DESIGNING RIGHTS:
INVISIBLE HAND THEOREMS, COVERING AND MEMBERSHIP [1]

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In this essay we propose a notion of a *Rechtsstaat*, equipped with a *code of rights*, and investigate the prospects for designing its elements so as to impart to it an *invisible hand* property and a property of the *preservation of the best public interest*, paralleling the spirit of the First and the Second Theorems of Economics, respectively. After a characterization theorem and a sufficiency result for the invisible hand property, we offer a sufficient condition for the preservation of the best public interest. Following a brief revisit of ordinary trade economies as a direct application area from this viewpoint, we discuss *membership rights* to govern the formation of covers, bringing to bear a set of ethical axioms whose conjunction determines a *liberal ethic*, applied previously in the rehabilitation of labor-managed "firms" to workers' enterprises. We apply this approach to the formation of covers, such as "networks", taking a fresher look at *covering games* for which we propose a couple of Shapley-like *values.* In the same spirit we regard the formation of coalitions, such as partnerships, subject to well-designed *membership rights*, as reflected in *membership markets* for workers' enterprises.

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1. Introduction

We start by agreeing to consider a set A of *agents* such as i,j ∈ A and a *state space* S, comprising the set of all possible *states* such as r, s, t ∈ S which these agents, or coalitions of these agents, can be confronted with. By a *coalition* we mean any subset of A, and we denote $\mathbf{A} = 2^{A}$ for the space of all coalitions, so that $\underline{\mathbf{A}} = 2^{\mathbf{A}}$ is the space of all *families* of coalitions, while $2^{\underline{\mathbf{A}}}$ is the space of *families of families* of coalitions of agents. Given any two coalitions B and C with B ⊂ C (⊂ A), we denote [B, C] for the "*interval*" [B, C] = {D ∈ $\mathbf{A}$| B ⊂ D ⊂ C} of all coalitions containing B and contained in C. We also define the *roster* function ?: $\underline{\mathbf{A}} \to \mathbf{A}$ at any family B ∈ $\underline{\mathbf{A}}$ of coalitions by ?(B) = ∪{C C∈ B}. Thus, by the roster of a family of coalitions we mean the set of all the members of the coalitions belonging to the family.

Given A (and $\mathbf{A}$ and $\underline{\mathbf{A}}$ and $2^{\underline{\mathbf{A}}}$) and S as above, we propose to regard a Rechtsstaat[3] as any ordered triplet s = <α, β, γ >, where α, β and γ are certain functions,

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[3] In an English-language text, why use the German term, 'Rechtsstaat'? Simply due to the difficulty of finding a good counterpart of 'Rechtsstaat' in English, and because the terms in other languages that the

called an *ability*, a *benefit* and a *code*, respectively, as we describe right away. Each of these functions is defined on S × S, the space of "*alterations*" of states.

For any alteration (s, t) ∈ S × S, the ability α gives us the family α(s, t) ∈ **A** (or α(s, t) ⊂ **A**) of coalitions which are able to bring about the alteration (s, t) of state s into state t. This ability α: S × S → **A** is given to us as an externally determined, physical or technological capability, which we have to reckon with as an unalterable constant[4]. The ability α thus summarizes all information about precisely which coalitions are able (or capable) of altering which states of the world into which, and we will have to take this datum as a constraint in all our design efforts. We will soon agree on certain sensible axioms that such an ability should satisfy, but first we run through the main idea of the next two functions.

Denoting ℜ for the set of real numbers, the next function β in the specification of a Rechtsstaat indicates the benefit $\beta^C(s, t) \in \Re$ accruing to each coalition C ∈ **A** at any alteration (s, t) ∈ S × S. As a function β: S × S → $\Re^{\mathbf{A}}$, its value $\{\beta^C(s, t) | C \in \mathbf{A}\}$ at any alteration (s, t) may be interpreted cardinally or just ordinally for each coalition C ∈ **A**. Generally, however, $\beta^C(s, t)$ is positive when the coalition C benefits from the alteration of s to t, in which case we say that C is *willing* to alter s to t. We say that a coalition C ∈ **A** *approves* an alteration (s, t) iff $\beta^C(s, t) = 0$, and that C *disapproves* the alteration iff $\beta^C(s, t) < 0$. We will discuss certain axioms that seem reasonable for such a *benefit* function β to satisfy, depending among other matters on whether we are able to reckon with cardinal benefits or just ordinal ones. To the extent that the designer can prescribe the relative benefits of some of the states of the world, such as by setting the rules and payoffs of certain games or by designing and implementing certain mechanisms to that effect, the benefit function falls within our scope as a social design variable. But our main focus for a design variable in the present inquiry will be on the third element in the specification of a Rechtsstaat s, namely the *code of rights*, or simply the *code*, ?, appearing as the third item in s.

By a *code of rights* or simply a *code* we mean a function ?: S × S → $2^{\mathbf{A}}$ specifying, for any alteration (s, t) of a state s to a state t, the family ?(s, t) ⊂ **A** of families of coalitions who are given the right to interfere in that alteration, in a sense implying that the alteration can be enacted only if there is a *"code family of coalitions"* for the alteration, i.e. a family C ∈ ?(s, t), which *approves*, by which we mean that every coalition C ∈ C approves. Otherwise, that is if there is no code family C ∈ ?(s, t) each of

present author is aware of [for instance, 'estado de derecho' (Spanish, i.e. Castellano), 'état de droit' (French), 'hukuk devleti' (Turkish), 'stato di diritto' (Italian)], fail to be single words.

[4] Of course, mankind has always shown an interest in altering the technology delineating feasible from infeasible, and has often succeeded in improving this boundary much to the benefit of some of its members, possibly at the cost of other members. All this, however, is beyond the scope of *social* design, our domain of discourse here. Thus, we follow the tradition of much of economic theory, taking a certain technology, in this case our "ability", as given to us and beyond our power of modification within our present discourse and analysis.

whose member coalitions $C \in \mathcal{C}$ approves, equivalently every code family owns a disapproving coailition, the alteration (s, t) cannot be enacted, even if there is an able coalition $B \in \alpha(s, t)$ which is willing to alter s to t, i.e. $\beta^{B}(s, t) > 0$.

Example: Often we design the code so that any majority of the members of a committee (such as a parliament) has the right to “pass” an alteration. In our model this is done by setting ?(s, t) to be the family of all families $\mathcal{C}$ consisting of a sufficient number of singleton coalitions: $\#\mathcal{C} = n/2$ if by “majority” we mean simple majority.

Again, we will very soon discuss what might be some sensible axioms to expect to be fulfilled by a code, but for the moment suffice it to say that this element of a Rechtsstaat is the critical one for our present investigation. We aim to deduce some “architectural” principles” to guide the design of a code of rights, given an ability α and a benefit β, so as to fulfill an “invisible hand” criterion and a property concerning the “preservation of the best public interest”. To bring clarity to these terms, we need to formulate a notion of equilibrium and one of optimality, as we now turn to do.

Our concept of equilibrium is based on three underlying notions, ability, willingness and approval. We say that a state $s \in S$ is an *equilibrium* of a Rechtsstaat s = $<\alpha, \beta, \gamma>$ iff, for every alteration (s, t) of s to a state $t \in S$ where there is a willing able coalitio n $B \in \alpha(s, t)$, every code family $\mathcal{C} \in$?(s, t) owns some coalition $C \in \mathcal{C}$ disapproving the alteration, i.e. $\beta^{C}(s, t) < 0$. In other words, a state s is an equilibrium of a Rechtsstaat iff, for every state t affording a coalition able and willing to alter s to t, the alteration (s, t) is disapproved by some code coalition in every code family. Thus, underlying this notion of equilibrium is a setting where we the approval of all the coalitions in some code family is required for an alteration to be enacted. We denote E(s) for the set of all equilibria of a Rechtsstaat s.

A more restricted notion of equilibrium for a Rechtsstaat s = <a , b , g > is obtained by regarding as *strong equilibria* of s the states s such that, for every state t possessing a coalition B Î a (s, t) which is (able and) willing to have the alteration (s, t), *every* coalition C Î ?(s, t) disapproves the alteration, i.e. *no* coalition C Î ?(s, t) approves it. In contrast with the weaker notion of equilibrium defined immediately before, this corresponds to the case where the approval of any single code coalition suffices for an alteration to be enacted. We denote the set of all strong equilibria of a Rechtsstaat s by E(s). Clearly, we always have E Ì E. For a code ? appointing a single coalition C(s, t) Î ?(s, t) at every alteration (s, t), i.e. a code ? of the form ?(s, t) = {C(s, t)}at every (s, t) Î S ´ S, the two notions of equilibrium coincide, of course, and in such a case we have E = E. (We will see such a code in dealing with membership rights in further sections.)

Given an ability α and a benefit β, for a system $<\alpha, \beta>$ void of a code, a state $s \in S$ could naturally be considered to constitute an equilibrium so long as, for every state $t \in S$, every able coalition $B \in \alpha(s, t)$ is unwilling (or every willing coalition is unable) to alter s to t.[5] While every equilibrium in this sense is also a strong equilibrium of any Rechtsstaat s = <α, β, γ> with the mentioned ability and benefit functions accompanied by an arbitrary code, depending on the code specified, s may in fact possess further **strong equilibria. For a state s admitting willing and able coalitions to alter it to some state t will still be an equilibrium in our sense so long as every such alteration is "blocked" because it is disapproved by some coalition in ?(s, t), or even a strong equilibrium if it is disapproved by every coalition in ?(s, t).**

Finally, coming to the notion of optimality, we simply agree that a state s is *optimal for a coalition* C iff the benefit β^C(s, t) of no feasible[6] alteration (s, t) (with α(s, t) non-empty) is positive for C. We regard a state as *(socially) optimal* iff it is optimal for the "grand" coalition A. We denote the set of optimal states of a Rechtsstaat s by W(s).

