Free Trade Networks*

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Abstract

The paper examines formation of free trade agreements (FTAs) as a network formation game. We build an n-country model with quasi-linear utility and a continuum of differentiated industrial goods. Countries can differ in the size of the industrial goods industry and the purchasing power. We show that countries with similar characteristics are more likely to reach FTAs. When countries are symmetric and industrial commodities are not extremely substitutable from one another, we show that the complete global free trade network is the unique pairwise stable (Jackson and Wolinsky, JET, 1996). We also compare customs unions and free trade areas in the incentives to form FTAs with outsiders under symmetric and asymmetric countries.

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1 Introduction

The network of preferential trade agreements (PTAs) covers most countries in a complex way. The tendency towards "regionalism," a movement to form regional trade agreements, has been steadily growing especially since 1980s (Bhagwati, 1993). Since the Treaty of Rome established the European Economic Community (EEC) in 1957, European Union (EU) has been growing with the accession of new members. The North American Free Trade Agreement (NAFTA) starts negotiations with Latin American countries to form a Free Trade Area of the Americas. Japan has recently signed a free trade agreement with Singapore, and now seeks the possibilities separately with South Korea and Mexico. The website of the World Trade Organization (WTO) on regionalism provides us with an excellent introduction to this topic.

> Over 200 regional trade agreements (RTAs) have been notified to the GATT or WTO over time; currently over 150 agreements are in force, most of which have been concluded in the past 10 years. Since 1995, over 100 agreements covering trade in goods or services, or both, have been notified to the WTO.
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> The network of RTAs throughout the world is now highly complex and many countries are members of several agreements, sometimes with differing rules. Nearly 60 percent of the notified RTAs in force at the end of 2000 have been concluded among European countries. RTAs concluded among developing countries account for about 15 percent of the total. One of the most frequently asked questions is whether the growth of regional groups helps or hinders the development of the WTO's multilateral trading system.
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> (*the World Trade Organization website* `http://www.WTO.org`)

Whether PTAs serve as "building blocs" or "stumbling blocs" is a central question on this topic (Bhagwati, 1993). Of course, multilateral trade liberalization efforts and PTA formation interact with each other.[1] However, putting this feature aside for a while, another important question remains. Will successive PTA formation alone effectively achieve global

[1] Levy (1997) and Krishna (1998) show in their political economy models that PTA formation hinders multilateral trade liberalization. Freund (2000b) demonstrates that countries have more incentive to form PTAs as multilateral trade negotiations lower tariffs imposed by every country. See also Bagwell and Staiger (1997a,b), Bond, Syropoulos, and Winters (2001), and Ethier (1998).

free trade, or will the process stop prematurely so that the world is divided into several, mutually exclusive trading blocs? If PTA formation continues until global free trade network is achieved, we may conclude that PTAs are "building blocs." But otherwise, PTAs can be "stumbling blocs."[2]

Ohyama (1972) and Kemp and Wan (1976) demonstrate a positive result for this "dynamic" path problem. The so-called Kemp-Wan theorem states that member countries can appropriately adjust external tariffs and make internal transfers so that a newly formed customs union is Pareto-improving, not only to members themselves but also to all countries in the world.[3] Continuous application of this Kemp-Wan process implies that the customs union expansion continues until all countries in the world are covered.[4] Although the theorem looks promising, it should be taken as an existence theorem (of a Pareto-improving customs union expansion). In reality, it is extraordinarily difficult to adjust external tariffs such that each nonmember country's welfare is not reduced by the formation of a customs union. Indeed, as Viner (1950) taught us, adverse trade-diversion effects often prevents PTAs from being Pareto improving.[5] It is far from obvious that in reality, countries always have incentives to form PTAs so that we will eventually observe the complete global free trade network (global free trade). Indeed, Yi (1996) shows that the world would be divided into two customs unions of asymmetric size when the number of countries is a realistic number.

Customs unions are not the only form of PTAs. There are free trade areas such as NAFTA, and, more generally, there are many bilateral free trade agreements (FTAs) in the world.[6] Unlike customs union, countries choose their individual external tariffs without

[2]Bhagwati and Panagariya (1996) raises this "PTA time-path" question. The complete global free trade network may still be different from global free trade attained through multilateral trade negotiations, as Freund (2000a) demonstrates in a model where firms incur distribution network costs, for example. The global free trade network may be more complex and inefficient ("spaghetti bowl" phenomenon) than global free trade attained through multilateral trade negotiations, as Bhagwati and Panagariya (1996) claim.

[3]See Panagariya and Krishna (2002) for a free trade area version of the Kemp-Wan theorem.

[4]Baldwin (1993) demonstrates that as a regional trading bloc expands, outside countries have more incentive to join the bloc. Haveman (1996) and Goto and Hamada (1999) conduct interesting simulations that investigate welfare consequences of customs union expansion.

[5]Krugman (1991b) claims that if a "natural" trading bloc, within which a large share of trade takes place even in the absence of a PTA, is formed, the gains from trade creation are likely to outweigh the losses from trade diversion. Deardorff and Stern (1994) show the possibility that the situation where the world is divided into trading blocs is almost as beneficial as global free trade.

[6]A free trade area such as NAFTA can be considered as a collection of free trade agreements between any pair of members of the free trade. In this paper, an abbreviation "FTA" will represent a "(bilateral) free trade agreement." A "free trade area" will be written as it is.

consent of other member countries undera free trade area or an FTA.[7] An important consequence of this difference, which seems to be overlooked more or less in the literature, is that under an FTA, each member country (or a subset of member countries) can sign another FTA(s) with outside countries without consent of other member countries. Whereas in the case of customs unions, such as European Union, all member countries should be involved when an outside country tries to form an FTA with a member country of a customs union. Thus, FTAs and free trade areas are more flexible than customs unions: a hub-and-spoke system, for example, can be allowed.[8] We can distinguish customs unions and free trade areas by whether or not there are constraints over admissible FTAs to be signed: there are constraints over admissible FTAs under the regime of customs union, while there is none under the one of free trade area.

The main goal of this paper is to investigate whether the world-wide movement toward FTAs continue until the complete global free trade network is attained.[9] An FTA may be added to the current free trade network in the world in various ways, such as expansion of free trade area, and a hub-and-spoke system where a country in a free trade area signs a bilateral FTA with an outside country. An appropriate way to accommodate such various forms of FTAs is to model the problem as a network formation game developed by Jackson and Wolinsky (1996). Given any FTA configuration in the world, we examine whether a pair of countries have incentives to sign an FTA, and whether a country has an incentive to cut an existing FTA. A network that is immune to these two incentives is called *(pairwise) stable* (Jackson and Wolinsky, 1996).[10] Then we ask whether the complete global free trade network is stable, and if it is, we further ask whether it is a unique one. If the complete global

[7] Richardson (1993), Yi (2000) and Bond, Riezman, and Syropoulos (2002) show that each member country's optimal tariffs against nonmember countries decline as a free trade area expands. See Krugman (1991a) and Bond and Syropoulos (1996) in the case of customs unions.

[8] Kowalczyk and Wonnacott (1992) discuss the hub-and-spoke system in the argument for NAFTA. Mukunoki and Tachi (2001) investigate dynamic formation of bilateral FTAs in a three-country model. See also Puga and Venables (1997) and Wonnacott (1996)

[9] Driven by the same motivation, Freund (2000c) builds a model such that each country calls out the number of countries with which it wants to have FTAs, and shows that global free trade is effectively attained as a unique Nash equilibrium. However, she seems to assume implicitly that a bilateral FTA between two countries is made effective as long as one of the countries benefits from an agreement, even if the other strictly prefers not to sign the agreement. This "open membership" rule (see also Yi, 1996) does not seem to be appealing for discussions of FTAs. If FTAs require consent from both sides, then we will run into the multiplicity of Nash equilibria (see footnote 26).

[10] Coalition formation games such as Yi (1996, 2000) may not be rich enough to accommodate this complex feature of FTAs.

free trade network is a unique stable network, the world is likely to attain global free trade, building many bilateral FTAs (unless there are constraints over admissible set of FTAs).[11]

In order to find stable free trade networks, we need to analyze each country's incentive to sign or abandon an FTA. As Krugman (1991b) and Grossman and Helpman (1995) suggest, asymmetry of countries is an important factor when we assess countries' incentives for FTAs. Viner (1950), on the other hand, suggests that substitutability of commodities traded internationally is also an important factor. The model of this paper is so general that it enables us to observe how these factors play in countries' decisions as to whether to have an FTA with other countries.

Our model is described as follows. There are n countries in the world, and there is a numeraire good and a continuum of differentiated industrial commodities. Countries may be different in the purchasing power (population size) and the size of the industrial goods industry (measure of firms). Each of the differentiated commodities is produced at the same marginal cost by one firm that belongs to one of n countries. Each consumer has a common quasi-linear utility function, in which substitutability of industrial commodities is parameterized. Each country has a tariff schedule for imported industrial commodities, and an FTA between countries i and j simply means that countries i and j simultaneously eliminate tariffs on commodities imported from countries j and i, respectively.

With a quasi-linear utility function, we can capture important factors for welfare impacts of an FTA. When the utility function is quasi-linear, social welfare, which is merely a representative consumer's utility, can be decomposed into two parts: the consumer's gross utility and the (industrial) trade surplus that is defined as export profits minus import payments in the industrial goods sector. An FTA with another country is likely to raise the gross utility, although the second-best effect (Lipsey and Lancaster, 1956) may sometimes outweigh general benefits from tariff reduction.[12] Thus, whether an FTA enhances social welfare can

[11] To derive a definite prediction regarding the time-path to global free trade, we may need to build a dynamic network formation model with farsightedness. Mukunoki and Tachi (2001) show in a dynamic, symmetric, three-country model that under a certain parameter values, only one bilateral FTA may be signed in equilibrium so that global free trade is not attained. As Kennan and Riezman (1990) suggest, countries in a bilateral FTA may in some cases prefer the current situation to global free trade. Then, each member country may not sign a new bilateral free trade agreement with an outside country since it would induce an FTA between spoke countries, effectively attaining global free trade, in the future. However, extending Mukunoki and Tachi's (2001) analysis to the case of many countries is not an easy task.

[12] If tariffs have been imposed on a large portion of commodities, it may not be welfare-improving to get rid of tariffs for a small portion of commodities since it enlarges distortions between these commodities and

be judged in many cases by observing its impact on the (industrial) trade surplus.

The effect on a country's trade surplus of signing an FTA with another country can be further decomposed into two: one on the trade surplus between these two countries (the direct surplus effect) and that on the trade surplus with third countries (the third country effect). The latter effect is always positive, since the country's export to third countries is not affected by the FTA, while its imports from them decrease since their commodities become relatively more expensive by the agreement. In contrast, the direct surplus effect can go either way. The sign of the direct surplus effect depends on the two countries' characteristics such as purchasing power, the size of the industrial good industry, and their current partner countries. Let us consider, for example, an FTA between a highly-industrialized small country and a less-industrialized large country. The FTA increases trade flows from the former to the latter disproportionately, dramatically increasing the trade surplus of the small highly-industrialized country and decreasing that of the large less-industrialized country. The direct surplus effect for the large less-industrialized country is likely to be negative, and it may outweigh the third country effect. Consequently, the large less-industrialized country is likely to oppose to sign the FTA.[13] If two countries are similar in their characteristics, however, the direct surplus effects would be small in magnitude irrespective of their signs, and they are likely to benefit from signing an FTA.[14]

The main results of this paper are as follows. When all countries are symmetric in their purchasing powers and industry sizes, we show that the global free trade network, a network in which any pair of countries has an FTA, is pairwise stable (Proposition 1).[15] If

the ones with high tariffs.

[13]It is interesting to notice that countries in our model have a view that Krugman (1991b) calls GATT-think: (1) Exports are good, (2) Imports are bad, (3) and other things being equal, an equal increase in imports and exports is good. Our model gives an economic reasoning to this "enlightened mercantilism," as Bagwell and Staiger (1999) does in a model that examines the economic role of the GATT/WTO principles of reciprocity and nondiscrimination.

[14]This result is not specific to our segmented market assumption. Furusawa and Konishi (2002) shows that even in competitive market economy, similar countries tend to have incentives to sign an FTA since it allows these countries to substitute imports from the third countries by the ones from their partners.

[15]Goyal and Joshi (2001) focuses on the analysis of pairwise stable free trade network in a different model from ours: a symmetric country model in which each country's market is segmented, and firms are Cournot players. Assuming that prohibitive tariffs are charged for countries that have no FTAs with, Goyal and Joshi characterize all pairwise stable networks of their economy: either a complete graph (the global free trade) or an almost complete graph in isolation of one country ($n-1$ countries have complete graph, and one country has no FTA link with any other country). As an extension of their model, they also show that the global

commodities are highly substitutable from one another, however, there may also be other pairwise stable networks. Even though all countries are symmetric in the purchasing power and industry size, the difference in the number of free trade partners can create a large differential in the impacts of the direct surplus. Nevertheless, we show that if commodities are not highly substitutable from one another, the global free trade network is the unique pairwise stable network (Proposition 2).[16] On the other hand, if countries are asymmetric, the global free trade network may not be attained. In a special case of no substitution among commodities, a pair of countries sign an FTA if and only if the levels of industrialization (industry size divided by purchasing power) are in a certain range (Proposition 3). This proposition implies that developing and developed countries may form stumbling blocs with the groups. In the same framework, we also discuss the differences in incentives to form FTAs under the two regimes: free trade areas and customs unions (given other things equal). We find that if countries are symmetric, countries have less incentive to adopt another country to a customs union than to a free trade area unless commodities are highly substitutable (Proposition 4), and shows an example in which expansion of customs union stagnates while adding free trade networks enables the world to attain global free trade. However, if countries are asymmetric, then it may be possible that customs union may do better than free trade areas, since customs union can average its members' characteristics out. We illustrate this possibility by the case of nonsubstitution.

