} \geq \frac{|S|}{2}c + \frac{1}{|S|} \geq c_{b(z)}(S)$ holds (the first inequality is strict if and only if z is unconnected).

Step 2.1: Suppose $\mathcal{W}$ contains two coalitions of different sizes and $|z| \geq |S_{max}|$. Note that [6.1] is trivially fulfilled if $|S| = |S_{max}| = |z|$. Let $|S| = |S_{max}| < |z|$. Equation [6.1] becomes [6.2] $|S||z| \geq \frac{2}{c}$ which is fulfilled since $|z| > |S_{min}|$ and $|S_{min}||S_{max}| = \frac{2}{c}$. If $|S| = |S_{min}|$ we reach again [6.2] which is true again, since $|z| \geq |S_{max}|$.

Step 2.2: Suppose $\mathcal{W}$ contains two coalitions of different sizes and $|S_{max}| > |z| > |S_{min}|$. Assume furthermore that z is such that $l(z) = l(S)$ for some coalition S.

Step 2.2.1: Suppose there exists $b \in b(z)$ with $|S_b| = |S_{max}|$. Note that, since if $|S| = |S_{max}|$ then b can not be closer to $l(z)$ than she is to $l(S)$, we focus on $|S| = |S_{min}|$. For a secession proposal z of size $|z|$, denote by $|z| + u$ the line segment between its lowest and highest agent. Note that if z is connected, then $u = 0$. Suppose the position of b lies between $l(z)$ and $l(S)$ (otherwise his distance increases in z). Agent b rejects z if and only if $c_b(z) = \frac{|z|+u}{2}c + \frac{1}{|z|} \geq \frac{|S_{max}|-|z|-u-|S_{min}|}{2}c + \frac{1}{|S_{max}|} = c_{b(z)}(S)$ which implies that $|z|c + \frac{1}{|z|} \geq \frac{|S_{max}|-2u-|S_{min}|}{2}c + \frac{1}{|S_{max}|}$. Hence it is sufficient that $min_{|z|}\{|z|c + \frac{1}{|z|}\} \geq \frac{|S_{max}|-|S_{min}|}{2}c + \frac{1}{|S_{max}|}$. Since the solution to the minimization problem is $\frac{1}{\sqrt{c}}$, it is enough to show that $2\sqrt{c} \geq \frac{|S_{max}|}{2}c + \frac{1}{|S_{min}|}$. By PMP $2\sqrt{c} \geq \frac{2}{|S_{min}|} \geq \frac{|S_{max}|}{2}c + \frac{1}{|S_{min}|}$, where the last inequality is holds by $|S_{min}||S_{max}| = \frac{2}{c}$.

Step 2.2.2: Suppose that for at least one $b \in b(z)$ holds $|S_b| = |S_{min}|$ or that $\mathcal{W}$ contains only coalitions with size $|S_{min}|$. Denote $|z| = |S_{min}|+m$ with $0 < m < 1$. Condition [6.2] $|S||z| \geq \frac{2}{c}$ can be written as $|S_{min}|^2+|S_{min}|m \geq \frac{2}{c}$. If $m \geq |S_{min}|$ this is true by PMP. Suppose now $0 < m < |S_{min}|$.
Note that if $|S| = |S_{max}|$, then there exists an agent who rejects z if $\frac{|S_{min}|+|S_{max}|}{2}c + \frac{1}{|z|} \geq \frac{1}{|S_{min}|}$. This gives $|z||S_{min}| \geq \frac{1}{c}\frac{2(|z|-|S_{min}|)}{|S_{min}|+|S_{max}|}$ and is fulfilled by PMP since $4|S_{min}| > 2|z|$.
Define the interval between both border agents in z as $n + |S_{min}|$. We have that z is connected if and only if $n = m$. Let $n < |S_{min}|$. Equation [6.1] can be written as [6.3] $c_{b(z)}(z) = \frac{|S_{min}|+n}{2}c + \frac{1}{|S_{min}|+m} \geq \frac{|S_{min}|-n}{2}c + \frac{1}{|S_{min}|} = c_{b(z)}(S)$. This gives $\frac{n}{m}|S_{min}|(|S_{min}| + m) \geq \frac{1}{c}$, which is true because of PMP, $\frac{n}{m} \geq 1$ and $|S_{min}|m \geq 0$. Let $2|S_{min}| > n \geq |S_{min}|$. Equation [6.3] becomes [6.4] $c_{b(z)}(z) = \frac{|S_{min}|+n}{2}c + \frac{1}{|S_{min}|+m} \geq (|S_{min}| - \frac{n}{2})c + \frac{1}{|S_{min}|} = c_{b(z)}(S)$. From this we get $(n - \frac{|S_{min}|}{2})c \geq \frac{|S_{min}|}{2}c \geq \frac{m}{(|S_{min}|+n)|S_{min}|}$ or $|S_{min}|^2\frac{|S_{min}|+m}{2m} \geq \frac{1}{c}$. This is true since PMP and $|S_{min}| + m > 2m$. It is clear that z with $n \geq 2|S_{min}|$ are also rejected.

Step 2.2.3: Note that by construction both border agents in z reject z. Therefore, for other locations $l(z) \neq l(S)$ there is always one border agent closer to $l(S)$ than in the numerical expressions above. This implies that such a z does not block $\mathcal{W}$.