Having defined our concepts of equilibrium and optimality, we are now ready to give precision to the idea of the invisible hand property and that of the implementability (or decentralizability) property for a Rechtsstaat. We say that a Rechtsstaat s possesses **the *invisible hand (IH) property* iff E(s) Ì W(s), the *strong invisible hand (SIH) property* iff E(s) Ì W(s). An "invisible hand theorem" giving sufficient conditions for a** Rechtsstaat to possess an invisible hand property would parallel the celebrated "First Theorem" of Economics, whereby certain (Walrasian) equilibria are optimal in a certain (Edgeworthian) sense under certain sufficient conditions. For the IH property is clearly an extension of the well-studied property of competitive private ownership economies, as heralded by the familiar First Theorem. With a modest stretch of our usual definitions, an invisible hand theorem would also be an implementation (or in an earlier sense of Hurwicz, "decentralization") theorem for the optimality we have in mind, in the sense that it provides sufficient conditions for all the equilibrium outcomes of our system to be optimal[7]. The implementation here would not necessarily be "full", as not every optimal outcome need be supported by an equilibrium. Full implementation would be the case where, in addition to the invisible hand property we have the reverse containment W(s) ⊂ E(s) as well, so that every optimum is also an equilibrium. That every optimum is supported by an equilibrium, on the other hand, is the familiar message of the "Second Theorem" of Economics, at least in the sense that for every optimum there is a way of re-allocating initial endowments in, for instance, a trade economy, so that the optimum in

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[5] Just as such a state would be an equilibrium of a *trivial* Rechtsstaat whose coalitions with the right to approve always approve of every alteration, i.e. whose benefit function is non-negative (β^C(s, t) = 0) at every alteration (s, t) for every coalition C belong to any code family C ∈ ?(s, t). According to a certain axiom (B1) which we are about to adopt for a benefit function, this will trivially be the case when ? is the constant function with ?(s, t) = {{∅}} at every alteration (s, t).

[6] We regard an alteration (s, t) as *feasible* iff α(s, t) is non-empty. Note that α(s, t) would be non-empty even if it owned just the empty coalition. However, we will agree, as expressed in the axiom A3 below, that it if it owns the empty coalition, then it actually owns every coalition.

[7] The usual definitions need to be stretched here, for we are not implementing via a solution of a normal form game. Instead the solution is the equilibrium correspondence just defined, and this is not a solution for any normal-form game that we have defined. The idea of implementation need not be constrained, however, to the usual definitions we have been employing so far. For instance, we might very well want to ask whether we can implement certain desirable allocations via bargaining. (For this, see Sertel and Yilmaz (xxx).)

question will be the allocation at some (Walrasian) equilibrium of the "new" economy determined by the re-allocation of initial endowments with preferences intact. Although the containment W(s) ⊂ E(s) would be quite strong, stronger than the "Second Theorem" dares to claim, a weaker property is nevertheless of interest from the viewpoint of our system somehow preserving the best public interest, i.e. optimality. For although some optimal states need not be equilibria, i.e. rest points, it might be that the only states to which they ever get altered are again optimal states. This is the case where every alteration of an optimal state that can in fact be enacted under the given ability, benefit and code is again to some optimal state, never to a sub-optimal state, and the *set* W(s) of optima is thus at rest. In this case we would say that the Rechtsstaat s *preserves the best public interest*. We would regard a theorem providing conditions for s to have such a property as a relative of the Second Theorem, calling it an "*optimality preservation theorem*".

For certain classes of Rechtsstaat we have a simple characterization theorem, giving necessary and sufficient conditions for the **strong invisible hand property to hold. After this characterization theorem for a Rechtsstaat with transferable (cardinal) benefit, we also record an impossibility for every Rechtsstaat with a certain type of ordinal benefit, as well as a strong invisible hand theorem for every Rechtsstaat with either of** two other focal types of ordinal benefit. All of this in Section 3.

Section 4 studies the prospects of designing a Rechtsstaat - i.e. its code in relation to its benefit, given its ability - so that it preserves the best public interest.

All of the above notions and theory are then applied in Section 5 to two central areas. One of these is the classical world of a private ownership economy. The other is that of covering games with its own application areas, notably coalition formation, in particular network formation, and the theory of workers' enterprises as salient examples, all studied in the presence of membership rights. These applications occupy us for the rest of the paper until our final Section 6 of Closing Remarks, where we touch upon sundry matters remaining.

In order to be able to develop our theory, in the next section we now study some alternative structures for a Rechtsstaat, depending on which axioms the ability, the benefit and the code satisfy.

2. The Structure of a Rechtsstaat

The structure of a Rechtsstaat depends on the forms of its constituent elements, the ability, the benefit and the code of rights. We examine each of these in turn.

2.1. The Ability

Certain axioms for an ability α: $S \times S \rightarrow \underline{\mathbf{A}}$ are natural. We adopt the following four right away:

(A1) for every $s \in S$ there exists $t \in S$ with some non-empty coalition $B \in \alpha(s, t)$;

(A2) for every $s \in S$, $\emptyset \in \alpha(s, s)$;

(A3) for all $s, t \in S$, if $B \in \alpha(s, t)$, then $[B, A] \subset \alpha(s, t)$;

(A4) for all $r, s, t \in S$, if $B \in \alpha(r, s)$ and $C \in \alpha(s, t)$, then $B \cup C \in \alpha(r, t)$.

Our first axiom, A1, here ensures that no state is unalterable, or that every state s is alterable in some way to some state by some "able" coalition in $\alpha(s, t)$. Unalterable states would be rest points regardless of the benefit function with which the ability of a Rechtsstaat can be combined, and it would then be futile to seek designs imparting invisible hand properties to the Rechtsstaat if such states existed, since the benefit could render such a state sub-optimal just as well as it could render it optimal. We therefore assume that every state of our world is alterable ("in at least some aspect"), simply disregarding states that are unalterable.

As to A2, it expresses the idea that the "idle alterations" (s, s), preserving any status quo $s \in S$, are trivially feasible. Combined with the next axiom, A3, it tells us that *even* the empty coalition is able to bring about the idle alterations. For A3 ensures that ability is inherited by supersets: if a coalition is able to alter a state s to a state t, then every coalition containing it, in particular the "grand coalition" A, is also able to do so.

Finally, A4 arises from a certain "triangular relation"[8] in the logic of ability: if B is able to alter r to s and C is able to alter s to t, then $B \cup C$ can alter r to t (e.g., by B first altering r to s and then C altering s to t).

As a consequence for our notion of optimality, we wish to stress right away that, for a Rechtsstaat $s = \langle \alpha, \beta, \gamma \rangle$, by A3, we have $A \in \alpha(s, t)$ iff the alteration (s, t) is at all *feasible*, i.e. there is some coalition which is able to alter s to t. Thus, a state s is *optimal* for a coalition C iff there is no state t with $A \in \alpha(s, t)$ such that $\beta^C(s, t) > 0$. In particular, W(s) consists of the states s for which there is no $t \in S$ with $A \in \alpha(s, t)$ and $\beta^A(s, t) > 0$.

2.2. The Benefit

The benefit $\beta: S \times S \rightarrow \Re^{\mathbf{A}}$ can be cardinal or just ordinal. In any case we agree that it should satisfy the following axioms:

(B1) for all $s, t \in S$, $\beta^{\emptyset}(s, t) = 0$;

(B2) for all $s \in S$ and $C \in \mathbf{A}$, $\beta^C(s, s) = 0$;

8 As in the "triangle inequality" for reckoning distances.

(B3) for all s, t ∈ S and C ∈ **A**, β^{C}(s, t) is positive, zero or negative, resp., accordingly as $\beta^{\{i\}}$ (s, t) is so for every i ∈ C.

Thus, by B1, the empty coalition always reaps nil benefit. By B2, the idle alterations impart nil benefits to all coalitions. Finally, B3 expresses an unanimity condition in the logic of benefit: if all members of a coalition receive positive (resp., zero, negative) benefit from an alteration, then the coalition does so too.

The ordinal case is often that of a *trivalent* benefit where we pay attention only to the sign of β, and in this case, with no loss of generality, we restrict the range of β to the tripleton set ***3*** = {-1, 0, 1}:

$$\beta(S \times S) \subset \mathit{3}^{\mathbf{A}}.$$

Three types of trivalent ordinal benefit are particularly interesting to focus upon, namely β?, β^ and β?^. Of these, for β? we agree on the property that

(B?) whenever B = C ∪ D is the union of two disjoint coalitions C and D of which at least one is non-empty, we have

β?^{B} = Max {β?^{C}, β?^{D}}.

(See the "Multiplication Table", below.) Thus, with a benefit reckoned in this manner, a coalition is willing (whether or not able) to have an alteration iff this positively benefits some member (or singleton sub-coalition) regardless of how it affects other members of the coalition, and a coalition will approve an alteration (whether or not it has the right to do so) as long as some sub-coalition would do so. As the empty coalition always approves, reckoning with this benefit, admitting the empty coalition as a code coalition and sufficing with the approval of a single code coalition will be tantamount to admitting a version of the "law of the jungle" as a Rechtsstaat. For no matter what form of ability is operant, if a state can be altered by some willing coalition (with some member benefiting from an alteration), then it will be altered, approval being granted anyway. While it may be premature at this early stage to dwell upon this matter, it would seem rather apparent that utilizing this particular benefit in our design could easily jeopardize our architecture from the very foundation.

In contrast, we define the trivalent benefit β^ through the property that

(B^) whenever B = C ∪ D is the union of two disjoint coalitions C and D of which at least one is non-empty, we have

β^^{B} = Min {β^^{C}, β^^{D}}.

(Again, see the "Multiplication Table", below.) Thus, with a benefit reckoned in this manner, at any (s, t) ∈ S × S, a coalition B is willing (whether or not able) to have the alteration (s, t) if and only if every non-empty sub-coalition is willing, and B would approve the alteration (whether or not it has the right to interfere in the matter) if and only if every sub-coalition would do so. A state is optimal for a coalition iff, reckoning with the benefit β^, no unanimously improved alteration from that state (positively benefiting all members of the coalition) is possible. Thus, reckoning with the benefit β^, our notion of optimality of a state for a coalition (that no alteration from that state gives positive benefit to the coalition) coincides with the classical notion of weak Pareto optimality for that coalition.