A simplification assumption adopted throughout this paper is that countries do not adjust their tariff rates on commodities from third countries when they sign FTAs. It is well-known that the optimal tariff rates charged by the members of customs unions or free trade areas may change as the sizes of groups become large.[17] This tariff rate adjustments potentially affect the incentives to form FTAs later in the process of FTA formation. We adopted this assumption by two reasons.[18] First, given the existence of the GATT multinational tariff negotiations, it is unclear if countries are charging optimal tariffs to the countries which they

free trade network is pairwise stable when tariffs are endogeneously determined (*c.f.* our Proposition 2 and Remark 2).

[16] In fact, we have effectively shown an even stronger result: The world free trade is always attained in the long run from any initial free trade network, if countries myopically make decisions on formations of FTAs. Given the complications of the free trade networks, myopic behaviors by countries may be justified.

[17] See papers in footnote 7.

[18] One additional reason is that there is literature that assumes fixed tariff rates in the process of FTA formation. **(papers?)** Note also that even if tariff rates are exogeneously fixed, there are still market powers created by forming a group. Indeed, in Example 2 below, we show a customs union benefit by excluding several countries from their coalition.

do not have FTAs with. Second, more practically, if tariff rates are adjusted optimally at each step of FTA formation process, calculations to check the incentives to form FTAs seem too complicated to obtain analytical results given that we allow complicated FTA networks.

Our paper is organized as follows. In Section 2, we present the model and derive the equilibrium in the presence of tariffs. Then, we propose a convenient way to decompose a country's social welfare, which plays an important role in the subsequent analysis. We also derive the optimal tariff. In Section 3, we analyze a country's incentive to sign an FTA, and then derive our main results in the symmetric country case. Section 4 compares incentives to sign the two types of PTAs: free trade areas and customs unions. Section 5 concludes the paper.

2 The Model

2.1 Overview

Let N be the set of n countries ($n \geq 2$), each of which is populated by a continuum of identical consumers who consume a continuum of horizontally differentiated commodities that are indexed by $\omega \in [0,1]$, as well as a numeraire good. A differentiated commodity can be considered as a variety of an industrial good. Each industrial commodity ω is produced by one firm which is also indexed by the same ω. Each firm is owned equally by all domestic consumers who receive equal shares of all firms' profits. The numeraire good is produced competitively, on the other hand. Each consumer is endowed with l units of labor, which is used for production of the industrial and numeraire goods. Each unit of labor produces one unit of the numeraire good, so that the wage rate equals 1. We also assume that industrial commodities are produced with a linear technology, and normalize the unit labor requirement to be equal to 0 for each industrial commodity, without loss of generality. Alternatively, we can interpret the model such that each consumer is endowed with l units of the numeraire good, which can be transformed by a linear technology into industrial commodities.[19]

In country i ($i = 1, 2, \cdots, n$), measure μ^i of consumers and measure s^i of firms that produce industrial commodities are located. Thus, country i produces s^i industrial commodities, which are consumed in every country in the world. We assume that the markets

[19] Due to the normalization, however, production of industrial commodities does not require any numeraire good as an input in our model.

are segmented so that firms can perfectly price discriminate among different countries. We normalize the size of total population so that $\sum_{k=1}^{n} \mu^k = 1$ as well as $\sum_{k=1}^{n} s^k = 1$. The ratio s^i/μ^i measures country i's industrialization level. The higher the ratio, the higher the country's industrialization level. This ratio plays an important role later in our analysis. Country i imposes a specific tariff at a rate of t_j^i on the imports of the industrial commodities that are produced in country j. For simplicity, we assume that every commodity produced in country j faces the same tariff rate t_j^i.[20] Since we assume that there is no commodity tax, we have $t_i^i = 0$. We also assume that the countries do not impose tariffs on the numeraire good which may be traded internationally to balance the trade. Tariff revenue is redistributed equally to domestic consumers.

2.2 Consumer Demands

A representative consumer's utility is represented by the following quasi-linear utility function:

$$U(q, q_0) = \alpha \int_0^1 q(\omega)d\omega - \frac{\beta}{2}\int_0^1 q(\omega)^2 d\omega - \frac{\delta}{2}\left[\int_0^1 q(\omega)d\omega\right]^2 + q_0, \tag{1}$$

where $q : [0, 1] \to \Re_+$ is an integrable consumption function, and q_0 denotes the consumption level of the numeraire good.[21] The second last term represents the substitutability among differentiated commodities, which may become clearer if we notice $\left[\int_0^1 q(\omega)d\omega\right]^2 = \int_0^1 \int_0^1 q(\omega)q(\omega')d\omega' d\omega$. Letting y denote the consumer's income, the budget constraint can be written as

$$y = \int_0^1 \tilde{p}(\omega)q(\omega)d\omega + q_0, \tag{2}$$

where $\tilde{p} : [0, 1] \to \Re_+$ denotes the consumer price function. The first order condition for the consumer's maximization problem gives us the inverse demand function for each good ω:

$$\tilde{p}(\omega) = \alpha - \beta q(\omega) - \delta \int_0^1 q(\omega')d\omega'.$$

Integrating over $[0, 1]$, we obtain

$$\alpha - \beta \int_0^1 q(\omega)d\omega - \delta \int_0^1 q(\omega)d\omega = \int_0^1 \tilde{p}(\omega)d\omega,$$

[20] In our simple model, country i's optimal tariff rates are the same across all commodities imported from country j.

[21] This utility function is a continuous-goods version of the ones of Shubik (1984) and Yi (1996, 2000) who analyze the case where there are only finitely many differentiated commodities. Our continuum of commodity setup is based on Ottaviano, Tabuchi, and Thisse (2002).

or

$$\int_0^1 q(\omega)d\omega = \frac{1}{\beta+\delta}\left(\alpha - \tilde{P}\right),$$

where $\tilde{P} = \int_0^1 \tilde{p}(\omega)d\omega$. Substituting this equation back into the first order condition, we obtain

$$q(\omega) = \frac{\alpha}{\beta} - \frac{1}{\beta}\tilde{p}(\omega) - \frac{\delta}{\beta(\beta+\delta)}\left(\alpha - \tilde{P}\right).$$

2.3 Equilibrium in Country i

Letting $p^i(\omega)$ and $\tilde{P}^i$ denote the producer price for commodity ω sold in country i, and the average consumer price in country i, respectively, a representative consumer's demands in country i for a commodity ω produced in country j can be written as[22]

$$q^i(\omega) = \frac{\alpha}{\beta} - \frac{1}{\beta}(p^i(\omega) + t^i_j) - \frac{\delta}{\beta(\beta+\delta)}(\alpha - \tilde{P}^i). \tag{3}$$

The firm ω in country j chooses $\{p^i(\omega)\}_{i=1}^n$ in order to maximize its profits $\pi(\omega) = \sum_{i=1}^n \mu^i p^i(\omega) q^i(\omega)$. The first order condition for this maximization gives us

$$p^i(\omega) = \frac{1}{2}\left[\frac{\alpha\beta}{\beta+\delta} - t^i_j + \frac{\delta}{\beta+\delta}\tilde{P}^i\right], \tag{4}$$

for any i. Notice that $p^i(\omega)$ does not vary with ω. Prices charged by firms depend only on the import country's tariff policies. For simplicity, we henceforth suppress the argument ω.

It follows from (4) that country i's average consumer price is calculated as

$$\begin{aligned}\tilde{P}^i &= \sum_{j=1}^n s^j(p^i + t^i_j) \\ &= \frac{1}{2}\left[\frac{\alpha\beta}{\beta+\delta} + \frac{\delta}{\beta+\delta}\tilde{P}^i\right] + \frac{1}{2}\bar{t}^i,\end{aligned}$$

where $\bar{t}^i \equiv \sum_{j=1}^n s^j t^i_j$. Thus, country i's average consumer price $\tilde{P}^i$ is given by

$$\tilde{P}^i = \frac{\alpha\beta}{2\beta+\delta} + \frac{\beta+\delta}{2\beta+\delta}\bar{t}^i. \tag{5}$$

Substituting (5) into (4) yields the equilibrium producer price that each firm in country j charges for the market of country i as a function of country i's tariff vector $\mathbf{t}^i = (t^i_1, ..., t^i_n)$:

$$p^i_j(\mathbf{t}^i) = \frac{\alpha\beta}{2\beta+\delta} - \frac{1}{2}t^i_j + \frac{\delta}{2(2\beta+\delta)}\bar{t}^i.$$

[22] In country i, consumer price of commodity ω produced in country j is $\tilde{p}^i(\omega) = p^i(\omega) + t^i_j$.

Then it follows from (3) that a representative consumer's demand in country i for a commodity produced in country j is

$$q_j^i(\mathbf{t}^i) = \frac{\alpha}{2\beta+\delta} - \frac{1}{2\beta}t_j^i + \frac{\delta}{2\beta(2\beta+\delta)}\bar{t}^i. \tag{6}$$

Notice that $p_j^i(\mathbf{t}^i) = \beta q_j^i(\mathbf{t}^i)$ holds for any tariff vector $\mathbf{t}^i$.

2.4 Social Welfare

Under the world tariff vector $\mathbf{t} = (\mathbf{t}^1, ..., \mathbf{t}^n)$, each firm in country i earns the profits:

$$\pi_i(\mathbf{t}) = \sum_{k=1}^{n} \mu^k p_i^k(\mathbf{t}^k) q_i^k(\mathbf{t}^k) = \sum_{k=1}^{n} \mu^k \beta q_i^k(\mathbf{t}^k)^2. \tag{7}$$

Country i's *per capita* tariff revenue is

$$T^i(\mathbf{t}^i) = \sum_{k=1}^{n} t_k^i s^k q_k^i(\mathbf{t}^i) = \sum_{k=1}^{n} s^k t_k^i q_k^i(\mathbf{t}^i). \tag{8}$$

A representative consumer's income in country i is the sum of labor income l, redistributed tariff revenue $T^i(\mathbf{t}^i)$, and the profit shares of the firms in country i, $s^i\pi_i(\mathbf{t})/\mu^i$:

$$y = l + T^i(\mathbf{t}^i) + \frac{s^i \pi_i(\mathbf{t})}{\mu^i}. \tag{9}$$

Then it follows from (2) that

$$\begin{aligned} q_0^i(\mathbf{t}) &= l + T^i(\mathbf{t}^i) + \frac{s^i\pi_i(\mathbf{t})}{\mu^i} - \sum_{k=1}^{n} s^k [p_k^i(\mathbf{t}^i) + t_k^i] q_k^i(\mathbf{t}_k^i) \\ &= l + \sum_{k=1}^{n} s^k t_k^i q_k^i(\mathbf{t}^i) + \frac{s^i}{\mu^i}\sum_{k=1}^{n} \mu^k p_i^k(\mathbf{t}^k) q_i^k(\mathbf{t}^k) - \sum_{k=1}^{n} s^k [p_k^i(\mathbf{t}^i) + t_k^i] q_k^i(\mathbf{t}^i) \\ &= l - \sum_{k \neq i} s^k p_k^i(\mathbf{t}^i) q_k^i(\mathbf{t}^i) + \frac{s^i}{\mu^i} \sum_{k \neq i} \mu^k p_i^k(\mathbf{t}^k) q_i^k(\mathbf{t}^k), \end{aligned}$$

where $q^i(\omega) = q_k^i(\mathbf{t}^i)$ if ω is produced in country k.

Substituting this demand function for the numeraire good into (1) and letting $\mathbf{q}^i(\mathbf{t}^i)$ represent country i's equilibrium consumption plan function under tariff $\mathbf{t}^i$, i.e., $q^i(\mathbf{t}^i) = (q_j^i(\mathbf{t}^i))_{j \in N}$, we obtain a representative consumer's utility in country i as a function of the world tariff vector, which can be considered as country i's *per capita* social welfare:

$$W^i(\mathbf{t}) = U(q^i(\mathbf{t}^i), q_0^i(\mathbf{t})) = V^i(\mathbf{t}^i) + X^i(\mathbf{t}^{-i}) - M^i(\mathbf{t}^i). \tag{10}$$

where

$$V^i(\mathbf{t}^i) = U(q^i(\mathbf{t}^i), l), \tag{11}$$

$$M^i(\mathbf{t}^i) = \sum_{k\neq i} s^k p^i_k(\mathbf{t}^i) q^i_k(\mathbf{t}^i) = \sum_{k\neq i} \beta s^k q^i_k(\mathbf{t}^i)^2, \tag{12}$$

$$X^i(\mathbf{t}^{-i}) = \frac{s^i}{\mu^i} \sum_{k\neq i} \mu^k p^k_i(\mathbf{t}^k) q^k_i(\mathbf{t}^k) = \frac{s^i}{\mu^i} \sum_{k\neq i} \beta \mu^k q^k_i(\mathbf{t}^k)^2, \tag{13}$$

with $\mathbf{t}^{-i} = (\mathbf{t}^1, \cdots, \mathbf{t}^{i-1}, \mathbf{t}^{i+1}, \cdots, \mathbf{t}^n)$. The functions $V^i(\mathbf{t}^i)$, $M^i(\mathbf{t}^i)$, and $X^i(\mathbf{t}^{-i})$ represent a consumer's gross utility, (industrial) import payments, and (industrial) export profits, respectively.[23] Country i's social welfare consists of a consumer's gross utility $V^i(\mathbf{t}^i)$, and the per-capita industrial trade surplus $X^i(\mathbf{t}^{-i}) - M^i(\mathbf{t}^i)$. Country i's tariffs affect social welfare through the effects on $V^i(\mathbf{t}^i)$ and $M^i(\mathbf{t}^i)$. Other countries' tariffs also affect country i's social welfare through the effect on $X^i(\mathbf{t}^{-i})$.

