

Information channels in labor markets. Personal connections vs. certification.*

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Abstract

Economists and sociologists disagree over markets' potential to take over functions typically performed by networks of personal connections. First among them is the reliable transmission of information. In this paper we begin from a model of labor markets where social ties are stronger between similar individuals, and thus firms employing productive workers prefer to rely on personal referrals by their employees than to hire on the open anonymous market (Montgomery (1991)). However, we allow workers in the anonymous market to engage in a costly action that has the potential to signal their high productivity. We study the extent to which the possibility of signalling reduces the reliance on the network. We find that the network is remarkably resilient - only for a small minority of parameter values does the network disappear. The problem is that to be effective signalling must fulfill two contradictory requirements: unless the signal is extremely precise, it must be expensive, or it is not informative; but it must be cheap, or the network can undercut it.

1 Introduction

Personalized networks, systems of personal connections that function as privileged channels of information and trust, are part of daily experience. In situations where the reliability of information is particularly important - when applying for a job, when needing fresh capital for a new enterprise, when moving to a new country, when substituting for formal enforcement - their role becomes often crucial, either as means of entering formal markets, or indeed as substitutes for these markets. Hence the ethnic enclaves, both residential and professional, in New York City; the economic weight of the Overseas Chinese

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in their countries of residence; the success of Medieval networks of merchants, organized along ethnic or religious lines (the Armenians, the Italians, the Jews, the Dutch..). All of these phenomena have in common the essential role played by the personal network, with its rapidity and its freedom from the constraints of unwieldy procedures.¹

As the function of the network is recognized, an important policy question is the extent to which planned interventions or market mechanisms can substitute for personal connections. The question is important both because the networks are often very successful, and thus it would be good to be able to copy them, and because they are by their nature exclusionary, and thus tend to generate resentment and opposition among those excluded. If the networks could be replicated these delicate distributional problems could be faced.

Economists and sociologists usually disagree on the potential for artificial replication of the networks. Not surprisingly, economists tend to be more optimistic, believing that appropriate market mechanisms, encouraged and supported by appropriate policy where necessary, can substitute for the missing personal channels. Sociologists on the other hand, see the personal, non-anonymous link as the essence of the relation, the fundamental, inescapable reason not only for the truthful transmission of information, but for the “trust” that accompanies the exchange, that trust that can never be approximated by fear of punishment (e.g Tienda and Raijman (2001), discussing Rauch (2001)).

In this paper, we approach the issue by focusing on one channel of information transmission that seems a plausible alternative to personalized exchanges taking place in a network: the possibility of signalling. Individuals who are not part of the network can take a costly action that signals high productivity, the only relevant information in our model. The question we ask is how effective the availability of the signal is in weakening the reliance on the network. We find that the network is extremely resilient - for the great majority of our parameter space, reliance on the network continues and is profitable, even though signalling does take place in the market and typically leads to more precise information than is being revealed in the network. The reason is, ex post at least, obvious: when signalling is informative, it is caught in two contradictory requirements. On one hand, it must be effective in separating the different types - it must induce the more productive type to signal while the less productive type cannot profitably do so. To induce this separation, either the signal is extremely precise, or it must be costly. But at the same time, signalling is competing with reliance on the network - if signalling is costly enough to differentiate among types, then the network is likely to be profitable because it can undercut the cost of information in the market. Indeed, the savings can be enough to expand the network beyond what the acquisition of useful information would dictate, where firms prefer hiring through the network, even when that implies a more than average probability of hiring a less productive worker.

In the next section, we describe the model; in section 3 we describe the

¹See, among many others, Braudel (1982), Greif (1993), Kotkin (1992), Redding (1990), Rauch (2001)

main properties of the network that will be exploited repeatedly in solving the model; section 4 characterizes the equilibria of the model, first without and then with signalling; section 5 summarizes the equilibria, focussing on the effect of signalling on firms' reliance on the network; section 6 concentrates on wages and profits, and section 7 concludes. The Appendix contains some of the proofs.

2 The Model

A model allowing us to study the relative performance of personal connections versus signalling must be very flexible: it must include both a network and a market, which must differ in some substantive measure, and it must capture the equilibrium effects linking the two; it must allow workers and firms to choose between networking and signaling, and it must be tractable enough to allow us to study the results' sensitivity to different costs of signaling and different precisions of both mechanisms. Montgomery (1991) proposed a simple, beautiful model that satisfies all our requirements. We start from Montgomery's set-up and adapt it for our purposes.

There is a large number of identical infinitely-lived firms and, at any period in time, two overlapping generations of workers, each composed of a number of individuals equal to the number of firms. Everything we write below will apply to the limit of this number becoming very large. Workers live two periods and work in the second period of their life. Each firm employs at most one worker. In each generation, half of the workers are productive and produce one unit of output when employed (H workers), and half are unproductive and produce no output (L workers). The two types of workers cannot be distinguished ex ante, and wages cannot be made conditional on production. With some abuse of notation we will call " H firms" firms whose current employee is an H worker, and similarly for " L firms".

Young workers, who are not yet employed, can establish a connection to an older employed worker at no cost. Employed workers' types are not observable outside the firm, but following Montgomery and building on sociologists' concept of "in-breeding", we assume that a young individual will have a higher probability of establishing a link to an older worker of his own type. More precisely, each young individual will be connected to an employed worker of his own type with a probability $\alpha > 1/2$ which is common knowledge. The links are otherwise random. Personal connections can be valuable because firms have the option of hiring their new workers through referrals from their current employees, whose own productivity is known and who, through α , are more likely than not to have connections to young workers of their own type.

If a firm chooses to hire through referrals, its employee transmits the offer to one of the young workers he is connected to (choosing randomly if he has several connections). If the young worker accepts the offer, the contract is concluded and he is hired for the next period. Young workers who either reject the referral offer or do not receive any, will need to find employment in the anonymous market. Before entering the market, however, each young worker has the option

of engaging in a costly action that may announce publicly that he is of type H - for example a worker can attempt to be certified through an exam. The action costs λ and the probability of success is $\beta > 1/2$ if the individual is of type H , and $(1-\beta) < 1/2$ if he is of type L . We call this option "signalling". Workers can borrow λ at no cost against their future labor earnings.² There are two central differences between personal networks and signalling in this model, and we will return to them repeatedly. First, signalling is costly while personal connections are not. Second, personal connections reveal information only locally - i.e. only to the firm employing the referring worker - while signalling reveals information globally, because certification is publicly observable (and verifiable).

Finally, the market opens, and all young workers who have not been hired through referrals, certified and uncertified, offer their labor. Firms still looking for their next employee offer market clearing wages and hire. Because firms can enter the labor market freely, the possible impending entry of new competitors bring their expected profits from hiring in the market to zero. The old generation of workers retires and a new generation of young agents, not yet working, is born.

Our model differs from Montgomery's in three aspects. First, and most important, is the possibility of signalling. The workers' option will affect the equilibrium, and we will be able to address our central question on the robustness and relative performance of personal networks when signalling is available. Second, relative to Montgomery's original model, we have reversed the direction of the personal ties: originating from the young workers and randomly connecting each of them to an older worker, rather than the opposite. The result is that each young worker who chooses to establish a tie - indeed all young workers, since the ties have no cost and can be valuable - will have exactly one connection to an older worker. But because the ties are random, some older workers will have several ties, while some will have none. The difference matters because it affects the competition among firms in making referral offers. We have chosen our modelling strategy because it allows us to move naturally to the case where networking too is costly (Casella, Hanaki, 2004), but it has the added advantage of leading to a very clean analysis where, contrary to Montgomery, we can always derive closed-form solutions. Finally, we have stream-lined the original two-period model, occasionally cumbersome, by adopting the simpler overlapping generations set-up that its logic suggests. Again, this is mostly a matter of taste, although it is also true that the possibility of referrals generates a dynamic link across periods: if having hired an H worker increases the probability of hiring another H worker next period, the value of an H worker goes beyond his immediate productivity. But because this is true whether the H worker is hired in the market or through referrals, the final results are not very sensitive to this change in assumptions.³

Because establishing a personal connection entails no cost and does not prevent the option of signalling later, doing so is a weakly undominated strategy:

²More generally, λ is the amount that will need to be repaid out of future wages if a worker decides to signal. Hence it could include borrowing costs, and a decline in λ could be interpreted as, for example, more generous conditions on loans financing extra schooling.

³A statement we have verified by solving the two-period model with signalling.

if the young worker does not receive a referral offer, or if the offer is inferior to the market wage, he can access the market.⁴ Thus we will study scenarios where all young workers establish a personal connection. This is an immediate implication of the assumption that personal links are created freely, as a natural side-effect of daily social interaction. Different hypotheses are sensible too, and will distinguish the type of personal connection in question: spontaneous and social in the case we study here, v.s deliberate and typically professional in the case of costly links. One important effect of zero-cost ties is that there is no equilibrium where only young H workers network. Unless $\alpha = 1$, in which case the two types are effectively segregated, personal connections cannot lead to a separation of types: it will always be the case that some L workers will be hired through referrals. This stands in contrast to the transmission of information through signalling which, as we shall see, can support equilibria with full separation of types. It is this asymmetry that lends interest to our question: will firms still choose to rely on personal connections, when the information transmitted through signalling can be much more precise?

