Potential Maximization and Coalition Government Formation*

Rod Garratt[†], James E. Parco[‡], Cheng-Zhong Qin[†], and Amnon Rapoport[#]

[†]University of California, Santa Barbara
Department Economics, Santa Barbara, CA 93106

[‡]United States Air Force Academy
Department of Management, Colorado Springs, CO 80840

[#]University of Arizona
Department of Management and Policy, Tucson, AZ 85721

August 14, 2003

Corresponding author:

Rod Garratt
garratt@econon.ucsb.edu
Phone: (805) 893-2849
Fax: (805) 893-8830
Homepage: http://www.econ.ucsb.edu/~garratt/faculty/garratt.htm
Grand Coalition web site: http://www.grandcoalition.com

*We would like to thank Maya Rosenblatt for help in data collection. Amnon Rapoport would like to acknowledge financial support of this research by a grant from the Hong Kong Research Grants Council (Project No. CA98/99.BM01) awarded to the Hong Kong University of Science and Technology. We thank Anne van den Nouweland and seminar participants at HKUST, University of North Carolina, and the 2001 Econometric Society Summer meetings for helpful comments.

Abstract

A model of coalition government formation is presented in which inefficient, non-minimal winning coalitions may form in Nash equilibrium. Predictions for five games are presented and tested experimentally. The experimental data support potential maximization as a refinement of Nash equilibrium. In particular, the data support the prediction that non-minimal winning coalitions occur when the distance between policy positions of the parties is small relative to the value of forming the government. These conditions hold in games 1, 3, 4 and 5, where subjects played their unique potential-maximizing strategies 91, 52, 82 and 84 percent of the time, respectively. In the remaining game (Game 2) experimental data support the prediction of a minimal winning coalition. Players A and B played their unique potential-maximizing strategies 84 and 86 percent of the time, respectively, and the predicted minimal-winning government formed 92 percent of the time (all strategy choices for player C conform with potential maximization in Game 2). In Games 1, 2, 4 and 5 over 98 percent of the observed Nash equilibrium outcomes were those predicted by potential maximization. Other solution concepts including iterated elimination of dominated strategies and strong/coalition proof Nash equilibrium are also tested.

JEL Classification: C72, C78, D72

KEYWORDS: Coalition formation, potential maximization, Nash equilibrium refinements, experimental study, minimal winning.

1. Introduction

This paper applies recent theoretical advances in the theory of potential games to coalition formation, derives predictions, and tests them experimentally. Our goal is to ascertain to what extent potential maximization (a refinement of Nash equilibrium) is a useful guide in predicting coalition formation behavior. We consider three-player games. This is the simplest setting in which interesting coalitional structures can emerge. Moreover, the theoretical predictions for the three-player case are unambiguous and reduce to few enough cases that exhaustive experimental testing is feasible.

The framing of our study is coalition government formation. This provides concreteness when discussing aspects of the coalition formation process and gives a context to the experimental subjects. The model we consider is highly stylized but it captures fundamental aspects that apply to government formation and other applications (e.g., corporate merges, cartels). Namely, increasing the size of the government beyond the minimum requirement increases the need to compromise and reduces the surplus to being in power; different members of coalitions have more power than others and hence extract more surplus; decisions made in the coalition formation process affect payoffs of winning members.

Players are political parties that are differentiated by exogenously given policy positions. Policy positions are points on the Real line. This is **not** essential, as the theory could be redone using the Euclidean distance between points in multiple dimensions. Assuming a one-dimensional policy space is convenient because it allows us to classify predictions for all three-player games in terms of just two distances. This reduces the number of cases to test experimentally and is appropriate for the purpose at hand.

The game begins after the election is over. To avoid trivial cases, it is assumed that no party controls a majority of the votes, and hence any two parties (or all three) control a majority. The coalition that forms the government is the one controlling a majority of the votes cast. The members of the government are entitled to share a surplus, which is interpreted as the value of forming the government. The amount of the surplus depends on the policy positions of the government members and the policy choice of the government. Assuming distinct policy positions, adding more parties to a coalition government lowers the value of forming the government as a greater deal of compromising is required to accommodate a wider spectrum of beliefs. The technical implication is that the characteristic function of the coalitional game is not superadditive.

The surplus of the coalition government is divided according to the Myerson value (Myerson, 1977). The Myerson value is a generalization of the Shapley value (Shapley, 1953) that reflects the cooperation (or link) structure within each coalition (see also Aumann & Myerson, 1988). If all the members of a coalition are linked, the Myerson value is the same as the Shapley value; otherwise, more weight is given to players that hold special positions in the link structure.

The attachment of more weight to players with a special position in the link structure makes sense in this context if links are interpreted as representing favorable, bilateral relationships between the parties. In the model, two players are linked if and only if each of them independently expresses a desire to cooperate with the other. It is likely that such players will work together toward mutually beneficial outcomes in the political process. The absence of a link between two players means that at least one player excludes the other in her proposal for the coalition government. It is easy to imagine that there will be sore feelings between unlinked players and that such players will draw less benefit from being in the government. Hence, it is natural to assign less weight to players with missing links.

By using the proposed method of dividing the surplus, the government formation game is a potential game, as defined by Monderer and Shapley (1996). A useful fact about potential games is that only a subset of the Nash equilibria of potential games coincides with the set of strategy profiles that maximize the potential. This renders potential maximization useful as a refinement tool.[1]

In the theoretical part of this study, we describe the predictions of potential maximization and a variety of applicable solution concepts. Only two of the possible cooperation structures survive potential maximization, and generically the cooperation structure selected by potential maximization is unique: the possibilities are either a two-party government or a three-party government with full cooperation (i.e., every pair of players is linked in the cooperation structure). We provide conditions for either type of government. These are expressed in terms of the size of the government surplus and the distances between parties on the policy line.

In the empirical part of this study, potential maximization is competitively tested against other solution concepts including iterated elimination of dominated strategies and strong/coalition proof Nash equilibrium. The data comes from a collection of experiments conducted using student subjects at the University of Arizona. The theory neatly divides the parameter space into three categories. We tested five games that were based on parameter selections from each one (including three selections from one category to test the importance of symmetry.) We find potential maximization outperforms other refinements of Nash equilibrium in predicting individual behavior and outcomes.

[1] See Monderer and Shapley 1996, Section 5.

Experimental studies of coalition formation have a long history in the social sciences. Some of the early experiments were conducted by political scientists (Laing & Morrison, 1973; Riker, 1971, 1972), psychologists (Kelley & Arrowood, 1960; Vinacke & Arkoff, 1957), sociologists (Gamson, 1961, 1964), economists (Murnighan & Roth, 1977, 1978), and game theoreticians (Kalisch et al., 1954: Maschler, 1965). Many of the early studies focused on psychological aspects of coalition formation behavior, whereas others (e.g., Kalisch et al., 1954; Maschler, 1965) were designed to test theoretical predictions of coalition formation modeled as games in characteristic function form (see Kahan & Rapoport, 1984, for a review of this early experimental literature).

We follow the later studies by focusing on games that are both nonzero sum and non-superadditive. The nonzero-sum assumption is in accordance with Riker (1967) who states "the greater portion of political activity in forming coalitions is nonzero sum in the sense that different minimal winning coalitions win different amounts and the loss to the loser may not equal the gain to the winner." Our focus on non-superadditive games reflects the idea mentioned above that adding new members to an already winning coalition might decrease the value of the coalition. Unlike Riker and Maschler, we do not assume that adding a new member to a winning coalition depreciates its value to zero. Instead, we assign the grand coalition (which is the only non minimal winning coalition in our design) a positive value. This allows for the selection of non-minimal winning coalitions in our model; a feature that is borne out in the experimental results reported below.