Now, we examine the effects of tariff changes on the three components of social welfare: $V^i(\mathbf{t}^i)$, $X^i(\mathbf{t}^{-i})$, and $M^i(\mathbf{t}^i)$. We notice from (11)-(13) that an increase in a tariff rate affects these components only through the changes in consumption of industrial commodities. We see from (6) that the consumption of an industrial commodity depends on the tariff rate imposed on that commodity and the average tariff rate, i.e., $q^i_k(\mathbf{t}^i) = \tilde{q}^i_k(t^i_k, \bar{t}^i)$. Thus, we can write, for example, $V^i(\mathbf{t}^i) = \tilde{V}^i(\tilde{q}^i_1(t^i_1, \bar{t}^i), \cdots \tilde{q}^i_n(t^i_n, \bar{t}^i))$. An increase in t^i_j directly affects q^i_j, and indirectly affects through an increase in the average tariff rates of country i on all industrial goods consumed in country i, q^i_k $(k = 1, 2, \cdots, n)$. These changes in consumption affect $V^i(\mathbf{t}^i)$ and $M^i(\mathbf{t}^i)$, in turn. As for the effect on $V^i(\mathbf{t}^i)$, for example, we have

$$\frac{\partial V^i}{\partial t^i_j} = \sum_{k=1}^{n} \frac{\partial \tilde{V}^i}{\partial \tilde{q}^i_k} \left(\frac{\partial \tilde{q}^i_k}{\partial t^i_j} + \frac{\partial \tilde{q}^i_k}{\partial \bar{t}^i} \frac{\partial \bar{t}^i}{\partial t^i_j} \right).$$

An increase in another country's tariff rate on country i's commodity affects the export profits $X^i(\mathbf{t}^{-i})$ in a similar fashion. We can easily obtain the following lemma that shows the effects of raising a tariff rate on the three components of social welfare.

Lemma 1 *The first order effects of raising t^i_j on V^i and M^i and the effects of raising t^j_i on*

[23] We should notice that utility derived from the numeraire good consumption in $V^i(\mathbf{t}^i)$ corresponds to the case where the entire amount of the numeraire good produced from an individual's labor endowment is consumed. A consumer may not exactly demand l units of the numeraire good and hence $V^i(\mathbf{t}^i)$ may not represent actual gross utility in any situation. But, since l is a fixed number, this possible discrepancy does not affect any decision of any agent.

X^i *are:*

$$\frac{\partial V^i}{\partial t^i_j} = s^j \left[-\frac{\alpha}{2\beta+\delta} + \frac{\delta}{2(2\beta+\delta)} \sum_{k=1}^{n} s^k q^i_k(\mathbf{t}^i) + \frac{1}{2} q^i_j(\mathbf{t}^i) \right],$$

$$\frac{\partial X^i}{\partial t^j_i} = \frac{\mu^j s^i}{\mu^i} \beta \left[q^j_i(\mathbf{t}^j) \left(-\frac{1}{\beta} + \frac{\delta}{\beta(2\beta+\delta)} s^i \right) \right],$$

$$\frac{\partial M^i}{\partial t^i_j} = s^j \beta \left[q^i_j(\mathbf{t}^i) \left(-\frac{1}{\beta} + \frac{\delta}{\beta(2\beta+\delta)} s^j \right) + \sum_{k \neq i,j} q^i_k(\mathbf{t}^i) \frac{\delta}{\beta(2\beta+\delta)} s^k \right].$$

It may appear that an increase in a tariff rate of country i, say t^i_j necessarily decreases the domestic consumer's gross utility V^i. Each consumer in country i reduces the consumption of country j's commodities as a consequence, which is detrimental. However, each agent consumes other commodities more than before at the same time, which tends to increase the consumer's gross utility. The latter indirect effect may outweigh the former so that an increase in a tariff rate may increase the domestic consumer's gross utility, if the industrial commodities are highly substitutable. Similarly, an increase in a tariff rate may not always decrease the import payments. If the industrial commodities are highly substitutable, the resulting decrease in q^i_j may be outweighed by increases in q^i_k for $k \neq i, k$. It is easy to see from the lemma that an increase in another country's tariff decreases the domestic profits obtained from the export to that country. The next section discusses the effects of a change in a tariff rate on the three components of social welfare in more detail.

Now, let us derive the optimal tariffs. Country i's optimal tariff maximizes $V^i(\mathbf{t}^i) - M^i(\mathbf{t}^i)$ since $X^i(\mathbf{t}^{-i})$ does not depend on $\mathbf{t}^i$. Let $C_i = \{j \in N | t^i_j = 0\}$ represent the set of countries that produce commodities on which country i does not impose tariffs. (Notice that C_i includes country i itself since $t^i_i = 0$.) We consider here the situation in which country i has signed FTAs, rather than CUs, with all other countries in C_i. Therefore, country i chooses its external tariffs without any coordination with other countries in C_i. The optimal tariff given country i's free trade network C_i is described as follows. As the following lemma shows, a country's optimal tariff rates only depend on its own characteristics.[24] In particular, they do not depend on other countries' characteristics at all due to the separability of a consumer's utility function.

Lemma 2 *Country i's optimal tariff rate τ^i is a function of s^i, $s^{C_i} (\equiv \sum_{j \in C_i} s^j)$, and pa-*

[24] Somewhat surprisingly, they do not depend on country i's own market size μ^i.

rameters α, β and δ:

$$\tau^i(s^i, s^{C_i}; \alpha, \beta, \delta) = \frac{4\alpha\beta(\beta + \delta s^i)}{3(2\beta+\delta)^2 - \delta(1-s^{C_i})[4(2\beta+\delta) - \delta(1-2s^i)]} > 0.$$

Moreover, it is increasing in s^i, and decreasing in s^{C_i}. Thus, a country of size s^i has the highest optimal tariff rate when $s^{C_i} = s^i$, or $C_i = \{i\}$, and a more industrialized country, whose s^i tends to be high, has a higher optimal tariff rate given that s^{C_i} is constant.

3 Free Trade Agreements

3.1 Incentives to sign an FTA

Let us consider an FTA between countries i and j, such that they eliminate all tariffs imposed on commodities imported from each other, but not the tariffs imposed on other commodities. We assume for simplicity that they keep the tariffs on other commodities imported from outside countries unchanged. For an FTA to be signed, both countries i and j must benefit from the agreement. That is, we must have both (i) $W^i(t^i_j, \mathbf{t}^i_{-j}; t^j_i, \mathbf{t}^j_{-i}; \mathbf{t}^{-\{i,j\}}) \leq W^i(0, \mathbf{t}^i_{-j}; 0, \mathbf{t}^j_{-i}; \mathbf{t}^{-\{i,j\}})$ and (ii) $W^j(t^j_i, \mathbf{t}^j_{-i}; t^i_j, \mathbf{t}^i_{-j}; \mathbf{t}^{-\{i,j\}}) \leq W^j(0, \mathbf{t}^j_{-i}; 0, \mathbf{t}^i_{-j}; \mathbf{t}^{-\{i,j\}})$. The condition (i), for example, can be written as

$$V^i(0, \mathbf{t}^i_{-j}) + X^i(0, \mathbf{t}^j_{-i}; \mathbf{t}^{-\{i,j\}}) - M^i(0, \mathbf{t}^i_{-j}) \geq V^i(\mathbf{t}^i) + X^i(\mathbf{t}^j; \mathbf{t}^{-\{i,j\}}) - M^i(\mathbf{t}^i),$$

or

$$\Delta V^i(\mathbf{t}^i) + \left[\Delta X^i(\mathbf{t}^j; \mathbf{t}^{-\{i,j\}}) - \Delta M^i(\mathbf{t}^i)\right] \geq 0, \tag{14}$$

where $\Delta V^i(\mathbf{t}^i) \equiv V^i(0, \mathbf{t}^i_{-j}) - V^i(\mathbf{t}^i)$, $\Delta X^i(\mathbf{t}^{-i}) \equiv X^i(0, \mathbf{t}^j_{-i}; \mathbf{t}^{-\{i,j\}}) - X^i(\mathbf{t}^j; \mathbf{t}^{-\{i,j\}})$, and $\Delta M^i(\mathbf{t}^i) \equiv M^i(0, \mathbf{t}^i_{-j}) - M^i(\mathbf{t}^i)$. A consumer's gross utility is likely to increase as a result of the tariff reduction, unless the industrial goods are highly substitutable. (For more precise conditions, see Lemma 3 below.) Since the FTA increases the country's export profits and is also likely to increase the import payments, the FTA has an ambiguous impact on country i's industrial trade surplus, whose change is expressed by the terms in the square brackets.

We analyze each term in order. First, let us investigate the sign of $\Delta V^i(\mathbf{t}^i)$. The next lemma shows that an FTA increases a consumer's gross utility either if the substitutability among the industrial commodities is low or if the original tariff rate is small.

Lemma 3 *A bilateral FTA increases a consumer's gross utility, i.e.,* $\Delta V^i(\mathbf{t}^i) > 0$*, if either one of the following conditions is satisfied:*

$$(i)\ 4\beta(\beta+\delta) - \delta^2(1 - 2s^{C_i} - s^j) \geq 0,$$

$$(ii)\ t^i < \frac{8\alpha\beta^2}{\delta^2(1-2s^{C_i}-s^j) - 4\beta(\beta+\delta)}.$$

In particular, in the case where $s^k = 1/n$ *for any* k *and the original tariff rate does not exceed the optimal tariff rate that is obtained when* k *does not have any FTA* ($C_i = \{i\}$)*, i.e.,* $\tau(n) = \tau^i(1/n, 1/n; \alpha, \beta, \delta)$*, it is sufficient that* $\delta \leq 10\beta$ *for condition (ii) to be satisfied.*

Remark 1 *Note that condition (i) is satisfied if* $1-2s^{C_i}-s^j \leq 0$*, or equivalently* $s^{C_i} + \frac{1}{2}s^j \geq \frac{1}{2}$*. This corresponds to the second best effect: In an economy with distortions, removing tax distortions partially may reduce efficiency (see Dixit, 1975, and Hatta, 1977). When a tariff on a commodity is eliminated, distortions between this commodity and untaxed commodities shrink, whereas distortions with taxed commodities expand. Thus, if there are more untaxed commodities than taxed commodities, the second best theory tells us that a consumer's gross utility is likely to rise. The condition* $s^{C_i} + \frac{1}{2}s^j \geq \frac{1}{2}$ *matches exactly to this observation.*[25] *Turning to condition (ii), the right-hand side is positive only when* $s^{C_i} + \frac{1}{2}s^j < \frac{1}{2}$*. Thus, condition (ii) is meaningful only when (i) is not satisfied, and it shows that the detrimental second best effect is negligible if the rate of the tariffs, which cause distortions in the first place, is small.*

Next, we turn to investigating the effect of an FTA between countries i and j on the industrial trade surplus. Letting M_k^i and X_k^i be country i's (per capita) import from country k and country i's (per capita) export to country k, respectively: i.e.,

$$\begin{aligned} M_k^i(\mathbf{t}^i) &= \beta s^k q_k^i(\mathbf{t}^i)^2, \\ X_k^i(\mathbf{t}^k) &= \frac{s^i}{\mu^i}\beta\mu^k q_i^k(\mathbf{t}^k)^2 \ \left(= \frac{\mu^k}{\mu^i} M_i^k(\mathbf{t}^k)\right), \end{aligned}$$

we can rewrite country i's industrial trade surplus as follows.

$$X^i(\mathbf{t}^{-i}) - M^i(\mathbf{t}^i) = \sum_{k \neq i} \left(X_k^i(\mathbf{t}^k) - M_k^i(\mathbf{t}^i)\right).$$

[25] The term $\frac{1}{2}s^j$ corresponds to an average effect of distortions in the process of reducing tariffs from t_j^i to zero.