The solution of the model is straightforward, once the stochastic properties of the network have been characterized. We begin then by studying the network, and in particular the density of connections at each node that determines the probabilities of contacts between young and old workers of the two types.

3 The Network

As we mentioned above, the stochastic nature of the connections implies that some young workers will find themselves linked to an older worker who has several other links, while some older workers will have none. Consider then the probability that a young worker of type i ($i = H, L$) will receive a referral offer if the firm employing the older worker he is connected to chooses to hire through referrals - a probability we call p_i .

With a very large number of workers and firms, the density of connections at each old worker node can be studied independently of the rest of the network. Suppose that the size of each generation of workers is $2N$ and equals the number of firms. Any young H worker has probability (α/N) of connecting to any specific old H worker, while the same probability equals $(1 - \alpha)/N$ for any young L worker. For large N , the number of ties connecting any individual old worker to young workers of either type is described by a Poisson distribution: the probability that an old H worker has x ties to young H workers, for example, is given by $\frac{\alpha^x}{x!}e^{-\alpha}$. It follows that the probability that the old H worker i is connected to has k additional connections, a probability we denote by $\gamma_{k,H}$, can

⁴This eliminates equilibria that seem rather trivial: where no networking takes place, because of pessimistic off-equilibrium beliefs on the part of the firms on the quality of any networking worker, or where young workers randomize over networking with just the probabilities that lead H firms to expect a referral worker to be equivalent to a market worker (thereby offering the market wage, and justifying workers' indifference).

be written as:

$$\gamma_{k,H} = \sum_{j=0}^k \left[\frac{\alpha^{k-j}}{(k-j)!} e^{-\alpha} \frac{(1-\alpha)^j}{j!} e^{-(1-\alpha)} \right] = \sum_{j=0}^k \left[e^{-1} \frac{\alpha^{k-j}(1-\alpha)^j}{(k-j)!j!} \right] = \frac{e^{-1}}{k!} \quad (1)$$

where we have used:⁵

$$\frac{1}{k!} = \sum_{j=0}^k \frac{\alpha^{(k-j)}(1-\alpha)^j}{j!(k-j)!}$$

Therefore:

$$\begin{aligned} p_H &= \lim_{N \rightarrow \infty} \left[1 - \left(\frac{2N-1}{2N} \gamma_{2N-1,H} + \frac{2N-2}{2N-1} \gamma_{2N-2,H} + \dots + \frac{1}{2} \gamma_{1,H} \right) \right] = \\ &= \lim_{N \rightarrow \infty} \left(1 - \sum_{k=1}^{2N-1} \frac{k}{k+1} \gamma_{k,H} \right) = 1 - e^{-1} \left[\lim_{N \rightarrow \infty} \left(\sum_{k=1}^{2N-1} \frac{k}{(k+1)!} \right) \right] \end{aligned}$$

or:

$$p_H = 1 - e^{-1} \quad (2)$$

The details of the derivation are in the Appendix (where in particular we show that the error introduced by evaluating the expressions in the limit, as N approaches infinity, is negligible). As intuition suggests, if all young workers network the probability of receiving a referral, conditional on the firm using referrals, is independent of the parameter α : the value of α affects the composition of the pool of other young workers with which i must compete for a referral, but not their expected number. Thus it is also the case that $p_H = p_L$, and in what follows we will identify both terms by $p \equiv 1 - e^{-1}$. But recall once again that these probabilities are conditional on the firms using referrals - and H and L young workers will expect to be connected to firms of different types, generally making different decisions about their reliance on referrals. The characterization of the equilibria of the model, below, will make this observation precise.

The stochastic nature of the network determines two further probabilities that will be exploited repeatedly. It is convenient to derive them here. We just observed that some old workers will have no ties to any young worker. Thus, from the point of view of the firm, what is the probability that their current employee of type i is able to recommend at least one worker for possible hiring? In other words, what is the probability that the firm's current employee has at least one connection? Call such a probability ϕ_i where i is the current employee's type. An old H worker is linked to a young H with probability α/N (since there are N old H workers). Thus the probability that he has no connection to any young H is given by $(1 - \alpha/N)^N$, the exponent now reflecting the fact that there are N young H workers. The probability that he has no connection to either a

⁵Since: $\sum_{j=0}^k \binom{k}{j} \alpha^{(k-j)} (1-\alpha)^j = \sum_{j=0}^k \frac{k!}{j!(k-j)!} \alpha^{(k-j)} (1-\alpha)^j = 1$

young H or a young L is then $(1 - \alpha/N)^N [1 - (1 - \alpha)/N]^N$ and thus ϕ_H , the probability that he has at least one connection, can be approximated by:

$$\phi_H = \lim_{N \rightarrow \infty} \left[1 - \left(1 - \frac{\alpha}{N}\right)^N \left(1 - \frac{1 - \alpha}{N}\right)^N \right] = 1 - e^{-1} \quad (3)$$

Similarly:

$$\phi_L = 1 - e^{-1} \quad (4)$$

and again we will identify both ϕ_H and ϕ_L by $\phi \equiv 1 - e^{-1}$.⁶

Finally, conditional on having at least one connection, what is the probability that an old H worker making a random referral will choose a young worker of type H ? Suppose that the old H worker has k connections, and call such a probability ζ_k . Recall that the probability of having k connections can be approximated by γ_{kH} in equation (1). Hence, for very large N , we can write:

$$\begin{aligned} \zeta_k &= \left(\frac{1}{\gamma_{kH}} \right) \sum_{j=0}^k \left[\binom{k-j}{k} \frac{\alpha^{k-j}}{(k-j)!} e^{-\alpha} \frac{(1-\alpha)^j}{j!} e^{-(1-\alpha)} \right] = \\ &= \sum_{j=0}^k \left[\binom{k-j}{k} \frac{\alpha^{k-j}}{(k-j)!} \frac{(1-\alpha)^j}{j!} \right] k! \end{aligned}$$

Using:

$$\sum_{j=0}^k \left[(k-j) \frac{\alpha^{k-j}}{(k-j)!} \frac{(1-\alpha)^j}{j!} \right] = \frac{\alpha}{(k-1)!}$$

the expression simplifies to:

$$\zeta_k = \left(\frac{\alpha}{k(k-1)!} \right) k! = \alpha \quad \forall k \quad (5)$$

4 The Equilibria of the Model

Given our focus on equilibria where all workers establish personal connections, the only decision workers have to take is whether to attempt certification if they are not hired through referrals. The firms, on the other hand, have to decide whether or not to attempt to hire through referrals of their current employee, and if so, what wage to offer. Each worker i 's strategy is the probability with which he chooses to signal, s_i , while each firm j 's strategy is the probability with which the firm chooses to hire through referrals (conditional on its current employee being connected to at least one young worker) r_j and the referral wage w_{rj} . We focus on symmetrical equilibria where all workers of the same type and all firms employing the same type of workers follow the same strategy.

⁶Notice that there is no reason why p and ϕ should be equal in general, and indeed they would differ if the either group of workers did not network with probability 1. (See Casella and Hanaki, 2004).

In addition, given the stationarity of our set-up, we restrict attention to steady state strategies that remain unchanged over time.⁷ If we use the terminology " $\forall i \in H$ " to indicate all workers of type H , and " $\forall j \in H$ " to indicate all firms employing workers of type H (and similarly for L), then: $s_{Hi} = s_H \forall i \in H$, $s_{Li} = s_L \forall i \in L$, $r_{Hj} = r_H$ and $w_{rHj} = w_{rH} \forall j \in H$, $r_{Lj} = r_L$ and $w_{rLj} = w_{rL} \forall j \in L$. We neglect the time subscript to emphasize that these strategies hold for all times. Call w_C and w_U the wage for certified and uncertified workers in the anonymous market. An equilibrium is a set of strategies $\{\sigma_H, \sigma_L, r_H, r_L, w_{rH}, w_{rL}\}$, a pair of market clearing wages $\{w_C, w_U\}$ and a set of beliefs about the workers' types such that no worker and no firm can strictly gain from choosing a strategy different from that assigned to his or its type, the labor market clears, and all beliefs are rational.

4.1 Equilibrium without Signalling

There is always an equilibrium where none of the workers signals: off-equilibrium beliefs on the part of the firms according to which anybody deviating must be an L type are sufficient to support the workers' rational decision not to signal. Because this equilibrium replicates Montgomery's original focus, we derive it in some detail, as an example of the basic methodology that we extend later to equilibria where signalling occurs. As in Montgomery's analysis, in this equilibrium only firms employing H workers choose to use referrals and obtain positive profits. More precisely, we can state:

Lemma 1. *If in equilibrium $s_H = s_L = 0$, then $r_H = 1$ and $r_L = 0$.*

Proof. We show here that the Lemma describes an equilibrium, and leave to the Appendix the proof that the equilibrium is unique.