Instead of β? or β^, we can imagine a trivalent benefit of a third and hybrid functional form. We define the benefit β? in this manner through the property

(B?) whenever B = C ∪ D is the union of two disjoint coalitions C and D of which at least one is non-empty, we have

$$\beta?^{B} = \begin{cases} \text{Max } \{\beta?^{C}, \beta?^{D}\} \text{ if } \beta?^{C} \geq 0 \leq \beta?^{D} \\ \text{Min } \{\beta?^{C}, \beta?^{D}\} \text{ otherwise.} \end{cases}$$

(Once again, see the "Multiplication Table", below.) Now reckoning with the benefit β?, a state is optimal for a coalition iff no Pareto-improving alteration from that state (positively benefiting some member without hurting any other members of the coalition) is possible. Thus, reckoning with the benefit β?, our notion of optimality of a state for a coalition (that no alteration from that state gives positive benefit to the coalition) coincides with the classical Paretian notion of optimality. And revisiting the conditions of willingness and approval, we see that under β? a coalition is willing to have an alteration if and only if it gives positive benefit to some member without giving negative benefit to any other member, while an alteration is approved by a coalition if and only if every (e.g., singleton) sub-coalition approves it.

At this point it may be useful to summarize the behavior of the three canonical trivalent functional forms of ordinal benefit, β^, β? and β?, by means of the following "multiplication tables":

β?^{D} \ β?^{C}	-1	0	1
1	1	1	1
0	0	0	1
-1	-1	0	1

β^^{D} \ β^^{C}	-1	0	1
1	-1	0	1
0	-1	0	0
-1	-1	-1	-1

β?^{D} \ β?^{C}	-1	0	1
1	-1	1	1
0	-1	0	1
-1	-1	-1	-1

Table 1. "Multiplication Table" for types b^, b? and b?ˇ of trivalent ordinal benefit (Shows values of b^B, b?B and b?ˇB for B = C È D Ì A with C Ç D = Æ.)

The case of a non-trivalent ordinal benefit admits the *derived* form where we have a function b: S → $\mathfrak{R}^A$ describing the benefit $b^C(s)$ of every state s to every coalition C ∈ **A**, and at every alteration (s, t) ∈ S × S and every C ∈ **A** the benefit for C of the alteration is derived as $\beta^C(s, t) = b^C(t) - b^C(s)$. For a derived benefit β, the following *inversion condition* clearly obtains:

(B4) at every (s, t) ∈ S × S and every C ∈ **A**, we have

$$\beta^C(s, t) = -\beta^C(t, s).$$

When for a derived benefit we furthermore have the *additive* form b^C = ? {b^i i ∈ C} with A finite, we say that the benefit is *(cardinal and) transferable*. We denote a (cardinal) transferable benefit by $\underline{\beta}$. Such a benefit clearly obeys the additive form

$\underline{\beta}^C(s, t)$ = ? {$\underline{\beta}^{\{i\}}(s, t)$ i ∈ C} = ? { $b^i(t)$ - $b^i(s)$ i ∈ C}

for benefits from alterations (s, t) ∈ S × S for every C ∈ **A**,

2.3. The Code of Rights

For a code ?: S × S → $\underline{\mathbf{A}}$ we adopt the following axioms:

(C1) for every s ∈ S, ∅ ∈ ? (s, s);

(C2) for all r, s, t ∈ S, if B ∈ ? (r, s) and C ∈ ? (s, t), then there exists D ⊂ B ∪ C such that D ∈ ? (r, t).

To briefly discuss our last couple of axioms, it is clear that together with the axiom B1 for benefits, C1 renders the idle alterations all approved by at least the empty coalition, as the empty coalition is given the right to approve or disapprove every such alteration (or preservation of status quo) and, by B1, always in fact approves every alteration. C2 is an axiom of consistency that the code designer has to abide by. We may require the approval of all or simply at least one of the code coalitions at an alteration in order for that alteration to be enactable by some willing able coalition. In any case, if we give a coalition B the right to approve the alteration of a state r to a state s and we also give a coalition C the right to approve the alteration of s to a state t, we simply cannot have denied all the sub-coalitions D ⊂ B ∪ C the same right regarding the alteration of r directly to t, for at least the coalition B ∪ C itself has already been given the said right. On the other hand, if for every alteration we require the approval of all code coalitions in order for the alteration to be enacted, then the same "triangular" discipline should govern in the matter at hand: given that the approval of coalition B is requisite for an alteration (r, s) and the approval of coalition C is requisite for an alteration (s, t), the approval of neither may be requisite for the direct alteration (r, t), but certainly one would require the approval of the empty coalition for this, as this is granted anyway by the axiom (B1) governing benefits.

3. A Restriction (NONE) on Externalities, and Two Strong Invisible Hand Theorems

We now introduce a condition limiting "negative" externalities and build upon it to obtain a theorem characterizing the strong invisible hand property for a Rechtsstaat with cardinal benefit $\underline{\beta}$. Then we regard the three focal cases where the benefit is ordinal, and we show that the strong invisible hand property fails when β has the functional form of β? but always obtains when β has a functional form consistent with βˆ or one consistent with β?̂.

Given any subspace $Q \subset S \times S$, we say that a Rechtsstaat[9] $s = <\alpha, \beta, \gamma>$ obeys ***No Overwhelming Negative Externalities (NONE)*** on Q iff, at every $(s, t) \in Q$, whenever $\beta^C(s, t) > 0$ for some $C \in$? (s, t) and $\beta^B(s, t) < 0$ for the complementary coalition $B = A \setminus C$, we have $|\beta^B(s, t)| = \beta^C(s, t)$. Thus, NONE holds on $Q \subset S \times S$ iff every negative benefit accruing to the complementary coalition of a code coalition positively benefiting from an alteration in Q is weakly dominated in absolute value by the benefit of the code coalition. Note that this says nothing about the case where the complementary coalition of the code coalition is indifferent to the alteration in question (the case of no externality here) or reaps a positive benefit from it, in other words exhibiting a "positive" externality. Nor does it say anything about the case where the code coalition fails to benefit positively from the alteration in question. Nor does it concern alterations beyond the limits indicated by the domain Q claimed for the absence of overwhelming negative externalities. A particularly interesting subspace of $S \times S$ in this context will be the set defined, for any Rechtsstaat $s = <\alpha, \underline{\beta}, \gamma>$, as

$$A(s) = \{(t, s) \in S \times E(s) \mid A \in \alpha(s, t)\}.$$

Thus, $(t, s) \in A(s)$ iff s is an equilibrium of s and can be altered to t by some coalition (and, hence, by the grand coalition A).

CHARACTERIZATION THEOREM (Strong Invisible Hand): A Rechtsstaat $s = <\alpha, \underline{\beta}, \gamma>$ (with transferable benefit) has the strong invisible hand property, i.e. $E(s) \subset W(s)$, if and only if it obeys NONE on A(s).

Proof: Let $s = <\alpha, \underline{\beta}, \gamma>$ be a Rechtsstaat with cardinal benefit.

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[9] Actually, it is clearly just the ordered pair $<\beta, \gamma>$, rather than the entire Rechtsstaat $s = <\alpha, \beta, \gamma>$, which obeys or disobeys NONE on Q.

(*ad* "if") To prove the sufficiency of NONE on A(s) for s to possess the strong invisible hand property, we take any s $\in$ E(s), we suppose that s $\notin$ W(s) and show that this negates NONE on A(s). If s $\notin$ W(s), then there exists t $\in$ S with A $\in \alpha$(s, t), so that (t, s) $\in$ A(s) and $\underline{\beta}^A$(s, t) > 0, i.e. A is able and willing to alter s to t and, by the condition (B) above for cardinal benefits, we have $\underline{\beta}^A(t, s) = -\underline{\beta}^A(s, t) < 0$. Since s $\in$ E(s), however, there exists C $\in$? (s, t) such that $\underline{\beta}^C(s, t) < 0$, hence $\underline{\beta}^C(t, s) = - \underline{\beta}^C(s, t) > 0$ by (B) again, whereby also C $\neq$ A, and so the complement B = A \ C $\neq \emptyset$. Again by (B), we now have $\underline{\beta}^B(t, s) + \underline{\beta}^C(t, s) = \underline{\beta}^A(t, s) < 0$, while $\underline{\beta}^C(t, s) > 0$, so we must have, not only $\underline{\beta}^B(t, s) < 0$, but also $|\underline{\beta}^B(t, s)| > \underline{\beta}^C(t, s)$, contradicting NONE on A(s). This proves "if".

(*ad* "only if") To prove the necessity of NONE on A(s) for s to possess the strong invisible hand property, we now suppose that NONE fails on A(s) and show that this contradicts that E(s) $\subset$ W(s). To this end, suppose that NONE fails on A(s), so that there exists (t, s) $\in$ A(s) with $\underline{\beta}^C(t, s) > 0$ for some coalition C $\in$? (s, t), while $\underline{\beta}^B(t, s) < 0$ and $|\beta^B(t, s)| > \beta^C(t, s)$ for the complement B = A \ C. Then $\beta^A(t, s) = \beta^B(t, s) + \beta^C(t, s) < 0$, so s $\notin$ W(s), although s $\in$ E(s) since (t, s) $\in$ A(s) $\subset$ S $\times$ E(s). This contradicts that E(s) $\subset$ W(s), proving "only if" and completing the proof of our theorem. **?**

Our characterization theorem above tells us that one can design a code of rights for a Rechtsstaat s with transferable benefit so as to give it the strong invisible hand property just so long as the condition NONE can be satisfied on A(s). Clearly, this is a matter of striking a particular harmony between the benefit and the code of rights, and this with regard to possible alterations of the equilibria, themselves involving all three elements of the Rechtsstaat, namely the ability, the benefit and the code.