An FTA between i and j only involves changes in $\mathbf{t}^i$ and $\mathbf{t}^j$ so that it does not affect $X_k^i(\mathbf{t}^k)$ for any $k \neq i, j$. Consequently, a change in country i's industrial trade surplus can be written as

$$\begin{aligned}
\Delta\left[X^i(\mathbf{t}^{-i}) - M^i(\mathbf{t}^i)\right] &= \Delta\left[X_j^i(\mathbf{t}^j) - M_j^i(\mathbf{t}^i)\right] + \sum_{k\neq i,j} \Delta\left[X_k^i(\mathbf{t}^k) - M_k^i(\mathbf{t}^i)\right] \\
&= \underbrace{\Delta\left[X_j^i(\mathbf{t}^j) - M_j^i(\mathbf{t}^i)\right]}_{\text{direct surplus effect}} + \underbrace{\left(-\sum_{k\neq i,j} \Delta M_k^i(\mathbf{t}^i)\right)}_{\text{third country effect}}.
\end{aligned}$$

The third country effect, represented by the terms in the parentheses, is always positive since the reduction of t_j^i makes commodities imported from country j relatively cheaper, and country i's imports from these third countries decrease, i.e., $\Delta M_k^i(\mathbf{t}^i) < 0$. Although the third country effect works positively for countries i and j, it implies that the FTA between them hurts all other countries.

Having shown that the third country effect is positive, let us now investigate the direct surplus effect, which can be rewritten as follows from the definitions of $M_i^j(\mathbf{t}^j)$ and $M_j^i(\mathbf{t}^i)$.

$$\begin{aligned}
\underbrace{\Delta\left[X_j^i(\mathbf{t}^j) - M_j^i(\mathbf{t}^i)\right]}_{\text{direct surplus effect}} &= \Delta\left[\frac{\mu^j}{\mu^i} M_i^j(\mathbf{t}^j) - M_j^i(\mathbf{t}^i)\right] \\
&= \Delta\left[\frac{\mu^j}{\mu^i} \beta s^i q_i^j(\mathbf{t}^j)^2 - \beta s^j q_j^i(\mathbf{t}^i)^2\right] \\
&= \mu^j \beta \Delta\left[\theta^i q_i^j(\mathbf{t}^j)^2 - \theta^j q_j^i(\mathbf{t}^i)^2\right],
\end{aligned}$$

where $\theta^i = s^i/\mu^i$ is the *level of country i's industrialization* (per capita number of firms in country i). The higher θ^i and the lower θ^j, the more the industrial trade surplus increases. Countries with different industrialization levels tend to have asymmetric incentives to form an FTA. Given other things being equal, the more industrialized country is enthusiastic, while the less industrialized country is reluctant. The intuition is clear. The more industrialized country derives a large benefit from the opening of the partner's relatively large market. In addition, opening its own market for the partner's firms does not significantly increase the import payments since the resulting penetration by the partner's firms to the relatively small domestic market is small. Another important factor that affects the incentives to form an FTA is the difference in the original tariff rates. It is easy to see that if t^j is higher than t^i, for example, $\Delta q_i^j(\mathbf{t}^j)$ is greater than $\Delta q_j^i(\mathbf{t}^i)$. Country i's export to country j increases more than its import from country j, and hence the FTA between i and j tends to be more beneficial to country i.

3.2 Stable Free Trade Networks

An FTA that involves more than two countries can be considered as a collection of bilateral FTAs between member countries when costs to form bilateral FTAs are negligible. Therefore, it is convenient to describe FTAs in terms of simple graph theory terminologies. An FTA between countries i and j is described by a *link*, which is an unordered pair of two countries. A graph is a collection of links among the countries in the world. An *FTA graph* is a nondirected graph, (N, Γ) that is a pair consisting the set of countries N and a (free trade) *network* Γ that is a collection of links. The set of *partners* of $i \in N$ in a network Γ is $C_i(\Gamma) = \{i\} \cup \{k \in N : (i, k) \in \Gamma\}$. As we have already described, we include i in the set of partners of i just for notational simplicity. This set describes the set of countries with which country i has FTAs. Country i imposes a tariff if and only if that commodity is imported from a country that is outside of these countries. We continue to write it as C_i without confusion, if a network Γ is fixed.

If tariff rates are exogenously determined, or if they are determined uniquely by an existing free trade network Γ (such as the case where all countries set their individual optimal tariffs given the prevailing network Γ), then country i's payoff (social welfare) can be written uniquely by $u_i(\Gamma)$. Consequently, the set of countries N and their payoff functions define a *network formation game*.

Network formation games are first studied by Jackson and Wolinsky (1996). A *pairwise stable network* is a network Γ^* such that (i) for any $i, j \in N$ with $i \neq j$ and $(i, j) \in \Gamma^*$, $u_i(\Gamma^*) \geq u_i(\Gamma^* \backslash (i, j))$ and $u_j(\Gamma^*) \geq u_j(\Gamma^* \backslash (i, j))$, i.e., for any pair of linked countries, neither country has an incentive to cut the link, and (ii) for any $i, j \in N$ with $i \neq j$ and $(i, j) \notin \Gamma^*$, either $u_i(\Gamma^*) \geq u_i(\Gamma^* \cup (i, j))$ or $u_j(\Gamma^*) \geq u_j(\Gamma^* \cup (i, j))$, i.e., for any pair of non-linked countries, at least one of them has no incentive to form a link with the other.[26]

We are particularly interested in the situation where the entire world consists of a large free trade area (the global free trade). This situation can also be described by a graph theory terminology. A *complete* graph is a graph (N, Γ^{comp}) that contains all possible links: i.e., for

[26] The readers may be tempted to formulate a strategic form game in which each player (country) announces names of players she wants to be linked, and a link is formed if and only if both sides of the link announce the other players' names. However, if we use Nash equilibrium as a solution concept of such a game, there are too many Nash equilibria including an outcome without any link. It is because a player has no incentive to announce another player's name who does not announce her name. For a refinement of Nash equilibrium in such games (coalition-proof Nash equilibrium), see Dutta and Mutuswami (1997).

any $i, j \in N$ with $i \neq j$, $(i,j) \in \Gamma^{comp}$. We call Γ^{comp} a *complete network*. The global free trade is a complete graph of the free trade network formation game.

3.3 Symmetric Countries

We say that countries i and j are *symmetric* if $s^i = s^j$ and $\mu^i = \mu^j$. We derive our main results confining ourselves to the case in which the world consists of n symmetric countries so that $s^i = \mu^i = 1/n$ for any $i \in N$. In this case, the effects on country i's industrial trade surplus can be simplified as follows,

$$\begin{aligned}
\Delta\left[X^i(\mathbf{t}^{-i}) - M^i(\mathbf{t}^i)\right] &= \Delta\left[X^i_j(\mathbf{t}^j) - M^i_j(\mathbf{t}^i)\right] + \left(-\sum_{k\neq i,j}\Delta M^i_k(\mathbf{t}^i)\right) \\
&= \mu^j\beta\Delta\left(\frac{s^i}{\mu^i}q^j_i(\mathbf{t}^j)^2 - \frac{s^j}{\mu^j}q^i_j(\mathbf{t}^i)^2\right) + \left(-\sum_{k\neq i,j}\Delta s^k\beta q^i_k(\mathbf{t}^i)^2\right) \\
&= \underbrace{\frac{\beta}{n}\left[\Delta(q^j_i(\mathbf{t}^j)^2) - \Delta(q^i_j(\mathbf{t}^i)^2)\right]}_{\text{direct surplus effect}} + \underbrace{\frac{\beta}{n}\left(-\sum_{k\neq i,j}\Delta(q^i_k(\mathbf{t}^i)^2)\right)}_{(\text{third country effect})\geq 0}.
\end{aligned}$$

Countries' cooperation structure affects the impact of the FTA between i and j on country i's industrial trade surplus through its effects on commodity demands. This effect is particularly important under symmetry: What matters to country i is the size of C_i and the size of C_j. We define $c_k = |C_k|$ for notational simplicity. Under symmetry, for example, we then have $s^{C_k} = c_k/n$.

Let us say that countries i and j are *completely symmetric* if they are symmetric and $c_i = c_j$. If the original tariffs are the same between completely symmetric countries i and j, i.e., $t^i = t^j = t$, then $\bar{t}^i = \bar{t}^j$ and $q^j_i(\mathbf{t}^j) = q^i_j(\mathbf{t}^i)$, and hence we have $\Delta q^j_i(\mathbf{t}^j) = \Delta q^i_j(\mathbf{t}^i)$. Thus, the direct surplus effect disappears if countries i and j are completely symmetric and their original tariffs are the same at t. An increase in country i's export to country j and an increase in country i's import from county j are completely canceled out.[27] On the other hand, the third country effect is present and is positive, if there are third countries. Thus, we have $\Delta\left[X^i(\mathbf{t}^{-i}) - M^i(\mathbf{t}^i)\right] \geq 0$ if countries i and j are completely symmetric.[28]

[27] This implies that if $n = 2$, then the industry trade surplus stays the same (though $\Delta V^i > 0$ by Remark 1).

[28] This argument can be related to Bagwell and Staiger's (1999) argument that reciprocal trade liberalization between two countries leaves each country's terms of trade unchanged and reduces negative terms-of-

Therefore, completely symmetric countries always have incentives to sign an FTA as long as one of the conditions in Lemma 3 is satisfied. One important case is that all pairs but (i, j) have already formed free trade links. Since most tariffs are already eliminated, the trade reform between i and j reduces distortions (condition (i) of Lemma 3), and hence enhances a consumer's gross utility in these countries ($\Delta V^i > 0$). Thus, these two countries can improve social welfare by signing an FTA, which leads to our first proposition.

Proposition 1 *Suppose that there are n symmetric countries in the world, and that their external tariff rates are the same if they are imposed. Then, the global free trade (a complete network Γ^{comp}) is a stable network.*

Proof. If all pairs but (i, j) have already formed free trade links, i.e., the free trade network is $\Gamma^{comp} \backslash (i, j)$, then countries i and j are completely symmetric. As a result, we know from the above calculations that each country's industrial trade surplus does not decrease by signing an FTA. Moreover, since $s^{C_i} = 1 - \frac{1}{n}$ and $s^j = 1/n$, and similarly for country j, when the free trade network is $\Gamma^{comp} \backslash (i, j)$, we have $s^{C_i} + \frac{1}{2} s^j = 1 - \frac{1}{2n} > 1/2$ for all $n \geq 2$. Then, it follows from Lemma 3 and Remark 1 that a consumer's gross utility in countries i and j also strictly increases. Therefore, we have

$$
\begin{aligned}
u_i(\Gamma^{comp}) &> u_i(\Gamma^{comp} \backslash (i, j)), \\
u_j(\Gamma^{comp}) &> u_j(\Gamma^{comp} \backslash (i, j)),
\end{aligned}
$$

implying that Γ^{comp} is a stable network.

Q.E.D.

Remark 2 *Note that this proposition holds even if we assume that each country adjusts its tariff rate optimally according to the current free trade network. Starting at the complete graph, if a pair of countries cut their FTA, they would impose the same tariff rates. Thus, the assumptions of the above proposition are satisfied even if tariff rates are endogenously determined.*

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trade externalities, and hence beneficial. In our economy, prices and quantities change proportionally. An FTA between two completely symmetric countries fits their argument in that it leaves the bilateral (industrial commodity) terms of trade unaffected and eliminate bilateral terms-of-trade externalities. In addition, each country's bilateral terms of trade against a third country improves as a decrease in $q_k^i(\mathbf{t}^i)$ and hence a decrease in $p_k^i(\mathbf{t}^i)$ show. By signing an FTA, a country can also benefit from the third countries.

A natural question now is whether or not the complete graph is a unique stable network. Unfortunately, it is not the case in general. If $q_i^j(\mathbf{t}^j)$ is significantly smaller than $q_j^i(\mathbf{t}^i)$ and hence $\Delta q_i^j(\mathbf{t}^j)$ is significantly smaller than $\Delta q_j^i(\mathbf{t}^i)$, the direct surplus effect is negative and it may outweigh the third country effect. This can occur when country j has many FTAs with other countries, while country i has a small number of FTAs. The following numerical example illustrates this. As a benchmark, we set each country's tariff rate at the optimal level without FTAs ($s^i = s^{C_i} = 1/n$), i.e., $\tau(n) \equiv \tau^i(1/n, 1/n; \alpha, \beta, \delta)$.

Example 1 *Suppose that countries are symmetric,* $n = 12$ *and* $\delta = 12\beta$. *Suppose further that* $t^i = \tau(n)$ *for any* $i \in N$. *In this case, graph* $\Gamma^{-1} = \{(j,k) : j,k \neq 1\}$ *(country 1 does not have any FTA, while all other countries have FTAs with one another) is also pairwise stable. The reason why* Γ^{-1} *is stable is that the isolated country* 1 *does not have an incentive to make a bilateral FTA with any other country, although each of countries 2,...,12 has an incentive to sign a bilateral FTA with country* 1.

Notice that under the graph Γ^{-1}, country 1 and other countries become most asymmetric to each other, thus country 1 has the least incentive to form FTAs with others. The intuition for country 1 having no incentive to form a bilateral FTA with any other country is as follows. When δ is large, consumer demands are price sensitive. This means that, in the absence of an FTA, country 1 does not import much of industrial commodities, and most of industrial commodities consumed are domestically produced. However, once country 1 signs an FTA with country 2, say, much of (about a half of) domestic commodity consumption is substituted by commodities produced in country 2 since they all have the same price after the FTA. Therefore, country 1 experiences a dramatic increase in its import payments. In contrast, country 2 has opened up its market for all but country 1 before the FTA. Consequently, the FTA with country 1 does not increase its import much even if δ is large. Therefore, the direct surplus effect of country 1 is negative and is large in its magnitude, which outweighs the third country effect and the effect on $\Delta V^i(\mathbf{t}^i)$ in Example 1.

