Inequality and growth clubs

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Abstract

The non-convergence of the growth processes is a well-established yet puzzling fact. In this paper, we offer a theoretical investigation of this feature, based on coalition theory and a simple model of endogenous growth. We prove that inequality generates a permanent segmentation of society into growth clubs. We then study the relationship between the distribution of individual endowments and the entire pattern of growth of these clubs. We relate the characteristics of the divergence process to the characteristics of this distribution and we prove that in most cases, an increase in inequality increases divergence in growth rates.

KEYWORDS : Growth clubs, Inequality, Core, Social segmentation, endogenous growth.

JEL Classification : D3, D71, H40, H41.

1 Introduction

In the past ten years, an important debate in the economics of growth has been related to the so-called "convergence" issue. There is more and more evidence that differences in growth rates and therefore relative per capita output across nations tend to persist. This has been thoroughly documented in the survey by Durlauf and Quah on "why do growth rates differ" (Durlauf and Quah, 1998).[1]

In particular Quah (1996,1997) has characterized a stylized fact in the pattern of growth of the world economy over the post-WWII period. The per capita income cross-section distribution for the world economy tends to evolve into a twin-peaks distribution which makes clear that no-convergence process is taking place at the scale of the world economy. Rather convergence tends to take place among subsets of nations, meaning that "convergence clubs" form over time but that they do not encompass the entire economy.

This stylized fact contradicts the standard neo-classical growth theory based on Solow (1956) which predicts that countries (economies) will tend to convergence to the same steady state, if they share the same technology, irrespective of the existing differences in their initial conditions. Several lines of research have been followed up to now so as to reconcile theory with the empirics of non-systematic and global convergence in growth rates and per capita income.

Quah has suggested that the convergence debate is related to the dynamics of inequality. Taking as given the distribution of individuals "economies" or "clubs", he emphasized the link between the aggregate human capital of an economy and its growth rate in an endogenous growth model. Therefore economies with different aggregate human capital will experience different growth rates and are unlikely to converge. However Quah does not explain the crucial point which is why "economies" form with different aggregate human capital. In other words, if the segmentation into clubs plays a crucial role in the dynamics of growth and the properties of the pattern of growth for the entire set of agents, a proper theory of growth has to offer a explanation for the segmentation itself.

This is precisely the aim of this paper. We offer a theory of the segmentation of society into growth clubs, based on coalition theory, as we assume that agents voluntarily form clubs so as to share some resources that are necessary for growth (because they finance a productive

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[1] See also Durlauf (1996).

club good). This segmentation depends on the relative ability of agents to contribute to the club good, and therefore depends on the initial distribution of endowment over individuals. We then relate inequality to the pattern of growth, since it depends on the segmentation of society into clubs, as already remarked by Quah. We can therefore trace the divergence in growth rates (the "twin peaks" or "multi-peaks" phenomenon) to inequality in an explicit and micro-founded way.

The plan of the paper is as follows. In the next section, we develop a simple endogenous growth model which requires that agents form coalitions for growth, that is growth clubs. In section 3, we characterize the core partition of the economy into growth clubs and we develop the properties of this partition. In section 4 we provide the link between inequality in the initial distribution of endowment over the set of agents and the growth properties of the various clubs in the core partition. We prove that the characteristics of this initial endowment play a determining role in the pattern of growth and for explaining the non convergence feature. Then we study the impact of an increase in inequality over the pattern of growth and the convergence issue. We prove that it is likely that such an increase leads to more divergence between growth clubs. Section 5 contains the conclusion.

2 The model

2.1 The economy

We consider a “society” $S = \{1, ..., N\}$ of infinitely-lived individuals. This society is constituted at time $t = 0$. Each individual enters society with an initial individual endowment a_i strictly positive. a_i is a scalar as there is one good. This good may be alternatively saved, privately consumed, or used to finance a public good which is used in the production process. Agents are ordered so that $a_1 > a_2 > .... > a_N$. The aggregate level of endowment given by $A = \sum_{i \in S} a_i$.

This society is unequal insofar as the initial endowments differ. This inequality is described by means of an inequality schedule as follows. Denoting $\lambda_{i,j}$:

$$\lambda_{i,j} = \frac{a_i}{a_j}, \tag{1}$$

we offer the following

Definition 1 A society S is characterized by an inequality schedule
$S = \{\lambda_{1,2}, \dots, \lambda_{i,i+1}, \dots, \lambda_{N-1,N}; A\}$.

Agents may willingly form clubs. A club C_j is a collection of individuals belonging to S:

$$C_j = \{\dots, i, \dots\}, i \in S. \tag{2}$$

We denote $n_j = card(C_j)$ the number of individuals belonging to C_j.

Agents are willing to form or join a club because a club provides a productive club good formed from voluntary individual contributions made by its members. This club good positively affects the productive capacity of the club and therefore the intertemporal flows of income for its members. We make an important assumption on the club good. It is formed at the beginning of time once and for all. There is no obsolescence as it keeps its productive capacities forever. In other words, once it is supplied in quantity G_j in club C_j, it will contribute to the productive process over the infinite growth path. Admittedly extreme, it is not without empirical content. Many club goods are formed at the foundation of the club and last “forever”: think about the location of the capital, the setting of public institutions, the definition of the legal apparatus, including property rights, the writing of a constitution, the discovery of a technological process.

The growth process is of the simple AK variety of endogenous growth models. The autonomous component A depends positively on the size of the productive club good and negatively on the number of members in C_j. This last assumption reflects the presence of congestion costs which explain why individual may not be willing to belong to a too large club. When membership is decided, an additional member has the positive effect of an increase in the financing of the club good but the negative effect of increasing congestion. Intuitively this trade-off shapes the sizes of clubs. Formally, for club C_j, we write:

$$Y_j(t) = A(G_j, n_j)K_j(t) \tag{3}$$

with

$$K_j(t) = \sum_{i \in C_j} k_i(t) \tag{4}$$

where $k_i(t)$ denotes level of productive capital accumulated at t by individual i. There is no depreciation of capital. We assume the following properties for function $A(G_j, n_j)$:

$A_1 > 0; A_{11} < 0; A_2 < 0$: Moreover we assume that a larger size does not make the marginal contribution fo the public good to production more productive. $(A_{12} \leq 0)$.

Importantly, given the founding property of the club good, it is immediate to note that a club is formed at the beginning of history and forever. No one has any possibility to re-open the case of the limits of clubs.

For simplicity we assume that:

$$G_j = \sum_{i \in C_j} g_i \tag{5}$$

Once it is formed a club will start a growth process as its members will save and accumulate the club capital over time. This process will reflect the membership of the club, in particular the affluence of members and its size. It is important to remark that once it is formed, each club is a productive autarky. There is economic interdependence within a club over time, but not between clubs as there is no economic relationship between existing clubs. However there is interdependence between all agents of society S at the formation stage as we shall see. This allows us to focus on the impact of inequality of the formation of clubs and its resulting consequences of the growth processes.

2.2 Clubs and partition

The resulting segmentation of society into clubs is called a partition:

Definition 2 The partition in J clubs of S is denoted $\mathcal{C} = \{C_1; \ldots; C_J\}; C_j \subset S$; and is such that:

$$\begin{aligned} \bigcup_{j=1} C_j &= S \\ C_j \bigcap C_{j'} &= \emptyset \qquad \text{for } j \neq j' \end{aligned}$$

As we have seen, given a club is established "forever", the initial partition of society lasts forever too.

