Towards Stable Routines Within a Group :Improving and Satisficing Enough by Exploration-Exploitation on an Unknown Landscape

or: **Organizational Routines and Resistances to Change Within a Group**: **the "Worthwhile Decision to Move Approach** ".

by Antoine Soubeyran (Greqam) and Bruno Soubeyran (Institut Fourier,Grenoble)

Summary:

Our main question is :do agents or groups of agents optimize in real life? If not, what do they do?

We consider the problem when there are resistances to change within a group, ie costs to choose, when choosing is an action, a move.

This paper generalizes for a group and all its subgroups the dynamic "Worthwhile To Move Decision Process" of Attouch-Soubeyran (2004) where at each stage, an agent chooses to move when it is "worthwhile to do so", ie when estimated incremental advantages to move are greater than estimated costs to move. These agents face an unknown landscape (goal , utility function) that they can only discover step by step, by exploration around at some costs. They explore around a state (performance) until they find some new state (performance) which, first, improve the goal and consequently exhibits incremental advantages to move, and then generates costs to move lower than incremental advantages.A stable routine is a position where it is not worthwhile to move around .It is a maximal element of the "Worthwhile to Move Relation". It offers resistance to change, moving from it to an other position, for at least one subgroup.

Our Process of Exploration-Exploitation modelizes a Punctuated Dynamic, ie a succession of static periods of exploitation (temporary routines) and exploration, followed by a dynamic period to move from a temporary routine to an other one.

We examine convergence of a worthwhile process towards stable routines and their existence. High local costs to move, exploring and improving enough locally, and no unexpected upward jumps (upper semi continuity) are the main three hypothesis which drive our results. No compacity assumptions are needed.

We show how our Dynamic "Worthwhile To Move Process" generalizes and give a Cognitive Version of important theorems in Approximate Optimization and Fixed Point Theory, like the celebrated Ekeland Epsilon Variational Theorem and the Carisiti Fixed Point Theorem.

We give an application to Potential Games where stable Routines appear to be equilibrium.

Resistance to Change and Inertia

Resistance to change within a Group or an Organization represents some form of Inertia within a dynamic decision process. Agents and even more groups change only with difficulty. But why change is difficult ? Organizational Ecology defines Inertia as change which is slow relative to environment (Hannan and Freeman, 1984,Rumelt). This comes from specialized investments in physical assets and social structures. Organizations have ”cores” which are very difficult to change relative more peripheral elements. Evolutionary Economics explains Inertia via Bounded Rationality, Routines, and Tacitness.

(Nelson-Winter,1982). Routines bound skills and capabilities. They are the skill set and the memory of the agent. In Management Sciences the main sources of Inertia (Resistance to Change) are five. Dulled Motivation-Failed Creative Response-Political Deadlocks-Action Disconnects and Distorted Perception.

1) Dulled Motivation describes the lack of sufficient motivation to change, due to the abandonment of costly sunk specific investments.

2) Failed Creative Responses concerns the difficulty to choose a direction because of the complexity of the choice, the speed of change (things happen too fast), inhibition and reactive mind-set (problems are natural and inevitable), or inadequate strategic vision.

3) Political Deadlocks come from the three main sources of disagreement among agents: difference in personal interest, difference in beliefs, and difference in fundamental values.

4) Action Disconnects comes from leadership inaction (lack of vision, lack of leading by example, attachment to statu quo, embedding in routines in complex process with great tacit aspects, where changing one part of the process requires to change a lot of other parts).

5) Distorted Perception comes from Myopia, (the unability to look into future with clarity), Denial (a defensive behavior, which is the rejection of information that is contrary to what is desired or what is believed to be true). Denial mays stem from hubris or from fear. Information Filtering rejects information which is unpopular, unpleasant or contrary to doctrine. Grooved Thinking rejects information which deviate too much from orthodoxy and mental habits.

Uncertainty, complexity, sparce informations, divergences of interests represent bounds which make decision making not very easy in an organization (Rojot, 2004). Substantive Rationality supposes that the agent or a group knows the whole set of possible choices, and is able to anticipate all the consequences of his choices. Then the agent or the group has to selects the best solution within the whole set of possible choices. He maximizes. Simon (1987) has objected that an agent or a group does not know his landscape around and cannot calculate all things. Agents have limited physical and intellectual capabilities. They examine solutions sequentially and stop to try to improve their choices when they have founded a solution which give them a minimal degree of satisfaction, but not an optimun. For an agent and even more for a group, his choice set and his goal are not given. They must be built step by step, after several experimentation (exploration), by a process of cognitive simplification and concept elaboration which evolves along the process of choice and decision making. The formulation of problems are linked to their process of resolution. A decision is a move, ie a process which starts from the identification of a problem, first searchs for solutions among those which are known, then searchs for new solutions and tries to evaluate their feasibility, chooses of one of them, including to change nothing and to repeat the previous action.