If δ is not very large, on the other hand, even when country 1 has no FTA with other countries, the share of domestic commodities is limited. Then, opening its market for country 2 does not have a large negative effect on country 1's industrial trade surplus. Thus, for a lower value of δ, it seems likely that country 1 has an incentive to have an FTA with country 2 given a positive third country effect.

In the following, we seek a parameter restriction on the values of δ and t that guarantees that every pair of countries has incentive to form an FTA regardless of their existing FTAs. In such a case, it is obvious that the complete graph (global free trade) becomes the unique pairwise stable network.

Lemma 4 *Suppose that countries i and j are symmetric and that the external tariffs are the same, i.e., $t^i = t^j = t$. Then, regardless of free trade networks of these two countries with the rest of the world, $\Delta(X^i - M^i) > 0$ holds if the following condition is satisfied.*

$$t \leq \frac{2\alpha\beta}{4\beta + (1 + \frac{1}{n})\delta}.$$

If the optimal tariff rate, derived in Lemma 2, is smaller than $\frac{2\alpha\beta}{4\beta+(1+\frac{1}{n})\delta}$, we can conclude that a bilateral FTA between symmetric countries i and j increases the industrial trade surplus for these countries regardless of the network structure, under the mild condition that their external tariffs are not greater than their optimal tariffs. The next proposition states that it is indeed the case if the industrial commodities are not highly substitutable.

Proposition 2 *Suppose that there are n symmetric countries in the world, and that their external tariff rates are the same at t that is not greater than the optimal tariff rate without any FTA, i.e., $\tau(n) = \tau^i(1/n, 1/n; \alpha, \beta, \delta)$. Suppose further that $\delta \leq 6\beta$. Then, under any network Γ, any pair of countries i and j without a bilateral FTA has incentives to form a free trade link. As a result, the global free trade (a complete network Γ^{comp}) is the unique stable network.*

The significance of this proposition is that it applies regardless of these two countries' existing FTAs (with other countries).[29] Consider the case where there exist several free trade areas, possibly different in the size. If the industrial commodities are not extremely substitutable, any pair of countries from different free trade areas has incentives to form an FTA. As far as trading blocs take the form of free trade areas, as opposed to customs union, they are likely to be "building blocs" instead of "stumbling blocs" towards the global free trade when countries are symmetric.

[29] Proposition 2 suggests that as far as countries myopically make decisions as to whether or not they have FTAs with other countries, the world free trade network will eventually reach the complete network such that global free trade is effectively attained.

3.4 Asymmetric Countries

Now, let us turn to an asymmetric country case. Under general values of substitution parameter δ, unfortunately, it is very hard to obtain any analytical result. Thus, in this subsection, we assume $\delta = 0$ in order to simplify the analysis. This is definitely not an ideal assumption, since it makes each commodity demand completely independent, and, as a result, the existing FTA network becomes irrelevant for country i's incentive to sign an FTA with country j. The third country effects vanish, too. Despite of all of these, it is a convenient assumption to illustrate how asymmetry of countries affect the resulting FTA network. Since country i's optimal tariff rate is constant irrespective of s^i, μ^i, and C_i, we can naturally assume that each country imposes the optimal tariff rate, $t = \frac{\alpha}{3}$ for any country i. This implies $p^i = \frac{\alpha}{3}$ for commodities with tariffs. In this special case of no substitution, we can explicitly calculate social welfare of each country easily. Since commodity demand is independent of each other when $\delta = 0$, we can calculate

$$V^i(\mathbf{t}^i) = \sum_{j \in C_i} s^j v(0) + \sum_{k \notin C_i} s^k v(t),$$

$$X^i(\mathbf{t}^i) = \frac{s^i}{\mu^i}\left[\sum_{j \in C_i \setminus \{i\}} \mu^j p(0)q(0) + \sum_{k \notin C_i} \mu^k p(t)q(t)\right],$$

$$M^i(\mathbf{t}^i) = \sum_{j \in C_i \setminus \{i\}} s^j p(0)q(0) + \sum_{k \notin C_i} s^k p(t)q(t),$$

where $v(t)$ and $q(t)$, ($v(0)$ and $q(0)$) are per capita utility and per capita consumption of a commodity with tariff (without tariff). Producer prices of commodities with and without tariff are denoted by $p(t)$ and $p(t)$, respectively. As is easily seen from Figures 1 and 2,

$$v(t) = \frac{5}{2} \times \frac{\alpha^2}{9\beta} = \frac{5}{18} \times \frac{\alpha^2}{\beta},$$

$$v(0) = \frac{3}{2} \times \frac{\alpha^2}{4\beta} = \frac{3}{8} \times \frac{\alpha^2}{\beta},$$

$$p(t)q(t) = \frac{\alpha}{3} \times \frac{\alpha}{3\beta} = \frac{\alpha^2}{9\beta},$$

and

$$p(0)q(0) = \frac{\alpha}{2} \times \frac{\alpha}{2\beta} = \frac{\alpha^2}{4\beta}.$$

Thus, social welfere of country i can be written as,

$W^i(\mathbf{t})$

$$
\begin{aligned}
&= V^i(\mathbf{t}^i) + X^i(\mathbf{t}^{-i}) - M^i(\mathbf{t}^i) \\
&= \frac{\alpha^2}{\beta}\left[\left(\frac{3}{8}s^{C_i} + \frac{5}{18}(1-s^{C_i})\right) + \frac{s^i}{\mu^i}\left(\frac{1}{4}(\mu^{C_i}-\mu^i) + \frac{1}{9}(1-\mu^{C_i})\right) - \left(\frac{1}{4}(s^{C_i}-s^i) + \frac{1}{9}(1-s^{C_i})\right)\right].
\end{aligned}
$$

If countries i and j sign an FTA, then j joins C_i. Thus, the impact on country i's welfare is,

$$
\begin{aligned}
\Delta W^i &= \frac{\alpha^2}{\beta}\left[s^j\left(\frac{3}{8}-\frac{5}{18}\right) + \frac{s^i}{\mu^i}\times\mu^j\left(\frac{1}{4}-\frac{1}{9}\right) - s^j\left(\frac{1}{4}-\frac{1}{9}\right)\right] \\
&= \mu^j \times \frac{\alpha^2}{\beta}\left[\frac{7}{72}\theta^j + \frac{5}{36}\left(\theta^i - \theta^j\right)\right].
\end{aligned}
$$

Recall $\theta^i = s^i/\mu^i$ is the level of country i's industrialization (per capita number of firms in country i). The first term of the contents of the above bracket corresponds to ΔV^i, which is always positive (see Lemma 3 and Remark 1), and the latter corresponds to the direct surplus effect, $\Delta X^i_j - \Delta M^i_j$. As we discussed in Section 3.1, the direct surplus effect depends on two countries' levels of industrialization. If countries are symmetric, our Proposition 2 applies, and ΔW^i is always positive.[30] By rearranging the above formula, we obtain,

$$
\Delta W^i = \frac{\mu^j\alpha^2}{72\beta}\left(10\theta^i - 3\theta^j\right).
$$

Therefore, country i has an incentive to sign an FTA with country j if and only if θ^j does not exceed 10/3 times θ^i. This implies that the levels of indiustrialization are not too different, two countries sign an FTA. The following proposition is a trivial consequesnce of this observation.

Proposition 3 *Suppose that $\delta = 0$. Suppose that there are n countries in the world, and that their external tariff rates are the optimal tariff rate $t = \frac{\alpha}{3}$. Then, countries i and j forms a link if their levels of industrialization satisfy $\theta^i \le \frac{10}{3}\theta^j$ and $\theta^j \le \frac{10}{3}\theta^i$. As a result, the stable network is a collections of all links between such pair of countries, and is generically unique.*

This proposition gives us an interesting prediction. Suppose that there are groups of developed countries (the levels of industrialization are high and similar) and developing countries (the levels of industrialization are low and similar), and that the level of industrialization of the developed countries are far larger (more than 10/3 times) than the one of the

[30] When $\delta = 0$, the third country effect vanishes.

developing countries. Then, the process of FTA formation leads to a stable network in which within each group all countries are linked with each other, while there is no link between groups. Thus, we can say that the FTA formation process may end up with two trade blocs (stumbling blocs) if the levels of industrialization of the two groups are very different from each other.

4 Customs Unions vs. Free Trade Areas

The literature has usually focused on the difference between customs unions and free trade areas in member countries' external optimal tariff rates. In contrast, we investigate in this section the difference in member countries' incentives to sign a new FTA emphasizing the fact that a customs union requires all members' consent when a member country wants to sign an FTA with an outside country. Customs union impose constraints on admissible FTAs to be signed. Our main goal of the paper is to assess how far the process of PTAs continues and whether or not global free trade is effectively attained as a complete world-wide web of FTAs. The analysis in this section may possibly tell us which form of PTAs, customs union or free trade area, should be encouraged for the purpose of facilitating more FTAs in the world. In order to focus on the difference between free trade areas and customs unions in the aspect that a country in a customs union must involve all other member countries to sign a new FTA with an outside country, we assume that tariff rates are fixed and the same in both cases.

First, we examine incentives for country i having FTAs with other countries in C_i to have a new FTA with country $j \notin C_i$, and compare the incentives between two cases: the case where C_i forms a customs union and the case where C_i forms a free trade area (or more weakly the case where country i has FTAs individually with other countries in C_i). Let us begin with investigating the impact on a consumer's gross utility V^i. As we have seen in Section 3.1, the impact on V^i is ambiguous in both cases. However, these effects are exactly the same between the two cases, since V^i only depends on $\mathbf{t}^i$ and changes in $\mathbf{t}^i$ are the same between the two cases. Thus, the difference in changes of the industrial trade surplus between these two cases will determine whether or not country i's incentive to have an FTA with country j is higher in the case where C_i is a customs union rather than a free trade area. Here, we decompose the third country effect into two:

$$\Delta X^i - \Delta M^i = \underbrace{\left(\Delta X^i_j - \Delta M^i_j\right)}_{\text{direct surplus effect}} + \underbrace{\sum_{k\in C_i\backslash\{i\}} \left(\Delta X^i_k - \Delta M^i_k\right)}_{\text{member country effect}} + \underbrace{\sum_{l\notin C_i\cup\{j\}} \left(\Delta X^i_l - \Delta M^i_l\right)}_{\text{nonmember country effect}},$$

where country k is a representative partner of i, i.e., $k \in C_i\backslash\{i\}$, and country l is a representative outsider of i, i.e., $l \notin C_i \cup \{j\}$. The following table compares these two cases item by item.[31]

	Free Trade Area		Customs Union
ΔV^i	?	=	?
ΔM^i_j	+	=	+
ΔX^i_j	+	>	+
ΔM^i_k	−	=	−
ΔX^i_k	0	>	−
ΔM^i_l	−	=	−
ΔX^i_l	0	=	0

We start with M^i_j, country i's import payments from country j. Since country i's import is solely determined by $\mathbf{t}^i$ and country i's post-FTA tariff vectors are the same between the two cases, the effects are exactly the same. This effect is positive since country i lowers its tariff rate for commodities imported from country j. In contrast, the effects on $X^i_j = X^i_j(\mathbf{t}^j)$ are different especially when $|C_i\backslash\{i\}|$ is large. It is because country j eliminates tariffs against all countries in C_i in the case of the customs union while it eliminates tariffs only for commodities imported from country i in the case of free trade area. Since industrial commodities are substitutes among themselves, it is obvious that an increase in X^i_j is smaller in the case of customs union. Consequently, the direct surplus effect is lower in the case of customs union than in the case of free trade area.

Next, we investigate the effects on country i's industrial trade surplus with a member country $k \in C_i\backslash\{i\}$. As before, the effects on $M^i_k = M^i_k(\mathbf{t}^i)$ are the same in both cases. However, the effects on $X^i_k = X^i_k(\mathbf{t}^k)$ are different again. In the case of free trade area, $\mathbf{t}^k$ is unaffected and hence X^i_k does not change. In the case of customs union, on the other hand, country k also eliminates tariffs against country j, and country i's export to country k is reduced due to the substitution effect. Country i's industrial trade surplus with a member

[31] We include ΔV^i for convenience. Signs of the effects of having an FTA with country j are not known, which are described by "?" in the table. The observation that these effects are the same is described by "=."

country k is again lower in the case of customs union. Finally, it is easy to see that the third country effect with nonmembers is the same in both cases. Import payments from country l decreases by the same amount due to the tariff reductions for commodities imported from country j, and country i's export to country l stays the same in both cases since $\mathbf{t}^k$ is not affected.

We have shown that the impacts of a new FTA on consumer's gross utility are the same between the two cases, but the effect on the industrial trade surplus is unambiguously lower in the case of customs union. We record this result as a lemma.