The number and boundaries of clubs are not given ex ante but will be endogenously determined according to a special process which has three stages:

1. the partition stage. The very first decisions by agents are to form clubs. The characteristics of these decisions are that each individual voluntarily joins a club where she

is unanimously accepted. In other words, a club is formed when no individual or group of individuals wants to defect and join any possible coalition of agents.

2. the contribution stage. Once a club is formed, each individual decides upon her voluntary contribution of the productive club good. This decision is taken in a non-cooperative manner, each member in a club taking as given the contributions made by all other members.
3. the accumulation stage. Then, using her endowment net of her contribution to the club good, each agent decides upon saving. This decision is made at time 0 and then is renewed at each period. Given that there is no ambiguity, it is as if each individual decides at 0 upon her saving plan over the entire infinite future.

The two first stages of this partition process are similar to the process in a static environment used by Barham et al. et (1997, later BBMP) and Jaramillo et al. (2003, later JKM). The first stage corresponds to a cooperative game without side payments, the second one corresponds to a non-cooperative game on contribution and the third one to a non-cooperative game on savings. This amounts to say that there is no commitment technology available such agents have to commit to the amount of good that she will contribute before entering a club and / or such that members of a club can punish one of them who does not fulfill her promise to contribute or to save.

We shall now turn to the study of the two last stages of this process before characterizing the resulting partition.

2.3 Capital accumulation and growth for a given club

We consider the last stage, when clubs have been formed and club goods have been funded and established.

Individuals are identical and differ only in their initial endowments. We assume the following utility function:

$$U_i = \int_0^{\infty} e^{-\rho t} \ln c_i(t) dt \tag{6}$$

where ρ denotes the discount factor and $c_i(t)$ consumption of individual i at date t: Capital is remunerated at its marginal productivity. But because clubs are autarkies, there is no equilibrating process for capital returns and different clubs entail (a priori) different capital returns. We denote $r_j(t)$ the rate of return of capital at time t for club C_j and we can write:

$$r_j(t) = A(G_j; n_j)$$

The maximization problem for individual $i \in C_j$ is therefore :

$$\max_{c_i;t} \int_0^{\infty} e^{-\rho t} \ln c_i(t) dt$$

subject to :

$$\begin{cases} \dot{k}_i = r_j(t)k_i(t) - c_i(t) \\ a_i - g_i = k_i(0) \\ \lim_{t \to +\infty} e^{-\int_0^{\infty} r_j(z)dz} k_t \geq 0 \end{cases}$$

The solution is such that:

$$\frac{\dot{c}_i}{c_i} = r_j - \rho = A(G_j; n_j) - \rho \qquad \forall i \in C_j \tag{7}$$

As usual in this sort of models, the capital accumulation rates and the growth rates are constant over time and equal to this value:

$$\frac{\dot{k}_i}{k_i} = r_j - \rho = A(G_j; n_j) - \rho \qquad \forall i \in C_j \tag{8}$$

$$\gamma_j \equiv \frac{\dot{Y}_j}{Y_j} = A(G_j; n_j) - \rho \tag{9}$$

This last equation makes clear that each club is characterized by a specific growth rate which is likely to differ from rates for other clubs. Moreover it depends on the decisions regarding the membership and the contributions to the clubs. In this respect it is clear that the various growth processes are interdependent insofar as the formation of clubs is an integrated process.

>From (7) we deduce:

$$c_i(t) = c_i(0)e^{(A(G_j;n_j)-\rho)t} \qquad \forall i \in C_j \tag{10}$$

which gives the discounted value of intertemporal level of utility denoted by :

$$U_i = \frac{1}{\rho} \ln c_i(0) + \frac{1}{\rho^2}[A(G_j; n_j) - \rho] \qquad \forall i \in C_j \tag{11}$$

2.4 The choice of individual provision

Turning to the contribution stage, once her club has been formed, an agent i belonging to C_j has to decide over g_i. From (??) and the value of $r_j(t)$, we know that:

$$c_i(0) = \rho k_i(0) = \rho(a_i - g_i). \tag{12}$$

Agent i therefore solves:

$$\max_{g_i} U_i = \frac{1}{\rho}\ln\rho(a_i - g_i) + \frac{1}{\rho^2}[A(G_j; n_j) - \rho].$$

given (5) and g_j given for any $j \in C_j$.

The solution of this non-cooperative stage is characterized by

$$g_i = a_i - \frac{\rho}{A_1(G_j; n_j)}. \tag{13}$$

The richer an individual in a club, the more she contributes to the club good. However the richer the other members are, the less an individual contributes. This expresses a free-riding behavior in the club. Finally, the larger is a club, the less an individual contributes (when $A_{12} \leq 0$), because the efficiency of a marginal contribution to the club good declines with membership and congestion.

It immediately follows that the club good at the non-cooperative equilibrium of this stage equals:

$$G_j^* = \sum_{i \in C_j} a_i - \frac{n_j \rho}{A_1(G_j^*; n_j)} \tag{14}$$

which is equivalent to:

$$G_j^* = \Gamma\left(\sum_{i \in C_j} a_i; n_j\right) \tag{15}$$

with:

$$\Gamma_1 > 0 \quad \text{and} \quad \Gamma_2 < 0. \tag{16}$$

For a given size, the richer (in aggregate) is the club the larger is the club good: this reflects the increased financing ability of members, despite the negative free-riding effect. The larger is a club for a given financing ability, the smaller is the club good. This reflects the aggregate efficiency effect of size (congestion) on the marginal provision of the club good ($A_{12} \leq 0$).

Remark that since individual consumptions are equal and growing at the same rate over time, this economy is characterized by extreme intra-club convergence. All members of a club share the same intertemporal sequence of consumption and hence the same utilities.

It is easy to compute the present value of utility of agent i belonging to club C_j; which we denote $V_i(C_j)$:

$$V_i(C_j) = \frac{1}{\rho}\ln\rho^2 - \frac{1}{\rho}\ln A_1(G_j^*;n_j) + \frac{1}{\rho^2}\left[A(G_j^*;n_j) - \rho\right]: \qquad (17)$$

This formula makes clear some interesting properties of a growing club.

1. First $V_i(C_j)$ does not depend on the individual endowment of i: All members of the growing club C_j benefit from the same intertemporal utility level. This is due to a characteristic well-known in the case of voluntary provision of public goods, first noticed by Bergstrom et al. (1986): when all members of a club have identical preferences, their individual arbitrage leads them to appreciate the same basket of goods and they give all their endowment in excess of a fixed amount left for private consumption. Here we find this property in the case of growing clubs. Because of (13) and initial individual budget constraint, what is left for private purposes for i is her individual amount of capital $k_i(0)$ and it is independent of the individual initial endowment of i:

   $$k_i(0) = \frac{\rho}{A_1(G_j;n_j)} \qquad \forall i \in C_j: \qquad (18)$$

   This implies an equal amount of private consumption over time for all members of C_j:

   $$c_i(t) = c_{i'}(t) \qquad \forall i, i' \in C_j: \qquad (19)$$

2. For a given size, the present value of utility for members of a club is increasing in the aggregate endowment of the club $\sum a_i$. Altogether despite the free-riding effect, a richer club generates a higher utility level because members contribute more to the club good.