But what of the cases where the Rechtsstaat s has only ordinal benefit? Concentrating on our three focal cases of trivalent ordinal benefits, here we have a very clear dichotomy. For the case where β has the form of β? there is no hope of guaranteeing even that E (s) $\subset$ W(s), let alone E(s) $\subset$ W(s), even with NONE on the entire space S $\times$ S of possible alterations[10]. For imagine the case of a Rechtsstaat s where there are at least two agents. Now it is quite possible, for some state s$\in$ S, that $\beta?^A(s, t) > 0$ for some t $\in$ S, meaning that $\beta?^{\{i\}}(s, t) > 0$ for some i $\in$ A, while possibly $\beta?^{\{j\}}(s, t) < 0$ for some {{j}}= ? (s, t). In such a case, even if there are many agents in A \ {i}, all of whom receive negative benefits from the alteration of s to t, even if s satisfies NONE on S $\times$ S, s would nevertheless be an equilibrium with s $\in$ E (s) $\subset$ E(s), while it would certainly fail to be optimal according to β?: s $\notin$ W(s).

Working with either of the other focal cases, βˆ and β?, of ordinal benefit which we have defined, however, our prospects as designer are significantly better. For both of these latter two types of benefit, we have the strong invisible hand property satisfied

[10] And this can be regarded as an *impossibility* result discouraging any hopes for guaranteeing an invisible hand in the case of the benefit β?.

directly, and all this now with no need at all to resort to a condition, such as NONE, disciplining externalities.

THEOREM (Strong Invisible Hand): Let s be a Rechtsstaat with trivalent ordinal benefit. If the benefit has the form of either $\beta\hat{}$ or $\beta\Upsilon$, then s has the strong invisible hand property.

Proof: (*ad* $\beta\hat{}$): Assume that β has the form of $\beta\hat{}$, and take any $s \in S$. We show that if $s \notin W(s)$, then $s \notin E(s)$. To that end, assume that s is not optimal (for A), i.e. that $\beta\hat{}^{A}(s, t) > 0$ for some $t \in S$. By the form of $\beta\hat{}$, we must therefore have $\beta\hat{}^{C}(s, t) > 0$ for every non-empty $C \subset A$, in particular $\beta\hat{}^{C}(s, t) \geq 0$ for every $C \in$? (s, t), so s cannot be an equilibrium of s : $s \notin E(s)$, as to be shown.

(*ad* $\beta\Upsilon$): The proof for this case is similar to the above. Now assume that β has the form of $\beta\Upsilon$ and take any $s \in S$. Again we show that if $s \notin W(s)$, then $s \notin E(s)$. To that end, assume once again that s is not optimal (for A), i.e. that now $\beta\Upsilon^{A}(s, t) > 0$ for some $t \in S$. By the form of $\beta\Upsilon$, we must then have $\beta\Upsilon^{C}(s, t) \geq 0$ for every $C \subset A$, in particular for every $C \in$? (s, t), so again s cannot be an equilibrium of s : $s \notin E(s)$, as to be shown. **?**

4. A Stronger Restriction (SNONE) on Externalities, and Preservation of the Best Public Interest

In this section we glance at the first principles determining the prospects for designing a Rechtsstaat s so that every optimal state $s \in W(s)$ is guarded against alterations (s, t) with negative social benefit ($\beta^{A}(s, t) < 0$) by the disapproval of every code coalition $C \in$? (s, t). We identify a sufficient condition, SNONE, which is a strengthened version of NONE, whose satisfaction on a subspace $B(s) \subset W(s) \times W'(s)$, where $W'(s) = S \setminus W(s)$, now guarantees the preservation of the best public interest.

To state our theorem, first we compose a couple of definitions. Given a Rechtsstaat $s = <\alpha, \beta, \gamma>$, we say that s *strongly preserves the best public interest* iff, at every feasible alteration (s, t) where s is optimal and t is not, every code coalition $C \in$? (s, t) disapproves the alteration. Finally, we say that s obeys the *strong condition of NONE*, abbreviated ***SNONE*** on a subspace $Q \subset S \times S$ iff $|\beta^{B}(s, t)| = \beta^{C}(s, t)$ whenever $\beta^{C}(s, t) \geq 0$ and $\beta^{B}(s, t) < 0$ for a code coalition $C \in$? (t, s) of any alteration $(s, t) \in Q$ and its complement $B = A \setminus C$. The most interesting space on which to assume SNONE is the set $B(s) = \{(s, t) \in W(s) \times W'(s) | A \in \alpha(s, t)\}$ of all feasible alterations of (from) optimal to sub-optimal states.

THEOREM (Optimality Preservation): A Rechtsstaat $s = <\alpha, \underline{\beta}, \gamma>$ (with transferable benefit) strongly preserves the best public interest iff SNONE holds on B(s).

Proof: (*ad* "if") Let $s = <\alpha, \underline{\beta}, \gamma>$ be a Rechtsstaat (with cardinal benefit) and take any s $\in$ W(s). Suppose that t $\in$ W'(s) with some coalition D $\in$ α(s, t). Now (s, t) $\in$ B(s) and $\underline{\beta}^A(s, t) < 0$. Suppose that $\underline{\beta}^C(s, t) \geq 0$ for some C $\in$? (s, t). Regarding the complement B = A \ C, we cannot have $\underline{\beta}^B(s, t) \geq 0$, for then we would have $\underline{\beta}^A(s, t) = \underline{\beta}^B(s, t) + \underline{\beta}^C(s, t) \geq 0$, contrary to our supposition that $\underline{\beta}^A(s, t) < 0$. Thus, we must have $\underline{\beta}^B(s, t) < 0$. But then, assuming SNONE on B(s) we have $|\beta^B(s, t)| = \beta^C(s, t)$, implying that $\underline{\beta}^A(s, t) = \underline{\beta}^B(s, t) + \underline{\beta}^C(s, t) \geq 0$, again a contradiction. We conclude that with SNONE on B(s) we cannot have any code coalition C $\in$? (s, t) approving the alteration (s, t), i.e. that every code coalition disapproves (s, t). This shows that s strongly preserves the best public interest so long as SNONE obtains on B(s), proving "if".

(*ad* "only if") If a Rechtsstaat $s = <\alpha, \underline{\beta}, \gamma>$ strongly preserves the best public interest, then $\underline{\beta}^B(s, t) \geq 0$ for no code coalition C $\in$? (s, t) of any feasible alteration from an optimal to a sub-optimal state, i.e. of any alteration in B(s), whereby SNONE on B(s) is trivially satisfied. This shows "only if" and completes the proof. **?**

5. A Brief Revisit of Economics

We have already announced that our invisible hand and our preservation theory parallel the first and the second theorems, respectively, of economics. Of course, there is little point in re-inventing either of these fundamental theorems, and that certainly was not the guiding motivation for constructing our theory above. Rather, we wanted to extend the basic idea in the design of a private ownership economy to try the limits of our prospects for obtaining publicly desirable outcomes in a social system where behavior is selfish, focusing on the code of rights to be adopted with this purpose in mind. Our next section should offer us some clearer light to illuminate our way in search for a good design of rights in the architecture of covers, networks, coalition structures, partnerships and their membership rights and associated institutions, such as membership markets. The present section is meant simply to briefly illustrate how some of our central theory in economics can be seen to carry little surprise value in view of some very basic facts which we have recorded concerning a Rechtsstaat.

Let us take a pure trade economy s with agent set A, where each agent starts with an initial endowment in the form of a positive real vector in some finite-dimensional commodity space of unambiguous goods (which are the more the better for each agent), and set our state space S to be the space of all possible allocations of these initial endowments among our agents in A leaving no agent with a negative amount of any good (thus, remaining within a generalized Edgeworth box). Now allow every coalition of agents to be able to alter any state (i.e., feasible allocation) s in S to any state t in S, so that the ability is the constant function $\alpha = \mathbf{A} = 2^A$ with (constant) value $\mathbf{A}$, but give the agents *private property rights*, i.e. in designing the code of rights allow every agent the right to block an alteration of a state if this involves that agent's receiving less of any good (whether or not this makes the agent happier, as when accompanied with a more than compensatory increase of the agent's receipts of other goods). Technically speaking, adopt the trivalent ordinal benefit β ˆ or β ?ˆ derived from individual utility functions which are increasing in the typical individual's own possession of goods and blind to those of others, as in the typical textbook case (with simply no externalities). Thus, a coalition disapproves of an alteration of an allocation if any member does so. Now we can apply our Strong Invisible Hand Theorem for β ˆ and β ?ˆ , and be assured that every equilibrium is optimal, i.e. $E(s) \subset W(s)$. Note that the set W(s)of optima under either of these benefit functions actually corresponds here to Edgeworth's solution, the core.

In fact, if we were to spell out the "nuts and bo lts" of such an exchange economy with private ownership, we would first note that every code coalition at any alteration is also an able coalition for the alteration, so as long as the maximal code coalition (of all agents whose ownership of some good decreases) benefits, i.e. every non-empty sub-coalition benefits, from an alteration, i.e. a re-allocation, this re-allocation will take place. Thus, no allocation is an equilibrium if it permits a "wedge" of allocations lying above the indifference levels of some agents and below those of none. The only allocations this leaves is those on the "contract curve", which is clearly just W(s), so all the equilibria are optimal, as our invisible hand property would promise. As to the *preservation of the best public interest*, we check that no movement from an allocation on the contract curve is approved by all agents, as there is always some singleton disapproving such a move, and so no movement away from an allocation in W(s) will receive the approval of all code coalitions. This is, in fact, a stronger property than just the preservation of the best public interest, as every optimal allocation here is guarded against all alterations by the disapproval of some code coalition, rendering the result that $W(s) \subset E(s)$. This sandwiches the optima of our Rechtsstaat of a private ownership exchange economy between the equilibria of the Rechtsstaat: $E(s) \subset W(s) \subset E(s)$, so the equilibria and the optima coincide: $E(s) = W(s)$, a rather strong relation between equilibrium and optimum.