2. Model

Consider a parliamentary system with three parties, where each party has an exogenously given policy position represented by a point on the real line. The policy position of each party $i \in N = \{A,B,C\}$ is denoted $p_i \in \Re$, where $p_A \le p_B \le p_C$. Let $p=(p_A, p_B, p_C)$ denote a vector of policy positions. Each party controls a number of seats that it won in an election. Seat shares are assumed to be such that any two parties (or all three parties) can combine to control a majority of the seats, but no single party has a majority. There is a surplus of G that is received by the members of whichever coalition government forms. However, this surplus is reduced by an amount determined by the distances between the policy positions of the coalition government members and the position, y, chosen by the government. Specifically, the value to any coalition is given by the characteristic function $v(p,G)$ where

$$v(S;p,G) = \begin{cases} \max_{y \in \Re^N} \left\{ G - \sum_{i \in S} (y - p_i)^2 \right\} & if \quad |S| \ge 2 \\ 0 & otherwise \end{cases} \tag{1}$$

The characteristic function specified in (1) is the same as that used in Austin-Smith and Banks (1988). We use it to define a government formation game in which parties A, B, and C are the players. We restrict attention to parameter values for which $v(S;p,G) > 0$ whenever $|S| \ge 2$. This ensures that there is always some positive value to being able to form the government.

The strategy set of player i is denoted by $\Pi_i = \{S \subseteq N \setminus i\}$. The empty set is included in Π_i. A strategy $\pi_i \in \Pi_i$ is a set of parties with whom player i wishes to form a government. For instance, the strategy set of player A is $\Pi_A = \{\emptyset, B, C, \{B,C\}\}$, signifying that player A can choose to cooperate with nobody, party *B*, party *C*, or both parties *B* and *C*. Let $\Pi = \times_{i \in N} \Pi_i$.

Each strategy profile $\pi = (\pi_A, \pi_B, \pi_C) \in \Pi$ determines a cooperation (link) structure $g(\pi)$ in the following way. Given $\pi \in \Pi$, a link between parties *i* and *j* forms if and only if the desire to cooperate is mutual, i.e., if and only if $i \in \pi_j$ and $j \in \pi_i$. Denote an undirected bilateral link between parties *i* and *j* by $i:j$. Then $g(\pi) = \{i:j \mid i \in \pi_i \ \ and \ \ j \in \pi_j\}$. All parties that are linked (directly or indirectly) in the cooperation structure that results from some strategy profile played in the government formation game are assumed to form the coalition government.

Payoffs to each player under a given strategy profile are determined by the Myerson values. The Myerson values for members of a coalition depend upon the characteristic function $v(p,G)$ and the graph $g(\pi)$ resulting from the selected strategy profile, π. Given a cooperation structure $g(\pi)$ and a coalition $S \subseteq N$, let $S / g(\pi)$ denote the partition of S into subsets of players who are connected (directly or indirectly) by $g(\pi)$ in S. To specify the Myerson value, let $v^{g(\pi)}(p,G)$ denote the characteristic function determined according to

$$v^{g(\pi)}(S;p,G) = \sum_{R \in S/g(\pi)} v(R;p,G), \qquad S \subseteq N.$$

The value $v^{g(\pi)}(S;p,G)$ reflects the fact that coalition *S* may not be able to form due to a lack of connectedness among all the members of *S*. We denote the Myerson values by the vector $\psi^{g(\pi)}(v(p,G)) = (\psi_A^{g(\pi)}(v(p,G)), \psi_B^{g(\pi)}(v(p,G)), \psi_C^{g(\pi)}(v(p,G)))$. Myerson (1977) shows that $\psi^{g(\pi)}(v(p,G)) = \phi(v^{g(\pi)}(p,G))$, where

$$\phi_i(v^{g(\pi)}(p,G)) = \sum_{S:S \ni i} \frac{(|S|-1)!(n-|S|)!}{n!}[v^{g(\pi)}(S;p,G) - v^{g(\pi)}(S \setminus i;p,G)]$$

is the Shapley value for player i in the game $v^{g(\pi)}(p,G)$. In summary, the government formation game consists of players' strategy sets Π_A, Π_B, and Π_C, and their payoff functions $\psi_A^{g(\pi)}(v(p,G))$, $\psi_B^{g(\pi)}(v(p,G))$, and $\psi_C^{g(\pi)}(v(p,G))$, $\pi \in \Pi$.

3. Theoretical Predictions

A variety of solution concepts apply to the government formation game.[2] We consider Nash equilibrium and some of its refinements. Because of the way payoffs are defined the government formation game is a potential game (see Qin 1996). Hence, one refinement that we consider is potential maximization. This refinement involves finding the set of strategy profiles for the game that maximizes the potential of the game. This set of strategy profiles is meaningful because it always constitutes a subset of the Nash equilibria for the game. Moreover, the potential has the property that its value increases with beneficial unilateral deviations by the players. Hence, individual actions taken to improve own welfare lead to higher values of the potential. [3]

Potential maximization is a particularly effective refinement in this game, because it always yields a unique prediction of the cooperation structure (and payoffs) that results from play of the game. We also examine strong Nash equilibria (SNE), Coalition proof Nash equilibria (CPNE), and solutions resulting from iterated elimination of weakly dominated strategies (IEWDS). SNE and CPNE are less desirable as refinements since they may not be unique and may not exist. Nevertheless, the comparisons are instructive.

[2] See Van den Nouweland (2003) for more on the refinements we discuss below.

3.1 Nash equilibria in weakly dominant strategies

The first result identifies parameter values for which each party has a weakly dominant strategy to cooperate with the other two parties. The result is followed by a corollary that identifies the parameter set for which full cooperation emerges as a Nash equilibrium in (weakly) dominant strategies.

Proposition 1. *Party A has a (weakly) dominant strategy to cooperate with parties B and C if and only if* $\psi_A^{\{A:C,B:C\}}(p,G) \geq 0$ *and* $\psi_C^{\{A:B,A:C\}}(p,G) \geq 0$.

Party B has a (weakly) dominant strategy to cooperate with parties A and C if and only if $\psi_A^{\{A:B,B:C\}}(p,G) \geq 0$ *and* $\psi_C^{\{A:B,B:C\}}(p,G) \geq 0$.

Party C has a (weakly) dominant strategy to cooperate with parties A and B if and only if $\psi_A^{\{A:C,B:C\}}(p,G) \geq 0$ *and* $\psi_C^{\{A:B,A:C\}}(p,G) \geq 0$.

See Appendix A for proof. Four of the six individual conditions obtained from applying Proposition 1 to each of the three parties are redundant. Hence, to check that all three parties have a dominant strategy to cooperate fully it is sufficient to check only two conditions.

Corollary 1. *All three parties have a (weakly) dominant strategy to cooperate with both of the other parties if* $\psi_A^{\{A:C,B:C\}}(p,G) \geq 0$ *and* $\psi_C^{\{A:B,A:C\}}(p,G) \geq 0$.

[3] Further justification of potential maximization as a refinement of Nash equilibrium is provided in Garratt and Qin (2003b).

Graphical representation: The characteristic function *v(p,G)* specified in (1) is invariant to translations of the party positions. In other words, the relative policy positions of the parties matter, whereas the absolute policy positions do not. For this reason, and because we assume $p_A \leq p_B \leq p_C$, the three parameters p_A, p_B, and p_C can be replaced with two distance parameters $d_1 = p_B - p_A$ and $d_2 = p_C - p_B$. Fix G. The conditions $\psi_A^{\{A:C,B:C\}}(p,G) \geq 0$ and $\psi_C^{\{A:B,A:C\}}(p,G) \geq 0$ describe a rotated hyperbola in *(d₁, d₂)* space. The intersection of the two graphs in $\Re_+^2$ defines the set of parameter values for which every party has a dominant strategy to cooperate with both of the other parties (See Fig. 1).