3. On the other hand, the direct effect of the size n_j of club C_j on the present value of utility is ambiguous for a given aggregate endowment. The congestion effect affects negatively this utility level; however the increase in the size decreases the free-riding effect on individual contributions as members are more equal when size increases. The net effect is ambiguous.

3 The core and growth clubs

In this section, we consider that the partition of the society in growth clubs is given by the core, given the cooperative nature of the formation of these clubs. Therefore we need to characterize the core partition and comment on its consequences on growth characteristics of the independent clubs.

First we shall define the pattern of growth associated with a given partition.

Definition 3 The pattern of growth associated to a given partition C is the set of growth rates characterizing the various clubs of this partition $\{\ldots, \eta_j, \ldots\}, j = 1, \ldots, J.$

Obviously two different partitions are likely to induce different patterns of growth, since the decisions for the financing of the club good and therefore for capital accumulation by a given individual depend on the size and the membership of the club to which this individual belongs. An agent with a given endowment opts for a different level of voluntary contribution according to the membership of the club to which he belongs, because of the non-cooperative nature of financing individual decisions. By generalization, given the non-cooperative nature of the individual decisions, the entire pattern of growth depends on the partition of society for a given inequality schedule.

We define a core partition as follows:

Definition 4 For a given inequality schedule, a core partition $C^E = \{C_1^E, \ldots, C_j^E, \ldots\}$ is such that:

$$\nexists \varpi \subset S \text{ such that } \forall i \in \varpi,\ V_i(\varpi, S, \alpha) > V_i\left(C^E, S, \alpha\right) \tag{20}$$

where $V_i\left(C^E, S, \alpha\right)$ denotes the utility for agent i associated with partition C^E, an inequality schedule S and a congestion parameter α.

A core partition derived from inequality schedule S and a congestion parameter α is such that there is no desirable defection to another possible coalition from anyone or group of agents, from any club in the core partition. The following proposition states that such a partition exists:

Proposition 1 For a given inequality schedule, there exists a core partition.

Proof. See Appendix. ■

An immediate property of the core partition is that for any club of the core partition, any member contributes a strictly positive amount to the club good. This directly comes from the presence of congestion effects. Each member to a club must represent a net gain from all other members. He inflicts some harm on them because of the increase in congestion. Therefore this must be balanced by some advantage: this can only come from a positive contribution to the club good.

The following proposition develops important general properties of the core partition.

Proposition 2 Let $C^E = \left\{C_1^E; ...; C_j^E; ...\right\}$ be a core partition associated with S and α. C^E satisfies the following properties:

i) The core partition is unique.

ii) The core partition is consecutive, that is if i and $\tilde{i}$ belong to C_j^E, $i > \tilde{i}$, then $\forall i^*$, $i > i^* > \tilde{i}$, $i^* \in C_j^E$.

iii) Welfare ordering: Consider two clubs C_j^E and $C_{j'}^E$ such that $\forall i \in C_j^E$, $\forall i' \in C_{j'}^E$, $i < i'$, then $V_i\left(C_j^E; S; \alpha\right) = V\left(C_j^E; S; \alpha\right) > V_{i'}\left(C_{j'}^E; S; \alpha\right) = V\left(C_{j'}^E; S; \alpha\right)$.

Proof. See Appendix. ■

Basically this proposition extends the properties of the core partition from the static case studied by JKM to the dynamic case with capital accumulation. These properties allow us to characterize the core partition by means of a sequence of pivotal agents. A pivotal agent is the poorest member of a given club. We normalize the indexing of clubs in such a way that a lower index corresponds to a more affluent club (with richer members):

$$\text{for any } j < j' \text{ and } i; i' \text{ such that} \tag{21}$$

$$i \in C_j^E; i' \in C_{j'}^E \tag{22}$$

$$\text{then } i < i' \tag{23}$$

Of course any individual would like to belong to the richest club, C_1: But the richest individuals may not be willing to accept a too poor agent: given their own contributions, he contributes too little to cover the congestion costs he inflicts to the others. That is why the first club is formed of the richest individuals only up to the first pivotal agent. Then substracting this club from society, the same reasoning explains why the second club is formed of the richest individual after the first pivotal agent, up to the second pivotal agent, etc... The last club of the partition, formed of the poorer agents is called a residual club.

The fundamental message of these properties is that the segmentation of society into growth clubs is uniquely determined by the initial inequality schedule. Inequality matters because it affects the relative contributing power of an individual which is the fundamental factor of her unanimous acceptance in a club. It therefore has a lasting influence on the pattern of growth because one of the engines of growth in this economy is the supply of a productive club good. We now turn to this point.

4 Inequality, the core partition and growth

What then are the consequences of inequality on the pattern of growth via the core partition of society? Is inequality a cause of non-convergence of growth rates between clubs? Are there some relations between the characteristics of inequality and the pattern of growth of this society?

To answer these questions, we shall use an explicit form of the autonomous component to growth $A(G_j, n_j)$. We shall assume:

$$A(G_j, n_j) = \ln\left(\frac{G_j}{e^{\alpha n_j}}\right). \tag{24}$$

This formulation satisfies the constraints imposed on the $A(\cdot)$ function in the previous section. It implies the following explicit values for the endogenous variables of interest:

$$G_j = \frac{\sum_{i\in C_j} a_i}{1+n_j\rho} \tag{25}$$

$$\gamma_j = \ln G_j - \alpha n_j - \rho = \ln\left(\sum_{i\in C_j} a_i\right) - \ln(1+n_j\rho) - \alpha n_j - \rho \tag{26}$$

$$\begin{aligned} V\left(C_j^E, S, \alpha\right) &= C + \left(\frac{1}{\rho}+\frac{1}{\rho^2}\right)\ln\sum_{i\in C_j} a_i \\ &\quad - \left[\left(\frac{1}{\rho}+\frac{1}{\rho^2}\right)\ln(1+n_j\rho) + \frac{1}{\rho^2}\alpha n_j\right] \end{aligned} \tag{27}$$

As we shall elaborate on these formulas, some comments are necessary. There are three effects on the growth rate of a given club. $\ln\left(\sum_{i\in C_j} a_i\right)$ expresses the positive impact of aggregate endowment on growth: given equal size, a richer club growth at a higher rate. $\ln(1 + n_j\rho)$ expresses the aggregate effect of free-riding: for a given aggregate endowment, the larger is a club, the more free-riding there is, and this depresses the rate of growth for this club. Finally, αn_j expresses the congestion effect and the direct depressing effect of size on the rate of growth of a given club. The impacts of these three effects on welfare are then immediate.

We shall reason on the formulas for answering these questions. We shall ask two questions:

1. How the characteristics of the inequality schedule impact on the pattern of growth through the core partition?
2. Is an increase in inequality leading to a more segmented society and more divergences in club growth rates?

4.1 Inequality characteristics and growth

We first tackle the first question. What we want to answer is whether the segmentation process we assume can lead to a “twin peaks” or a “many peaks” phenomenon, that is generate a non-convergence process of output levels or growth rate?