Consider the problem from the perspective of the firms. The type of worker they hire in any given period is valuable both because of his productivity and because of his ties to younger workers in the future which will enable the firm to hire through referrals, if advantageous. Call V_H the value of hiring a H worker, and Π_H the firm's expected profits from referrals from a current H employee (and similarly for V_L and Π_L). Then:

$$\begin{aligned} V_H &= 1 + \max\{0, \delta\Pi_H\} \\ V_L &= \max\{0, \delta\Pi_L\} \end{aligned} \tag{6}$$

where δ is the rate with which expected profits in the next hiring cycle are discounted. Keeping in mind that profits from hiring in the market must be zero, expected profits from referrals must equal the probability of hiring a worker

⁷Workers live only two periods and make each decision - whether to establish a personal connection and whether to signal - only once. Firms on the other hand are infinitely-lived and must decide whether or not to hire through referrals every period. By restricting attention to steady states, we reduce the set of strategies for a firm of each type to a single probability, ignoring time.

whose value, combining productivity and future referrals, is larger than the referrals wage. If we call h_{LH} the probability of hiring a L worker through referrals from a current H employee (and similarly for the other types), then expected profits from referrals are:

$$\Pi_H = h_{HH}(V_H - w_{rH}) + h_{LH}(V_L - w_{rH}) \quad (7)$$

$$\Pi_L = h_{HL}(V_H - w_{rL}) + h_{LL}(V_L - w_{rL})$$

where the results of the previous section imply:

$$h_{HH} = h_{LL} = \alpha\phi = \alpha(1 - e^{-1}) \quad (8)$$

$$h_{LH} = h_{HL} = (1 - \alpha)\phi = (1 - \alpha)(1 - e^{-1})$$

The strategies described in the lemma are an equilibrium if $\Pi_H \geq 0$ and $\Pi_L \leq 0$ (and thus $V_L = 0$).

Both referral and market wages, and finally profits from hiring through referrals, depend on the relative frequency of the two types of workers in the anonymous market, and thus on the firms' strategies. Only H firms use referrals, and because our modelling assumptions assign all bargaining power to the firms, set the referral wage at the level of the market wage: $w_{rH} = w_U$.⁸ The firms' zero profit condition for market hirings ensures that the market wage reflects the market workers' expected value, and since $V_L = 0$, $w_U = h_{HU}V_H$, where h_{HU} denotes the probability of hiring a worker of type H in the market. Π_H simplifies to:

$$\Pi_H = (1 - e^{-1})V_H(\alpha - h_{HU}) \quad (9)$$

The probability of hiring a H worker in the market, h_{HU} , reflects the relative frequency of H workers in the market:⁹

$$h_{HU} = \frac{1 - \alpha p}{1 - \alpha p + 1 - (1 - \alpha)p} = \frac{e - \alpha(e - 1)}{1 + e} \quad (10)$$

It can now be verified immediately that $\alpha > h_{HU}$ for all $\alpha > 1/2$, and thus, recalling $V_H \geq 1$, $\Pi_H > 0$: attempting to hire through referrals is indeed a best response strategy for a firm employing an H worker, when all other H firms also use referrals, while none of the L firms do.

To verify that $\{r_H = 1, r_L = 0\}$ are equilibrium strategies, consider the potential for deviation for a L firm. The firm can consider a once-for-all deviation - where the firm makes a referral offer in the current period, but no

⁸It is possible to think of the referral relationship as a bilateral bargaining problem and study different rules according to which the firm and the new hire share the surplus (see Casella and Hanaki, 2004). The possibility of multiple ties to a single old worker makes the assumption in the text the most natural, but the qualitative results of the paper are not sensitive to plausible alternatives.

⁹ $h_{HU} = \text{prob}(H|U) = \frac{\text{prob}(U|H)\text{prob}(H)}{\text{prob}(U|H)\text{prob}(H) + \text{prob}(U|L)\text{prob}(L)}$ where $\text{prob}(H) = \text{prob}(L) = 1/2$.

referrals will take place in the next hiring cycle if the next employee is again a L type; a steady state deviation - where the firm expects always to use referrals from L employees; or in fact deviation over any subset of future periods. Given our focus on stationary equilibria and our environment, where a single firm is negligible and rationally ignores any reaction to its own change of strategy, the gain from repeated deviations is only the appropriately discounted sum of the gain from a once-for-all deviation. Ruling out the latter is thus sufficient to rule out any other pattern of deviation.¹⁰ Profitable once-for-all deviation requires $\Pi_L > 0$, where Π_L is evaluated at $V_L = 0$ (the value of hiring a L worker is 0, since referrals will not be used again). If a L firm makes a referral offer it must offer the market wage, $w_{rL} = w_U = h_{HU}V_H$, unaffected by the deviation of a single firm. Thus:

$$\Pi_L = (1 - e^{-1})[V_H(1 - \alpha - h_{HU})]$$

Since $V_H > 1$, $\Pi_L > 0$ requires $1 - \alpha > h_{HU}$, impossible for all $\alpha > 1/2$. Deviation is not profitable, and hiring in the market is indeed the best response strategy for a firm employing a L worker when all H firms use referrals and no other L firm does: $\{r_H = 1, r_L = 0\}$ are equilibrium strategies. \square

Substituting (9) and (10) in (7) we can derive the equilibrium value of hiring a H worker, V_H :

$$V_H = \frac{1 + e}{(1 + e) - \delta(e - 1)(2\alpha - 1)} \quad (11)$$

and thus the wage at which all workers are hired:

$$w_U = w_{rH} = \frac{e - \alpha(e - 1)}{(1 + e) - \delta(e - 1)(2\alpha - 1)} \quad (12)$$

Expected profits are zero for L firms, who hire on the open market, but are positive for H firms and equal to expected profits from referral hiring:

$$\Pi_H = \frac{(e - 1)(2\alpha - 1)}{(1 + e) - \delta(e - 1)(2\alpha - 1)} \quad (13)$$

As expected, the value of a H worker is higher than his personal productivity ($V_H > 1$) for all $\alpha > 1/2$ because employing a H worker leads to a higher than random chance of hiring a H worker in the following period. This effect is more important the higher is α - the higher the probability that connected agents are of the same type - and the higher is δ - the less the future is discounted. The wage on the other hand is declining in α : it would equal $1/2$ if α equalled $1/2$

¹⁰To compare the potential for once-for-all vs. steady-state deviation, consider Π_L where we allow $V_L > 0$ (if it is advantageous to use referrals in the future, the value of hiring a L worker is positive):

$$\Pi_L = (1 - e^{-1})[\alpha V_L + V_H(1 - \alpha - h_{HU})]$$

or, since $V_L = \delta \Pi_L$ if $\Pi_L > 0$:

$$\Pi_L [1 - (1 - e^{-1})\delta\alpha] = (1 - e^{-1})[V_H(1 - \alpha - h_{HU})]$$

because referral hiring by H firms would then not affect the average productivity of the market pool, but its value falls monotonically at higher α values reflecting the increased selection of young H workers out of the market. Because the wage reflects not only the probability but also the value of hiring a H worker in the market, it is increasing in δ - it is higher the less the future is discounted. Note that the expected wage is identical for H and L workers, a consequence of our granting all bargaining power to the firm when hiring through referrals.¹¹ The profit from referral hiring accrues entirely to the firm and is given by Π_H , an expression increasing in α and δ , like V_H and for the same reasons.

With the only noticeable difference that the direction of the network links we have posited strengthens the bargaining position of the firms, this equilibrium essentially replicates Montgomery's analysis and results. How sensitive these conclusions are to the absence of signalling is the question addressed in the remainder of the paper.

4.2 Equilibria with Signalling

Consider a scenario where firms extend referral offers with probabilities r_H and r_L , and workers who have not been hired through referrals attempt certification with probabilities s_H and s_L . We can describe equilibrium wages and firms' profits for generic values of these probabilities. Given wages and profits, different equilibria can be posited and the firms and workers' incentive compatibility constraints identify the range of $\{\alpha, \beta, \lambda, \delta\}$ values for which each equilibrium exists.

The value of hiring a worker of type H or L continues to be given by (6), and the profits from referrals by (7). Any firm which attempts to hire through referrals offers the lowest wage acceptable to a H worker, thus $w_{rH} = w_{rL} \equiv w_r$.¹² But now, when some of the workers attempt certification, two different markets exist, a market for certified workers which clears at wage w_C , and a market for uncertified workers, which clears at wage w_U . The extent to which the referral wage reflects the wage for uncertified or certified workers (net of certification costs) depends on the strategy followed by H workers left in the market. Since a H worker attempting certification is successful with probability β , the lowest referral wage he would accept must be:

$$w_r = \begin{cases} \beta w_c + (1 - \beta)w_U - \lambda & \text{if } s_H = 1 \\ w_U & \text{otherwise} \end{cases} \quad (14)$$

The market wages reflect the probabilities and the values of hiring workers of either type. Defining, as earlier, h_{HU} as the probability of hiring a H worker

¹¹If workers hired through referrals had more bargaining power than our model grants them, equilibrium strategies would be unchanged but their own wage would differ from the market wage, and they would be able to appropriate part of the surplus from their personal connection to a currently employed H employee. The market wage would be lower, reflecting the lower value to the firm of hiring a H worker, but it would again depend negatively on α .