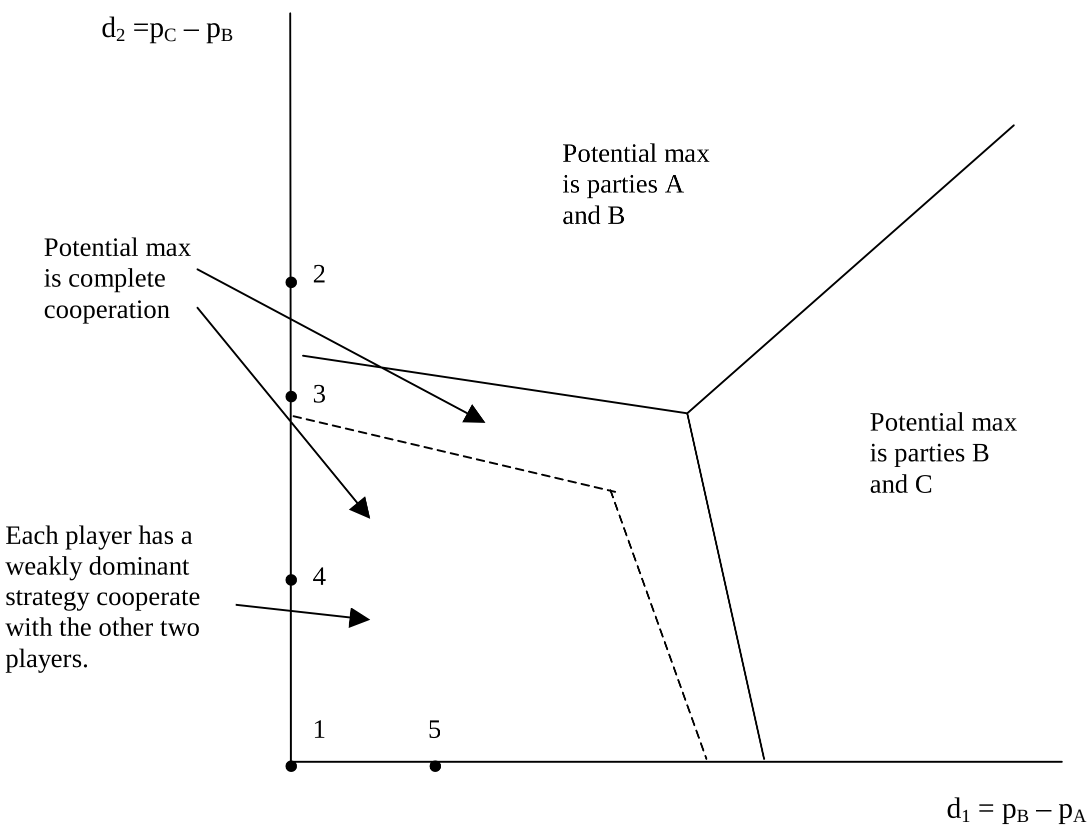


Figure 1. Dominant strategies, potential maximization, and the five games.

3.2 Potential maximization

We adapt the definition of potential from Monderer and Shapley (1996). A potential for a government formation game (with parameter values p, G) is a function $P : \Pi \to \Re$ such that for any $i \in N$, $\pi \in \Pi$, and $\pi_i^{'} \in \Pi_i$, $\psi_i^{g(\pi_i^{'},\pi_{-i})}(p,G) - \psi_i^{g(\pi)}(p,G) = P(\pi_i^{'},\pi_{-i}) - P(\pi)$.

The predictions of potential maximization are described below.

Proposition 2. *A three-party government with complete cooperation will maximize the potential if and only if* $\phi_A(v(p,G)) \geq 0$ *and* $\phi_C(v(p,G)) \geq 0$*. The two-party government {A,B} (respectively {B,C}) will maximize the potential if and only if* $\phi_A(v(p,G)) \leq 0$ *(respectively* $\phi_C(v(p,G)) \leq 0$*).*

Proof is in Appendix A.

Graphical representation: Fix *G*. Then each of the conditions $\phi_A(v(p,G)) = 0$ and $\phi_C(v(p,G)) = 0$ describe a rotated ellipse in (d_1,d_2) space. The intersection of the two ellipses in $\Re_+^2$ defines the set of parameter values for which the potential maximizing outcome is full cooperation (See Fig. 1).

The second statement in Proposition 2 is not surprising. It should be taken merely as an indication of the reasonableness of the solution concept. At the same time, it is worth noting that some well-known, sequential processes of coalition formation, such as the method described in Austin-Smith and Banks (1988), do not have this property. Conversely, it is perhaps surprising that the magnitude of G matters in determining whether the government will be two-party (minimal winning) or three-party (non-minimal winning). This occurs because the Myerson value averages marginal contributions that occur when a party enters a coalition that is already winning with cases where the coalition is not already winning. Whenever a party joins a coalition that is already winning, it reduces the surplus available to the coalition members. Whether or not such instances are overridden by cases where the party is instrumental in causing a coalition to be winning depends on the magnitude of G. The practical implication is that we expect larger governments to be more prevalent in instances where the gains to being in power are large, and the distances between policy positions are small.

4. Treatment Parameters

We specified five games to be studied experimentally. The parameters selected for the five games are shown graphically in Fig. 1. Games 1, 4, and 5 lie in the intersection of the set of parameter values for which each party has a weakly dominant strategy to cooperate with the other two parties and the set of parameter values for which the unique potential maximizing strategy profile is complete cooperation. These games differ in terms of the degree of symmetry: all three parties are seen to have the same party position in Game 1, only parties A and B are the same in Game 4, and all three parties have different policy positions in Game 5. Game 2 lies in the set of parameters for which potential maximization predicts a two-party coalition government including parties A and B. Game 3 lies in the set of parameter values for which the unique potential maximizing strategy profile is complete cooperation, but unlike games 1, 4, and 5, it lies outside the set of parameter values for which each party has a weakly dominant strategy to cooperate with the other two parties.

With three parties there are eight possible cooperation structures. Appendix B illustrates the eight possible cooperation structures for each of these five games and the associated payoffs for each of the three players. Numbers in parentheses in Appendix B indicate negative payoffs (losses). Observe that in Games 1-4 the efficient, minimal winning government is the coalition $\{A,B\}$, whereas in Game 5 it is the coalition $\{B,C\}$. Potential maximization predicts complete cooperation (which is inefficient) in all but Game 2. Hence, in all but Game 2, the occurrence of potential maximizing outcomes cannot be interpreted as a desire by subjects to maximize total surplus.

4.1 Other solution concepts

We evaluate other solution concepts that apply to the games in Fig. 1 on a case-by-case basis. These include SNE and CPNE, two well-known refinements of Nash equilibrium. CPNE is defined recursively. As compared to SNE, there is an additional requirement that a deviation by a coalition must be valid in the sense that no proper subcoalition would want to deviate from the deviation. In this case, any proper subcoalition of a two-player coalition is a singleton. Thus, any deviation by a two-player coalition is valid provided that deviation guarantees individual rationality. In our games, single-player coalitions receive a payoff of zero and all two-player coalitions yield positive payoffs for both players in the coalition (see Appendix B). Thus, SNE and CPNE coincide for our games.

There are no strong or coalition proof Nash equilibria in Games 1, 4, and 5. All strategy profiles in Game 2 that produce the cooperation structure $A{:}B$ are both potential maximizing and strong/coalition-proof. In Game 3, SNE/CPNE produces a subset of the Nash equilibria that does not include the potential maximizing strategy profile: only Nash equilibrium strategy profiles that pair parties *A* and *B* (without *C*) are both strong and coalition-proof.

In addition, we consider solutions obtained by IEWDS. The profile ({*B*,*C*},{*A*,*C*},{*A*,*B*}) is the sole survivor of IEWDS in Games 1, 4, and 5. It is one of the eight Nash equilibria of these games. There are four strategy profiles in Game 2 that survive IEWDS, namely, (*B*,*A*,∅), (*B*,{*A*,*C*},∅), ({*B*,*C*},*A*,∅), and ({*B*,*C*},{*A*,*C*},∅). Each of these four strategy profiles is a Nash equilibrium. Moreover, each gives rise to the cooperation structure *A*:*B*, which yields the same payoff of 270 to parties *A* and *B* (Fig. 2b). Party *C* cooperates with nobody in each of the strategy profiles that survive IEWDS. There are eight strategy profiles in Game 3 that survive IEWDS. Of these eight profiles, five are Nash equilibria; namely (*B*,*A*,∅), (*B*,{*A*,*C*},∅), ({*B*,*C*},*A*,∅), (*B*,*A*,{*A*,*B*}), and ({*B*,*C*},{*A*,*C*},{*A*,*B*}), and three are not; namely *(B, {A,C}, {A,B}), ({B,C}, A, {A,B}), ({B,C}, {A,C}, Ø)*. Four of the Nash equilibrium strategy profiles that survive IEWDS in Game 3 give rise to the cooperation structure *A*:*B* (with symmetric payoffs to players A and B, see Fig. 2c), and the fifth gives rise to the cooperation structure *A*:*B*,*B*:*C*,*A*:*C*.