Now the above scenario of a trade economy is Edgeworthian, in the sense that it permits every exchange (in fact, every re-allocation) of goods, but we can also turn it into a Walrasian story in order to obtain the classical First Theorem that every Walrasian equilibrium is Edgeworthian (i.e. in the core) simply by restricting the feasible re-allocations to obey, at some positive price vector, individual optimality subject to each agent's budget constraint while making excess demand vanish (while continuing to satisfy material balances, i.e. "remaining within the Edgeworth box"). Again define the ability a so as to render every coalition able to make any feasible alteration[11], and again adopt the same benefits as in the case above. It is precisely when the price is Walrasian that the individual demands are in such quantities that excess demand vanishes. Here we have simply made sure that the ability is restricted so that alterations can happen only along price hyperplanes passing through the initial allocation and to which all the agents' indifference curves are ("back-to-back") tangential at the new allocation. It is clear that

[11]This is in the new sense of "feasible", as just agreed on. Thus, every coalition, in particular some artificial singleton serving as the "market helmsman" or "auctioneer", or the coalition of agents involved in any re-allocation because it alters their individual possessions, is "physically" able to re-allocate the goods in a manner which satisfies individual optimality subject to budget constraints arising from a common price vector at the same time as it satisfies nil excess demand.

they can reach Walrasian allocations. Moreover, they will reach *only* Walrasian allocations, for the allocations reached in this manner will fail to be equilibria unless they allocate in a manner which is individually optimal for each agent subject to the budget constraint determined by the prices being used, since otherwise coalitions will be able and willing to trade further along the same price hyperplane with positive benefit to all (members of) code coalitions. These equilibria form a subset $E^*(s) \subset E(s)$ of those in the above scenario. For, in addition to being members of E(s), they furthermore have to be reachable from the initial allocation by price-taking exchange along a price hyperplane that brings the initial state (allocation) to the equilibria in question. By the same Invisible Hand Theorem used above, with benefits βˆ and β ?ˆ as derived from individual benefits coinciding with the usual individual utilities for own possession of goods, we see that these equilibria are optimal, again in a sense coincident with the core, repeating the First Theorem of Economics. But the set of (Walrasian) equilibria does not contain W(s), as $E(s) = W(s)$ but the inclusion $E^*(s) \subset E(s)$ is typically strict. Nevertheless, the preservation of the best public interest is a property that is inherited directly from the fact, already seen above in the "Edgeworthian scenario", that movements away from optima are always disapproved by some code coalition.

6. Cover Formation

The formation of covers of the agent space A is central to much of social life, including the economic and the political. Sometimes we restrict attention to the formation of partitions of A, i.e. to covers with non-overlapping non-empty coalitions, or to covers with coalitions of restricted cardinality. To recall some examples, the formation of uniform customs unions involves partitioning the space of agents where the "agents" are whole economies, although non-uniform customs unions may overlap and generally form covers. [12] As to an interesting case of covers with coalitions of restricted cardinality, we immediately note that of "networks", where we restrict the covering coalitions, called "links", to be doubleton or singleton (degenerate links where an agent is linked to itself). The central problem in the formation of covers is, we think, the discrepancy between individual and collective interests – nothing new in social matters. In this sense, the topic falls well within our general scope of interest, and so we shall try to treat it, at least to a suggestive extent, in our present context. We will investigate the prospects for designing a code of rights for a Rechtsstaat, where the states are certain covers of the agent space and the alterations of these obey certain feasibility requirements, in such a way that the

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[12] Following Sertel and Yildiz (1996), we call a customs union "uniform" iff, for any good or service to be imported, any two economies belonging to the union impose the same tariffs on imports from a non-union economy. While this is what is normally regarded as a customs union pure and simple in the usual theory, the distinction brought by the qualifier 'uniform' is significant. For as long as arbitrage is not free, relaxing uniformity in tariffs imposed on outsiders is not only feasible, but may in fact be better for the union. Thus, uniformity is generally a restriction standing in the way of optimal design for customs unions. Lifting this restriction also makes it possible for an economy to belong to more than one customs union, so that the collection of such unions forms a cover, but not necessarily a partition, of the set of members. Sertel and Yildiz (1996) give a subgame-perfect equilibrium of a game of customs union formation in which the customs unions overlap.

equilibrium under individual pursuit of own interest leads to social optima – the invisible hand property.

In studying the formation of covers and the prospects for invisible hand properties in designing the code of rights to govern in this context, what we take as our states are covers of the agent space A. Denoting K(A) for the space of all covers of A and $K_m(A)$ for the subspace of covers by coalitions of cardinality no greater than m (any positive integer), we typically take a state space of the form S = K(A) or $S = K_m(A)$. On the space K(A) × K(A) we define the *symmetric difference* ? in the usual manner: for any two covers s, t ∈ K(A) we set ? (s, t) = (s \ t) ∪ (t \ s). Thus, ?(s, t) is the collection of all the coalitions which are present in s but absent in t or absent in s but present in t. In the alteration (s, t) of a cover s to a cover t, ?(s, t) thus gives us the set of all the coalitions that were dissolved or formed. In particular, for instance, if two distinct coalitions in a cover s unite but cease to exist separately, then they both disappear and their union appears, so all three occur in and belong to the difference of s with the newly formed cover; if either of the two uniting coalitions in s continues to exist, on the other hand, then it is missing in the difference of s with the new cover.

Finally, on the space K(A) × K(A) we define the *"roster"* D(s, t) of any alteration (s, t), of a cover s to a cover t, through D(s, t) = ?(?(s, t)), i.e. D(s, t) = ∪{B| B ∈ ?(s, t)} = {i ∈ A| i ∈ B for some B∈ ? (s, t)}. Thus, D(s, t) is just the roster of the symmetric difference ?(s, t), the set of agents whose membership has changed in the alteration (s, t), that is the set of agents who belong to a coalition that has dissolved or formed in that alteration.

In the design of a Rechtsstaat where the state space consists of covers of the agent space, we focus on *membership rights* as the central matter of concern. In this regard, we consider certain *ethical* principles to guide our design of a code of rights in the presence of some typical forms of ability and benefit function.

To introduce the notions we want to employ in such designs, first we concentrate on the case of "elementary" alterations of a cover. We call an alteration (s, t) ∈ K(A) × K(A) *elementary* iff it consists of a single non-empty coalition joining or leaving an already existing non-empty coalition: ?(s, t) ⊂ {B, C, B ∪ C} for some non-empty B, C ∈ s. An elementary alteration (s, t) ∈ K(A) × K(A) is called *primitive* iff the same inclusions obtain for some singleton coalition C = {i). It is clear that every alteration of a finite cover of a set A can be written as a finite concatenation of elementary alterations and that one can arrive from any state s in S = K(A) to any other state t in S by chaining a finite number of suitable such elementary alterations. And in case A is finite, any cover of A can be altered to any other by concatenating a finite number of primitive alterations.[13]

[13] Concerning the willingness or approval of coalitions for alterations, however, we will have to decline any attempt to consider any sort of "far-sighted" or "forward" reckoning where agents or coalitions contemplate possibilities and consequences of further alterations when calculating their benefit from a given alteration. The business of modeling a "far-sighted" or "forward" calculus appears to involve some deep questions far beyond our scope or intellectual preparation here.

The ethical axioms that we have in mind presenting here were first expressed in the context of primitive alterations of covers (Sertel, 1982, 1987, 1991) and later (Fehr and Sertel, 1993; Sertel and Toros, 1999) also more generally for elementary alterations, albeit of partitions in any case.[14] Here we wish to introduce them both for primitive alterations, as in their original context, as well as in the extended sense, applying generally to elementary alterations. Our central question is, for elementary alterations of a cover of A, which coalitions should be consulted for their approval, and which should be given blocking rights? The question can be addressed from an ethical viewpoint as well as with the intent of imparting an invisible hand property to the Rechtsstaat at hand. These need not be consistent, however, as there may exist no solution to satisfy both approaches. For certain ethical philosophies, i.e. conjunctions of certain lists of ethically desirable axioms concerning whose permission we should set to be requisite for an alteration of (memberships in) covers, it may be that they cannot be guaranteed to impart an invisible hand property to the Rechtsstaat to be designed in accordance with the ethical principles of these philosophies. Now here we are not searching for impossibility results banning classes of ethical philosophy as incompatible with the invisible hand property. In contrast, we are in quest of at least one reasonable "ethical" philosophy that is compatible with imparting an invisible hand property to our Rechtsstaat of concern.

As our most primitive ethical principle, as our *Primitive Axiom **(P)***, let us propose that every agent joining or leaving a coalition in any primitive alteration of a cover should have the right to block the alteration. Accordingly, for any alteration $(s, t) \in K(A) \times K(A)$ with $?(s, t) \subset \{B, \{i\}, B \cup \{i\}\}$, we must have $\{i\} \in ?(s, t)$. Acceptance of this axiom seems rather natural if we wish to avoid treating agents as slaves or peons or objects and, instead, give them the most basic right of being able to reject joining or leaving a coalition in any cover of the agent space.