¹²Note that even when L firms choose to use referrals, and thus $V_L > 0$, they must be driven by the goal of hiring a H worker since L workers are not productive.

in the market for uncertified workers (and similarly h_{LU}), and h_{HC} and h_{LC} as the corresponding probabilities in the market for certified workers:

$$w_U = h_{HU}V_H + h_{LU}V_L = h_{HU}V_H + (1 - h_{HU})V_L = h_{HU}(V_H - V_L) + V_L$$

$$w_C = h_{HC}V_H + h_{LC}V_L = h_{HC}V_H + (1 - h_{HC})V_L = h_{HC}(V_H - V_L) + V_L$$

These probabilities are given by the relative frequencies of workers of either type. If we define $prob(C|H)$ ($prob(C|L)$) as the probability of being in the market for certified workers conditional on being a H type (a L type), then:

$$h_{HC} = \frac{prob(C|H)prob(H)}{prob(C|H)prob(H) + prob(C|L)prob(L)}$$

Note that $prob(C|H)$ is the joint probability of not having been hired through referrals and being certified, conditional on being a H worker (and correspondingly for $prob(C|L)$). Thus:

$$prob(C|H) = [1 - r_H\alpha p - r_L(1 - \alpha)p](s_H\beta)$$

$$prob(C|L) = [1 - r_H(1 - \alpha)p - r_L\alpha p]s_L(1 - \beta)$$

Therefore:

$$w_C = \frac{[1 - r_H\alpha p - r_L(1 - \alpha)p](s_H\beta)(V_H - V_L)}{[1 - r_H\alpha p - r_L(1 - \alpha)p](s_H\beta) + [1 - r_H(1 - \alpha)p - r_L\alpha p]s_L(1 - \beta)} + V_L \quad (15)$$

Similarly:

$$h_{HU} = \frac{prob(U|H)prob(H)}{prob(U|H)prob(H) + prob(U|L)prob(L)}$$

and:

$$prob(U|H) = [1 - r_H\alpha p - r_L(1 - \alpha)p](1 - s_H\beta)$$

$$prob(U|L) = [1 - r_H(1 - \alpha)p - r_L\alpha p][1 - s_L(1 - \beta)]$$

Therefore:

$$w_U = \frac{[1 - r_H\alpha p - r_L(1 - \alpha)p](1 - s_H\beta)(V_H - V_L)}{[1 - r_H\alpha p - r_L(1 - \alpha)p](1 - s_H\beta) + [1 - r_H(1 - \alpha)p - r_L\alpha p][1 - s_L(1 - \beta)]} + V_L \quad (16)$$

Recall from (2) that $p = (1 - e^{-1})$.

We can now write the incentive compatibility constraints for firms and workers. Firms will use referrals only if it profitable to do so, or, taking (7) and (8) into account:

$$r_H > 0 \Leftrightarrow \Pi_H = (1 - e^{-1})[\alpha V_H + (1 - \alpha)V_L - w_r] \geq 0 \quad (17)$$

$$r_L > 0 \Leftrightarrow \Pi_L = (1 - e^{-1})[(1 - \alpha)V_H + \alpha V_L - w_r] \geq 0$$

Workers attempt certification if the cost of doing so is compensated by the difference in the wages, or, given the different probabilities of success for H and L workers:

$$\begin{aligned}
 s_H &> 0 \Leftrightarrow \beta w_C + (1 - \beta)w_U - \lambda \geq w_U \\
 s_L &> 0 \Leftrightarrow (1 - \beta)w_C + \beta w_U - \lambda \geq w_U
 \end{aligned}
 \tag{18}$$

The characterization of the economy is complete: the three different wages are given by (14), (15) and (16); the firms' profits from referrals by equations (7); the value to the firm of hiring a worker of either type by (6) and finally the incentive compatibility constraints by (17) and (18). If all incentive compatibility constraints hold with strict inequality, the equilibrium is in pure strategies, and the probabilities $\{r_H, r_L, s_H, s_L\}$ assume only 0 or 1 values; otherwise mixed strategies are possible.

Once a candidate set of equilibrium strategies is posited, the incentive compatibility constraints, evaluated at the correct wages, identify the range of parameter values for which the strategies are indeed an equilibrium, if one such range exists. Different equilibria can exist for different parameter values, or indeed multiple equilibria can occur over the same range of parameters. The number of candidate equilibrium regimes is large,¹³ but some combinations of strategies can be ruled out ex ante. We can state:

Lemma 2. (i) If $r_L > 0$, then $r_H = 1$; (ii) If $s_L > 0$, then $s_H = 1$; (iii) if $s_H \in (0, 1)$, then $r_H = 1$.

The lemma is proved in the Appendix, but its intuition is clear: it is always better to hire a H worker than a L worker, and since the probability of doing so through referrals is always higher for H firms, the incentive to use referrals must always be strictly higher for H firms. Similarly, if any worker incurs the positive costs of attempting certification, the market wage for certified workers must always be strictly higher than the wage for uncertified workers, and since the probability of success, upon attempting certification, is always higher for H workers, the incentive to signal must be strictly higher for H workers. Finally, we know from Lemma 1 that H firms always rely on referrals in the absence of signalling. If $s_H \in (0, 1)$, then $s_L = 0$ (by remark (ii)), and firms can offer as referral wage the market wage for uncertified workers. Because some H workers, and only some H workers, exit the uncertified workers pool, the referrals wage is lower than in the case of no signalling, while the expected productivity of a referral hire for H firms remain constant at α . If referrals were profitable in the absence of signalling, they must be profitable when $s_H \in (0, 1)$.

It is possible to analyze systematically each candidate equilibrium and the range of parameter values that support it, but given the equations derived above the procedure is mechanical and rather tedious. We limit ourselves to solving

¹³Without imposing any constraint, there are 3^4 combinations of candidate equilibrium strategies. Of these, 2^4 correspond to pure strategy equilibria and the remainder have at least one set of agents randomizing.

one example in this section, and focussing on the more interesting results in the next.

The question motivating us is the role of signalling when personal connections provide an alternative channel for information transmission. Because networking comes at no cost, all workers recur to it, and for all $\alpha < 1$ information cannot be transmitted precisely: there will always be some referral offers made to young L workers. Thus a natural starting point, and an interesting example to study, is the existence of a separating equilibrium in the case of signalling: can our model support an equilibrium where only H workers attempt to be certified? With $\beta < 1$ some H workers will fail, but all certified workers will necessarily be H , and firms can, if they so choose, guarantee themselves a new H worker. If workers differentiate themselves through signalling, will firms choose not to use referrals?

Consider then a candidate equilibrium where $s_H = 1$, $s_L = 0$, $r_H = 0$, $r_L = 0$.¹⁴ In such an equilibrium, where no profits are available to firms through referral hirings, $V_H = 1$ and $V_L = 0$. The wages for certified and uncertified workers are given by (15) and (16) above: because all certified workers are of type H and their value is fully reflected in the market wage, $w_C = 1$, while $w_U = (1 - \beta)/(2 - \beta)$. The incentive compatibility constraints for the workers (18) impose a first set of constraints on parameters:

$$\begin{aligned} s_H = 1 &\Rightarrow \frac{\beta}{2 - \beta} \geq \lambda \\ s_L = 0 &\Rightarrow \frac{1 - \beta}{2 - \beta} \leq \lambda \end{aligned} \tag{19}$$

With $\beta > 1/2$, there is a non-empty range of λ values for which the two constraints can both be satisfied.

Consider the potential for deviation by the firms. As earlier, it is sufficient to focus on once-for-all deviations. Any firm willing to hire through referrals would need to offer a wage acceptable to H workers, who prefer to attempt certification. Hence, from (14), $w_r = \beta + (1 - \beta)^2/(2 - \beta) - \lambda$. If firms do not use referrals, $V_L = 0$, and $V_H = 1$, or $\Pi_H = (1 - e^{-1})[\alpha - w_r]$ and $\Pi_L = (1 - e^{-1})[(1 - \alpha) - w_r]$. Ruling out deviation to referrals by a H firm is sufficient to rule out deviation by a L firm. Thus equilibrium requires:

$$r_H = 0 \Rightarrow \lambda \leq \beta + \frac{(1 - \beta)^2}{2 - \beta} - \alpha = \frac{1 - 2\alpha + \beta\alpha}{2 - \beta} \tag{20}$$

Because (19) and (20) must both be satisfied, this equilibrium can exist only if $(1 - \beta) \leq (1 - 2\alpha + \beta\alpha)$, or $\beta \geq 2\alpha/(1 + \alpha)$.