Ambiguity is always present in an organization. Goals are more or less fuzzy and coherent, and links between actions and consequences are not clear.

Following Lindblom (1959) two aspects are fundamental for decision making in an organization:

i) The ”**Instrumentalism Principle**”: within a Bounded Rational Process the ways of doing and the dynamic process used to reach the initial goals modify the goals during the Process. Ways of doing and goals are interrelated.

ii) The ”**Incrementalism Principle**” :starting from a given position, the agent or the Group moves step by step, exploring around this given position, to reach a new position, and start again exploration around this new position. Choices or decisions are incremental and anchored , each step, to a given position, the previous one. Only a limited number of alternatives are examined around at each step, and a lot of alternatives are ignored. Choosing which alternatives not to explore is an important part of the decision process. Sub-groups play a major role to built the goals of the organization which are negociated.

To be able to modelize such a Difficulty to Change for an agent or a group, we will adopt a framework based on Bounded (Procedural) Rationality assumptions.

We propose an original Punctuated Dynamic model of Exploration-Exploitation where the group moves from one static temporary routine phase to a new static one, switching via a dynamic moving phase (see Attouch-Soubeyran, 2004). In the present paper we extend this model to the case of a Group and its Subgroups , adding Dynamic Rewarding and Sharing Rules to the model.

To help the reader we list in this introduction the basic ingredients of this ”Simple Model of Bounded Rationality”.The basic model for ”One Agent” ω or for a Group ”as a whole” Ω is the following :

1) An ”Initial Concept Elaboration” phase where the group clarifies his list of unsolved problems P, (his needs), the list or vector of criteria $i \in I$ (or aspects, caracteristics, influences) necessary to define the various degrees of resolutions of all these problems. The vector $x \in X$ of degrees of resolution is the state of the model. The feasible choices are the feasible actions or moves, from one state of resolution $x \in X$ to an other one $y \in X$ .They are $\alpha = (x \to y) \in \Lambda$.

2) The initial goal, or utility, or valence $g(x,\Omega) \in V$, where $V = R$ or $V = R^n$,or more generally a Complete Lattice Banach space.This goal is supposed to be known, once some initial performance $x \in X$ have been reached. Starting, each step, from a new given degree of resolution (performance) $x \in X$, the landscape around x, ie $y \in N(x) \subset X \to g(y,\Omega) \in V = R$ is unknown. This is a fundamental point to be noticed. It drives the rest of the model. Usually agents and groups do not know their utility function. They must discover it, step by step.

3) Several unsatisfied needs, (ie unsolved problems , partial or imperfect resolution) which generate some frustration ; This frustration gives the motivation to improve the goal from $g(x,\Omega)$ to some higher level $g(y,\Omega) > g(x,\Omega)$.

4) The necessity to decide to spend some costs $K(x) \geq 0$ to explore around x, within a local exploration subset $E(x,K(x)) \subset X$ whose size depends of $K(x)$.

5) The ability to compare advantages and costs to move $A(x,y,\Omega)$ and $C(x,y,\Omega)$ for all new performance $y \in E(x,K(x))$ within the local exploration set .

6) A Punctuated Dynamic made of a succession of static phases of exploration -exploitation (x) , (y) of lenght $t(x)$, $t(y)$ where the agent or the group both exploits his ability to get the gain $g(x,\Omega)$ or $g(y,\Omega)$ and explores to be able to improve the next period from $g(x,\Omega)$ to $g(y,\Omega) > g(x,\Omega)$. These static phases of exploitation -exploration are linked by dynamic moving phases (x,y) from x to y. During the static phases exploration consists in estimation, simulation, essays -errors, but decision to move from one performance x to a new performance y are post poned to moving periods.

6) The evaluation of advantages to move (anchoring effects) as incremental advantages $A(x,y,\Omega) = t(y)[g(y,\Omega) - g(x,\Omega)]$, where $t(y)$ is the lenght of the next exploitation phase.

7) The evaluation of costs to move $C(x,y,\Omega)$, time spend to move, $t(x,y) \geq 0$, as well as unit costs to move $c(x,y) = C(x,y)/t(x,y)$.