We are able to offer the following

Proposition 3 The core partition and the pattern of growth associated to it are related to the endowment sequence in the following way:

1. Whatever $j \in \{1, ..., J-1\}$, $n_j^E = n_{j+1}^E$ and $\eta_j^E > \eta_{j+1}^E$ if and only if $\lambda_{i;i+1} = \lambda$, $\forall i \in S$.
2. Whatever $j \in \{1, ..., J-1\}$, $n_j^E \leq n_{j+1}^E$ and $\eta_j^E > \eta_{j+1}^E$ if and only if $\lambda_{i;i+1} \geq \lambda_{i+1;i+2}$, $\forall i \in S$.
3. Whatever $j \in \{1, ..., J-1\}$, $n_j^E \geq n_{j+1}^E$ if and only if $\lambda_{i;i+1} \leq \lambda_{i+1;i+2}$, $\forall i \in S$; but the rate of growth of the $j-$club η_j^E is an ambiguous function of j.

Proof. see Appendix. ■

This proposition conveys information both on the segmentation of society and of its consequences on growth. The segmentation of society is related to the relative sizes of clubs n_j^E. This part of the proposition is strictly identical to the characteristics of segmentation obtained in the static case by JKM and the explanations therefore parallel. Basically what matters in this dynamic environment is the relative importance of the endowment of the pivotal agent of a club and the aggregate endowment of the entire club, that is on the whole distribution of the inequality ratios $\lambda_{i;j}$ between members of a club. In the case of constant ratios, since each club is sequentially in the same position, this explains why they have a constant size. When these ratios are decreasing, the relative importance of endowment for an agent with the same ranking in a club tends to increase with the ranking of a club. In a poorer club, the $k-$th agent in the club (when ranking depends on decreasing endowment) is relatively richer that the k-th agent in a richer club. That explains why the poorer club is larger. The reverse explanation is true when the inequality ratios increase.

The consequences on growth are substantial and Proposition 3 relates the characteristics of the pattern of growth to the characteristics of the inequality schedule. When the inequality schedule is such that the inequality ratio $\lambda_{i;i+1}$ is at most constant when i increases, then the rates of growth of the richer clubs are bigger than those of the poorer club. When the inequality ratio increases with the ranking of agents, then there is an ambiguity in the sign of the variation of growth rates.

This can be explained as follows. Consider the case where inequality ratios are equal and therefore clubs in the core partition have the same size. As we saw before (see (26)), the growth rate depends on three effects: the aggregate wealth of a club, a congestion effect and a free-riding effect. The congestion effect and the free-riding effect are the same for two clubs in the core partition, since both depend on the size of club. Two clubs then only differ because of the aggregate wealth effect and therefore a richer club grows at a faster rate than a poorer club.

Then consider the case where the inequality ratios decline with the ranking of individuals. This means that the endowment ratio is larger between two rich successive agents than between two poorer successive agents. The relative capacity to contribute to the club good declines with the ranking of agents. As we have just noted, this implies that a richer club is smaller than a poorer club. Then the three effects on the growth ratio work in the same direction. There is less congestion in a richer club than in a poorer club, since its size is smaller, there is less free-riding effect because it is also related to size and each agent contributes more in a richer club because she is richer than agents in a poorer club. Altogether the aggregate amount of productive club good is higher in a richer club than in a poorer one. Consequently a club formed of richer agents when the inequality schedule is such that inequality ratios decline with the ranking of agents, experiences a higher growth rate than a club formed of poorer agents.

However this is not true when the inequality ratios increase with the ranking of individuals. This corresponds to the case where the endowment leap is smaller between two rich individuals than between two poor ones. Then we have just seen that the size of a club tend to decrease with the ranking of this club: a club formed of richer agents is bigger than a club formed of poorer agents. Then the three effects on the growth rate work in opposite direction: a richer club represents a higher aggregate financing capability than a poorer one, but since it is larger, there are more congestion and free-riding which have a depressing effect on the growth rate. Altogether there is ambiguity and we cannot conclude that a richer club experiences a higher growth rate. It may be characterized by a lower growth rate than a poorer club, because of the negative effects due to a higher size.

With respect to the issue of convergence, it is clear that in the case of decreasing or equal inequality ratios, a poorer club is unable to catch-up with a richer club. No convergence either in levels or even in growth rates can be observed. The clubs will be continuously drifting apart. It is only in the case of increasing inequality ratios that a convergence in levels may be observed: there may be a date where two growth clubs will experience the same aggregate level of output, because the poorest club benefits of a higher growth rate than the richest one. But we should realize that this convergence if it exists will not last since after this period the initially poorest club becomes richer than the initially richest club. This of course does not contradict the ranking in welfare which is established at the beginning of history with a positive discount factor.

The general implication that we can draw from Proposition (3) is that the pattern of growth and the answer to the convergence debate rely on the characteristics of the initial inequality schedule.

4.2 Difference in inequality and the pattern of growth

An important issue widely discussed by economists, political scientists, moral philosophers and policymakers is whether more inequality fosters "more" growth or not. This is at the root of the main antagonisms in democracy. However it has been impossible to answer this question in a positive sense through analytical models of market economies where agents do belong to political jurisdictions and share public goods and values but are autonomous and free to make their accumulation decisions. Here we shed some light on this debate by addressing the following issue: How does an increase in inequality affect the pattern of growth associated to the core partition in our economy?

We know that the different impacts of different inequality schedules on the pattern of growth comes through a different segmentation of the society in clubs. In order to highlight this link, we need the following:

Definition 5 A society S is weakly (stricly) more segmented than a society $\tilde{S}$ if the number of non-residual clubs in the core partition $\tilde{C}^E$ associated with $\tilde{S}$, $\tilde{J}-1$, is at least equal to (bigger than) the number of non-residual clubs in the core partition C^E associated with S, $J-1$, and the j-th club in $\tilde{C}^E$ is never larger than the j-th club in C^E, for $j<J$.

Consider two societies with an equal number of identical agents, S and $\tilde{S}$. First we make two assumptions about the inequality schedules S associated with S and $\tilde{S}$ associated with $\tilde{S}$:

(A1) both societies have an identical aggregate (average) endowment A;

(A2) $S=\{\lambda_{1,2},\ldots,\lambda_{i,i+1},\ldots,\lambda_{N-1,N};A\}$ and $\tilde{S}=\left\{\tilde{\lambda}_{1,2},\ldots,\tilde{\lambda}_{i,i+1},\ldots,\tilde{\lambda}_{N-1,N};A\right\}$ are such that $\tilde{\lambda}_{i,i+1}>\lambda_{i,i+1},\forall i.$

To make relevant comparisons, we assume an equal aggregate initial endowment as we want to single out the impact of an increase in inequality. The second assumption allows us to state that $\tilde{S}$ is more unequal than S as S Lorenz-dominates $\tilde{S}$.[2]

Then additional more restrictive assumptions can be made about the ranking of successive λ. In particular we shall use the two following ones:

(A3) $S=\{\lambda_{1,2},\ldots,\lambda_{i,i+1},\ldots,\lambda_{N-1,N};A\}$ and $\tilde{S}=\left\{\tilde{\lambda}_{1,2},\ldots,\tilde{\lambda}_{i,i+1},\ldots,\tilde{\lambda}_{N-1,N};A\right\}$ are such that: $\lambda_{i,i+1}=\lambda_{i+1,i+2},\forall i$ and $\tilde{\lambda}_{i,i+1}=\tilde{\lambda}_{i+1,i+2},\forall i.$

(A4) $S=\{\lambda_{1,2},\ldots,\lambda_{i,i+1},\ldots,\lambda_{N-1,N};A\}$ and $\tilde{S}=\left\{\tilde{\lambda}_{1,2},\ldots,\tilde{\lambda}_{i,i+1},\ldots,\tilde{\lambda}_{N-1,N};A\right\}$ are such that: $\lambda_{i,i+1}>\lambda_{i+1,i+2},\forall i$ and $\tilde{\lambda}_{i,i+1}>\tilde{\lambda}_{i+1,i+2},\forall i.$

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[2] See Foster and Ok (1999).