We can thus conclude with three observations. The first is that, for a suitable range of parameter values, the candidate equilibrium indeed exists: signalling

¹⁴The only other candidate equilibrium where signalling leads to separation has strategies $s_H \in (0, 1)$ and $s_L = 0$. But then we know by Lemma 2 that $r_H = 1$: if an equilibrium exists, referrals must take place.

provides separation of the workers' types, in the limited sense that all certified workers must be of type H , and firms refrain from using referrals and prefer to hire in the market, an option they would not take in the absence of signalling. Signalling provides information and supplants the use of personal referrals. Notice that hiring in either market, for certified or uncertified workers, leads to zero firm profits, and thus firms are indifferent between paying the premium attached to certification, and guaranteeing themselves a H worker, or hiring a cheaper uncertified worker with a lower but positive probability of being a H type. But hiring an uncertified worker in this equilibrium is not equivalent to hiring an uncertified worker when no signalling takes place: the more precise information reflected in the wage makes this option superior to referrals in this equilibrium but inferior when no signalling takes place.

The second observation, however, is that the range of parameter values for which the equilibrium exists is surprisingly limited. Since $\alpha > 1/2$, β cannot be smaller than $2/3$, and λ must fall in a correspondingly narrow interval. For example, if $\alpha = 3/4$, the equilibrium requires $\beta \geq 6/7$ and the acceptable range for λ goes from the unique value $\lambda = 1/8$ when $\beta = 6/7$ to the range $\lambda \in [0, 1/4]$ when $\beta = 1$.

Finally, the tight range of acceptable parameter values is mostly dictated by the incentive compatibility constraints ensuring that firms refrain from making referrals. In their absence, all values of $\beta > 1/2$ could sustain the equilibrium, and, for comparison, the acceptable range of λ would go from $\lambda \in [1/8, 3/4]$ when $\beta = 6/7$ to $\lambda \in [0, 1]$ when $\beta = 1$. This suggests two conjectures: first it seems likely that other equilibria exist, for ranges of parameter values that may be different or overlapping the range identified here, where signalling provides separation between the workers' types but firms still choose to make referral offers.¹⁵ Second, notice that the firms' incentive compatibility constraints select values of λ close to the lower bound of the interval satisfying the workers' incentive constraints. A higher λ would induce firms to deviate. This is an interesting point that begins to suggest the resilience of referrals in our model: when β is not too close to unity, signalling can be informative only if λ is high enough to induce separation of workers' types. But if λ is high, signalling is informative but also expensive: referral hiring can undercut it and be less precise but preferable. We make this conjecture more precise in the next section.

5 Reliance on Personal Connections when Signalling is Informative.

Having verified that signalling can come to eliminate the recourse to personal referrals, we now focus on the opposite question. Under what conditions is the availability of a signalling mechanism compatible with continued reliance

¹⁵The existence of such equilibria does not follow immediately from a violation of the firms' incentive compatibility constraints in the candidate equilibrium studied. The composition of the markets, the wages and thus all constraints would reflect the different firms' strategies.

on referrals? And if reliance on referrals continues to exist, does signalling affect referrals indirectly, by influencing the quality of referral hiring, i.e. the percentage of referrals falling on workers of different type?

Some scenarios are trivial: if the cost of signalling λ is very low, and the probability of success β not very different across types (β close to $1/2$), then all workers may choose to signal, but certification is uninformative and H firms continue to prefer hiring through referrals (and the more so the higher is α). The question becomes interesting if we restrict attention to equilibria where signalling is informative, where we use the strict criterion that only H workers attempt to signal: $s_H > 0$, $s_L = 0$ - by hiring certified workers firms can guarantee themselves H employees, something they can never do when relying on referrals (but for the extreme case $\alpha = 1$).

The first result is summarized in the following proposition:

Proposition 1. *For all $\alpha \in (1/2, 1)$, there exist equilibria where signalling is perfectly informative and firms strictly prefer to hire through referrals.*

The proof, in the Appendix, amounts to showing the existence of one such equilibrium. In particular, we focus on strategies $\{s_H = 1, s_L = 0, r_H = 1, r_L = 1\}$ - H workers in the market prefer to signal, L workers do not, and all firms prefer to hire through referrals - in the case $\beta = \alpha$, and show that for all $\alpha \in (1/2, 1)$ there exists a non-empty range of λ values for which such strategies are indeed equilibrium strategies. The case $\beta = \alpha$ is chosen primarily because it simplifies the algebra and the presentation of the results, but it is also a natural reference point: the exogenous precision of the personal connections equals the exogenous precision of the signal. The assumption does not imply that personal connections and signalling are constrained to be equally informative because the signalling mechanism has one additional element, the cost λ , which leads to self-selection in the decision to engage in signalling - hence the possibility that signalling be perfectly informative even for β very close to $1/2$, as in the equilibrium described here. Because we are evaluating equilibria over the whole range of possible λ value, the restriction $\beta = \alpha$ is compatible with a large number of scenarios, while leaving the model ex ante unbiased. We will use repeatedly as our reference case.

Although the proof selects one particular example, in fact there are several equilibrium regimes where signalling is informative and firms prefer to use referrals - equilibria where only H firms use referrals, or where a share of them do so while L firms hire in the market, or where all H firms and a share of L use referrals, or equilibria where only a share of H workers signal, while L workers do not. The range of parameter values for which referral hiring takes place and is imprecise, while firms could guarantee themselves H workers is far from limited or special. Figure 1 shows equilibrium strategies in $\lambda - \alpha$ space in the case $\beta = \alpha$ and $\delta = 0.90$. The model is obviously very stylized, but we can read parameter values keeping in mind that the unit of time is the hiring cycle, or more precisely the length of employment of a worker at a single firm. The difference in productivity between a productive and unproductive worker over that cycle is normalized to 1. Thus if we think of the time horizon as about five

years, $\delta = 0.90$ corresponds to a yearly discount rate of 2 percent.¹⁶ The parameter λ , the fixed cost of certification, can be thought of as the cost of college education, and should be read relative to 1: $\lambda = 1$ in our model represents the case where the cost of college education equals the total difference in productivity between a productive and a non-productive worker over 5 years. As we saw, in the presence of referrals the value of a worker to the firm includes the value of future referral hiring, and thus the premium that firms may be willing to pay to college educated workers may be well above the one-cycle difference in productivity, implying in turn that acceptable values of λ in our model may well be above 1.

Figure 1a depicts workers' strategies, and figure 1b firms' strategies. As mentioned earlier, there is always an equilibrium without signalling, supported by firms' negative off-equilibrium beliefs, and lemma 1 describes the unique equilibrium strategies for firms in that case. In Figure 1a, we have allowed signalling to take place whenever it can be supported in equilibrium, with rational beliefs.¹⁷

The figures show that there is a large area of the parameter space for which signalling takes place and is fully informative. Workers signal if the cost λ is not too large; if the precision of the signal (β , which in the figure equals α) is high, only H workers signal, and they do so for a large range of λ values; if on the contrary β is low, then signalling can occur only if λ is low, and for most of these values signalling is not informative because the incentives to signal are very similar for H and L workers.¹⁸ Unless β is high, informative signalling requires intermediate values for λ , low enough to be affordable by H workers, but high enough to discourage L workers. As for referrals, the immediate observation from Figure 1b is that there is no equilibrium where firms do not use referrals: over the entire parameters range, referrals never cease to be profitable for firms, whether signalling is informative or not, whether λ is high or low, whether α and β are high or low. The figure relies on $\alpha = \beta$, and thus, as made clear by the equilibrium with no referrals characterized in the previous section, the result is not general. But given our emphasis on the $\alpha = \beta$ case as plausible unbiased reference, we emphasize this conclusion in a separate proposition, proved in the Appendix:

Proposition 2. *Suppose $\alpha = \beta$. Then there exist equilibria where signalling is perfectly informative, but there are no equilibria where referrals are not used.*

Upon a moment of reflection, the reason is obvious: unless β is high, when signalling is informative it is also rather expensive (λ is high), expensive enough

¹⁶Our results are effectively insensitive to the specific value we assign to δ , for all $\delta \in (0, 1)$.

¹⁷No signalling is the unique equilibrium in the area left white: a worker of either type would not want to attempt certification even if firms, off-equilibrium, expected any certified worker to be of type H .

¹⁸When β is close to $1/2$, the probability of success upon signalling are very similar for H and L workers, but the wage premium for certified workers is small. At high β , the premium is high but the probability of success for a L worker is low. The incentive to signal for L workers is maximum for intermediate values of β .

to allow H workers to differentiate themselves. Market wages for certified workers compensate them for the cost of certification, and thus tend to be high. Firms may well prefer the less informative but cheaper reliance on referrals. In the trade-off between information and cost, cost becomes the deciding factor. The conclusion always holds when $\alpha = \beta$ because informative signalling can be cheap only when its precision is high, but if the precision of referral hirings is also correspondingly high, then the cost advantage, small as it may be, remains the dominant consideration.