Step 2.2.4: It remains to show that if $\mathcal{W}$ contains two coalitions of different sizes and $|S_{max}| > |z| > |S_{min}|$, all z that lie entirely in a coalition of size $|S_{max}|$ or between two of the same size are not successful. We use the results of AS which say that it suffices that $|S_{max}| \leq \frac{\sqrt{2}+2}{\sqrt{c}}$. We show that $\frac{2}{c} \leq \frac{\sqrt{2}+2}{\sqrt{c}}|S_{min}|$. Since $|S_{min}| \geq \frac{1}{\sqrt{c}}$, we need that $\frac{2}{c} \leq \frac{\sqrt{2}+2}{c}$, which is true. For unconnected z one can argue as in step 1. ■

Proof of Proposition **7**:

We show that the efficient coalition structure determined in AS (which we review in proposition 2) is always a STE. From the proof of proposition 1 in AS (page 1047) we know that for $c < 8$ the grand coalition is efficient. Note that in this case it is not possible to form a larger coalition. Hence the lower bound for STE does not apply. For this range of values of c the condition $1 \leq \frac{\sqrt{6}+2}{\sqrt{c}}$ is always fulfilled. For $c \geq 8$ take the efficient coalition structure consisting of N^* coalitions of the same size. We express the conditions for N^* to be a STE also in the number of coalitions and not in its size. We know that $N^* \in [max\{2, \frac{\sqrt{c}}{2} - 1\}, \frac{\sqrt{c}}{2} + 1)$. Note that $2 \geq \frac{\sqrt{c}}{2} - 1$ if and only if $c \leq 36$. For this values of c it is true that $\frac{\sqrt{c}}{\sqrt{2}+2} < 2$. On one hand we have $\frac{\sqrt{c}}{\sqrt{2}+2} < \frac{\sqrt{c}}{2} - 1$, which is also fulfilled for $c > 36$. Consider now the upper bound for N^*. We have that $\frac{\sqrt{c}}{2} + 1 < \sqrt{c}$ if and only if $c > 4$, which is also fulfilled. ■

Proof of Proposition **8**:

Consider strong Tiebout equilibria which are stable under rule A.

Step 1: Application of rule $B(\frac{2}{3})$ in order to increase the number of coalitions. A change from a coalition structure $\mathcal{W}$ with N coalitions of size $|S| = \frac{1}{N}$ to $\mathcal{W}'$ with $N+1$ coalitions of size $|S'| = \frac{1}{N+1}$ is carried out if there is a majority of $\frac{2}{3}$ in every coalition of $\mathcal{W}$. Hence this change is *not* carried out if there is a coalition with $\frac{1}{3}$ of its population against it.[23] Consider the first coalition in $\mathcal{W}$. If the change is carried out, coalitions get smaller, which implies that all agents have to pay more for public good provision. Denote the population share between the old local public good $l(S_1)$ and the new border $b(S_1', S_2')$ as P_1. Note that all agents in $P_1 = |S'| - \frac{|S|}{2}$ get unambiguously worse off by the change. We have that $P_1 \geq \frac{1}{3}|S|$ if and only if $N \geq 5$. Note that for $c > 50$, the grand coalition is not a STE. We are left with $N \in \{2, 3, 4\}$. Consider the change from $N = 2$ to $N' = 3$ and the first country. The agents who get unambiguously worse off are those located in the interval $[\frac{5}{24}, \frac{3}{8}]$. This is a total of $\frac{1}{6}$ or $\frac{1}{3}$ of the population of the first country. Consider the change from $N = 3$ to $N' = 4$ and the second country. The agents in $[\frac{7}{16}, \frac{9}{16}]$ get unambiguously worse off. Their cardinality is $\frac{1}{8}$ which is more than the necessary $\frac{1}{9}$. For the change from $N = 4$ to $N' = 5$ consider again the first country. The agents in $[\frac{1}{8}, \frac{17}{80}]$ get unambiguously worse off. Their cardinality is $\frac{7}{80}$ which is again more than the necessary $\frac{1}{12}$. Thus, there are always enough agents against the change.

Step 2: Application of rule $B(\frac{2}{3})$ in order to decrease the number of coalitions. From the analysis of AS (lemma 4, page 1053) we know that stability under rule A implies that there is always a majority (in at least one coalition) against the change. A majority is more than enough under rule B($\frac{2}{3}$).

Step 3: $B(\frac{2}{3})$-equilibria. Because of step 1 and 2 all STE which are stable under rule A are B($\frac{2}{3}$)-equilibria.

Step 4: $B(\frac{2}{3})$-stability. Note that for $c > 50$ there are multiple B($\frac{2}{3}$)-equilibria. Suppose a perturbation to any STE which is stable under rule

[23] We assume here that exactly $\frac{1}{3}$ is enough. This is motivated by the fact that in most countries $\frac{2}{3}$ of the parliament is not an integer and therefore strictly more than $\frac{2}{3}$ is needed. This assumption is not crucial for our result because the only change in which we have ties is the one from $N = 2$ to $N' = 3$.

A. Since the latter is a B($\frac{2}{3}$)-equilibrium (step 3), rule B($\frac{2}{3}$) is not applied and the system does not return to its initial position. Thus, there exists no B($\frac{2}{3}$)-stable coalition structure. ■