(A3) states that for each inequality schedule, the endowment ratios between two successive agents are equal. Hence, from Proposition 3, we know that in each society, clubs are of equal size and the richer clubs grow faster.

(A4) states that for each inequality schedule, the endowment ratios between two successive agents are decreasing with the ranking of agents. Hence, again from Proposition 3, we know that in each society, the size of a club increases with its ranking and that richer clubs grow faster.

It is likely but not necessary that an increase in inequality modifies the core partition of society as we have seen that the segmentation into growth clubs depends on the relative endowments of agents. Under the two first assumptions only, in the admittedly special case where segmentation is similar for the two inequality schedules $\tilde{S}$ and S, we are able to offer the following:

Proposition 4 Assume two sequences, S and $\tilde{S}$ satisfying (A1) and (A2) are such that segmentation is not modified $\left(C^E = \tilde{C}^E\right)$. Then:

i) for any C_j^E such that all its members are characterized by an endowment higher than the mean endowment, $\tilde{\theta}_j > \theta_j$.

ii) for any C_j^E such that all its members are characterized by an endowment lower than the mean endowment, $\tilde{\theta}_j < \theta_j$.

iii) if in addition (A3) is satisfied by S and $\tilde{S}$, then $\left(\tilde{\theta}_j - \tilde{\theta}_{j+1}\right) > \left(\theta_j - \theta_{j+1}\right), \forall j \leq J - 1$.

Proof. see Appendix. ■

Since segmentation is assumed to be identical, we get the same number of clubs for both inequality schedules and the same pivotal agents. Remark then that a higher λ leads to a higher (lower) ratio $\frac{a_i}{\bar{a}}$ where $\bar{a}$ denotes the constant mean endowment for any individual which is richer (poorer) than the average individual.[3] Then any club j, the members of which are richer (poorer) than the average individual is richer (poorer) in aggregate terms under $\tilde{S}$ than under S. Therefore it grows more (less) rapidly under $\tilde{S}$ than under S since the change in inequality schedule does not modify the congestion nor the free-riding effect when

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[3] We define the average individual as the individual that would be characterized by the average endowment $\bar{a}$. This individual may not actually exist in any society.

we assume a constant segmentation of society (and therefore no modification of size for clubs of same ranking). This explains i) and ii).

Moreover iii) states that an increase in inequality, under the assumption that segmentation with clubs of equal size remains constant, widens the difference in growth rates between two successive clubs. This means that the divergence in growth paths is increased.

Now we relax the constraint that both societies are characterized by the same segmentation in clubs. First we would like to understand the impact of an increase in inequality on the segmentation of society into growth clubs. We are able to offer the following:

Proposition 5 Assume two sequences S and $\tilde{S}$ satisfying (A1), (A2) and (A3) or (A4). Then $\tilde{S}$ is weakly more segmented than S:

Proof. See Appendix. ■

This proposition extends in the dynamic case the result obtained in the static case by JKM.[4] A more equal society (which is Lorenz-dominant) is weakly less segmented than a more unequal one: it has at most as many non-residual clubs and each of its non-residual club is at least as large as the corresponding non-residual club (with the same ranking index) in the more unequal society. Remark that under (A3), the first club $\tilde{C}_1^E$ is at most equal in size to C_j^E because its members are richer under $\tilde{S}$ than under S and the size of clubs is constant in each partition, because of Proposition 3. Hence each club in $\tilde{C}^E$ is at most as large as the corresponding club in C^E. Under (A4), the size of clubs increases with the ranking of club in each partition. Hence since the first club $\tilde{C}_1^E$ is at most equal in size to C_j^E because its members are richer under $\tilde{S}$ than under S, and $\tilde{n}_2^E$ is at most equal to $\tilde{n}_1^E$; this implies that $\tilde{n}_2^E$ is at most equal to n_2^E:

However we cannot conclude in the case where the endowment ratios increase with i; since then, it may happen that the first club is larger than the second club under $\tilde{S}$ and under S. This would violate the wek segmentation property.

Turning now to the impact of inequality on the growth pattern, we offer the following:

Proposition 6 Assume two sequences S and $\tilde{S}$ satisfying (A1), (A2) and (A3) or (A4) and such that $\tilde{S}$:is strictly more segmented than S. Then the growth rate of $\tilde{C}_j^E$ is higher than the growth rate of C_j^E for any club C_j^E, such that all its members are characterized by an endowment higher than the median / mean endowment.

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[4] see Proposition X in JKM.

Proof. See Appendix. ■

This proposition may be explained by reasoning sequentially on clubs. Consider the impact on the first (the richest) club of an increase of inequality such that the inequality ratios are at most equal when the ranking of individuals increases. The higher endowment leap between two successive individuals tends to decrease the number of members in this first club. This generates less congestion. It also leads to less free riding. Finally a higher endowment leap leads to more affluent members when clubs are formed with richer than average members, which are more able and more willing to pay for the club good. Therefore the first club grows at a higher rate under the more unequal inequality schedule $\tilde{S}$ than the first club of the partition associated with S. Then we may disrecard the first club. Looking at the second clubs under S and $\tilde{S}$, and assuming that their members are richer than the average individual, we remark that the richest agent of $\tilde{C}_2^E$ is richer than the agent in $\tilde{S}$ with the same ranking because of assumptions 1. and 2., who is richer than the richest agent of C_j^E; because of her lower ranking (since $\tilde{n}_1^E$ is at most than n_1^E). Since $\tilde{n}_2^E$ is at most than n_2^E; it implies that in aggregate members of $\tilde{C}_2^E$ are richer than members of C_2^E: Hence all three effects make the growth rate associated with $\tilde{C}_2^E$ higher than the growth rate associated with C_2^E: By repeating the reasoning on the following clubs, we obtain the same result as long as the club we consider is formed of individuals richer than the average individual. When this is not true, the aggregate endowment of a club of rank j under $\tilde{S}$ may be lower than the aggregate endowment of the club of same rank j under S. Then this has a depressing effect on the growth rate which may overcome the effects of a smaller size.

Finally we offer the following proposition on the impact of the congestion parameter α;

Proposition 7 An increase in α leads to a weakly more segmented society but has ambiguous effects on growth rates.

Proof. See Appendix. ■

An increase in the congestion parameter tends to decrease the size of clubs as each additional member has to overcome a higher marginal cost and be able to contribute more. However this implies that the first club is at most as rich as before and this depresses the aggregate ability to pay for the public good. Hence the smaller size of the first club and its lower aggregate endowment have conflicting effects on the formation of the club good and this leads to an ambiguous effect on the growth rate of the first club. This reasoning generalizes to any successive club.