The firms' equilibrium strategies are particularly interesting when β (and α) are low. There is then a range of parameter values where, if signalling occurs in equilibrium, *all* firms hire through referrals (the black area in figure 1b), including L firms, which never use referrals in the absence of signalling because referrals give them a more than even chance of hiring an unproductive worker. As stated in the following proposition (proved in the Appendix):

Proposition 3. *Signalling, including informative signalling, may induce L firms to hire through referrals.*

Once again, the reason is the market premium for certified workers: when α is close to $1/2$, the bias inherent in referral hiring is not too costly for L firms, while referrals allow all firms to avoid the premium for certified workers while still hiring workers of higher average productivity than in the uncertified market. What is remarkable about this equilibrium is that the existence of signalling, and in particular of informative signalling, leads to an *increase* in firms' reliance on referrals. In fact, if $\alpha = \beta$ signalling always weakly increases the expected proportion of workers hired through referrals: either referral hiring is used by H firms alone, as in the absence of signalling, or it is used by both H firms and some positive share of L firms (possibly all).

This observation allows us to address a question we raised at the beginning of this section: how does signalling affect the quality of referral hiring? The answer is an immediate corollary to Proposition 3:

Corollary to Proposition 3. *Signalling always weakly lowers the quality of referral hirings.*

Since only H firms recur to referrals in the absence of signalling, any expansion of referral hiring to L firms necessarily lowers the average productivity of workers hired through referrals. Paradoxically then, signalling not only does not eliminate referrals, but in fact may lead to increased reliance on personal connections, and lower expected productivity of referral hires.

We have phrased our discussion mostly in terms of the $\alpha = \beta$ case, but the results generalize predictably when we loosen this constraint. Because the only real complication remains generating transparent three dimensional figures, we specialize the model in a different direction. Figure 2 describes equilibrium strategies for workers (figure 2a) and firms (figure 2b) for arbitrary values of β and λ when we fix α at 0.75 (and δ at 0.90, as earlier). Figure 2a is very similar to figure 1a: signalling can occur in equilibrium only if the cost λ is not too high, where the highest acceptable λ increases with β ; if λ is low, both types of

workers have an incentive to signal.¹⁹ The difference is in figure 2b. First, of all, there are now equilibria without referrals, when β is high (higher than α) and λ low - the area left white in the lower right corner of the figure. This is the equilibrium with no referrals and informative signalling discussed in section 4.2, and a second equilibrium with no referrals and every worker signalling when λ is particularly low. The low λ makes the firms' savings from referral hirings negligible, and the high β makes signalling a good channel of information, even in the absence of full separation.²⁰ It is clear from the figure though that even in the area where $\beta > \alpha$ ($\beta > 0.75$), the absence of referrals remain a rather special case. Second, the equilibrium where both H and L firms rely on referrals now is concentrated mostly in an area of high β and λ values. With $\alpha = 0.75$, L firms are discouraged from using referrals unless the savings from doing so are substantial: a range of high λ where signalling still occurs, and a range of high β where the certification premium is high.

Armed with these figures, we can ask other questions. For example, what is the effect of making certification less expensive - reducing λ ? If college becomes more affordable to all and an objective measure of productivity is then made public, will personal connections become less important in labor markets? According to our model, the answer is not immediate, because it depends both on the precision of the information inherent in the personal network and on the standards used in certification. Only when certification remains substantially more informative than personal connections, even in the absence of the self-selection induced by high certification costs - when $\beta \gg \alpha$ - does the cheap availability of public information reduce the reliance on personal connections. But the opposite option of making certification *more* expensive - increasing λ - relying then on the information provided by self-selection, never reduces the use of personal referrals. On the contrary, it tends to increase the use of referrals, even when they are known to lead to a less than average probability of hiring productive workers. Similarly, what is the effect of increasing the standards for certification - increasing β ? Again, the answer depends on other factors too. Only when certification is both rigorous *and* cheap does it become a preferred option to personal referrals.

¹⁹The sensitivity of the acceptable λ range to β is reduced, relative to the case where $\alpha = \beta$: at low β but constant α (and H firms only using referrals), equilibrium market wages are lower than they would be if α equalled β , keeping certification valuable and raising the highest acceptable λ ; at high β , the effect is reversed: with constant α , smaller than β , market wages are now smaller and the incentive to signal is reduced, reducing in turn the highest acceptable λ .

²⁰When only H workers signal, all certified workers must be H types, but the composition of the uncertified labor pool and the difference in wages still depend on β . Similarly, when all workers signal, some L workers are expected to become certified, but the composition of the two labor pools, and the wages, remain significantly different if β is high enough.

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6 Appendix

Probability that a young worker receives a referral offer, conditional on firms using referrals.

The exact probability that an old worker of type H (of which N exist) has k connections in addition to i 's is given by:

$$\tilde{\gamma}_{k,H} = \sum_{j=0}^k \left[\binom{N-1}{c} \left(\frac{\alpha}{N}\right)^c \left(1 - \frac{\alpha}{N}\right)^{N-1-c} \binom{N}{d} \left(\frac{1-\alpha}{N}\right)^d \left(1 - \frac{1-\alpha}{N}\right)^{N-d} \right] \quad (\text{A1})$$

where

$$\begin{aligned} c &\equiv \min(k-j, N-1) \\ d &\equiv \min(j, N) \end{aligned}$$

The Poisson approximation $\gamma_{k,H}$

$$\gamma_{k,H} = \sum_{j=0}^k \left[\frac{\alpha^{k-j}}{(k-j)!} e^{-\alpha} \frac{(1-\alpha)^j}{j!} e^{-(1-\alpha)} \right] = \sum_{j=0}^k \left[e^{-1} \frac{\alpha^{k-j} (1-\alpha)^j}{(k-j)! j!} \right] = \frac{e^{-1}}{k!} \quad (\text{A2})$$

is the limit of $(.)$ as N tend to infinity and has well established bounds of error. For example, it can be shown (Feller, 1957, chapter 6) that

$$e^{-\alpha} \frac{\alpha^c}{c!} e^{-\frac{\alpha^2}{N-c} - \frac{\alpha^2}{N-\alpha}} < \binom{N}{c} \left(\frac{\alpha}{N}\right)^c \left(1 - \frac{\alpha}{N}\right)^{N-c} < e^{-\alpha} \frac{\alpha^c}{c!} e^{c\frac{\alpha}{N}}.$$

implying that:

$$\lim_{N \rightarrow \infty} \tilde{\gamma}_{k,H} = \lim_{N \rightarrow \infty} \gamma_{k,H}$$

and therefore:

$$\lim_{N \rightarrow \infty} \left(1 - \sum_{k=1}^{2N-1} \frac{k}{k+1} \tilde{\gamma}_{k,H} \right) = \lim_{N \rightarrow \infty} \left(1 - \sum_{k=1}^{2N-1} \frac{k}{k+1} \gamma_{k,H} \right)$$

- i.e. the error introduced by the approximation does not distort the sum.

The last part of the derivation in the text states without proof that:

$$\lim_{N \rightarrow \infty} \left(\sum_{k=1}^{2N-1} \frac{k}{(k+1)!} \right) = 1$$

To see this, begin by expanding e^x .²¹

$$\begin{aligned} e^x &= \sum_{k=0}^{\infty} \frac{x^k}{k!}, \\ &= 1 + x + \sum_{k=2}^{\infty} \frac{x^k}{k!}, \\ &= 1 + x + x \sum_{k=1}^{\infty} \frac{x^k}{(k+1)!}. \end{aligned}$$

²¹We thank Peter S. Dodds for this proof.

Differentiating both sides with respect to x :

$$(e^x)' = e^x = 1 + \sum_{k=1}^{\infty} \frac{x^k}{(k+1)!} + x \sum_{k=1}^{\infty} \frac{kx^k}{(k+1)!}.$$

or:

$$e^x = 1 + (e^x - x - 1)/x + x \sum_{k=1}^{\infty} \frac{kx^k}{(k+1)!}.$$

Setting $x = 1$:

$$e = 1 + (e - 1 - 1)/1 + \sum_{k=1}^{\infty} \frac{k}{(k+1)!},$$

or:

$$\sum_{k=1}^{\infty} \frac{k}{(k+1)!} = 1$$

□

Lemma 1. Proof of uniqueness.

When $s_H = s_L = 0$, we can write:

$$\Pi_H = \phi(V_H - V_L)(\alpha - h_{HU}) \tag{A3}$$

$$\Pi_L = \phi(V_H - V_L)(1 - \alpha - h_{HU})$$

and

$$h_{HU} = \frac{1 - r_H \alpha p - r_L(1 - \alpha)p}{2 - r_H p - r_L p} \tag{A4}$$

(see the derivation for arbitrary $\{r_H, r_L, s_H, s_L\}$ in section 4.2).