At the other extreme, what are the maximal coalitions that should have the right to interfere in an elementary alteration of a cover of A? If we are sufficiently generous in this matter - where it is easy for the designer to be generous but at the possible expense of those who are directly concerned, i.e. those who are to live in the Rechtsstaat we are designing – we could easily obtain a cheap invisible hand property by resort to the trivial solution declaring all coalitions as code coalitions: $?(s, t) = \mathbf{A}$ for all alterations (s, t). The challenge is, of course, to propose a *minimal* value for ? and still guarantee the invisible hand property in our design. Now a well-known way of thinking about social life is what is often called the "liberal" way of thinking, and this means, first and most, that only those who are in some sense "directly involved" should have the right to interfere in any alteration of socially relevant states, whatever the indirect consequences (through what economists call "externalities") may be for others. Here this would seem to mean that no coalitions owning agents who fail to belong to D(s, t), i.e. no coalitions intersecting the complement of this critical coalition, should be given the right to block an elementary alteration of a cover in K(A) with symmetric difference $?(s, t) \subset \{B, C, B \cup C\}$. What

[14] The covers in question here were actually partitions of a set of workers into "production coalitions", some of which were "workers' enterprises" and some "entrepreneurial firms". For details, please see our Historical Note at the end of the paper.

we should like to propose as the *Liberal Axiom **(L)*** is the dictum that tells us precisely this: for any such (elementary) alteration (s, t) of a cover s in K(A) to a cover t in K(A) with symmetric difference ?(s, t) ⊂ {B, C, B ∪ C}, the code of rights obeys ∪ ?(s, t) ⊂ D(s, t), i.e. D' ⊂ ?(? (s, t)), for every D' ∈ ?(s, t).

Now the conjunction of our Primitive Axiom with the Liberal Axiom already gives us an ethical philosophy: {i} ∈ ?(s, t) and ∪ ?(s, t) ⊂ B ∪ {i} for every primitive alteration (s, t) ∈ K(A) × K(A) with ?(s, t) = {B, {i}, B ∪ {i}}. But primitive alterations of covers are of two types, an agent joining a coalition (*entry*) or an agent leaving a coalition (*exit*), and now we want to refine our notions and propose some ethical principles guiding the design of membership rights for these two types of case.

Regarding entry, there are two extreme ethical principles, Free Entry and Approved Entry. *Free Primitive Entry **(FPE)*** is the case where, in every primitive alteration of a cover of A, any coalition of at least two[15] incumbents is denied the right to block the entry of an outsider ("newcomer"): B ∉ ?(s, t) for any (primitive) alteration (s, t) ∈ K(A) × K(A) with ?(s, t) ⊂ {B, {i}, B ∪ {i}}, t \ s = {B ∪ {i}}and #B = 2. More generally, *Free Entry **(FE)*** is the case where, in every elementary alteration of a cover of A where a coalition of at least two[16] incumbents is joined by a non-empty coalition, every subset of the former coalition is denied the right to block this union: B' ∉ ?(s, t) for any (elementary) alteration (s, t) ∈ K(A) × K(A) with ? (s, t) ⊂ {B, C, B ∪ C}, t \ s = {B ∪ C}, #B = 2, B' ⊂ B and C ? Ø. *Approved Primitive Entry **(APE)***, in contrast, is the case where, in every primitive alteration of a cover of A, every coalition of at least two[17] incumbents is given the right to block the entry of any outsider (" newcomer"): B ∈ ?(s, t) for any (elementary) alteration (s, t) ∈ K(A) × K(A) with ?(s, t) ⊂ {B, {i}, B ∪ {i}}, t \ s = {B ∪ {i}} and #B = 2. More generally, *Approved Entry **(AE)*** is the case where, in every elementary alteration of a cover of A, any coalition of at least two[18] incumbents is given the right to block the entry of any non-empty coalition of outsiders (" newcomers"): B ∈ ?(s, t) for any (elementary) alteration (s, t) ∈ K(A) × K(A) with ? (s, t) ⊂ {B, C, B ∪ C}, t \ s = {B ∪ C}, #B = 2 and C ? Ø.

Similarly, in the case of exit, there are again two extreme ethical principles, Free Exit and Approved Exit.[19] *Free Primitive Exit **(FPX)*** stipulates that, in every primitive

[15] We require this for coalitions *of at least two* incumbents in order not to contradict the Primitive Axiom **P**. For when two singleton coalitions join each other, it is arbitrary which of the two owns a member entering the other, so we have to accept both individuals as entering the singleton coalition of the other, hence, if P is to be operant, possessing the right to block the entry, i.e. union.

[16] Again, we require this for coalitions *of at least two* incumbents, in order not to contradict the Primitive Axiom **P**. For when a singleton coalition joins another non-empty coalition, it will have the right to block this union so long as **P** is operant.

[17] In this case we restrict the axiom to hold only for coalitions of at least two incumbents *so as not to imply* the Primitive Axiom **P**.

[18] Likewise, as explained in the previous footnote, here again we restrict the axiom to hold only for coalitions of at least two incumbents *so as not to imply* the Primitive Axiom **P**.

[19] In the treatment of exit we will again postulate conditions for a coalition of at least two incumbents without further notice or commentaire in footnotes, as the rationale is quite similar to that in the previous footnotes concerning this matter.

alteration of a cover of A, every subset of any coalition of at least two remaining incumbents is denied the right to block the exit of an incumbent ("deserter"): B' ∉ ?(s, t) for any (primitive) alteration (s, t) ∈ K(A) × K(A) with ? (s, t) ⊂ {B, {i}, B ∪ {i}}, s \ t = {B ∪ {i}}, #B = 2 and B' ⊂ B. *Free Exit* ***(FX)*** stipulates that, in every elementary alteration of a cover of A, every subset of any coalition of at least two remaining incumbents is denied the right to block the exit of any non-empty coalition of incumbents ("deserters"): B ∉ ?(s, t) for any (elementary) alteration (s, t) ∈ K(A) × K(A) with ?(s, t) ⊂ {B, C, B ∪ C}, s \ t = {B ∪ C}, #B = 2 , B' ⊂ B and C ? Ø. *Approved Primitive Exit* ***(APX)***, in contrast, stipulates that, in every primitive alteration of a cover of A, any coalition of at least two remaining incumbents is given the right to block the exit of any incumbent ("deserter"): B ∈ ?(s, t) for any (elementary) alteration (s, t) ∈ K(A) × K(A) with ?(s, t) ⊂ {B, {i}, B ∪ {i}}, s \ t = {B ∪ {i}} and #B = 2. *Approved Exit* ***(AX)*** stipulates that, in every elementary alteration of a cover of A, any coalition of at least two remaining incumbents is given the right to block the exit of any non-empty coalition of incumbents ("deserters"): B ∈ ?(s, t) for any (elementary) alteration (s, t) ∈ K(A) × K(A) with ⊂ ?(s, t) ⊂ {B, C, B ∪ C}, s \ t = {B ∪ C}, #B = 2 and C ? Ø.

At this point we are able to define the *Primitive Liberal Ethic* ***(PLE)*** as the conjunction **LE = P & L & APE & APX** of the Primitive and the Liberal Axiom with the axioms of Approved Primitive Entry and Approved Primitive Exit. Generally, we understand the *Liberal Ethic* ***(LE)*** as the conjunction **LE = P & L & AE & AX** of the Primitive and the Liberal Axiom with the axioms of Approved Entry and Approved Exit As a liberal philosophy, the Liberal Ethic is "full", in the sense of owning both the coalition of incumbents viewing the admission of a coalition of newcomers as well as the coalition of the remaining members when a coalition of members breaks away. Is this formula or its "primitive" version just right as an ethical philosophy? Of course, an ethical philosophy can be judged in its own domain of discourse, Ethics, and this author would do best to claim incompetence in this domain. But we can also regard it from the viewpoint of its prospects for imparting desirable properties of design, and this constitutes our focus here.

First we look into the business of forming networks, which is a case where S = $K_2(A)$, so that the covers of A are restricted to consist of "links", i.e. non-empty subsets of A with no more than two elements. Then we regard the theory of workers' enterprises.

6.1 Network Formation

There are many models of the important phenomenon of network formation, the relevance of which extends beyond economics into other areas of social life and interaction, as well as into the life sciences, physics and chemistry[20]. For a good source leading into the economically most relevant literature on networks and their formation, we refer the reader to the forum of the *Review of Economic Design*, where B. Dutta and M.O. Jackson edited a Symposium on the Formation of Groups and Networks (Dutta and Jackson, 2000).

[20] To lead the reader into the physical-chemical literature, O.Okay (2000) should serve as a good source.

In much of this literature, notably in the work of Jackson and Wolinsky (1996) and, the formation of networks can typically be construed as happening through concatenations of elementary alterations of "networks", i.e. covers $s \in S = K_2(A)$. Jackson and Wolinsky (1996) study, e.g. a "connections" model which we would interpret as a Rechtsstaat with an ability where all elementary alterations and only elementary alterations are feasible, each singleton consisting of either agent in a forming or dissolving link being able to effect the elementary alteration in question, where there is a certain cardinal transferable benefit and a code of (membership) rights subscribing to the ethic determined by the conjunction **P & L & AE & FX**, differing from the Liberal Ethic **LE** above in that exit here is free (**FX**) whereas it is approved (**AX**) in **LE**. Thus, the formation of a link between two distinct agents is subject to the approval of both, while either agent can sever such a link unilaterally. In this model of Jackson and Wolinsky (1996), the direct benefit of becoming linked to eachother is d-c for both (distinct) link-forming agents, where d is a positive gain and c a positive loss, while each agent benefits d^k indirectly from becoming linked indirectly via a shortest path of length k to another agent (with k-1 other agents in between on the chain of links connecting to it). (Thus, with $d < 1$, indirect links are less beneficial the greater is the length of the shortest path.) Here the cost of establishing or maintaining a link is borne by the pair of agents in the link, but others can benefit costlessly. Constraining the code by a liberal axiom will thus impart positive externalities so long as linking is good for at least one of the pair of agents directly linked. In severing a link, however, one of such a pair may benefit positively but less than the (absolute value of the) total loss of the rest of the agents affected by the severance (by losing costless but beneficial indirect connections). In the latter case of severance, we will thus observe *overwhelming negative externalities* so long as we stick to a liberal axiom. Now if only those coalitions containing both agents of a link are able to sever and we give up on the liberal axiom, by declaring the coalition of agents in the links forming a maximal connected component of the network as the code coalition for such alterations, at least for networks where the benefits are transferable (and "component-additive" and "component-balanced", in the terminology of Jackson and Wolinsky (1996), as they will necessarily be with the transferable benefit assumed in our modified version of the "connections model"), the connections model will have been rehabilitated to possess the invisible hand property, thanks to our characterization theorem. It can be checked to possess the invisible hand property for the Rechtsstaat we have just proposed as its counterpart in our present framework.