5. Experiment

To competitively test the different solution concepts, we devised a computer-controlled experiment with payoffs contingent on performance where groups of three players each played repeatedly the five different games identified in Appendix B. Both player roles (party *A*, party *B*, or party C) and group membership were varied from one period to another to prevent reputation effects. In what follows, we describe the experimental procedure and present the results. Each game separately does not allow a competitive test of all the theoretical predictions, but the aggregate of the five games does allow this test.

5.1 Method

Subjects. Sixty undergraduate students from the University of Arizona participated in the experiment. The players participated in four separate sessions each consisting of 15 members. To allow for generality of the results, we chose subjects with different levels of sophistication or knowledge. Sessions 1 and 2 included inexperienced undergraduate students from the school of Business and Public Administration who volunteered to take part in a decision making experiment for monetary reward contingent on performance. Session 3 included undergraduate students who had some familiarity with the basic notions of non-cooperative game theory and considerable experience in playing computer-controlled strategic games. Session 4 included graduate students who have taken at least one course on non-cooperative game theory. Payoffs were stated in terms of a fictitious currency called "francs" that at the end of the session were converted into US dollars at the rate $1.00 = 200 francs for Sessions 1, 3, and 4 and $1.00 = 125 francs for Session 2. In addition to their earnings in the experiment, players of Sessions 1 and 2 received a $5.00 show-up fee, whereas players of Sessions 3 and 4 received course credit in lieu of the show-up fee. Total earnings for Sessions 1 through 4 were $235.00, $319.00, $311.00, and $321.00, respectively.

Procedure. All the four sessions were conducted in the Economics Science Laboratory (ESL) at the University of Arizona. At the beginning of each session, the players drew poker chips from a bag to randomly determine their seat assignment. Players were individually seated in the ESL, which consists of 40 networked computer workstations in separate cubicles. Each cubicle contains a computer monitor, keyboard, mouse, paper and pencil, and a set of written instructions.[4] When every player in the session completed reading the instructions, the supervisor entertained a brief question and answer period to ensure that everyone understood the operation of the computers, game design, and payoff function. Very few questions were asked. Five different three-player games were then presented to the players. Each game was iterated either three (Sessions 1 and 2) or six (Sessions 3 and 4) times. The order of presentation of the games was randomized within five-trial blocks. The same ordering of the games was used in all four sessions to facilitate comparison between them. On each trial, the fifteen subjects in each session were divided into five groups of three players each. Both player role and group membership were varied from trial to trial to prevent reputation effects and provide each player the same opportunity to participate in all three roles. A between-subject randomized design was used to assign the players to the three roles in the game. The same assignment schedule was used for each session.

Prior to starting the experiment, all the players in a session completed a self-paced two-part computerized test to ensure their understanding of the game. During the first part of this test, players were presented with the eight possible cooperation structures. For each of their four individual strategies they were asked to identify which cooperation structure could potentially result, given that they did not know the strategies played by the other players. The purpose of the first part of the test was to make it clear to each player that her individual outcome depended not only on her decision, but also on the decisions of the two other players in her group. If a question was answered incorrectly, the player was informed of this fact and then received the correct answer. After a player completed the first part of the test, she immediately proceeded to the second part.

---

[4] To view the instructions go to http://www.eller.arizona.edu/~map/research/

In the second part of the test each player was presented with scenarios identifying all three of the players' strategies, including their own. They were then asked to state which cooperation structure would (necessarily) result from the combination of these strategies. Players were shown a non-repeating sequence of scenarios and were required to provide three consecutive correct responses. The experiment started once all the players in the session passed the test satisfactorily.

The pre-experimental manipulation check served two purposes. The first was to ensure that each player fully understood the government formation game and learned how to register his or her responses. The second and equally important purpose was to ensure that complete understanding of the game by all players in the session was common knowledge.

Because of delays incurred during the test and inexperience with strategic thinking, players in Sessions 1 and 2 completed only 15 trials each. The more "sophisticated" players in Sessions 3 and 4 successfully answered the test questions considerably faster, made their decisions quicker, and subsequently succeeded in completing 30 trials in the two-hour session.

At the beginning of each trial, all the players in a session viewed the same screen illustrating the eight possible cooperation structures and their associated payoffs. Below the diagram, a statement appeared identifying their player role for that trial. Additionally, the relevant payoffs were individually highlighted for each player role by changing the color to a bright green. To register a choice, a player had to 'click' on one of four computerized buttons representing her four possible strategies. Once selected, a 'commit' button appeared for the player to confirm her decision. Players were allowed to change their strategy choices as often as they wished without penalty or without revealing their decision to the two other members of their group. Once all three players confirmed their decisions, they viewed an earnings summary that graphically depicted the resulting cooperation structure and identified the individual payoffs. Any form of communication during the experiment was strictly forbidden.

Players were only informed of the outcome of the group in which they participated. Information about other groups in the session was not provided. Each player could review the outcomes of all the previous trials in which she had actually participated. Once a player completed reviewing her results, if she chose doing so, she exited the Results screen and waited for all other players to complete the trial. When the last player in the session completed reviewing the Results screen, the experiment advanced to the next trial, if it was not the last.

6. Results

Previous experience in playing computerized games and "sophistication" had significant effects on the outcomes. We found no significant difference (t=0.31, p<0.76) in the percentage of choice of undominated strategies (summed across the 15 trials) between Sessions 1 and 2. Similarly, we found no significant difference (t=1.61, p<0.12) in the percentage of choice of undominated strategies (summed across 30 trials) between Sessions 3 and 4. However, the null hypothesis of no effect for experience in playing computer-controlled games and sophistication in game theory was rejected (t=2.70, p<0.009) when we compared the combined frequencies in Sessions 1 and 2 with the combined frequencies in Sessions 3 and 4. As might be expected, the subjects in Sessions 3 and 4 chose undominated strategies significantly more often. Therefore, in many of the analyses below we differentiate between the "unsophisticated" (Sessions 1 and 2) and "sophisticated" (Sessions 3 and 4) groups.

We found no significant difference (t=1.70, p<0.09) in the strategy choices or game outcomes between trials 1-15 and 16-30 in Sessions 3 and 4. Therefore, the results were combined across trials. With 15 trials in Sessions 1, we have a total of 225 strategy choices for each of these two sessions. The number of strategy choices per session in Sessions 3 and 4, each including twice as many trials, is 450. When the results are combined across sessions, the analyses of strategy choices reported below are based on a total of 1350 data points, 270 for each of the five games. Although both group membership and player roles were varied from trial to trial, trials within sessions cannot be considered as statistically independent. Because players realized that they might be matched with one or more players more than once, interdependencies cannot be ruled out.

Game Outcomes. Table 1 lists the frequencies of the game outcomes across the four sessions by game and cooperation structure. The modal cooperation structure in Games 1, 4, and 5 is {*A*:*B*,*A*:*C*,*B*:*C*}, whereas the one in Games 2 and 3 is {*A*:*B*}. Non-minimal winning governments formed 98, 52, 93, and 94 percent of the time for Games 1, 3, 4, and 5, respectively. The minimal winning government formed 83 percent of the time for Game 2.

Game	None	A:B	A:C	B:C	A:B,B:C	A:C,B:C	A:B,A:C	A:B, A:C, B:C	Totals
1	0	2	0	0	9	3	9	67	90
2	2	83	0	0	3	0	1	1	90
3	1	41	1	0	17	0	16	14	90
4	0	4	1	1	13	2	18	51	90
5	0	1	2	2	25	4	4	52	90
Totals	3	131	4	3	67	9	48	185	450

Table 1. Frequencies of game outcomes by cooperation structure and game.

Frequencies of Strategy Choices. Across the three player roles and four sessions, undominated strategies were chosen with relative frequencies of 0.911, 0.903, 0.944, 0.822, and 0.844 in

Games 1- 5, respectively. However, as mentioned earlier, the "sophisticated" players of Sessions 3 and 4 chose undominated strategies significantly more often than the "unsophisticated" players of Sessions 1 and 2. The mean (and standard deviation) of these proportions were 0.83 (0.116) for the unsophisticated players and 0.91 (0.047) for the sophisticated players.