Lemma 5 *Country i has less incentive to have an FTA with country $j \notin C_i$ when C_i forms a customs union rather than a free trade area (or the case where country i has FTAs with other countries in C_i), unless the industrial goods are independent from one another, i.e., $\delta = 0$, in which case the incentives are the same.*

In the symmetric country case with low subsititution parameter δ, we can make a strong statement. Proposition 2 says that free trade areas are more preferable in that sense if all countries are symmetric, and δ is not very high. In such a situation, country j has an incentive to have an FTA with any country, in particular with country i alone and with an integrated economy consisting of all countries in C_i. Therefore, country j wants to have an FTA with country i whether C_i forms a customs union or free trade area. Combining this observation together with Lemma 5, we have shown the following proposition.

Proposition 4 *An FTA is less likely to be signed if a country involved is a member of a customs union rather than a free trade area if all countries are symmetric, imposing the same external tariffs rate, and $0 < \delta \leq 6\beta$, i.e., the industrial commodities are not highly substitutable among themselves.*

The intuition for this result can come from the prisoners' dilemma game: customs union has a power to coordinate the actions (not to sign FTAs with outsiders) among the members and can attain a cooperation outcome, while free trade area cannot force a member to defect by signing an FTA with an outsider. In the following, we show by an example that these two regimes may generate very different predictions in stable allocations. As we have seen in Proposition 2, as long as countries are symmetric and commodities are not highly substitutable with one another ($\delta \leq 6\beta$), the network formation process in FTAs results in global free trade, i.e., the complete graph is the unique pairwise stable network.

How about the customs union case? A customs union formation problem may be best described as a coalition formation problem. Yi (1996) analyzes customs union formation problem among symmetric countries by employing a coalition bargaining game analyzed by Bloch (1996) and Ray and Vohra (1999).[32] Coalition bargaining game is an extension of the Rubinstein bargaining problem to an n-player game with endogenous coalition structures. In each period an exogenously chosen proposer calls a coalition to form, and then the members of the called coalition choose to accept or reject the offer in order. If all members accept, then the coalition is formed, and the rest of players play the same game among them. In the (Markov) perfect equilibrium, the first proposer chooses a coalition that maximizes her payoff among the ones that are acceptable to the members by foreseeing which coalitions are formed afterwards by the rest of the players. In the context of customs union formation, Yi (1996) shows that in equilibrium, at most two customs unions with different size are formed for reasonable numbers of countries. Although our model is different from Yi's in details, we can obtain qualitatively similar outcomes. This implies that equilibrium (or stable) outcomes in PTA formation processes can be very different depending on the rules: whether PTAs are customs unions or free trade areas. The following example illustrates this point.

Example 2 *Suppose that countries are symmetric, $n = 50$ and $\delta = 5\beta$. Suppose further that each country sets its tariff rate at $\tau(n)$, the optimal level without any FTA. In this case, Proposition 2 suggests that a unique stable network in the FTA regime is the complete graph Γ^{comp}. However, in the customs union regime, two customs unions $\{43, 7\}$ are formed in the equilibrium allocation (the first proposer calls a 43 country coalition). Members of the larger union obtain higher social welfare in this allocation than in global free trade. (Members of the large and the small customs unions attain 0.081928 and 0.072397, respectively. Under global free trade, each country obtains 0.081633.)*[33]

In the above, we considered the case where outsiders always have incentives to sign FTAs with the members of free trade group. Let us turn to the investigation whether or not country j's incentive to have an FTA with country i is lower when C_i forms a customs union

[32] See Bloch (1996) and Ray and Vohra (1999) for details. Note that we cannot utilize typical solution such as the core in cooperative game theory. Our problem involves externalities across customs unions (coalitions), although typical cooperative games (characteristic function form games) do not allow such externalities.

[33] This implies that global free trade cannot be the equilibrium outcome of the coalition bargaining game. In contrast, Proposition 2 implies that the above allocation with two customs unions is not pairwise stable: two customs unions have incentives to form an FTA, leading to global free trade.

rather than a free trade area. The difference between the two cases in our terminology is that country j adds only one link with country i in the case of free trade area whereas in the case of customs union country j adds c_i links simultaneously with all individual countries in C_i. To see the difference in country j's incentives, we first notice that the latter case is effectively equivalent to the case where country j has an FTA with an integrated economy that consists of all countries in C_i. That is, country j's potential partner produces s^{C_i} industrial commodities rather than s^i, and its population is $\mu^{C_i}(\equiv \sum_{k\in C_i} \mu^k)$ rather than μ^i, in the case of customs union (thus, $\theta^{C_i} = s^{C_i}/\mu^{C_i}$ matters).

By this observation, we can again analyze an asymmetric country case but with a no substitution assumption $\delta = 0$. Under this assumption, customs unions lose benefit from coordination, as a result, a member of group (either customs union or free trade area) would not have incentive difference (see Proposition 3). However, in this case, customs unions may work as building blocs. Imagine the situation where n countries are ordered by their levels of industrialization, θ^is ($i = 1, 2, ..., n$, and $\theta^1 \geq \theta^2 \geq ... \geq \theta^n$). From Proposition 3, we know that if $\theta^1 > \frac{10}{3}\theta^n$, then countries 1 and n won't sign an FTA, and the process of bilateral FTA formation would never attain the global free trade. Now, on the other hand, consider a customs union $C(k) = \{1, 2, ..., k\}$ among countries with high θ^is. As long as $\frac{10}{3}\theta^{k+1} > \theta^{C(k)}$, country $k + 1$ wants to sign an FTA with $C(k)$, or to join the union. This is true even if $\theta^1 > \frac{10}{3}\theta^{k+1}$. Obviously, every member of $C(k)$ welcomes country $k+1$, and the the union is expanded. Customs union averages the members' levels of industrializations out, and it may encourage a country with low level of industrialization. Thus, if the condition $\frac{10}{3}\theta^{k+1} > \theta^{C(k)}$ is satisfied for each $k = 1, ..., n - 1$, customs union may work as "building blocs".[34]

[Taiji-san, Do you think we need the following part of the paper?]

To see the effect of C_j being a customs union, we calculate $\partial\Delta V^i/\partial s^j$ from (17) in the proof of Lemma 3 in the Appendix to obtain

$$\frac{\partial\Delta V^i}{\partial s^j} = \frac{t^i}{8\beta(2\beta+\delta)^2}\{8\alpha\beta^2 + [4\beta(\beta+\delta) - (1 - 2(s^{C_i} + s^j))\delta^2]t^i\},$$

which is more likely to be positive if $s^{C_i} + s^j$ is large as we can infer from the second best theory. Whether or not consumer's gross utility increases more (or decreases less) in the case of customs union is generally ambiguous, however. The same is true in the impacts on

[34]However, we should note that history of customs union expansions may matter. It might be possible for the customs union expansion process stumbles if there are two unions fromed by developed countries only and developing countries only. If two unions' levels of industrialization differ too much, then the process stumbles

the industrial trade surplus. We have shown in Section 3.1 that an increase in country i's industrial trade surplus is more likely to be positive and large if the partner country is less industrialized. Thus, the less industrialized and the more populated countries in $C_j \setminus \{j\}$, the more likely that an increment in country i's industrial trade surplus is larger in the case where C_j is a customs union than in the case where it is a free trade area.

Lemma 6 *Whether or not country i's incentive to have an FTA with country j is lower when C_j forms a customs union rather than a free trade area is ambiguous. However, country i's incentive is more likely to be lower if $s^{C_i} + s^{C_j}$ is small and countries in $C_j \setminus \{j\}$ is more industrialized on average than country j.*

It follows immediately from Lemma 6 that which forms of PTA should be encouraged in order to facilitate FTA formation is generally ambiguous.

5 Concluding Remarks

We have introduced a very general analytical framework that is suitable to the investigation of PTAs and shown how countries' incentives vary with the country size, industrialization level, substitutability among industrial commodities, etc. We have found that if all countries are symmetric, a complete global free trade network is pairwise stable and that it is a unique stable network if industrial commodities are not highly substitutable from one another. We also have shown that in such a situation, countries have less incentives to have an FTA if one of the countries is a member of a customs union rather than a free trade area, and shown an example in which expansion of customs union stagnates while adding free trade networks enables the world to attain global free trade.

We must note that all of such results are obtained under the assumption that external tariffs are fixed when countries form PTAs. Since Lemma 2 implies that a country's optimal tariffs decrease as the country have more free trade links, countries would lower their external tariffs as they form more free trade links. It is interesting to know how these tariff adjustments affect countries incentives to form PTAs. We leave this extension for future research.[35]

[35] It is indeed difficult to obtain an analytical result for this case in general free trade network formation games. Our initial numerical analyses on n symmetric country case show that even if the optimal tariffs are

Appendix

Proof of Lemma 1. Direct calculations using $\partial\bar{t}^i/\partial t^i_k = s^i$ shows the following.

$$\begin{aligned}
\frac{\partial V^i}{\partial t^i_j} &= \sum_{h=1}^{n} \frac{\partial \tilde{V}^i}{\partial \tilde{q}^i_h}\left(\frac{\partial \tilde{q}^i_h}{\partial t^i_j} + \frac{\partial \tilde{q}^i_h}{\partial \bar{t}^i}\frac{\partial \bar{t}^i}{\partial t^i_j}\right) \\
&= \sum_{h=1}^{n} s^h \left(\alpha - \beta q^i_h(\mathbf{t}^i) - \delta \sum_{k=1}^{n} s^k q^i_k(\mathbf{t}^i)\right)\frac{\delta}{2\beta(2\beta+\delta)} s^j \\
&\quad - \left(\alpha - \beta q^i_j(\mathbf{t}^i) - \delta \sum_{k=1}^{n} s^k q^i_k(\mathbf{t}^i)\right)\frac{1}{2\beta} s^j \\
&= \left(\alpha - (\beta+\delta)\sum_{k=1}^{n} s^k q^i_k(\mathbf{t}^i)\right)\frac{\delta}{2\beta(2\beta+\delta)} s^j - \left(\alpha - \beta q^i_j(\mathbf{t}^i) - \delta\sum_{k=1}^{n} s^k q^i_k(\mathbf{t}^i)\right)\frac{1}{2\beta} s^j \\
&= s^j[-\frac{\alpha}{2\beta+\delta} + \frac{\delta}{2(2\beta+\delta)}\sum_{k=1}^{n} s^k q^i_k(\mathbf{t}^i) + \frac{1}{2} q^i_j(\mathbf{t}^i)].
\end{aligned}$$

Since country j's tariffs on country i's commodities only affect demands for those commodities in country j, we obtain

$$\begin{aligned}
\frac{\partial X^i}{\partial t^j_i} &= \frac{s^i}{\mu^i}\mu^j 2\beta q^j_i(\mathbf{t}^j)\left(\frac{\partial \tilde{q}^j_i}{\partial t^j_i} + \frac{\partial q^j_i}{\partial \bar{t}^j}\frac{\partial \bar{t}^j}{\partial t^j_i}\right) \\
&= \frac{s^i}{\mu^i}\mu^j \beta q^j_i(\mathbf{t}^j)\left(-\frac{1}{\beta} + \frac{\delta}{\beta(2\beta+\delta)} s^i\right),
\end{aligned}$$

$$\begin{aligned}
\frac{\partial M^i}{\partial t^i_j} &= \sum_{k\neq i} s^k \beta 2 q^i_k(\mathbf{t}^i)\left(\frac{\partial \tilde{q}^i_k}{\partial t^i_j} + \frac{\partial \tilde{q}^i_k}{\partial \bar{t}^i}\frac{\partial \bar{t}^i}{\partial t^i_j}\right) \\
&= 2\beta\left(s^j q^i_j(\mathbf{t}^i)(-\frac{1}{2\beta}) + \sum_{k\neq i} s^k q^i_k(\mathbf{t}^i)\frac{\delta}{2\beta(2\beta+\delta)} s^j\right) \\
&= s^j\left(-q^i_j(\mathbf{t}^i) + \sum_{k\neq i} s^k q^i_k(\mathbf{t}^i)\frac{\delta}{2\beta+\delta}\right).
\end{aligned}$$

Q.E.D.