5 Conclusion

This paper tackles the theoretical issue of explaining "twin peaks" phenomena. We focus on the impact of inequality over the pattern of growth and we prove that the non-convergence phenomena may be ultimately related to the underlying structural inequality between agents. This is obtained by means of an explicit analysis of the formation of growth clubs, that is of clubs of agents that share a club good with continuing productive capacities over time. Using coalition theory, we prove that the segmentation of society into growth clubs depends on three effects: a contributing effect since the individual provision for the club good depends on individual endowment; a congestion effect since the increase in the size of a club is likely to have negative productive consequences; a free-riding effect common to clubs with voluntary contribution to the public good.

We are then able to relate the segmentation of society and consequently its pattern of growth to the initial endowment distribution over infinitely lived agents. We prove that clubs are not characterized by a identical growth rate and therefore that they do not converge to a unique steady state growing path. Under special assumptions of the inequality schedule a catching-up process may take place, as the poorer clubs may grow at a higher rate than richer clubs.

Our results depend on several restrictive assumptions. Their relaxation should allow us to further explore the links between inequality, economic and social segmentation and the characteristics of the pattern of growth and maybe uncover new dimensions worth empirical investigation. Many of them were made for simplicity and could be relaxed at the expenses of clarity. Two of them are particularly important.

First it is assumed that the provision of the club good is made at the beginning of history, once and forever. Obviously, this does not cover the more plausible case where the club good is provided period after period: think about R&D expenditures, public infrastructure which need to be repaired and improved over time. But taking this fact into account raises the annoying point that the core partition may be reassessed every period. Clearly this raises difficult technical issues that could not be tackled in the present paper.

Second it is assumed that once clubs are formed, there are no economic relationships between them. But again this is a gross simplification. Trade between economies can be interpreted as an enduring relationship between clubs. In other words, we exploit here externalities within clubs but not externalities between clubs. Such externalities however are likely to affect the growth pattern and the convergence properties of the growth dynamics. It should therefore be valuable to include inter-club relationships.[5]

We leave these extensions to further research. As it is, this paper proves the relationship between inequality and the characteristics of the growth process and offers a possible framework to study this relationship.

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[5] On this point, see Beaudry, Cahuc and Kempf (1997).

APPENDIX

1 Proof of Proposition 1

Proof. As voluntary individual financing yields the same level of indirect utility for individuals belonging to the same club, our coalition formation game turns out to be identical to the one developped by Farrell and Scotchmer in the case of partnerships. Our proof thus follows their existence demonstration which consists in constructing a core partition.

First, let us consider the club C_1^E that maximizes the utility function $V(.; S; \alpha)$. Those agents belonging to C_1^E thus get the highest possible level of utility. Second, we can restrict our attention to the subset $S \setminus C_1^E$ and form a club C_2^E of agents remaining in this subset that maximizes $V(.; S; \alpha)$. We repeat the process until the set $\left\{C_1^E; \ldots; C_j^E; \ldots\right\}$ constitutes a partition of S.

It then turns out that this partition $C^E = \left\{C_1^E; \ldots; C_j^E; \ldots\right\}$ belongs to the core of the coalition formation game. The reason is that no blocking coalition can contain any member of C_1^E as they obtain the highest possible level of utility. This argument is also true for those agents belonging to C_2^E and so on. ■

2 Proof of Proposition 2

2.1 Uniqueness

Proof. Suppose that there exist two core partitions $C^E = \left\{C_1^E; \ldots; C_j^E; \ldots\right\}$ and $C'^E = \left\{C_1'^E; \ldots; C_j'^E; \ldots\right\}$ with $C_1^E \subset C_1'^E$ and $V(C_1^E) = V(C_1'^E)$. We argue that this case is non-generic. After small perturbations of initial individual endowments, there will no longer be two coalitions that would give the same maximal level of utility.

Thus if the club C_1^E that maximises utility is unique it must appear in any core partition and so on. The core partition is then generically unique. ■

2.2 Consecutive structure

Proof. Let us consider by contradiction that the core partition $C^E = \left\{C_1^E; \ldots; C_j^E; \ldots\right\}$ is constituted by non-consecutive clubs. Without loss of generality, we suppose that $C_1^E =$

$\{1, \ldots, i, i+2, \ldots, n_1\}$ and $C_2^E = \{i+1, n_1+1, \ldots, n_1+n_2\}$ with $V(C_1^E) > V(C_2^E)$. If we form the new club $\Omega = (C_1^E \setminus \{n_1\}) \cup \{i+1\}$; as $a_{i+1} > a_{n_1}$, it is easy to show from the comparative statics on $V(C_j)$ that Ω is a blocking coalition, i.e.:

As,

$$\frac{\partial V(C_j)}{\partial \left(\sum_{i \in C_j} a_i \right)} = -\frac{1}{\rho} \frac{-_1 A_{11}}{A_1(G_j^*, n_j)} + \frac{1}{\rho^2} -_1 A_1 > 0$$

then,

$$V(\Omega) > V(C_1^E) \; \forall i \in \Omega$$

which contradicts the fact that C^E is a core partition. ■

2.3 Welfare Ordering

Proof. Let us consider by contradiction that the core partition $C^E = \left\{ C_1^E, \ldots, C_j^E, \ldots \right\}$ does not satisfy the welfare ordering property.

Without loss of generality, we suppose that the club C_1^E containing the richest members provides less utility than the club C_2^E, i.e.:

$$V(C_2^E) > V(C_1^E)$$

In this case any agent $i \in C_1^E$ has an incentive to propose the following group $\Omega = (C_2^E \setminus \{z\}) \cup \{i\}$ with $z \in C_2^E$ that satisfies:

$$V(\Omega) > V(C_2^E) > V(C_1^E)$$

contradicting the fact that C^E belongs to the core. ■

3 Proof of Proposition 3

Proof. First, let us provide a formal definition of a pivotal agent.

Definition 6 For a given core partition $C^E = \left\{ C_1^E, \ldots, C_j^E, \ldots \right\}$, a pivotal agent of a club C_j^E indexed by p_j is defined by the following inequalities:

$$a_{p_j} \geq \sum_{z=p_{j-1}+1}^{p_j-1} a_z \left(e^{\frac{\alpha}{1+\rho}} \frac{1+n_j^E \rho}{1+(n_j^E-1)\rho} - 1 \right) \tag{28}$$

and

$$a_{p_j+1} < \sum_{z=p_{j-1}+1}^{p_j} a_z \left(e^{\frac{\alpha}{1+\rho}} \frac{1+n_j^E\rho+\rho}{1+n_j^E\rho} - 1 \right) \tag{29}$$

In other words, the pivotal agent is such that the marginal benefit of her entry into a club covers the subsequent marginal cost but the marginal benefit of the entry of the agent immediately after him (indexed $p_j + 1$) in the inequality schedule does not cover the subsequent marginal cost.