Otherwise, seen in the fashion of the literature so eloquently surveyed by Jackson (2000), there is an infamous conflict in link formation and severance between individual incentives and the public interest or optimality. So blatantly disharmonious can the design of the membership rights be in some cases that, let alone imparting an invisible hand property or a property of the preservation of the best public interest, it can even render equilibrium (called "stability" in this literature) *incompatible* with social optimum, so that the set of equilibria and the set of optima are disjoint, even when both are non-empty.[21]

[21] Currarini and Morelli (2000) present a way out of the conflict, building networks "sequentially", but theirs is a different avenue, bearing a parallel with Sertel and Yildiz (1995).

6.2 Covering Games and Shapley-like Value

The study of a Rechtsstaat whose state space consists of covers is related to that of what we have elsewhere [(Sertel, 1991), (Sertel and Yildiz, 1996)] called covering games. Given an agent space A as above, by a *covering game*[22] we mean any function g:K(A) ? $\Re^{\mathbf{A}}$, assigning a real number $g^C(s)$ at every cover s to every coalition C (whether or not C ∈ s). For our purposes below we are most interested in the values $g^A(s)$, which we abbreviate to g(s). The implicit understanding in studying these games is that agents or coalitions are able to alter a cover by "moving", typically by engaging in an elementary alteration, and the code is such that the uniting of two coalitions is subject to them both being willing, while every coalition has both the ability and the right to break off from any other coalition. Thus, the ability and a liberal code obeying **L**, **AE** and **FX** (without **P**) are given, and the game determines the benefit. In this context the payoffs will determine whether we have an equilibrium, i.e. a cover which no able coalition is willing to alter through an elementary alteration subject to the approval of a coalition which it is joining or subject to nothing if it is leaving a greater coalition. A result in the network formation literature (Jackson, 2000) heralds a positive property of paying the agents in a Shapley-like manner (based on average marginal contributions) proposed by Myerson (1977) and used by Aumann and Myerson (1988), as it guarantees the existence of certain equilibria ("pairwise stable" networks) when S = $K_2(A)$. This "value", which we would like to refer to as the *(Shapley-)Myerson value*, is parallel to a "value" which we would like to announce here.[23]

Whatever might be the value of this "value"[24] that we wish to bring attention to, its definition requires notions of "restrictions" of a cover. There are possibly many sensible such notions, but here we bring attention to two notions, s(B) and š(B), of the "restriction" of a cover s ∈ K(A), as determined by a coalition B ⊂ A. Given any s ∈ K(A) and any B ⊂ A, we write s(B) = {C ∈ s| C ⊂ B}, and by the *disbanded restriction* s(B) of s to B we mean the cover consisting of coalitions in s that are contained in B and the (disbanded) singleton coalitions of agents who do not belong to them: s(B) = s(B) ∪ {{i}|i ∈ A \ ∪s(B)}. Given the same, by the *banded restriction* š(B) of s to B we mean the cover consisting of coalitions in s that are contained in B and the ("banded") coalition of agents who do not belong to them: š(B) = s(B) ∪ {A \ ∪s(B)}. We note that, for any cover s of A and any coalition B ⊂ A, s(B) and š(B) are also covers of A. Now we define two values for covering games, one employing the disbanded and the other employing the banded restriction of a cover. The *disbanded value* g is the function which assigns to each agent i ∈ A a real number $g_i(s)$ at each cover s of A, as determined by

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[22] We also called these "coalition-structural games", although they differed from games (studied, e.g., by Aumann and Drèze,) with similar names in that our coalition structures need not be partitions and the payoffs for games G to individuals or coalitions are not generally "orthogonal", i.e. the payoff to a coalition depends generally on the entire cover, not just the coalition.

[23] This value had been the subject of several homework and examination questions in courses (Mathematical Economics, and Microeconomic Theory) given at Bogaziçi Üniversitesi in the mid-1980s.

[24] And we are able to claim none just here.

$$g_i(s) = ?\ \{q(B, i)[g(s(B \cup \{i\}) - g(s(B))] \mid B \subset A \setminus \{i\}\},$$

where

$$q(B, i) = [(\#B)!\ (\#A - \#B - 1)!] / (\#A)!$$

is the classical coefficient corresponding to the “probability” of (i’s) encountering B, as in Shapley value or (the Shapley-like) Myerson value which we will see very shortly. And the *banded value* g is the function assigning to each agent $i \in A$ the real number $g_i(s)$ at each cover s of A, as determined by

$$g_i(s) = ?\ \{q(B, i)[g(\check{s}(B \cup \{i\}) - g(\check{s}(B))] \mid B \subset A \setminus \{i\}\},$$

where q(B, i) is the same as above.

The main idea behind these notions reflects the type of basis underlying Shapley value: in each case we are paying agents their average marginal contributions to the total payoff. But the contributions are reckoned differently in the disbanded calculus from that in the banded one. In the case of g, the disbanded value, for each parameter coalition B, we pretend that all the agents outside of $\cup s(B) \subset B$ are “disbanded”, meaning that they appear in our cover s(B) only as singletons. In contrast, the banded value g evaluates the covers š where these agents are all banded as a single coalition.

When the state space S is restricted to $K_2(A)$, the case of the so-called “networks” covering A, we encounter a parallel with a value proposed by Myerson (1977) and utilized by Aumann and Myerson (1988). In their case we are given a function f, regarded as a game or a “valuation” (Matthews, 2000), which assigns a real number to every cover $t \in K_2(B)$ of each subset B of A, whereas here we have only a function g, as a covering game, assigning real numbers to covers $s \in K_2(A)$. By means of our restrictions (the disbanded and the banded) above, we actually obtain an extension of g to assign real numbers to covers of subsets $\cup s(B) \subset B$ of A, however, as the disbanded and the banded restrictions of covers of A are again covers of A and, so, g has a value to ascribe to each of them. The Myerson (1977) value assigns to each agent a payoff which is the average marginal contribution of that agent to the value of a typical restriction of a network cover of A to a subset B, i.e. to the value assigned by f to a parametric $B \subset A$. [See (Matthews, 2000).] The Myerson (1977) value guarantees the existence of equilibrium (called a “pairwise stable network”) under a certain understanding of ability and code. [See (Matthews, 2000).] It might be of interest to investigate the fruits of reckoning with the disbanded or the banded value for related purposes.

6.3 Workers’ Enterprises

The theory of labor-managed "firms" [25](Ward, 1958; Domar, 1967; Vanek, 1970?) suffered several problems, basically due to the absence of a sensible notion of membership rights underlying the theory intended to treat the economics of such organizations. As a result, these "firms" experienced perversities, at least in the theory intended to treat them.[26] The present author proposed a rehabilitation by paying attention to membership and ownership rights, defining a workers' enterprise as a firm (production coalition) in which one is a worker if and only if one is a partner. These firms belong to their partners, as in any firm under normal commercial law, but membership (i.e. partnership) is restricted by charter to be equivalent to being a worker in the firm. As the partners run the operation, the (worker-) partners now have a natural authority over management, hence the firm is labor-managed, simply because it is partner-managed[27] as any firm is, but here the partners are the workers.

We envision a workers' enterprise, i.e. a firm whose workers coincide with its partners and are hence worker-partners, as a coalition of agents (workers) in a world in which there are possibly other firms, some of which are entrepreneurial, where the workers are paid a wage instead of the partnership income which they would receive as members of a workers' enterprise. In such a context a typical worker, constituting a singleton coalition in an unorganized ("competitive") market of workers employed in the entrepreneurial sector, is willing to pay a certain entrance fee, a *demand price* for membership, to enter a typical workers' enterprise. This is the excess of the monetary value of his welfare as a worker–partner over and above that as a simple worker in the entrepreneurial sector. On the other hand, for any entrance fee above some minimum, the incumbents in a workers' enterprise which he may be contemplating to join are willing to admit him as a worker-partner, increasing the work force but also the roster of partners to share the worker-partners' total income, i.e. value added by labor. The sum total of the individual contributions that they require in order to admit the applicant worker-partner is the *supply price* of the worker-partnership deed that the applicant needs to purchase for entry. Thus, we have a cardinal transferable benefit, measured in units of money, and we are speaking of primitive alterations of the cover of the set of workers (agents) and are postulating approved primitive entry, **APE**, conjoined with the primitive axiom, **P**. Entry occurs so long as there are workers willing to enter at a demand price no less than the supply price of the worker-partnership deed.

As to the exit of a worker-partner from a workers' enterprise, since the agent is a partner, again our primitive axiom, **P**, requires this agent's approval. Admitting also the axiom, **APX**, of approved primitive exit, and constraining all the membership rights by the liberal axiom, **L**, we end up in a particular world of membership rights according to the liberal ethic, regulating the flux of worker-partners in and out of workers' enterprises.

[25] We refer the reader to Kleindorfer and Sertel (1995) for the reason why we utilize quotation marks in uttering the word 'firms' in the Ward-Domar-Vanek context.
[26] It is not at all clear that the real-world approximations to the workers' enterprises which we are about to refer to actually exhibited these perversities. In fact, Holler (1999) and Dow (2001) will testify to a literature disclaiming such perversities in the real world of workers' enterprise as we see, e.g. in Mondragon.
[27] Not necessarily directly, but through an appointed management - appointed, as usual, by the general assembly of partners.