Table 2 breaks down the frequency of strategy choices by player role and game. The results are combined across the four sessions. For example, players *A* in Game 1 chose strategies {*B*}, {*C*}, {*B*,*C*}, and {∅}, 5, 1, 84, and 0 times, respectively. The frequencies of undominated strategies appear in boldface. The right-hand column of Table 2 presents the percentages of the choices that were undominated strategies across the four sessions. Table 2 shows that undominated strategies were chosen between 73.3% (player *C* in Game 2) and 100% (player *B* in Game 3) of the time. In Games 1, 4, and 5, where each player had a single (weakly) dominant strategy, these were chosen between 80% and 93.3% of the time. Although there was a general tendency for each player to cooperate with the other two players, the players clearly distinguished among the five games. Thus, the modal choice of player *A* in Game 2 was *B* (rather than {*B*,*C*}), and the modal choice of player *B* in the same game was A (instead of {*A*,*C*}). The third and second columns from the right display the results separately for the unsophisticated and sophisticated players, respectively. With only two exceptions, the percentages of choice of undominated strategies are higher for the latter than former players.

	Cooperated with					% of Undominated Strategies		
	Player A	Player B	Player C	Both Players	Neither Player	Sessions 1, 2	Sessions 3, 4	Across Sessions
Game 1								
Player A	---	5	1	**84**	0	83.3	98.3	93.3
Player B	6	---	2	**82**	0	83.3	95.0	91.1
Player C	4	4	---	**80**	2	80.0	93.3	88.9
Game 2								
Player A	---	**76**	0	**13**	1	100.0	98.3	98.9
Player B	**77**	---	0	**12**	1	96.7	100.0	98.9
Player C	8	10	---	6	**66**	56.7	81.7	73.3
Game 3								
Player A	---	**52**	2	**36**	0	93.3	100.0	97.8
Player B	**49**	---	0	**41**	0	100.0	100.0	100.0
Player C	8	5	---	**63**	**14**	85.6	90.0	83.3
Game 4								
Player A	---	16	2	**72**	0	76.7	81.7	80.0
Player B	13	---	2	**75**	0	76.7	86.7	83.3
Player C	12	2	---	**75**	1	76.7	86.7	83.3
Game 5								
Player A	---	12	1	**76**	1	73.3	90.0	84.4
Player B	4	---	5	**80**	1	73.3	96.7	88.9
Player C	2	16	---	**72**	0	83.3	78.3	80.0

Note: Undominated strategies appear in boldface.

Table 2. Frequency of strategy choices by player role and game.

Equilibrium Outcomes. Table 3 summarizes information about the predictive success of the various solution concepts. Columns 1 and 2 report the frequency of Nash equilibrium and potential maximizing strategy profiles for each game. From the data in columns 1 and 2 we can compute how often potential maximization selects the correct Nash equilibrium in each game. These numbers are reported in column 5. In Games 1, 2, 4, and 5, potential maximization is shown to select the correct Nash equilibrium 99, 100, 100 and 98 percent of the time. The potential maximizing strategy profile consists of (weakly) dominant strategies in Games 1, 4, and 5, but no player has a dominant strategy in Game 2. In Game 3 the potential maximization selects the correct Nash equilibrium only 27 percent of the time. While the percentage is considerably lower in Game 3 than the other four games, it is worth noting that in Game 3 the individual players *A*, *B*, and *C* played their unique, potential maximizing strategy 40, 46, and 70 percent of the time, respectively.

	NE	PotMax	IEWDS	SNE	PotMax|NE	SNE|NE
Game 1	68	67	67	--	98.5%	--
Game 2	83	83	65	83	100%	100%
Game 3	52	14	75	39	26.9%	75%
Game 4	51	51	51	--	100%	--
Game 5	53	52	52	--	98.1%	--

Table 3. Observed outcomes (across sessions).

Column 3 shows the number of times IEWDS strategy profiles were played in each game. In each of Games 1, 4, and 5 Nash equilibria occurred that were not IEWDS less than 2 percent of the time. Nash equilibria that are not IEWDS occurred 21 percent of the time in Game 2. In Game 3, where neither solution set is a subset of the other, Nash equilibria occur 62 percent of the time compared to 83 percent of the time for IEWDS. Deeper analysis of the data shows that IEWDS profiles occur that are not Nash equilibria 29 times, while Nash equilibria occur that are not IEWDS only 6 times.

Column 4 of Table 3 shows the number of times strong/coalition proof Nash equilibrium (simply labeled SNE) strategy profiles were played in each game. Column 6 shows how often refinement to SNE selects the correct Nash equilibrium in each game. The set of SNE is empty in Games 1, 4, and 5. In Game 2, the set of SNE coincides with the set of potential maximizers and hence picks the observed Nash equilibria the same number of times. In Game 3, the set of strong/coalition proof Nash equilibria includes eight strategy profiles and does not include the unique potential maximizer. SNE is correct 75 percent of the time, compared to 27 percent for potential maximization.

Overall, and taking into account the differential predictability of the different solution concepts (Table 1), there is strong support for potential maximization as a predictor of play in the government formation game. Potential maximization does the same or better than IEWDS in four of five games. As a refinement of Nash equilibrium it is equal to strong/coalition proofness in Game 2 but worse in Game 3. It does very well as a refinement of Nash in Games 1, 4, and 5, where the sets of SNE are empty. The support for potential maximization is significantly stronger for sophisticated players. In Games 1, 2, 4, and 5 potential maximization selected the Nash equilibrium played by sophisticated players 100 percent of the time. The percentage of potential-maximizing Nash equilibria selected by sophisticated players in Game 3 (27 percent) was the same as for the unsophisticated players.

In Game 1, all three players have the same policy position making it the only game that is superadditive. There is a large body of work that predicts the formation of the grand coalition in superadditive games.[5] Our experimental results for Game 1 are consistent with the findings of those studies.

6. Conclusion

We propose and experimentally test a model of government coalition formation that combines information about the distribution of votes among the parties and their position on the policy line. A major feature of the model is that it allows for the formation of either minimal winning or non-minimal winning coalitions. A second major feature is that it treats the formation of a coalition government as a non-cooperative (strategic) game. Consequently, various solution concepts for games in strategic form may be applied and competitively tested.

[5] See, for example, Qin (1996), Slikker (2001), Slikker et al. (2000).

The experimental results support the main theoretical predictions of the paper that are based on the premise that political parties select potential-maximizing Nash equilibria. The first prediction is that a three-party government with full cooperation between parties will form whenever the distance between policy positions of the parties is large relative to the value of forming the government. We conducted four games of this sort; Games 1, 3, 4, and 5. The theoretical predictions for Games 1, 4, and 5 are the same (see Table 1a): cooperating with both of the others players is a weakly dominant strategy for all three players, and everyone cooperating fully is potential maximizing as well as being the only strategy profile that survives IEWDS. The only difference between the three games is the degree of symmetry in the policy positions of the parties. In Game 1, all three parties are the same; in Game 4, two are different; and in Game 5, all three have different policy positions. These differences are reflected in the payoffs shown in Appendix B. The occurrence of the potential maximizing outcome was high in all three of these games: potential maximization performed very well as a refinement of Nash, being correct over 98 percent of the time in each case. This is despite the fact that in game 1 all graphs (but the one with no links) yield the same total welfare and in games 4 and 5 the potential maximizing solution does not maximize total welfare. There was no evidence that differences in the degree of symmetry mattered in these games.

Full cooperation in Game 3 is predicted by potential maximization and is consistent with IEWDS, but cooperating with both players is not a weakly dominant strategy for any of the players. Moreover, the strategy profile that leads to full cooperation is neither strong nor coalition proof. Here, potential maximization does poorly in terms of outcomes for the three-player game; however, the data show that individuals played their potential maximizing strategy, which is to cooperate with both of the other players 52 percent of the time (see Table 2). Hence, non-minimal winning governments were still observed most often in Game 3 (see Table 1).

The second prediction is that in cases where the distance between policy positions of the parties is small relative to the value of forming the government a minimal winning government will form that includes the two parties closest together on the policy line. We conducted one game of this sort, namely, Game 2. Here, potential maximization selected the observed strategy profile over 90 percent of the time despite the fact that there are no dominant strategies; as a refinement of Nash equilibrium potential maximization was perfect!