We rewrite (28) and (29) as follows:

$$1 \geq \left(e^{\frac{\alpha}{1+\rho}} \frac{1+n_j^E\rho}{1+(n_j^E-1)\rho} - 1 \right) \sum_{z=p_{j-1}+1}^{p_j-1} \left(\prod_{x=z}^{p_j-1} \lambda_{x;x+1} \right) \tag{30}$$

and

$$1 < \left(e^{\frac{\alpha}{1+\rho}} \frac{1+n_j^E\rho+\rho}{1+n_j^E\rho} - 1 \right) \sum_{z=p_{j-1}+1}^{p_j} \left(\prod_{x=z}^{p_j} \lambda_{x;x+1} \right) \tag{31}$$

Second, we focus on the link between the size of the non residual clubs and the inequality characteristics of S. Let us consider the inequality schedule such that $\lambda_{i;i+1} \geq \lambda_{i+1;i+2}$, $\forall i \in S$. Denoting $n(i)$ and $n(i')$ the optimal sizes of clubs when i (resp. i', $i' < i$) is the richest member. Hence, according to (30) and (31), $n(i)$ and $n(i')$ are the smallest integers such that the following inequalities are satisfied:

$$1 < \left(e^{\frac{\alpha}{1+\rho}} \frac{1+n(i)\rho+\rho}{1+n(i)\rho} - 1 \right) \sum_{z=i}^{i+n(i)-1} \left(\prod_{x=z}^{i+n(i)-1} \lambda_{x;x+1} \right)$$

and

$$1 < \left(e^{\frac{\alpha}{1+\rho}} \frac{1+n(i')\rho+\rho}{1+n(i')\rho} - 1 \right) \sum_{z=i'}^{i'+n(i')-1} \left(\prod_{x=z}^{i'+n(i')-1} \lambda_{x;x+1} \right)$$

As $\lambda_{x;x+1} \geq \lambda_{x+1;x+2}$, $\forall x \in S$ it turns out that:

$$\left(e^{\frac{\alpha}{1+\rho}} \frac{1+n(i)\rho+\rho}{1+n(i)\rho} - 1 \right) \sum_{z=i'}^{i'+n(i)-1} \left(\prod_{x=z}^{i'+n(i)-1} \lambda_{x;x+1} \right)$$
$$\geq \left(e^{\frac{\alpha}{1+\rho}} \frac{1+n(i)\rho+\rho}{1+n(i)\rho} - 1 \right) \sum_{z=i}^{i+n(i)-1} \left(\prod_{x=z}^{i+n(i)-1} \lambda_{x;x+1} \right)$$

which implies that $n(i') \leq n(i), \forall i' < i, i'$ and $i \in S$.

Now we show that $n(i') \leq n(i), \forall i' < i, i'$ and $i \in S$ implies that $\lambda_{x;x+1} \geq \lambda_{x+1;x+2}$, $\forall x \in S$. Given that $n(i') \leq n(i)$; we can write:

$$\left(e^{\frac{\alpha}{1+\rho}}\frac{1+n(i')\rho+\rho}{1+n(i')\rho}-1\right)\left(\sum_{z=i'}^{i'+n(i')-1}\prod_{x=z}^{i'+n(i')-1}\lambda_{x;x+1}\right)$$
$$\geq \left(e^{\frac{\alpha}{1+\rho}}\frac{1+n(i')\rho+\rho}{1+n(i')\rho}-1\right)\left(\sum_{z=i}^{i+n(i')-1}\prod_{x=z}^{i+n(i')-1}\lambda_{x;x+1}\right)$$

which leads to the following inequality:

$$\left(\sum_{z=i'}^{i'+n(i')-1}\prod_{x=z}^{i'+n(i')-1}\lambda_{x;x+1}\right) \geq \left(\sum_{z=i}^{i+n(i')-1}\prod_{x=z}^{i+n(i')-1}\lambda_{x;x+1}\right)$$

which is true when $\lambda_{x;x+1} \geq \lambda_{x+1;x+2}$, $\forall x \in S$. Thus we can conclude that for a given core partition, whatever $j \in \{1; \ldots; J-1\}$, $n^E_{j+1} \geq n^E_j$ if and only if $\lambda_{x;x+1} \geq \lambda_{x+1;x+2}$, $\forall x \in S$.

The proving is similar for $\lambda_{x;x+1} = \lambda$ and $\lambda_{x;x+1} \leq \lambda_{x+1;x+2}$, $\forall x \in S$.

Third, we now concentrate on the link between the pattern of growth and the inequality characteristics of a society. For a given core partition $C^E = \left\{C^E_1; \ldots; C^E_j; \ldots\right\}$ and according to (26) and (27), we have, whatever $j; j' \in \{1; \ldots; J-1\}$ and $j \neq j'$:

$$\eta_j - \eta_{j'} = \ln\left(\frac{\sum_{i\in C^E_j} a_i}{\sum_{i\in C^E_{j'}} a_i}\right) - \ln\left(\frac{1+n^E_j\rho}{1+n^E_{j'}\rho}\right) - \alpha(n^E_j - n^E_{j'}) \qquad (32)$$

and

$$V\left(C^E_j; S; \alpha\right) - V\left(C^E_{j'}; S; \alpha\right) = \left(\frac{1}{\rho}+\frac{1}{\rho^2}\right)\left[\ln\left(\frac{\sum_{i\in C^E_j} a_i}{\sum_{i\in C^E_{j'}} a_i}\right) - \ln\left(\frac{1+n^E_j\rho}{1+n^E_{j'}\rho}\right)\right]$$
$$-\frac{\alpha}{\rho^2}(n^E_j - n^E_{j'})$$

So when $n^E_{j+1} = n^E_j$ whatever $j \in \{1; \ldots; J-1\}$, we have:

$$\eta_j - \eta_{j+1} = \ln\left(\frac{\sum_{i\in C^E_j} a_i}{\sum_{i\in C^E_{j+1}} a_i}\right)$$

which is positive whatever $j \in \{1; \ldots; J-1\}$ given the consecutive structure property.

According to the welfare ordering property, if $j < j'$ we thus have:

$$V\left(C_j^E;S;\alpha\right) - V\left(C_{j'}^E;S;\alpha\right) > 0$$

which is equivalent to

$$\ln\left(\frac{\sum_{i\in C_j^E} a_i}{\sum_{i\in C_{j'}^E} a_i}\right) > \ln\left(\frac{1+n_j^E\rho}{1+n_{j'}^E\rho}\right) + \frac{\alpha}{\rho+1}(n_j^E - n_{j'}^E)$$

Given (32), it turns out that:

$$\gamma_j - \gamma_{j'} > \frac{-\alpha\rho}{\rho+1}(n_j^E - n_{j'}^E)$$

As a consequence when $n_{j+1}^E \geq n_j^E$ whatever $j \in \{1;\ldots;J-1\}$, we can easily conclude that $\gamma_j - \gamma_{j+1} > 0$, whatever $j \in \{1;\ldots;J-1\}$.

However, in the case where $n_j^E \geq n_{j+1}^E$ whatever $j \in \{1;\ldots;J-1\}$ it is impossible to conclude on the sign of $\gamma_j - \gamma_{j+1}$. ■

4 Proof of Proposition 4

Proof. Let us consider two sequences S and $\tilde{S}$ satisfying A1. and A2. We thus know that whatever $i \in S$ with $a_i > \frac{A}{N}$ (respectively $a_i < \frac{A}{N}$) we have $\tilde{a}_i > a_i$ (respectively $\tilde{a}_i < a_i$).