Adopting the understanding that every agent contemplating an entry or an exit is able to effect such an alteration and that the set of incumbent agents is also able, as a coalition, to do the same, we end up having defined a certain Rechtsstaat, given that the earnings of a firm are divided among its partners, say equally in a model of identical agents. It turns out that, at least with identical agents, the equilibria of a Rechtsstaat so defined duplicate those of the entrepreneurial counterpart where the workers are hired by entrepreneurs in reach of the same technology but guided by the profit motive, so that both types of firm are the same in employment, hence production, but may differ in the distribution of welfare (since workers in a workers' enterprise share the quantity that would have gone to the entrepreneur as profit in the entrepreneurial counterpart of their firm).[28]

This means that an economy with workers' enterprises need not satisfy an invisible hand property, for the entrepreneurial counterpart which it duplicates in physical statistics can also fail to do so – at least when competition is imperfect – even for the agents employed in the firms, disregarding the consumers. After all, for the partners in a given industry, the best solutions are those of a cartel (producing the monopolistic quantity of output) whose spoils are divided in some manner compatible with the continued adhesion of the "signatories" to the deal.

At this point we would like to turn to two final matters circumscribing the topic at hand. First we would like to visit a classroom example of an industry of workers' enterprises where equilibrium may or may not exist and where equilibria may disagree with optimality even when they do exist. This example will serve to illustrate the fact that liberal ethic membership rights do not guarantee an invisible hand property or the preservation of the best public interest. Next, and finally, we would like to regard the idea of membership markets, much in the style of the literature on workers' enterprises. After all, when market economies satisfy well-known properties concerning competition and information they give reason to expect efficiency of equilibria. Thus, membership markets may offer a channel toward reconciling the conflict or at least the disharmony between equilibrium and efficiency which we often see in social life, not only in the paradigm of a prisoners' dilemma but also, e.g., in the network formation literature.

So, at this point we envisage a set $A = \{1, 2, 3, 4, 5\}$ of five workers, each with a unit of labor to expend in the production of a good to be produced only in an industry employing these workers. For simplicity, workers care only about income, measured in units of output, and work causes no disutility. The output of any production coalition, i.e. workers' enterprise, is simply the sum of labor inputs of its members. The inverse demand for the good is the affine function giving price as $p = a - bX$, where X is the sum of outputs of the firms in the industry and a and b are positive real numbers. Each firm to form in this industry operates with the same technology, producing the good subject to the same linear cost with (constant) marginal cost c, a positive real number less than a. We normalize the units of output and price so that $b = 1$ and $a - c = 1$ as well, whereby $p = a - X$ and the Cournot output of each oligopolist with m firms ($m \in A$) in the industry becomes $x(m) = 1/(m+1)$, rendering total output $X(m) = m/(m+1)$ and hence each

[28] For all this, the reader may consult the survey by Kleindorfer and Sertel (1995), relevant parts of Holler (?), Dow (2001) or Sertel (1982, 1987, 1991, 199?) and Fehr and Sertel ( ) and Sertel and Toros ( ).h

oligopolist firm's profit $v(m) = 1/(m+1)^2$. The profit of each firm is shared equally among its members. In the case of the *discrete* partition $\{\{i\}|i \in A\}$of A, where the industry consists of five singleton workers' enterprises, the workers each receive 1/36, while the workers' total income as well as individual income is maximized at the *indiscrete* partition {A} with the grand coalition A serving as a monopolist, where each worker receives 1/20.[29] We now record the payoffs at other partitions in order to see which of these forms an equilibrium and which form optima, depending on the design of the Rechtsstaat to govern in this context. First of all, we are speaking of a Rechtsstaat whose state space is S = P(A), the space of partitions of A. Let us first Let us agree that the primitive axiom **P** obtains. Let us first see the consequences of adopting the free primitive exit axiom, **FPX**. As any individual breaking off from the grand coalition secures a payoff of 1/9 instead of 1/20 (while the other four remain together as a duopoly, each of whose members receives 1/36), the socially optimal partition {A} is no equilibrium. But such a partition, with a singleton and a four-element coalition, is not an equilibrium either, as now any member of the four element firm can depart and increase his payoff from 1/36 to 1/16 as one of the two singleton firms in the triopoly now materializing, while the three workers forming the remaining three-member firm each now receive 1/48. Again, this partition fails to be an equilibrium, as one of the three members of the three-worker firm can also break off and increase his payoff to 1/25 while decreasing the payoffs of his remaining two ex-partners to 1/50. Similarly, the partition thus formed, with three singleton firms and a doubleton firm, also fails to be an equilibrium, as now the remaining doubleton coalition also dissolves into singletons, so that we have the discrete partition. It can be checked that no two of the five singleton firms in this discrete partition will be willing to unite, so the discrete partition, minimizing total oligopoly profit, is a *"primitive equilibrium"*, i.e. a rest point when only primitive alterations are considered. It can also be checked that no other partition has this property under **FPX**. Thus, as in certain cases of network formation, elementary equilibrium and efficiency are incompatible under the membership rights just examined.

Under **P** and approved primitive exit, **APX**, on the other hand, {A} is a primitive equilibrium, since every departing coalition hurts the coalition of remaining ex-partners; and, in fact, even if compensations were to be used as an instrument of persuasion, the loss of the remaining incumbents could not be compensated by any departing coalition and the departure still be profitable to the departing coalition after the compensation. Even here, however, there may be alterations under the liberal ethic that decrease total payoff. Consider the partition with a singleton and a four-member firm. When one of the four members of the big firm here departs and the partition with two singletons and a tripleton forms, the total payoff of the four workers in the original four-worker firm increase their total payoff from 1/36 + 1/36 + 1/36 +1/36 = 1/9 to 1/16 + 1/48 + 1/48 + 1/48 = 1/8, and so with compensations allowed under the liberal ethic, **LE**, this alteration will take place, hurting the original singleton whose payoff decreases from1/9 to 1/16, thus decreasing total oligopoly profit from 2/9 to 3/16.

[29] The terms of the "discrete" and the "indiscrete" partitions are copied from standard terminology in topology.

Before totally leaving this educative example, let us modify it slightly to see a case where cycles in partition-formation may exist. Introducing a fixed cost for each firm, one can check that when this cost is in the open interval (14/800, 23/800), under **P & AE & FPX**, allowing all elementary alterations we obtain a cycle. Starting from any partition with a doubleton and three singletons, we move to one with two doubletons and a singleton (as two singletons unite), and then the two doubletons unite[30] to form a partition with a four-worker firms and a singleton, but from the four-element firm one of the workers now defects as a singleton, bringing about a partition with two singletons and a three-element coalition, which in turn alters into the original partition where we had started, one with a doubleton and three singletons.

7. Closing Remarks

We have toured quite a terrain of issues having to do with the design of a Rechtsstaat, concentrating on the interation of ability, benefit and rights concerning alterations of the states of the world that may be of relevance to the agents and coalitions which they can form. After formal definitions we established a characterization theorem for the invisible hand property to hold in a Rechtsstaat with cardinal, transferable benefit, and then a sufficiency theorem for the same property when the benefits are ordinal. Next we established a sufficiency result for the preservation of the best public interest. Then we turned to some applications, first revisiting trade economies and then the world of coalition formation. In particular we considered the formation of covers, e.g. networks, then proposing a couple of values for covering games. Finally, we concentrated on membership rights, revisiting axioms of the liberal ethic and the world of workers' enterprises. We were sometimes formal and rigorous, but sometimes sufficed with simply a suggestive rhetoric. The main purpose was to formulate and communicate some basic notions, in particular the idea that ethical principles in the design of rights may be tested for their quality to impart properties, such as the invisible hand property connecting equilibrium and optimality in a Rechtsstaat.

Historical Note: The development of the ideas presented in this essay has stretched over roughly a quarter of a century. To a large extent it all started with my trying to understand the economics of what I later[31] called "*workers' enterprises*" (WEs), a term which I proposed in order to distinguish them from the problematic so-called labor-managed "firms"[32] (Ward, Domar, Vanek). The key to understanding and properly designing these enterprises as viable production coalitions turned out be *membership property rights* along with a market where membership in these coalitions could be negotiated. The idea of *membership markets* makes sense only when membership in coalitions is not free, i.e. coalitions are not subject to *free entry* and *free exit*. Instead, when the axiom of *approved entry* is operant, i.e. entry requires the approval of, not only

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[30] Note that this is an elementary but not a primitive alteration.
[31] E.g., in (Sertel, 1982).
[32] For an explanation of why it is impossible to regard these as *firms* – hence the quotation marks in the text – see, e.g., Kleindorfer and Sertel, 19xx.

the potential entrant, but also the incumbent members of a coalition, the adhesion of a new member may require the payment of possibly non-zero prices, either by the entrant to the incumbent members or by the latter to the former. Similarly, retiring a member from a coalition may require a non-zero indemnity to be paid either by the exiting member to the remaining members or by the latter to the former, as when exit requires the approval of all these parties concerned – the case where *approved exit* is operant. I remember when I first worked this out to my satisfaction, at least for workers' enterprises: it was toward the end of 1977 when the Technion in Haifa had so generously hosted me. It appeared later in a lecture which I gave at the First Conference of IAFES, the International Association for the Economics of Self-Management, in Cavdat (Dubrovnik), 1978, and was further developed and extended in several other works so as to deal with the discrete and the continuous case of the worker population, imperfections in markets, financial issues, etc. (Sertel, 1982; EEA, JCE, Fehr & Sertel, Sertel and Steinherr, Sertel and Toros).

The study of industry structure under various membership rights, as illustrated in the five-worker example above, was part of regular class-room material at Boğaziçi University in the 1980s, and ideas relating to covering games again come out of the same class-rooms in the early 1990s. I have benefited indispensably from classes of bright and enthusiastic students and many outstanding co-authors over the last quarter century who have also been my worker-partners in research.

The first formal formulation from which the definitions of Rechtsstaat, ability, benefit and code here have descended was constructed when I was graciously asked to give the ASSET (Associacion SudEuropea por la Economia Teorica) Lecture in Bologna, 1998, which is where I first brought the idea of invisible hand properties and NONE and SNONE out of the classroom into the public. I am grateful to my hosts at the University of Bologna for their kind invitation that made this possible.

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