Appendix A

Proof of Proposition 1. First consider party *A*, and fix her strategy at {*B*,*C*}. In order for {*B*,*C*} to be a weakly dominant strategy for party *A*, the strategy *{B,C}* must earn a payoff at least as high as any other strategy for party A, regardless of the strategies played by the other two parties. To show this we use the following notation. Given any graph *g*, let $\Pi_g^{-A} = \{\pi^{-A} \in \Pi^B \times \Pi^C : g(\{B,C\}, \pi^{-A}) = g\}$ be the set of strategy pairs for *B* and *C*, which when combined with strategy {B,C} for party A produce the graph *g*. [6] When played against the strategy pairs in $\Pi^{-A}{}_{\phi}$, the strategy {*B*,*C*} is at least as good as any other strategy for party A. This is because in this case all strategies for player A earn a payoff of zero. Assuming players B and C play a strategy pair in $\Pi_{B:C}^{-A}$, {*B*,*C*} is at least as good as any other strategy for party *A* because, once again, all strategies for party A earn a payoff of zero.

Next, we establish necessary and sufficient conditions for *{B,C}* to be at least as good as any other strategy for party *A*, given that parties *B* and *C* play strategy pairs corresponding to the remaining six graphs. We then show that given our specification of the characteristic function in (1), these conditions are equivalent to those in Proposition 1. To this end, consider the set $\Pi_{A:B}^{-A}$. The only alternative that party *A* can achieve by unilateral deviation is to earn zero by changing her strategy to one that breaks the link with player *B*. It follows from the specification of the Myerson value that for *{B,C}* to be weakly dominant for party A, it is necessary and sufficient that

$$\tfrac{1}{2}v(\{A,B\}) \geq 0. \tag{2}$$

Likewise, the condition for *{B,C}* to be weakly dominant for party A against strategy pairs in the set $\Pi_{A:C}^{-A}$ is

$$\tfrac{1}{2}v(\{A,C\}) \geq 0. \tag{3}$$

Consider the set $\Pi^{-A}_{A:B,B:C}$. In this case, link between parties *B* and *C* remains regardless of the strategies for party *A*. Consequently, party *A*'s only alternative is to earn zero by changing her strategy to one that breaks the link with party *B*. Hence, in this case, for *{B,C}* to be weakly dominant for party A, it is necessary and sufficient that

$$\tfrac{1}{3}[v(N) - v(\{B,C\})] + \tfrac{1}{6}v(\{A,B\}) \geq 0 \tag{4}$$

Likewise, the condition for *{B,C}* to be weakly dominant for party A against strategy pairs in $\Pi^{-A}_{A:C,B:C}$ is

$$\tfrac{1}{3}[v(N) - v(\{B,C\})] + \tfrac{1}{6}v(\{A,C\}) \geq 0 \tag{5}$$

Consider the set $\Pi^{-A}_{A:B,A:C}$. Party *A* has three unilateral deviations that can change her payoff. She can earn zero by choosing a strategy that breaks the links to both players *B* and *C*, or she can break the link to just one of the two players. If she breaks the link to player *B* and remains linked to player *C*, she gets a payoff of $\tfrac{1}{2}v(\{A,C\})$. If she breaks the link to player *C* and remains linked to player *B* she gets a payoff of $\tfrac{1}{2}v(\{A,B\})$. Hence, in this case, for *{B,C}* to be weakly dominant for party A, it is necessary and sufficient that

$$\tfrac{1}{3}v(N) + \tfrac{1}{6}v(\{A,B\}) + \tfrac{1}{6}v(\{A,C\}) \geq \max\{\tfrac{1}{2}v(\{A,C\}), \tfrac{1}{2}v(\{A,B\}), 0\}. \tag{6}$$

Finally, consider the set $\Pi^{-A}_{A:B,A:C,B:C}$. Again, party *A* has three unilateral deviations that can change her payoff. She can earn zero by choosing a strategy that breaks the links to both players *B* and *C*, or she can break the link to just one of the two players. If she breaks the link to player B and remains linked to player C she gets a payoff of $\tfrac{1}{3}[v(N)-v(\{B,C\})]+\tfrac{1}{6}v(\{A,C\})$ (since players *B* and *C* are still linked). If she breaks the link to player *C* and remains linked to player *B*

[6] Note that we are using *g(.)* as a function mapping strategy profiles to graphs and *g* as a graph.

she gets a payoff of ${}^{1}/_{3}[v(N)–v(\{B,C\})]+{}^{1}/_{6}v(\{A,B\})$ (since players B and C are still linked). Hence, for *{B,C}* to be weakly dominant for party A against strategy pairs in $\Pi^{-A}_{A:B,A:C,B:C}$ it is necessary and sufficient that

$$
{}^{1}/_{3}[v(N) – v(\{B,C\})] + {}^{1}/_{6}[v(\{A,B\}) + v(\{A,C\})] \geq
$$

$$
\max\{{}^{1}/_{3}[v(N) – v(\{B,C\})] + {}^{1}/_{6}v(\{A,B\}), {}^{1}/_{3}[v(N) – v(\{B,C\})] + {}^{1}/_{6}v(\{A,C\}),0\}. \quad (7)
$$

Conditions (2) and (3) are satisfied automatically since we assume $v(S) \geq 0$ for all $S \subseteq N$. For the same reason, the first two elements in the set on the right-hand-side of (7) are no bigger than the left-hand-side. Moreover, conditions (4) and (5) imply the third element is no bigger than the left-hand-side. Hence (7) is satisfied.

The inequality is satisfied for the third element in the set on the right-hand-side of (6) since we assume $v(S) \geq 0$ for all $S \subseteq N$. The remaining conditions from (6) can be rewritten as

$$
{}^{1}/_{3}[v(N) – v(\{A,C\})] + {}^{1}/_{6}v(\{A,B\}) \geq 0 \quad (8)
$$

and

$$
{}^{1}/_{3}[v(N) – v(\{A,B\})] + {}^{1}/_{6}v(\{A,C\}) \geq 0 \quad (9)
$$

Given the specification of the characteristic function is (1) and the assumption $p_A \leq p_B \leq p_C$, $v(\{A,B\}) = G – 0.5(p_B - p_A)^2$, $v(\{A,C\}) = G – 0.5(p_C - p_A)^2$, $v(\{B,C\}) = G – 0.5(p_C - p_B)^2$, and $v(N) = G – (6(p_B - p_A)^2 + 6(p_C - p_B)^2 + 10(p_B - p_A)(p_C - p_B)/9$. Thus, $v(\{A,B\}) \geq v(\{A,C\})$ and hence (5) implies (4). Likewise, we know that $v(\{B,C\}) \geq v(\{A,C\})$ and hence (4) implies (8). The remaining conditions, (5) and (9), are the ones stated in the proposition.

The same reasoning shows that (5) and (9) are also necessary and sufficient for *{A,B}* to be a weakly dominant strategy for player *C*. We need to pay special attention to party *B*, however, since party B is in the middle. For *{A,C}* to be a weakly dominant strategy for party *B*, it is necessary and sufficient that

$$\tfrac{1}{2}\, v(\{A,B\}) \geq 0, \tag{10}$$

$$\tfrac{1}{2}\, v(\{B,C\}) \geq 0, \tag{11}$$

$$^{1}/_{3}[v(N) - v(\{A,C\})] + {}^{1}/_{6}v(\{B,C\}) \geq 0, \tag{12}$$

$$^{1}/_{3}[v(N) - v(\{A,C\})] + {}^{1}/_{6}v(\{A,B\}) \geq 0, \tag{13}$$

$$^{1}/_{3}[v(N) + {}^{1}/_{6}v(\{A,B\}) + {}^{1}/_{6}v(\{B,C\}) \geq \max\{\tfrac{1}{2}v(\{B,C\}), \tfrac{1}{2}v(\{A,B\}),0\}, \tag{14}$$

and

$$^{1}/_{3}[v(N) - v(\{A,C\})] + {}^{1}/_{6}[v(\{A,B\}) + v(\{B,C\})] \geq$$
$$\max\{{}^{1}/_{3}[v(N) - v(\{A,C\})] + {}^{1}/_{6}v(\{B,C\}), {}^{1}/_{3}[v(N) - v(\{A,C\})] + {}^{1}/_{6}v(\{A,B\}),0\}. \tag{15}$$