Proof of Lemma 2. First, we extend the results of Lemma 1 by substituting (6) into the formulae.

$$\frac{\partial V^i}{\partial t^i_j} = s^j[-\frac{\alpha}{2\beta+\delta} + \frac{\delta}{2(2\beta+\delta)}\sum_{k=1}^{n} s^k q^i_k(\mathbf{t}^i) + \frac{1}{2} q^i_j(\mathbf{t}^i)]$$

constantly adjusted in the process of free trade network formation, a variant of Proposition 2 holds: Social welfare improves by forming a new bilateral free trade agreement under any kind of free trade networks, if δ is not very high and n is reasonably large.

$$
\begin{aligned}
&= s^j[-\frac{\alpha}{2\beta+\delta} + \frac{\delta}{2(2\beta+\delta)}\left(\frac{\alpha}{2\beta+\delta} - \frac{1}{2\beta}\bar{t}^i + \frac{\delta}{2\beta(2\beta+\delta)}\bar{t}^i\right) \\
&\quad +\frac{1}{2}\left(\frac{\alpha}{2\beta+\delta} - \frac{1}{2\beta}t^i_j + \frac{\delta}{2\beta(2\beta+\delta)}\bar{t}^i\right)] \\
&= s^j\left[-\frac{\alpha\beta}{(2\beta+\delta)^2} + \frac{\delta^2}{4\beta(2\beta+\delta)^2}\bar{t}^i - \frac{1}{4\beta}t^i_j\right]. \qquad (15)
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial M^i}{\partial t^i_j} &= s^j\left(-q^i_j(\mathbf{t}^i) + \sum_{k\neq i} s^k q^i_k(\mathbf{t}^i)\frac{\delta}{2\beta+\delta}\right) \\
&= s^j[-\left(\frac{\alpha}{2\beta+\delta} - \frac{1}{2\beta}t^i_j + \frac{\delta}{2\beta(2\beta+\delta)}\bar{t}^i\right) \\
&\quad +\frac{\delta}{2\beta+\delta}\left(\frac{\alpha}{2\beta+\delta}(1-s^i) - \frac{1}{2\beta}\bar{t}^i + \frac{\delta}{2\beta(2\beta+\delta)}(1-s^i)\bar{t}^i\right)] \\
&= s^j\left[-\frac{\alpha}{2\beta+\delta} + \frac{\delta\alpha}{(2\beta+\delta)^2}(1-s^i) + \frac{1}{2\beta}t^i_j\right. \\
&\quad \left. -\frac{\delta}{\beta(2\beta+\delta)}\bar{t}^i + \frac{\delta^2}{2\beta(2\beta+\delta)^2}(1-s^i)\bar{t}^i\right]. \qquad (16)
\end{aligned}
$$

Noticing that X^i does not depend on t^i_j, we have

$$
\begin{aligned}
\frac{\partial W^i}{\partial t^i_j} &= \frac{\partial V^i}{\partial t^i_j} - \frac{\partial M^i}{\partial t^i_j} \\
&= s^j\left[\left(-\frac{\alpha\beta}{(2\beta+\delta)^2} + \frac{\delta^2}{4\beta(2\beta+\delta)^2}\bar{t}^i - \frac{1}{4\beta}t^i_j\right)\right. \\
&\quad -\left(-\frac{\alpha}{2\beta+\delta} + \frac{\delta\alpha}{(2\beta+\delta)^2}(1-s^i) + \frac{1}{2\beta}t^i_j - \frac{\delta}{\beta(2\beta+\delta)}\bar{t}^i\right. \\
&\quad \left.\left. +\frac{\delta^2}{2\beta(2\beta+\delta)^2}(1-s^i)\bar{t}^i\right)\right].
\end{aligned}
$$

Thus, $\partial W^i/\partial t^i_j = 0$ generates

$$
\begin{aligned}
\frac{3}{4\beta}t^i_j &= -\frac{\alpha\beta}{(2\beta+\delta)^2} + \frac{\delta^2}{4\beta(2\beta+\delta)^2}\bar{t}^i + \frac{\alpha}{2\beta+\delta} - \frac{\delta\alpha}{(2\beta+\delta)^2}(1-s^i) \\
&\quad +\frac{\delta}{\beta(2\beta+\delta)}\bar{t}^i - \frac{\delta^2}{2\beta(2\beta+\delta)^2}(1-s^i)\bar{t}^i.
\end{aligned}
$$

The right-hand side of this formula is common for any j. Thus, the value of t^i_j is independent of j, so that we may express $t^i_j = t^i$ for any $j \notin C_i$. Since $\bar{t}^i = \sum_{k=1}^n s^i t^i_i = (1-s^{C_i})t^i$, where $s^{C_i} = \sum_{k \in C_i} s^k$, we have

$$\begin{aligned} \frac{3}{4\beta} t^i &= \frac{\alpha(\beta+\delta)}{(2\beta+\delta)^2} - \frac{\delta\alpha}{(2\beta+\delta)^2}(1-s^i) \\ &\quad + \left[\frac{\delta^2}{4\beta(2\beta+\delta)^2} + \frac{\delta}{\beta(2\beta+\delta)} - \frac{\delta^2}{2\beta(2\beta+\delta)^2}(1-s^i)\right](1-s^{C_i})t^i, \end{aligned}$$

or

$$\begin{aligned} &\frac{1}{4\beta(2\beta+\delta)^2}\left\{3(2\beta+\delta)^2 - \left[4(2\beta+\delta)\delta - \delta^2 + 2\delta^2 s^i)\right](1-s^{C_i})\right\} t^i \\ = &\frac{\alpha}{(2\beta+\delta)^2}\left[(\beta+\delta) - \delta(1-s^i)\right]. \end{aligned}$$

Thus, the optimal tariff rate τ^i is obtained as a function of s^i, s^{C_i}, and parameters α, β and δ.

$$\tau^i(s^i, s^{C_i}; \alpha, \beta, \delta) = \frac{4\alpha\beta(\beta+\delta s^i}{3(2\beta+\delta)^2 - \delta(1-s^{C_i})[4(2\beta+\delta) - \delta(1-2s^i)]}$$

It is easy to see that $\partial\tau^i/\partial s^i > 0$ and $\partial\tau^i/\partial s^{C_i} < 0$.

Q.E.D.

Proof of Lemma 3. Using (6), we have

$$\begin{aligned} \sum_{k=1}^{n} s^k q_k^i(\mathbf{t}^i) &= \frac{\alpha}{2\beta+\delta} - \frac{1}{2\beta}\sum_{k=1}^{n} s^k t_k^i + \frac{\delta}{2\beta(2\beta+\delta)}\bar{t}^i \\ &= \frac{\alpha}{2\beta+\delta} - \frac{1}{2\beta}\bar{t}^i + \frac{\delta}{2\beta(2\beta+\delta)}\bar{t}^i \\ &= \frac{\alpha}{2\beta+\delta} - \frac{1}{2\beta+\delta}\bar{t}^i. \end{aligned}$$

By substituting this result and (6) into $\partial V^i/\partial t_j^i$ in Lemma 1, we obtain

$$\begin{aligned} \frac{\partial V^i}{\partial t_j^i} &= s^j\left[-\frac{\alpha}{2\beta+\delta} + \frac{\delta}{2(2\beta+\delta)}\left(\frac{\alpha}{2\beta+\delta} - \frac{1}{2\beta+\delta}\bar{t}^i\right) + \frac{1}{2}\left(\frac{\alpha}{2\beta+\delta} - \frac{1}{2\beta}t_j^i + \frac{\delta}{2\beta(2\beta+\delta)}\bar{t}^i\right)\right] \\ &= s^j\left[-\frac{\alpha\beta}{(2\beta+\delta)^2} + \frac{\delta^2}{4\beta(2\beta+\delta)^2}\bar{t}^i - \frac{1}{4\beta}t_j^i\right]. \end{aligned}$$

Let $\mathbf{t}(\gamma)$ denote the bilateral tariff reform schedule between countries i and j. This schedule satisfies $t_j^i(\gamma) = (1-\gamma)t^i$ and $t_i^j(\gamma) = (1-\gamma)t^j$, where $\gamma \in [0,1]$ and hence $t_j^i(0) = t^i$ and $t_j^i(1) = 0$, for example. All other tariff rates are kept unchanged, i.e., $t_k^i(\gamma) = t^i$ and $t_k^j(\gamma) = t^j$ for any k where $k \neq i, j$. Notice that $\bar{t}^i$ also changes in the course of tariff reform such that $\bar{t}^i(\gamma) = \sum_{k \notin C_i \cup \{j\}} s^k t^i + s^j(1-\gamma)t^i = (1 - s^{C_i} - \gamma s^j)t^i$, and similarly for $\bar{t}^j(\gamma)$. By substituting $\bar{t}^i(\gamma)$ and $t^i_j(\gamma)$ for $\bar{t}^i$ and t^i_j, respectively, and using $dt^i_j/d\gamma = -t^i$, we obtain

$$\frac{dV^i(\mathbf{t}^i(\gamma))}{d\gamma} = s^j t^i \left[\frac{\alpha\beta}{(2\beta+\delta)^2} - \frac{\delta^2}{4\beta(2\beta+\delta)^2}(1 - s^{C_i} - \gamma s^j)t^i + \frac{1}{4\beta}(1-\gamma)t^i \right].$$

By integrating over γ, the welfare change of country i due to the FTA with j becomes

$$\begin{aligned} \Delta V^i &\equiv V^i(0, \mathbf{t}^i_{-j}) - V^i(\mathbf{t}^i) \\ &= s^j t^i \left[\frac{\alpha\beta}{(2\beta+\delta)^2} - \frac{\delta^2}{4\beta(2\beta+\delta)^2}(1 - s^{C_i} - \frac{s^j}{2})t^i + \frac{1}{8\beta}t^i \right] \\ &= \frac{s^j t^i}{8\beta(2\beta+\delta)^2} \left\{ 8\alpha\beta^2 + \left[4\beta(\beta+\delta) - (1 - 2s^{C_i} - s^j)\delta^2 \right] t^i \right\}. \end{aligned} \tag{17}$$

The sufficient condition (i) immediately follows.

Let us suppose now that the condition (i) does not hold so that the terms in the square brackets of the last equation in the above are negative. Then the condition (ii) also follows immediately.

In order to derive the sufficient condition $\delta \leq 10\beta$ for (ii), we consider the case where $s^k = 1/n$ for any k. Since the original tariff rate t^i is less than or equal to the optimal tariff τ^i, we need only show that if $\delta \leq 10\beta$ condition (ii) holds when $t^i = \tau^i$.

Now, $\Delta V^i > 0$ if and only if

$$\frac{8\alpha\beta^2}{\delta^2(1 - 2s^{C_i} - s^j) - 4\beta(\beta+\delta)} > t^i. \tag{18}$$

Recall that we only consider the case where the denominator of the left-hand side is positive. As Lemma 2 shows, the optimal tariff decreases in s^{C_i}. To derive the condition under which (ii) holds for any s^{C_i}, therefore, we substitute $s^i = 1/n$, which is the smallest s^{C_i}, for s^{C_i}. Then, the optimal tariff rate that we substitute for t^i in the above inequality equals

$$\tau^i = \frac{4\alpha\beta(\beta + \frac{\delta}{n})}{3(2\beta+\delta)^2 - \delta(1 - \frac{1}{n})[4(2\beta+\delta) - \delta(1 - \frac{2}{n})]}.$$

Noticing that the left-hand side of (18) decreases by substituting s^i for s^{C_i}, the inequality that we need to verify can be written as

$$\frac{8\alpha\beta^2}{\delta^2(1 - \frac{3}{n}) - 4\beta(\beta+\delta)} \geq \frac{4\alpha\beta(\beta + \frac{\delta}{n})}{3(2\beta+\delta)^2 - \delta(1 - \frac{1}{n})[4(2\beta+\delta) - \delta(1 - \frac{2}{n})]},$$

or

$$2\beta \left\{ 3(2\beta+\delta)^2 - \delta(1 - \frac{1}{n})[4(2\beta+\delta) - \delta(1 - \frac{2}{n})] \right\} \geq (\beta + \frac{\delta}{n}) \left[(1 - \frac{3}{n})\delta^2 - 4\beta^2 - 4\beta\delta \right].$$

Further computation shows that this inequality can be reduced to

$$(\beta+\delta)^3\left[28\hat{\beta}^3+(12+\frac{20}{n})\hat{\beta}^2\hat{\delta}-(1-\frac{9}{n}-\frac{4}{n^2})\hat{\beta}\hat{\delta}^2-(\frac{1}{n}-\frac{3}{n^2})\hat{\delta}^3\right]\geq 0,$$

where $\hat{\beta}=\beta/(\beta+\delta)$ and $\hat{\delta}=\delta/(\beta+\delta)$. To simplify the expressions, we let $x=1/n$. Then, it follows from $\hat{\delta}=1-\hat{\beta}$ that the above inequality holds if the following function takes a nonnegative value for any $x\in(0,1/2]$ and any $\hat{\beta}\in[0,1]$.

$$\begin{aligned} g(x,\hat{\beta}) &\equiv 28\hat{\beta}^3+(12+20x)\hat{\beta}^2(1-\hat{\beta})-(1-9x-4x^2)\hat{\beta}(1-\hat{\beta})^2-(x-3x^2)(1-\hat{\beta})^3 \\ &= (4\hat{\beta}(1-\hat{\beta})^2+3(1-\hat{\beta})^3)x^2+(20\hat{\beta}^2(1-\hat{\beta})+9\hat{\beta}(1-\hat{\beta})^2-(1-\hat{\beta})^3)x \\ &\quad +28\hat{\beta}^3+12\hat{\beta}^2(1-\hat{\beta})-\hat{\beta}(1-\hat{\beta})^2 \\ &= (1-\hat{\beta})^2(3+\hat{\beta})x^2+(1-\hat{\beta})(10\hat{\beta}^2+11\hat{\beta}-1)x+\hat{\beta}(\hat{\beta}+1)(15\hat{\beta}-1). \end{aligned}$$

It follows immediately that if $\hat{\beta}\geq\frac{1}{15}$, then $g(0,\hat{\beta})\geq 0$. Moreover, we have

$$\frac{\partial g}{\partial x}(x,\hat{\beta})=2(1-\hat{\beta})^2(3+\hat{\beta})x+(1-\hat{\beta})(10\hat{\beta}^2+11\hat{\beta}-1),$$

which is increasing in x. Evaluating this derivative at $x=0$, we obtain

$$\frac{\partial g}{\partial x}(0,\hat{\beta})=(1-\hat{\beta})(10\hat{\beta}^2+11\hat{\beta}-1).$$

Thus, $\partial g/\partial x\geq 0$ for all $x\in(0,1/2]$ if $\hat{\beta}\geq\frac{-11+\sqrt{161}}{20}$, which in turn holds if $\hat{\beta}\geq 1/11$. Together with the fact that $g(0,\hat{\beta})\geq 0$ if $\hat{\beta}\geq\frac{1}{15}$, it implies that $g(x,\beta)\geq 0$ for any $x\in(0,1/2]$ and for any $\hat{\beta}\in[0,1]$ if $\hat{\beta}\geq 1/11$, or equivalently, $\delta\leq 10\beta$.