Let us now consider two core partitions $C^E = \left\{C_1^E;\ldots;C_j^E;\ldots\right\}$ and $\tilde{C}^E = \left\{\tilde{C}_1^E;\ldots;\tilde{C}_j^E;\ldots\right\}$ respectively associated with S and $\tilde{S}$ satisfying A1 and A2 and such that (i) for clubs C_j^E with $j = \left\{1;\ldots;\underline{j}\right\}$ we have $a_i > \frac{A}{N}\ \forall i \in C_j^E$, and (ii) for clubs C_j^E with $j = \left\{\bar{j};\ldots;J-1\right\}$; $\bar{j} > \underline{j}$, we have $a_i < \frac{A}{N}\ \forall i \in C_j^E$.

Given (32), the growth rates difference between clubs emerging under schedules S and $\tilde{S}$ can be expressed as follows:

$$\gamma_j - \tilde{\gamma}_j = \ln\left(\frac{\sum_{i\in C_j^E} a_i}{\sum_{i\in\tilde{C}_j^E} \tilde{a}_i}\right)$$

It is then straightforward to deduce that $\gamma_j - \tilde{\gamma}_j > 0$ for any $j = \left\{1;\ldots;\underline{j}\right\}$ and $\gamma_j - \tilde{\gamma}_j < 0$ for any $j = \left\{\bar{j};\ldots;J-1\right\}$. This proves items i) and ii) of Proposition 4.

For inequality schedules S and $\tilde{S}$ satisfying A1, A2 and A3, the growth rates difference between two successive clubs equals:

$$\eta_j - \eta_{j+1} = \begin{cases} \ln\left(\dfrac{\sum_{i\in C_j^E} a_i}{\sum_{i\in C_{j+1}^E} a_i}\right) & \text{under } S \\ \ln\left(\dfrac{\sum_{i\in \tilde{C}_j^E} \tilde{a}_i}{\sum_{i\in \tilde{C}_{j+1}^E} \tilde{a}_i}\right) & \text{under } \tilde{S} \end{cases}$$

Knowing that $n_j = n_{j+1} = n$ for all $j \in \{1, \ldots, J-1\}$, we obtain:

$$\eta_j - \eta_{j+1} = \begin{cases} n \ln \lambda & \text{under } S \\ n \ln \tilde{\lambda} & \text{under } \tilde{S} \end{cases}$$

Hence, $\eta_j - \eta_{j+1}$ is higher under $\tilde{S}$ than under S which completes the proof of Proposition 4. ■

5 Proof of Proposition 5

Proof. Denoting $n(i)$, respectively $\tilde{n}(i)$, the size of the consecutive club whose richest member is i and maximizes i's welfare under S, respectively under $\tilde{S}$, we thus have:

$$\left(e^{\frac{\alpha}{1+\rho}}\frac{1+n(i)\rho+\rho}{1+n(i)\rho}-1\right)\sum_{z=i}^{i+n(i)-1}\left(\prod_{x=z}^{i+n(i)-1}\tilde{\lambda}_{x,x+1}\right)$$

$$> \left(e^{\frac{\alpha}{1+\rho}}\frac{1+n(i)\rho+\rho}{1+n(i)\rho}-1\right)\sum_{z=i}^{i+n(i)-1}\left(\prod_{x=z}^{i+n(i)-1}\lambda_{x,x+1}\right)$$

Hence it is easy to deduce that $\tilde{n}(i) \leq n(i)$, $\forall i \in S$. This implies that the first pivotal agent under S has a lower index than under $\tilde{S}$, that is $\tilde{p}_1 < p_1$. Moreover, $\tilde{n}(\tilde{p}_1) \leq n(\tilde{p}_1)$. Under A3 (A4), using Proposition 3, we know that $n(\tilde{p}_1) = (<)\, n(p_1)$. Combining these two inequalities, we get that the second club is smaller under $\tilde{S}$ than under S. Repeating this reasoning completes the proof of Proposition 5. ■

6 Proof of Proposition 6

Proof. Let us consider two sequences S and $\tilde{S}$ satisfying A1, A2, and A3 or A4. We thus know that whatever i with $a_i > \frac{A}{N}$ (respectively $a_i < \frac{A}{N}$) we have $\tilde{a}_i > a_i$ (respectively $\tilde{a}_i < a_i$).

Let us now consider two core partitions $C^E = \left\{C^E_1; \ldots; C^E_j; \ldots\right\}$ and $\tilde{C}^E = \left\{\tilde{C}^E_1; \ldots; \tilde{C}^E_j; \ldots\right\}$ respectively associated with the sequences S and $\tilde{S}$ such that (i) for clubs C^E_j with $j = \left\{1; \ldots; \underline{j}\right\}$ we have $a_i > \frac{A}{N}\ \forall i \in C^E_j$, and (ii) for clubs C^E_j with $j = \left\{\bar{j}; \ldots; J - 1\right\}; \bar{j} > \underline{j}$, we have $a_i < \frac{A}{N}\ \forall i \in C^E_j$:

We now concentrate on the clubs C^E_j and $\tilde{C}^E_j$ with $j = \left\{1; \ldots; \underline{j}\right\}$. We want to show that the following difference is positive:

$$V\left(\tilde{C}^E_j; \tilde{S}; \alpha\right) - V\left(C^E_j; S; \alpha\right) = \left(\frac{1}{\rho} + \frac{1}{\rho^2}\right)\left[\ln\left(\frac{\sum_{i \in \tilde{C}^E_j} a_i}{\sum_{i \in C^E_j} a_i}\right) - \ln\left(\frac{1 + \tilde{n}^E_j \rho}{1 + n^E_j \rho}\right)\right] - \frac{\alpha}{\rho^2}(\tilde{n}^E_j - n^E_j)$$

Given that $\tilde{C}^E_1$ belongs to the core partition, we can write:

$$\nexists \mathcal{S} \subset \tilde{S} \text{ such that } \forall i \in \mathcal{S};\ V_i\left(\mathcal{S}; \tilde{S}; \alpha\right) > V_i\left(\tilde{C}^E_j; \tilde{S}; \alpha\right)$$

Therefore we deduce that

$$\forall i \in C^E_j;\ V_i\left(\tilde{C}^E_j; \tilde{S}; \alpha\right) > V_i\left(C^E_j; \tilde{S}; \alpha\right)$$

and as

$$V_i\left(C^E_j; \tilde{S}; \alpha\right) - V_i\left(C^E_j; S; \alpha\right) = \left(\frac{1}{\rho} + \frac{1}{\rho^2}\right)\ln\left(\frac{\sum_{i \in C^E_j} a_i}{\sum_{i \in C^E_j} a_i}\right) > 0$$

we can then deduce that:

$$V_i\left(\tilde{C}^E_j; \tilde{S}; \alpha\right) > V_i\left(C^E_j; S; \alpha\right)$$

which is equivalent to

$$\ln\left(\frac{\sum_{i \in \tilde{C}^E_j} a_i}{\sum_{i \in C^E_j} a_i}\right) > \ln\left(\frac{1 + \tilde{n}^E_j \rho}{1 + n^E_j \rho}\right) + \frac{\alpha}{\rho + 1}(\tilde{n}^E_j - n^E_j)$$

Given (32); it turns out that:

$$\tilde{\eta}_j - \eta_j > \frac{-\alpha\rho}{\rho + 1}(\tilde{n}^E_j - n^E_j)$$

According to Proposition 5, we have $\tilde{n}^E_j \leq n^E_j$: We thus deduce that:

$$\tilde{\eta}_j - \eta_j > 0$$

■

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