We proceed in two stages. First we show that $(V_H - V_L) > 0$ always. Then we show that $\Pi_H > 0$ and $\Pi_L < 0$ always. (1) If $r_L < 1$, $V_L = 0$ and $V_H \geq 1$, hence $(V_H - V_L) > 0$. If $r_L = 1$, there are two possibilities: (i) if $r_H < 1$, then $\Pi_H = 0$ and $V_H = 1$. In this case, $V_L = \delta\phi(1 - \alpha - h_{HU})/(1 - \delta\alpha) < 1$ for all $\delta \leq 1$. Hence $(V_H - V_L) > 0$. (ii) If $r_H = 1$ and $\Pi_H > 0$, then, by (6) and (A3) above, $(V_H - V_L) = 1 + \delta\phi(V_H - V_L)(2\alpha - 1) > 0$. Thus we can conclude that in all cases, $(V_H - V_L) > 0$. (2) From (A4), $\partial h_{HU}/\partial r_H < 0$, and $\partial h_{HU}/\partial r_L > 0$. Thus h_{HU} is maximal at $\{r_H = 0, r_L = 1\}$ and minimal at $\{r_H = 1, r_L = 0\}$: $h_{HU} \in [\underline{h}_{HU}, \overline{h}_{HU}]$, where $\underline{h}_{HU} \equiv (1 - \alpha p)/(2 - p)$ and $\overline{h}_{HU} \equiv [1 - (1 - \alpha)p]/(2 - p)$. But $\alpha > \underline{h}_{HU}$ and $(1 - \alpha) < \underline{h}_{HU}$ for all $\alpha > 1/2$. Since $(V_H - V_L) > 0$, we can then conclude that $\Pi_H > 0$ and $\Pi_L < 0$ always. If an equilibrium exists, it must have $r_H = 1$ and $r_L = 0$. □

Proof of Lemma 2. (i). Suppose $r_L \in (0, 1)$ and $\Pi_L = 0$. Then, by (6), $V_L = 0$; but since $V_H \geq 1$, $V_H > V_L$. By (7) then $\Pi_H > 0$ and $r_H = 1$. Suppose $r_L = 1$ and $\Pi_L > 0$. Could it be that $\Pi_H \leq 0$? If so, $V_H = 1$ and $V_L = \delta(1 - e^{-1})(1 - \alpha + \alpha V_L - w_r)$, or $V_L = \delta(1 - e^{-1})(1 - \alpha - w_r)/(1 - \delta\alpha) < 1$

for all $\delta \leq 1$ and $w_r \geq 0$. But if $V_H > V_L$ it must be that $\Pi_H > \Pi_L$ by (7), a contradiction. Hence if $r_L = 1$ and $\Pi_L > 0$, $\Pi_H > 0$ and $r_H = 1$. (ii) By (18), signalling by either type of workers requires $w_C > w_U$ for all $\lambda > 0$. But then, since $\beta > 1/2$, the incentive to signal is always strictly higher for a H worker than a L worker, and the conclusion follows. (iii) If $s_H \in (0, 1)$, then $w_r = w_U = h_{HU}(V_H - V_L) + V_L$ and $\Pi_H = \phi(V_H - V_L)(\alpha - h_{HU})$. By result (ii) in the Lemma, if $s_H \in (0, 1)$, then $s_L = 0$, hence

$$h_{HU} = \frac{(1 - r_H \alpha p - r_L(1 - \alpha)p)(1 - s_H \beta)}{(1 - r_H \alpha p - r_L(1 - \alpha)p)(1 - s_H \beta) + (1 - r_H(1 - \alpha)p - r_L \alpha p)} \quad (\text{A5})$$

It is then easy to verify that $\partial h_{HU} / \partial s_H < 0$, and since $\alpha > h_{HU}(s_H = 0)$, a fortiori $\alpha > h_{HU}(s_H > 0)$, $\Pi_H > 0$ and $r_H = 1$. \square

Proof of Proposition 1. To prove the proposition, it is sufficient to show the existence of some such equilibrium. Consider the candidate scenario $\{s_H = 1, s_L = 0, r_H = 1, r_L = 1\}$, where H workers in the market always signal but all firms always prefer hiring through referrals. Suppose $\beta = \alpha$. Equations (6) (7) (14) (15) and (16) yield $V_H, V_L, \Pi_H, \Pi_L, w_r, w_U$, and w_C . The incentive compatibility constraints (17) and (18) allow us to conclude that such scenario is an equilibrium if and only if $\lambda \in [\underline{\lambda}, \bar{\lambda}]$ where:

$$\begin{aligned} \underline{\lambda} &= \max\left\{\frac{(1 - \alpha)(1 - h_{UH})}{1 - \delta\phi(2\alpha - 1)}, \frac{2\alpha - 1 + (1 - \alpha)h_{UH}}{1 - \delta\phi(2\alpha - 1)}\right\} \\ \bar{\lambda} &= \frac{\alpha(1 - h_{UH})}{1 - \delta\phi(2\alpha - 1)} \end{aligned}$$

and, by (A5):

$$h_{UH} = \frac{1 - \alpha}{2 - \alpha}$$

The unique upper bound is given by the requirement that λ should be low enough for H workers to be willing to signal; and the lower bounds by the incentive compatibility constraints on the L workers and the L firms (λ should be high enough for L workers not to signal and L firms to prefer referrals)²² It is not difficult to verify that for all $\alpha \in (1/2, 1)$, $\bar{\lambda}$ is larger than either of the two lower bounds (the relevant lower bound is the first for $\alpha \leq 2 - \sqrt{2}$ and the second for higher α). The incentive compatibility constraints are satisfied and the scenario is an equilibrium for all $\alpha \in (1/2, 1)$. \square

Proof of Proposition 2. The existence of equilibria where signalling is perfectly informative has been established in Proposition 1. Consider now possible equilibria where referrals are not used. By Lemma 1, if $s_H = s_L = 0$, then $r_H = 1$; and by Lemma 2, if $s_H \in (0, 1)$, then $r_H = 1$, and if $s_L > 0$, then $s_H = 1$. Thus the only workers' strategies compatible with the absence of referrals are: $\{s_H = 1, s_L = 0\}$, $\{s_H = 1, s_L \in (0, 1)\}$, and $\{s_H = 1, s_L = 1\}$. The

²²If the incentive compatibility constraint ensuring that the L firms' strategy is a best response is satisfied, the equivalent constraint on H firms is satisfied automatically.

first case is the only one of the three possibilities where signalling is perfectly informative and referrals do not take place. It was studied in detail in Section 4.2, and we showed there that the equilibrium requires $\beta \geq 2\alpha/(1 + \alpha) > \alpha \forall \alpha \in (1/2, 1)$, and thus is ruled out if $\alpha = \beta$. Proving that the latter two candidate scenarios cannot be equilibria amounts once again to showing that the incentive compatibility constraints must be violated. Deriving such constraints requires working through the appropriate wage and profits equations for each scenario, but given equations (6), (7), (14), (15) and (16) the derivation of the constraints is trivial, and we leave the details to the reader. In the second scenario, $\{s_H = 1, s_L \in (0, 1), r_H = r_L = 0\}$, the two binding constraints are that L workers must be indifferent about signalling, while H firms must prefer not to use referrals, or:

$$\lambda = \frac{(1 - \beta)(h_{HC} - h_{HU})}{1 - \delta(1 - e^{-1})[\alpha - (2\beta - 1)h_{HC} - 2(1 - \beta)h_{UC}]} \quad (\text{A6})$$

$$\lambda < \beta(h_{HC} - h_{HU}) + h_{HU} - \alpha \equiv \bar{\lambda} \quad (\text{A7})$$

where:

$$h_{HC} = \frac{\beta}{\beta + (1 - \beta)s_L}$$

$$h_{HU} = \frac{1 - \beta}{1 + (1 - \beta)(1 - s_L)}$$

For any given λ , (A6) identifies the equilibrium s_L , as long as (A7) is satisfied. Substituting $\alpha = \beta$ and (A7) in (A6), we can write:

$$\lambda = \frac{(1 - \beta)(h_{HC} - h_{HU})}{1 + \delta(1 - e^{-1})[(1 - \beta)(h_{HC} - h_{HU}) - \bar{\lambda}]}$$

or:

$$\bar{\lambda} > \frac{(1 - \beta)(h_{HC} - h_{HU})}{1 + \delta(1 - e^{-1})[(1 - \beta)(h_{HC} - h_{HU}) - \bar{\lambda}]} \quad (\text{A8})$$