Conditions (10) and (11) are satisfied since $v(S) \geq 0$ for all $S \subseteq N$. For the same reason, the inequality is satisfied for the first two elements in the set on the right-hand-side of (15). Moreover, conditions (12) and (13) imply the inequality is also satisfied for third element. Hence, (15) is satisfied. The inequality is satisfied for the third element in the set on the right-hand-side of (14) since $v(S) \geq 0$ for all $S \subseteq N$. The remaining conditions from (14) can be rewritten as

$$^{1}/_{3}[v(N) - v(\{B,C\})] + {}^{1}/_{6}v(\{A,B\}) \geq 0 \tag{16}$$

and

$$^{1}/_{3}[v(N) - v(\{A,B\})] + {}^{1}/_{6}v(\{B,C\}) \geq 0 \tag{17}$$

Given the specification for the characteristic function in (1), we know that $v(\{A,B\}) \geq v(\{A,C\})$ and hence (17) implies (12). Likewise, we know that $v(\{B,C\}) \geq v(\{A,C\})$ and hence (16) implies (13). The remaining conditions (16) and (17) are the one stated in the proposition. ■

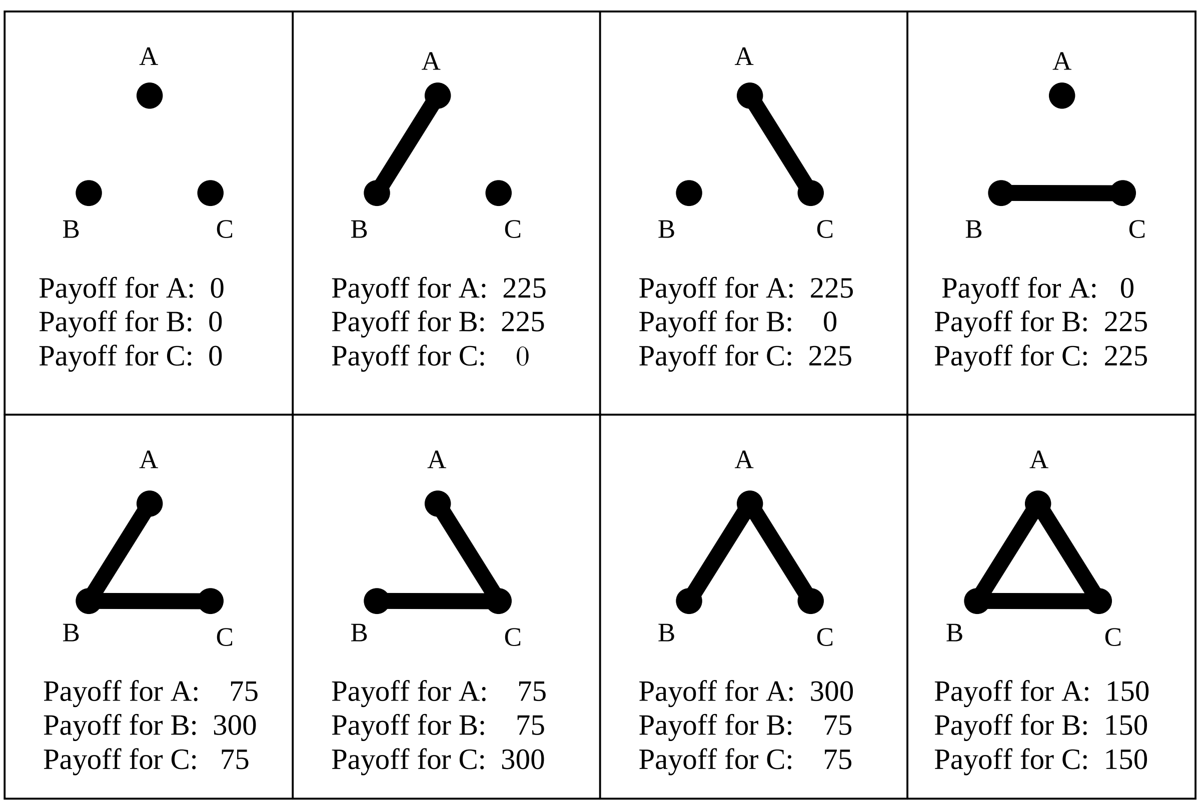
Proof of Proposition 2. Given the specification of the characteristic function in (1) and since $p_A \leq p_B \leq p_C$, $\phi_B(v(p,G)) \geq \phi_i(v(p,G))$ for $i = A,C$. Hence, $\phi_i(v(p,G)) \geq 0$ for $i = A,C$ implies $\phi_B(v(p,G)) \geq 0$. Statement 1 then follows from Remark 3(ii) of Garratt and Qin (2003a).

Now consider statement 2 of Proposition 2. Suppose $\phi_i(v(p,G)) < 0$ for some $i \in \{A,C\}$ so that some two-party government forms. Since $p_A \le p_B \le p_C$, it must be the case that either parties *A* and *B* are closest together on the policy line or parties *B* and *C* are closest together. Due to the obvious similarities we only prove the case where $(p_B - p_A) < (p_C - p_B)$. For any two-party coalition $\{i,j\}$, $i,j \in N$, the solution to the maximization problem in (1) is $y^* = (p_i + p_j)/2$. Consequently, $v(\{i,j\}) = G - \frac{1}{2}(p_i - p_j)^2$ for any $i,j \in N$. It is immediate that $v(\{A,B\}) = \max\{v(S) : |S| = 2\}$, and hence by Remark 3(i) in Garratt and Qin (2003a) a two-party government including parties *A* and *B* forms. ■

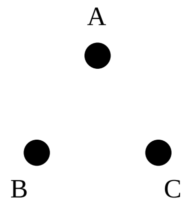
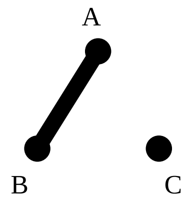
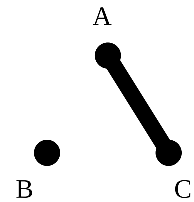
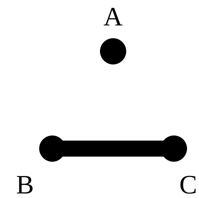
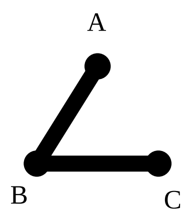
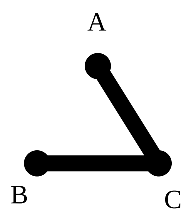
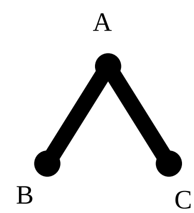
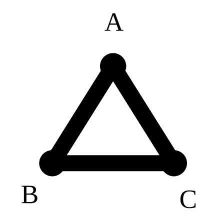
Appendix B

Cooperation structures and payoffs by game.

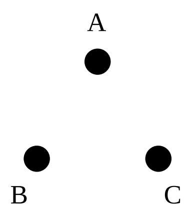
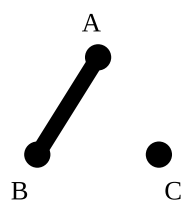
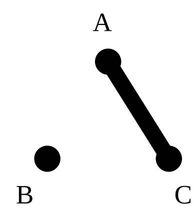
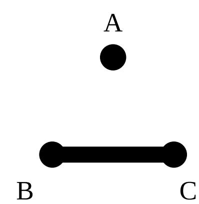
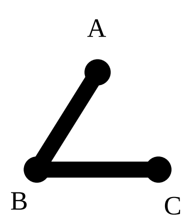
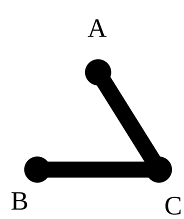
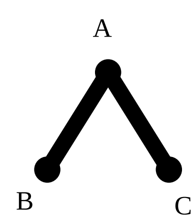
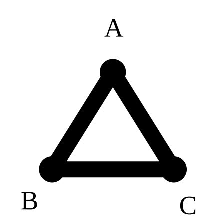
Game 1



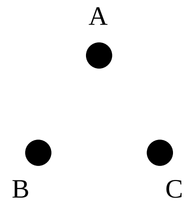
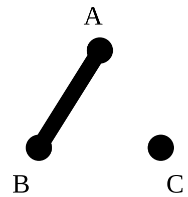
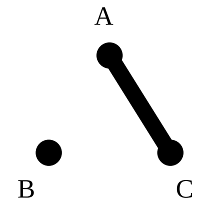
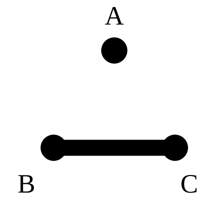
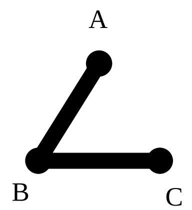
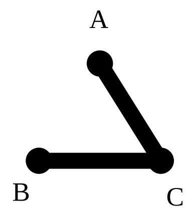
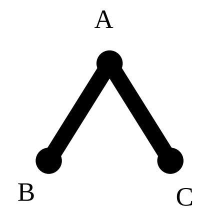
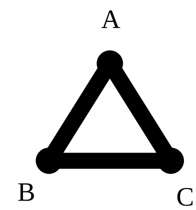
Game 2