7) The decision to move is adopted if it exists some explored performance $y \in E(x,K(x))$ such that estimated advantages to move are greater than estimated costs to

move: $A(x,y,\Omega) > C(x,y,\Omega)$.

7) For a group and its subgroups the model adds the definition of the family $\mathcal{F}$ of feasible subgroups $M \subset \Omega$ of the group Ω which can form. Then the model defines Rewarding and Sharing Rules for each subgroup M, ie a list or a map of advantages and costs to move.

For each feasible given potential move $(x,y) \in X \times X$, sharing or rewarding rules are the maps:

$M \in \mathcal{F} \to A(x,y,M) \in R$,

$M \in \mathcal{F} \to C(x,y,M) \in R_+$

$M \in \mathcal{F} \to g(y,M) \in R$.

They can be defined as vectors where the index set is the family of subgroups $M \in \mathcal{F}$: $A(x,y) = (A(x,y,M), M \in \mathcal{F})$, $C(x,y) = (C(x,y,M), M \in \mathcal{F})$ and $g(y) = (g(y,M), M \in \mathcal{F})$.

8) The definition of the"Worthwhile to Move Relation" for the group as a whole, or for the group and all

its subgroups :$y \geq_W x \Leftrightarrow A(x,y,M) > C(x,y,M)$, for all $M \in \mathcal{F} \Leftrightarrow A(x,y) > C(x,y)$.

The explored portion of the worthwhile to move relation is

$x \in X \to W^E(x) = \{y \in E(x,K(x)), A(x,y) \geq C(x,y)\}$.

9) A Stable Routine $x^* \in X$ is a maximal element of this relation, ie a performance where $W^E(x^*) = \{x^*\}$.

In the case of a group and its subgroup this means that for any "potential move" from x^* to a new one $y \neq x^*$, there is exists a feasible subgroup of agents $M = M(x^*, y) \in \mathcal{F}$ such that for this subgroup advantages to move are strictly lower than costs to move :$A(x,y,M) < C(x,y,M)$.For one agent the result is stronger: he prefers to stay there."No move are worthwhile" for him.

Part 1 examines very quickly the case of one agent or a group as a whole (see Attouch-Soubeyran, 2004). Part 2 extends the model to a group and its subgroups. The paper examines both "convergence towards" and existence of stable routines without traditional concavity and compactness assumptions.

The three main hypothesis which drive our existence and convergence results are:

i) A complete metric space of performances X (more generally a uniform complete space).

ii) A bounded from above and upper semi continuous hidden goal $g(y) \in V$, where V is a Riesz- Kakutani space ($V = R$, or $V = R^m$ for one agent, or $V = L$, the space of signed measures, for a group and its subgroups).

iii) Unit costs to move $C(x,y)/d(x,y)$ are high locally (ie C is continuous with respect to the distance).

and Costs to move are continuous with respect to y.

An "Improving Enough Process" for all agents drives the convergence process and the existence result.

This implies "enough exploration" and an "enough equitable rewarding and sharing rule".

In Part 2, as a by product we offer a generalization of the celebrated Ekeland Theorem in the multi dimensional setting of a Kakutani-Riesz space.A Riesz space is a complete Lattice Banach space.In this general context our model proposes also a new version of Potential Games using our generalization of the celebrated Carisiti theorem.

PART I

A" Worthwhile To Move Incremental

Process" For An Agent or A Group.

Consider a group of agents $\Omega = \{\omega_1, \omega_2, ..., \omega_n\}$ which have several problems to solve (needs to satisfy) $p \in P = \{1, 2, ..., m\}$ .These problems come from unsatisfaction "of not enough" and aspiration " to more" when some problems remain partially solved, ie when some needs are not fully satisfied and generate frustration.

To each problem p the group can attach a degree of resolution $x_i^p \in [0, \overline{x_i}^p]$ with respect to a list I of elementary "influences variables", or aspects (criteria, caracteristics, elements of evaluation) $i \in I$.

Let I the list of criteria $I = \{1, 2, ..., m\}$ which caracterizes the different points of vue or aspects $i \in I$ choosen to evaluate the degree of resolution of all these problems.

The degree of resolution of a problem $p \in P$ with respect to some criteria $i \in I$ is x_i^p.Let $x = (x_i^p)_{(i,p) \in I \times P} \in X$ the matrix of degree of resolution x_i^p of all problems p with respect to the choosen list of criteria $i \in I$. In Management Science such a matrix is close to a "**decision matrix**"if the x_i^p are the intensities of the influence variables $i \in I$. The space of such "decision matrix" is the state space X. The state of the decision process is $x \in X$.