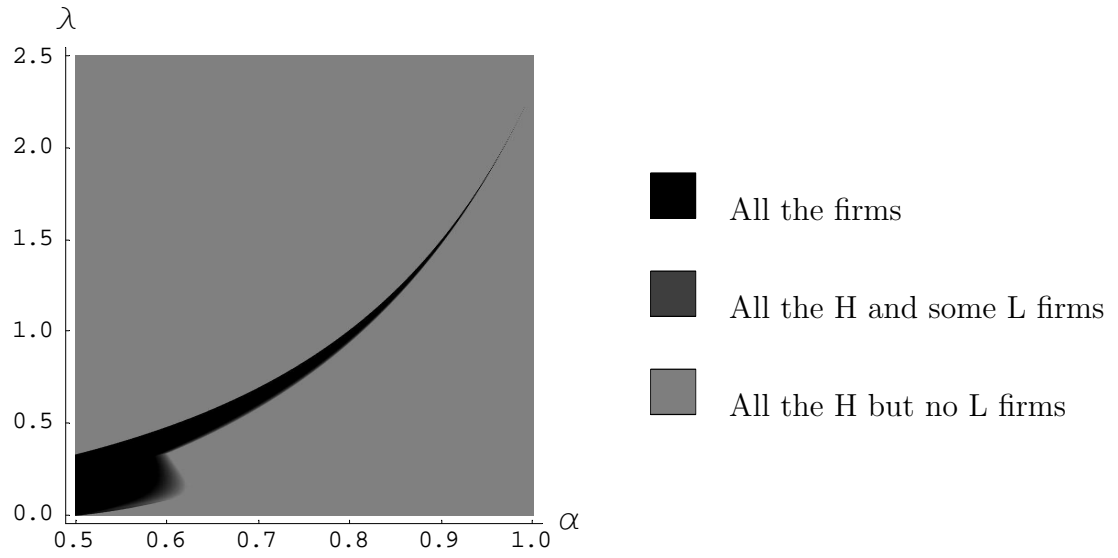
It is not difficult to verify that for all $s_L \in (0, 1)$ $(1 - \beta)(h_{HC} - h_{HU}) > \bar{\lambda}$. Hence (A8) implies

$$\delta(1 - e^{-1})[(1 - \beta)(h_{HC} - h_{HU}) - \bar{\lambda}]\bar{\lambda} > [(1 - \beta)(h_{HC} - h_{HU}) - \bar{\lambda}]$$

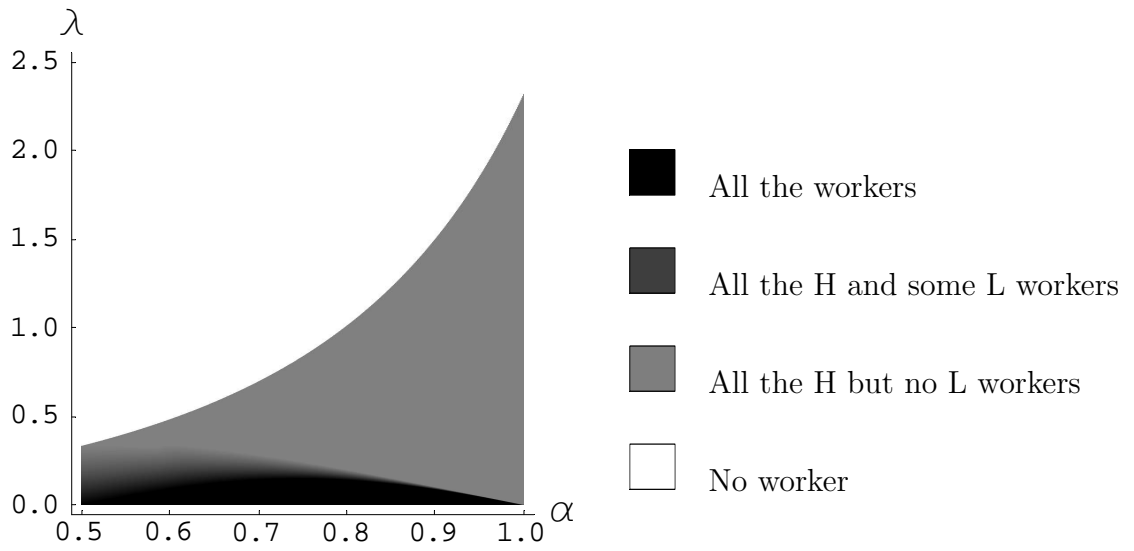
a condition that can only be satisfied for $\bar{\lambda} > 1$. But for all $s_L \in (0, 1)$ $h_{HC} > h_{HU}$, implying $\bar{\lambda} < h_{HC} - \beta < 1$. The scenario cannot be an equilibrium when $\alpha = \beta$. In the third scenario, $\{s_H = 1, s_L = 1, r_H = r_L = 0\}$, if H firms do not use referrals it must be that: $\lambda < \beta(2\beta - 1) + 1 - \beta - \alpha$. If $\alpha = \beta$ the constraint becomes $\lambda < (\beta - 1)(2\beta - 1) < 0$, which is violated for all $\lambda > 0$, $\beta \in (1/2, 1)$.

Proof of Proposition 3. The same argument used to prove Proposition 1 proves Proposition 3. \square

Equilibrium Referral

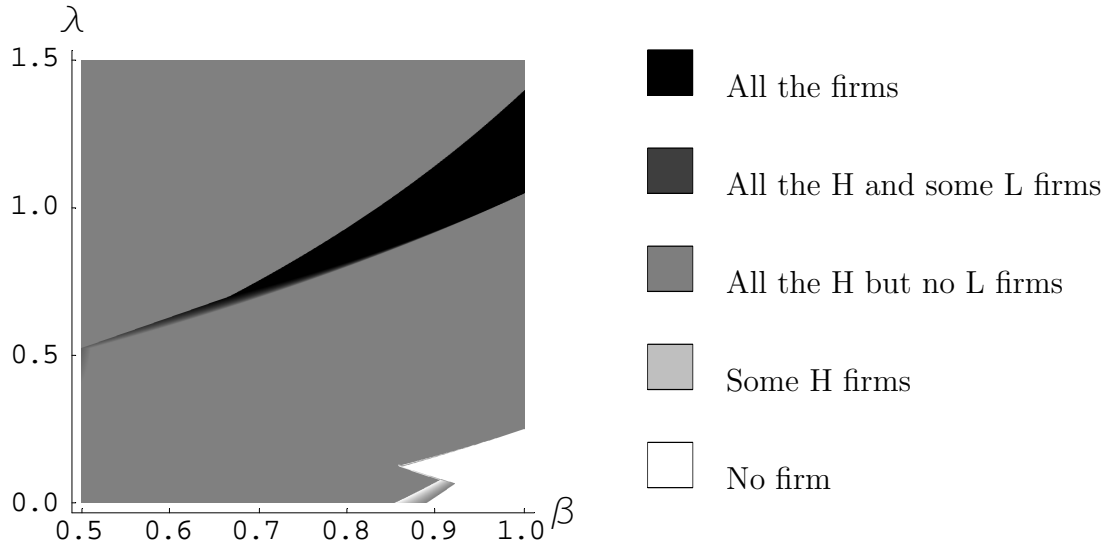


Equilibrium Signalling

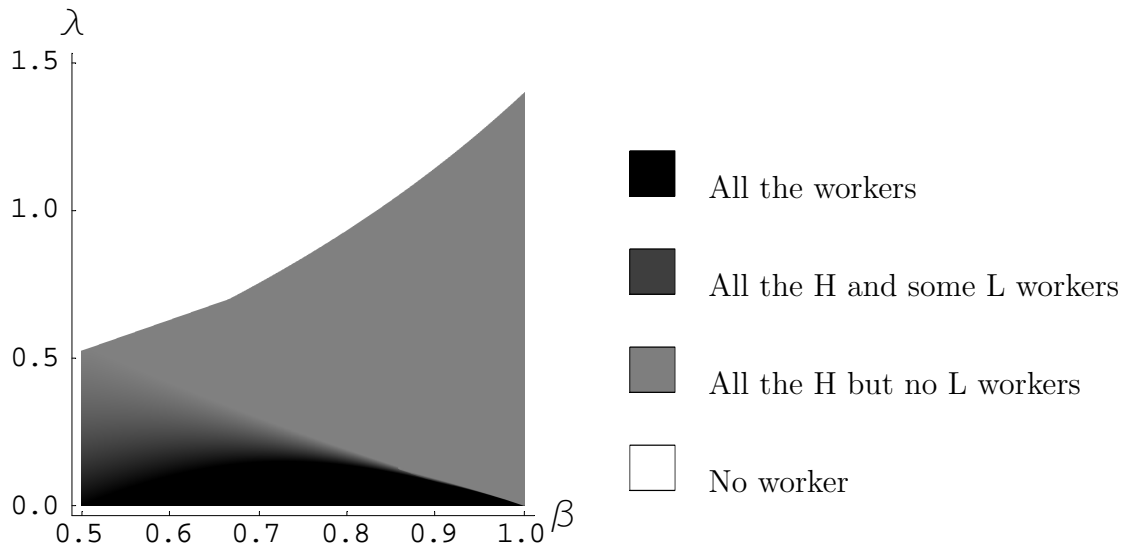


Equilibrium referral (top) and signalling (bottom) in $\alpha - \lambda$ space for $\alpha = \beta$ and $\delta = 0.90$

Equilibrium Referral



Equilibrium Signalling



Equilibrium referral (top) and signalling (bottom) in $\beta - \lambda$ space for $\alpha = 0.75$ and $\delta = 0.90$