 Payoff for A: 0 Payoff for B: 0 Payoff for C: 0	 Payoff for A: 270 Payoff for B: 270 Payoff for C: 0	 Payoff for A: 90 Payoff for B: 0 Payoff for C: 90	 Payoff for A: 0 Payoff for B: 90 Payoff for C: 90
 Payoff for A: 50 Payoff for B: 140 Payoff for C: (130)	 Payoff for A: (10) Payoff for B: (10) Payoff for C: 80	 Payoff for A: 140 Payoff for B: 50 Payoff for C: (130)	 Payoff for A: 80 Payoff for B: 80 Payoff for C: (100)

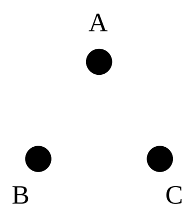
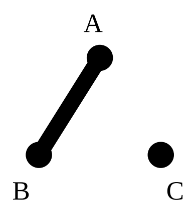
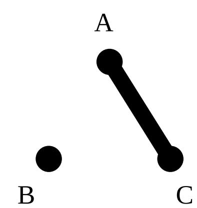
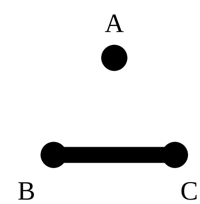
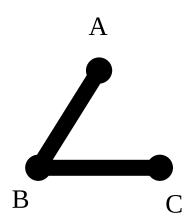
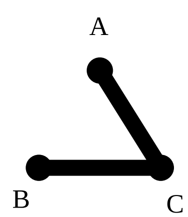
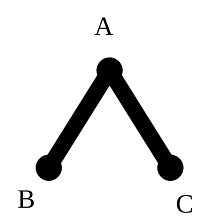
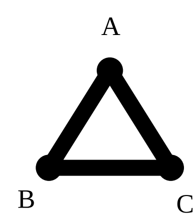
Game 3

 Payoff for A: 0 Payoff for B: 0 Payoff for C: 0	 Payoff for A: 270 Payoff for B: 270 Payoff for C: 0	 Payoff for A: 180 Payoff for B: 0 Payoff for C: 180	 Payoff for A: 0 Payoff for B: 180 Payoff for C: 180
 Payoff for A: 70 Payoff for B: 250 Payoff for C: (20)	 Payoff for A: 40 Payoff for B: 40 Payoff for C: 220	 Payoff for A: 250 Payoff for B: 70 Payoff for C: (20)	 Payoff for A: 130 Payoff for B: 130 Payoff for C: 40

Game 4

 Payoff for A: 0 Payoff for B: 0 Payoff for C: *0*	 Payoff for A: 198 Payoff for B: 198 Payoff for C: 0	 Payoff for A: 162 Payoff for B: 0 Payoff for C: 162	 Payoff for A: 0 Payoff for B: 162 Payoff for C: 162
 Payoff for A: 58 Payoff for B: 220 Payoff for C: 22	 Payoff for A: 46 Payoff for B: 46 Payoff for C: 208	 Payoff for A: 220 Payoff for B: 58 Payoff for C: 22	 Payoff for A: 112 Payoff for B: 112 Payoff for C: 76

Game 5

 Payoff for A: 0 Payoff for B: 0 Payoff for C: 0	 Payoff for A: 243 Payoff for B: 243 Payoff for C: 0	 Payoff for A: 222 Payoff for B: 0 Payoff for C: 222	 Payoff for A: 0 Payoff for B: 267 Payoff for C: 267
 Payoff for A: 43 Payoff for B: 310 Payoff for C: 67	 Payoff for A: 36 Payoff for B: 81 Payoff for C: 303	 Payoff for A: 295 Payoff for B: 73 Payoff for C: 52	 Payoff for A: 117 Payoff for B: 162 Payoff for C: 141

7. References

Aumann, R. and R. Myerson (1988). “Endogenous formation of links between players and of coalitions: An application of the Shapley value,” In: A. E. Roth (ed.) *The Shapley Value: Essays in Honor of Lloyd Shapley*. Cambridge University Press: Cambridge.

Austin-Smith, D. and J. Banks (1988). “Elections, coalitions, and legislative outcomes,” *American Political Science Review,* Vol. 82, 405-422.

Gamson, W. A. (1961). “An experimental test of a theory of coalition formation,” *American Sociological Review* Vol. 26, 565-573.

Gamson, W. A. (1964). “Experimental studies of coalition formation,” In: L. Berkowitz (ed.) *Advances in Experimental Social Psychology*. Academic Press: New York.

Garratt, R. and C.-Z. Qin (2003a). “On cooperation structures resulting from simultaneous proposals,” *Economics Bulletin*, Vol. 3, No. 5, 1-9.

Garratt, R. and C.-Z. Qin (2003b). “On potential maximization as a refinement of Nash equilibrium,” *Economics Bulletin*, Vol. 3 No. 12, 1-11.

Kahan, J. P. and Am. Rapoport (1984). “Theories of Coalition Formation,” Lawrence Erlbaum Associates: Hillsdale New Jersey.

Kalisch, G. K., J. W. Milnor, J. F. Nash and E. D. Nering (1954). “Some experimental n-person Games,” In: R. M. Thrall and C. H. Coombs (eds.) *Decision Processes,* Wiley: New York.

Kelley, H. H. and A. J. Arrowood (1960). “Coalitions in the triad: Critique and experiment,” *Sociometry,* Vol. 23, 231-244.

Laing, J. D. and R. J. Morrison (1973). “Coalitions and payoffs in three-person sequential games: Initial tests of two formal models,” *Journal of Mathematical Sociology,* Vol. 3, 3-26.

Maschler, M. (1965). “Playing an n-person game: An experiment,” Economic Research Program. Research memorandum No. 73, Princeton, New Jersey.

Monderer, D. and L. Shapley (1996). “Potential games,” *Games and Economic Behavior* Vol. 14, 124 –143.

Murnighan, J. K. and A. E. Roth (1977). “The effects of communication and information availability in an experimental study of a three-person game,” *Management Science* Vol. 23, 1336-1348.

Murnighan, J. K. and A. E. Roth (1978). “Large group bargaining in a characteristic function Game,” *Journal of Conflict Resolution* Vol. 22, 299-317.

Myerson, R. (1977). “Graphs and cooperation in games,” *Mathematics of Operations Research,* Vol. 2, 225-229.

Qin, C.-Z. (1996). “Endogenous formation of cooperation structures,” *Journal of Economic Theory,* Vol. 69, 218-226.

Riker, W. H. (1967). “Bargaining in a three-person game,” *American Political Science Review,* Vol. 61, 642-656.

Riker, W. H. (1971). “An experimental examination of formal and informal rules of a three-person game,” In: B. Lieberman (ed.) *Social Choice*. Gordon and Breach: New York.

Riker, W. H. (1972). “Three-person coalitions in three-person games: Experimental verification of the theory of games,” In: J. Herndon and J. L. Brend (eds.) *Mathematical Applications in Political Science*. University of Virginia Press: Charlottesville, VA.

Shapley, L. S. (1953). “A value for n-person games,” In: H. W. Kuhn and A. W. Tucker (eds.) *Contributions to the Theory of Games, Volume II, Annals of Mathematics Studies No. 28*. Princeton University Press: Princeton.

Slikker, M. (2001). "Coalition formation and potential games," *Games and Economic Behavior* Vol. 37, 436-448.

Slikker, M., B. Dutta, A. van den Nouweland and S. Tijs (2000). "Potential maximizers and network formation," *Mathematical Social Sciences* Vol. 39, 55-70.

van den Nouweland, A. (2003). "Models of network formation in cooperative games," mimeo.

Vinacke, W. E. and A. Arkoff (1957). "An experimental study of coalitions in the triad," *American Sociological Review* Vol. 22, 406-415.