A choice is an action, a move from one initial state x to an other one y (ie an edge between two nodes or states). Several actions can allow to move from one given state to an other one. This defines a network of decision making with nodes and arcs, moving from state to state via actions. Then each decision is anchored to the previous state of the process (Gilovitch-Griffin-Kahneman, 2002).

Let $g(x, \Omega) = \sum_{(i,p) \in I \times P} w_i^p g_i^p(x_i^p)$ the utility (valence) of the various degree of resolution of each problem p with respect to each criteria, where all $w_i^p \geq 0$ represent weights attached to each criteria and each problem. The function $g_i^p : [0, \overline{x_i}^p] \to R$ represent the partial utilities with respect to each criteria and each problem.

The group will try to move from a state, or performance, $x \in X$ to a new state, or performance $y \in X$, to try improve the degrees of resolution of all his problems to hope to solve them partially, step by step.Then the Group moves from a matrix of performances x to a new matrix y.

The group will improve his position if he is able to move from $x \in X$ to $y \in X$ such that $g(y, \Omega) > g(x, \Omega)$. If he can succeed to do that, he will enjoy this improvement during the lenght of time $t(y) > 0$ and will get the incremental (marginal) gain or advantages to move $A(x, y, \Omega) = t(y)[g(y, \Omega) - g(x, \Omega)] > 0$.

But moving is costly. Let $C(x, y, \Omega) \geq 0$ the costs to move from x to y. If $y = x, C(x, x, \Omega) = 0$ (no move, no cost of moving). The moving period takes the time $t(x, y) \geq 0$. Then the cost of moving per unit of time is $c(x, y, \Omega)$ such that $C(x, y, \Omega) = c(x, y, \Omega)\, t(x, y)$.

It is worthwhile to move from x to y if the advantages to move $A(x, y, \Omega)$ are greater than the costs to move $C(x, y, \Omega)$, ie when $A(x, y, \Omega) \geq C(x, y, \Omega)$ .In this case we will say that $y \in W(x, \Omega) \Leftrightarrow A(x, y, \Omega) \geq C(x, y, \Omega) \Leftrightarrow t(y)[g(y, \Omega) - g(x, \Omega)] \geq c(x, y, \Omega)\, t(x, y)$.

Let $t(y) = [1 + \gamma(x, y)]\, t(x, y)$ which defines the ratio between the lenght of the moving period $t(x, y) = h$ and the lenght $t(y)$ of the next exploitation-exploration period .Then

$y \in W(x, \Omega) \Leftrightarrow [1 + \gamma(x, y)][g(y, \Omega) - g(x, \Omega)] \geq c(x, y, \Omega)$

$\Leftrightarrow g(y, \Omega) - g(x, \Omega) \geq [1/(1 + \gamma(x, y))]\, c(x, y, \Omega)$.

Worthwhile Improving Process and Stable Routines

Suppose that the lenght $\gamma(x, y)$ of the exploitation period is bounded from above , $0 < \gamma(x, y) \leq \overline{\gamma}$. Then $1/(1 + \gamma(x, y)) \geq 1/(1 + \overline{\gamma})$ implies that

$y \in W(x) \Rightarrow g(y, \Omega) - g(x, \Omega) \geq [1/(1 + \overline{\gamma})]\, c(x, y, \Omega)$.

Note that $c(x, y, \Omega) = 0$ if $y = x$.

A Worthwhile To Move process of incremental improvements is defined by the dynamical system $x_{n+1} \in W(x_n, \Omega), n \in N$ (see Attouch-Soubeyran A, 2004). It will stop when it reachs a stable routine. We will say that $x \in X$ is a stable routine iff there exists no new state of resolution $y \neq x$ such that it is worthwhile to move from x to y, ie when $g(y,\Omega) - g(x,\Omega) < 1/(1+\overline{\gamma})\, c(x, y, \Omega)$ for all $y \neq x$ .Formally $x \in X$ is a stable routine iff $W(x, \Omega) = \{x\}$ .The agent or the group "prefers to stay than to move".

The Ability to be Able to Compare Advantages and Costs to Move :**Exploration Around**.

We will give later more details about the nature of the moving costs, as well as the exploration process which helps the agent to discover his unknown landscape or goal-utility function g around each state of resolution $x \in X$, to be able to compare (make an estimation of) advantages and costs to move (see Attouch-Soubeyran A, 2004 for an extensive discussion about these points). The need to explore, each step, to hope to improve and find a worthwhile move plugs our model within the Bounded Rational paradigm. But our process requires a minimun amount of rationality, ie to "explore enough" to be able to "improve enough".