Q.E.D.

Proof of Lemma 4. Recall from the proof of Lemma 3 the definition of the bilateral tariff reform schedule between countries i and j, denoted by $\mathbf{t}(\gamma)$ where $t^i_j(\gamma)=(1-\gamma)t$ and $\bar{t}^i(\gamma)=(1-s^{C_i}-s^j\gamma)t=(1-\frac{c_i}{n}-\frac{\gamma}{n})t$, and similarly for j, while $\mathbf{t}^k(\gamma)=\mathbf{t}^k$ for any $k\neq i,j$, and any $\gamma\in[0,1]$. Then, we can write

$$\begin{aligned} q^i_j(\mathbf{t}^i(\gamma)) &= \frac{\alpha}{2\beta+\delta}-\frac{1}{2\beta}t^i_j(\gamma)+\frac{\delta}{2\beta(2\beta+\delta)}\bar{t}^i(\gamma) \\ &= \frac{\alpha}{2\beta+\delta}-\frac{1}{2\beta}(1-\gamma)t^i+\frac{\delta}{2\beta(2\beta+\delta)}(1-\frac{c_i}{n}-\frac{\gamma}{n})t^i, \end{aligned}$$

$$\begin{aligned} q^i_k(\mathbf{t}^i(\gamma)) &= \frac{\alpha}{2\beta+\delta}-\frac{1}{2\beta}t^i_k(\gamma)+\frac{\delta}{2\beta(2\beta+\delta)}\bar{t}^i(\gamma) \\ &= \frac{\alpha}{2\beta+\delta}-\frac{1}{2\beta}t^i+\frac{\delta}{2\beta(2\beta+\delta)}(1-\frac{c_i}{n}-\frac{\gamma}{n})t^i. \end{aligned}$$

Consequently, we have

$$\begin{aligned}
\frac{dq_j^i}{d\gamma} &= \frac{1}{2\beta}t - \frac{\delta}{2\beta(2\beta+\delta)n}t, \\
\frac{dq_k^i}{d\gamma} &= -\frac{\delta}{2\beta(2\beta+\delta)n}t.
\end{aligned}$$

Now, we can rewrite a change in country i's industrial trade surplus.

$$\begin{aligned}
&\Delta\left[X^i(\mathbf{t}^{-i}) - M^i(\mathbf{t}^i)\right] \\
&= \int_0^1 \left(\frac{dX^i(\mathbf{t}^{-i}(\gamma))}{d\gamma} - \frac{dM^i(\mathbf{t}^i(\gamma))}{d\gamma}\right) d\gamma \\
&= \int_0^1 \left[\left(\frac{dX_j^i(\mathbf{t}^j(\gamma))}{d\gamma} - \frac{dM_j^i(\mathbf{t}^i(\gamma))}{d\gamma}\right) + \left(-\sum_{k\neq i,j}\frac{dM_k^i(\mathbf{t}^i(\gamma))}{d\gamma}\right)\right] d\gamma \\
&= \frac{\beta}{n}\int_0^1 \left[\left(2q_i^j(\mathbf{t}^j)\frac{dq_i^j}{d\gamma} - 2q_j^i(\mathbf{t}^i)\frac{dq_j^i}{d\gamma}\right) + \left(-\sum_{k\neq i,j} 2q_k^i(\mathbf{t}^i)\frac{dq_k^i}{d\gamma}\right)\right] d\gamma \\
&= \frac{2\beta t}{n}\int_0^1 \left[(c_i - c_j)\frac{\delta}{2\beta(2\beta+\delta)n}\left(\frac{1}{2\beta} - \frac{\delta}{2\beta(2\beta+\delta)n}\right)t\right. \\
&\quad \left. + \left(\sum_{k\neq i,j}\left(\frac{\alpha}{2\beta+\delta} - \frac{1}{2\beta}t_k^i + \frac{\delta}{2\beta(2\beta+\delta)}(1 - \frac{c_i}{n} - \frac{\gamma}{n})t\right)\right)\left(\frac{\delta}{2\beta(2\beta+\delta)n}\right)\right] d\gamma \\
&= \frac{\delta t}{n^2(2\beta+\delta)}\int_0^1 \left[(c_i - c_j)\left(\frac{1}{2\beta} - \frac{\delta}{2\beta(2\beta+\delta)n}\right)t\right. \\
&\quad \left. + \left((n-2)\frac{\alpha}{2\beta+\delta} - (n-2+(c_i-1))\frac{1}{2\beta}t + (n-2)\frac{\delta}{2\beta(2\beta+\delta)}\left(\frac{n-c_i-\gamma}{n}\right)t\right)\right] d\gamma.
\end{aligned}$$

The value of this formula is minimized when $c_i = 1$ and $c_j = n-1$. Thus,

$$\begin{aligned}
&\Delta\left[X^i(\mathbf{t}^{-i}) - M^i(\mathbf{t}^i)\right] \\
&\geq \frac{\delta t}{n^2(2\beta+\delta)}\int_0^1 \left[-(n-2)\left(\frac{1}{2\beta} - \frac{\delta}{2\beta(2\beta+\delta)n}\right)t\right. \\
&\quad \left. + \left((n-2)\frac{\alpha}{2\beta+\delta} - (n-2)\frac{1}{2\beta}t + (n-2)\frac{\delta}{2\beta(2\beta+\delta)n}(n-1-\gamma)t\right)\right] d\gamma \\
&= \frac{(n-2)\delta t}{n^2(2\beta+\delta)}\int_0^1 \left[\left(-2\frac{1}{2\beta} + \frac{\delta}{2\beta(2\beta+\delta)n}\times(n-\gamma)\right)t + \frac{\alpha}{2\beta+\delta}\right] d\gamma \\
&= \frac{(n-2)\delta t}{2n^2\beta(2\beta+\delta)^2}\int_0^1 \left[t\left(-2(2\beta+\delta) + \frac{n-\gamma}{n}\delta\right) + 2\alpha\beta\right] d\gamma.
\end{aligned}$$

The contents of the bracket is nonnegative for any γ if and only if

$$\begin{aligned}
t &\leq \frac{2\alpha\beta}{2(2\beta+\delta) - \frac{n-1}{n}\delta} \\
&= \frac{2\alpha\beta}{4\beta + \left(1+\frac{1}{n}\right)\delta}.
\end{aligned}$$

Indeed, they are strictly positive under this condition, except at $\gamma = 1$ in which case they becomes zero. Therefore, $\Delta\left[X^i(\mathbf{t}^{-i}) - M^i(\mathbf{t}^i)\right] > 0$ if this condition is satisfied.

Q.E.D.

Proof of Proposition 2. If $n = 2$, then $s^i = s^j = \frac{1}{2}$, and hence the industrial trade surplus does not change as we argue in Footnote 27. Then, it follows from Lemma 3 that if $\delta \leq 10\beta$, countries i and j have incentives to sign an FTA.

If $n \geq 3$, on the other hand, the upperbound for t in Lemma 4 becomes important. As Lemma 2 shows, the optimal tariff is highest when a country does not have any FTA, i.e., $s^{C_i} = s^i$. Thus, in the case of n (≥ 3) symmetric countries, the optimal tariff rate under any cooperation structures is bounded above by $\tau(n)$. Now, let us examine how $\tau(n)$ varies with n. To see this, we define the function τ^* by $\tau^*(s^i) = \tau(1/s^i)$, keeping in mind that $n = 1/s^i$ for any i when all countries are completely symmetric. Then, $\tau(n) \leq \tau(3)$ for any $n \geq 3$ if and only if $\tau^*(s^i) \leq \tau^*(1/3)$ for any $s^i \leq 1/3$.

Since $\tau^*(s^i) = \tau^i(s^i, s^i; \alpha, \beta, \delta)$ for a given set of α, β, and δ, we have $d\tau^*/ds^i = \partial\tau^i/\partial s^i + \partial\tau^i/\partial s^{C_i}$. Then, it follows from the optimal tariff formula in Lemma 2 that

$$\begin{aligned}
&\left\{3(2\beta+\delta)^2 - \left[4(2\beta+\delta)\delta - \delta^2(1-2s^i)\right](1-s^i)\right\}^2 \frac{d\tau^*}{ds^i} \\
&= 4\alpha\beta\delta\{3(2\beta+\delta)^2 - \left[4(2\beta+\delta)\delta - \delta^2(1-2s^i)\right](1-s^i)\} \\
&\quad -4\alpha\beta\{(\beta+\delta) - \delta(1-s^i)\}\{4(2\beta+\delta)\delta - \delta^2(1-2s^i) - 2\delta^2(1-s^i)\} \\
&= 4\alpha\beta\delta[4\beta^2 + 3\beta\delta - 4s^i\beta\delta - 2(s^i)^2\delta^2],
\end{aligned}$$

or

$$\frac{d\tau^*}{ds^i} = 4\alpha\beta\delta\frac{A(s^i)}{B(s^i)^2},$$

where

$$\begin{aligned}
A(s^i) &= 4\beta^2 + 3\beta\delta - 4s^i\beta\delta - 2(s^i)^2\delta^2, \\
B(s^i) &= 3(2\beta+\delta)^2 - \left[4(2\beta+\delta)\delta - \delta^2(1-2s^i)\right](1-s^i).
\end{aligned}$$

Now, we show that if $\delta \leq 9\beta$, then $A(s^i) > 0$ and $B(s^i) > 0$ for any $s^i \in [0, 1/3]$, which implies that τ^* is increasing on $[0, 1/3]$. First, it is easy to see that $A(0) > 0$ and $dA/ds^i = -4\beta\delta - 4s^i\delta^2 < 0$. We can also see $A(\frac{1}{3}) = \frac{1}{9}(36\beta^2 + 15\beta\delta - 2\delta^2) > 0$ if $\delta \leq (15 + \sqrt{513})/4$, which in turn is satisfied if $\delta \leq 9\beta$. Thus, $A(s^i) > 0$ on $[0, 1/3]$ if $\delta \leq 9\beta$. Turning to $B(s^i)$, it is also easy to see that $B(0) = 4\beta(3\beta + \delta) > 0$ and

$$\begin{aligned}\frac{dB}{ds^i} &= 4(2\beta+\delta)\delta - \delta^2(1-2s^i) - 2\delta^2(1-s^i) \\ &= 8\beta\delta + \delta^2(1+4s^i) > 0.\end{aligned}$$

Thus, we have $B(s^i) > 0$ for any $s^i \in [0, 1/3]$.

Now, we have found that $d\tau^*/ds^i$ is positive for any $s^i \in (0, 1/3]$ so that τ^* takes the largest value at $s^i = 1/3$. Thus, if $\delta \leq 9\beta$, then $\tau^*(s^i) \leq \tau^*(1/3)$ for any $s^i \leq 1/3$ and hence $\tau(n) \leq \tau(3)$ for any $n \geq 3$.

Now, we calculate the optimal tariff rate when $n = 3$.

$$\begin{aligned}\tau(3) &= \frac{4\alpha\beta\left(\beta + \frac{1}{3}\delta\right)}{3(2\beta+\delta)^2 - \left(4(2\beta+\delta)\delta - \frac{1}{3}\delta^2\right)\frac{2}{3}} \\ &= \frac{12\alpha\beta(3\beta+\delta)}{108\beta^2 + 60\beta\delta + 5\delta^2}.\end{aligned}$$

Since $2\alpha\beta/[4\beta + (1+\frac{1}{n})\delta]$ is increasing in n, we need only show $\tau(3) \leq 2\alpha\beta/[4\beta + (1+\frac{1}{3})\delta]$ to prove that the condition in Lemma 4 is satisfied. Now, we have

$$\begin{aligned}&\frac{2\alpha\beta}{4\beta + \frac{4}{3}\delta} - \tau(3) \\ &= \frac{3\alpha\beta}{2(3\beta+\delta)} - \frac{12\alpha\beta(3\beta+\delta)}{108\beta^2 + 60\beta\delta + 5\delta^2} \\ &= \frac{9\alpha\beta}{2(3\beta+\delta)(108\beta^2 + 60\beta\delta + 5\delta^2)}(6\beta - \delta)(2\beta + \delta).\end{aligned}$$

This value is nonnegative if and only if $\delta \leq 6\beta$. Thus, as long as $\delta \leq 6\beta$ (as we have seen before, $\delta \leq 9\beta$ guarantees that $\tau(n)$ is decreasing in n for $n \geq 3$), the condition in Lemma 4 is satisfied when $n \geq 3$. Moreover, if $\delta \leq 6\beta$, then $\delta \leq 10\beta$ is also satisfied, so that Lemma 2 implies that a consumer's gross utility increases by the FTA. Consequently, countries i and j have incentives to have an FTA if $\delta \leq 6\beta$.

Q.E.D.

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