Existence of Stable Routines

A "Worthwhile to Move Process" is "Worthwhile Enough", if, each step, the agent or the group explores a "large enough portion" of the worthwhile to Move set $W(x,\Omega)$ .The exploration process

$E : x \in X \mapsto E(x, K(x))$ which drives this "Procedural Rational Improving Process" gives the opportunity, each step, to be able to be rational in a limited (local) way, ie it offers the possibility to be able to compare locally advantages and costs to move.The costs of exploration that the group chooses to spend at position $x \in X$ are $K(x) = t(x)k(x)$ where $t(x) \geq 0$ is the lenght of the exploration phase and $k(x) \geq 0$ the cost of exploration per unit of time.Let, at each step $x \in X$, $W^E(x,\Omega) = W(x,\Omega) \cap E(x, K(x)) \subset X$ be the explored part of the worthwhile to move set $W(x,\Omega)$. Exploring enough means that the non explored portion $W(x,\Omega) - E(x,K(x))$ of the worthwhile to move set is "small enough" in a sense that $s^E(x) = \sup\{g(y), y \in W^E(x,\Omega)\} < +\infty$ is close to
$s(x) = \sup\{g(y), y \in W(x,\Omega)\} < +\infty$, ie

$0 \leq s(x) - s^E(x) \leq \varepsilon(x)$, where $\varepsilon(x_n) \to 0$, for $x_{n+1} \in W(x_n,\Omega), n \in N$ (see Attouch-Soubeyran, 2004).

We make here a list of simplifying assumptions to be able to show the convergence of a "Worthwhile Enough to Move Process" towards a Stable Routine.

Hypothesis.

Suppose that the space X of states (degrees of resolution) x of all problems is a complete metric space with the distance d.

Suppose that $g(y,\Omega) - g(x,\Omega) > 0 \Rightarrow c(x,y,\Omega) > c(y,x,\Omega)$, ie it is more costly to move from x to y than the reverse, if y improves with respect to x.

Let $\rho(x,y) = d(x,y)/t(x,y)$ the module of the vectorial speed of movement (which is not defined in the metric space X, but which can be defined in a normed linear space, ie a Banach space X) and let $e(x,y,\Omega)$ the effort or cost per unit of distance.

We have $C(x,y,\Omega) = c(x,y,\Omega)\, t(x,y) = e(x,y,\Omega)\, d(x,y) \Rightarrow c(x,y,\Omega) = e(x,y,\Omega)\, \rho(x,y)$

Suppose also that costs to move per unit of time $c(x,y,\Omega)$ are high for small movements around each $x \in X$. This means that efforts to move per unit of time $e(x,y)$ must be high, locally.:

$c(x,y,\Omega) = e(x,y,\Omega)\, d(x,y) \geq \theta d(x,y) \Leftrightarrow e(x,y,\Omega) \geq \theta > 0.$

Suppose that g is bounded above :$g(x,\Omega) \leq \overline{g} < +\infty$, for all $x \in X$.

Suppose that g is upper semi continuous (with no surprising jumps upward).

Suppose that the group (the agent) can explore enough around each step to improve enough and finds a new "worthwhile enough to move" performance, each step. Let

$x_{n+1} \in \widetilde{W}(x_n,\Omega), n \in N$ such a"Worthwhile Enough Process" starting from some $x_0 \in X$.

Existence of Stable Routines for a Worthwhile Enough to Move Process :

Then, such a "Worthwhile Enough Process" $x_{n+1} \in \widetilde{W}(x_n), n \in N$, starting from any $x_0 \in X$ converges to a stable routine $x^* \in X$, such that $W(x^*,\Omega) = \{x^*\}$.

Moreover, if the agent starts from a good enough state of resolution of his problems x_0, ie such that $\overline{g} - g(x_0,\Omega) \leq \varepsilon$, for an ε small enough, then

It exists $x^* \in X$, such that $g(x^*,\Omega) \geq g(x_0,\Omega) \geq \theta d(x_0,x^*)$ and $W(x^*,\Omega) = \{x^*\}$.

This result is a Cognitive Version of the celebrated Ekeland Theorem (1972) (see Attouch-Soubeyran A, 2004 for the Cognitive Model and proof).

Ekeland Theorem:

The usual formulation of the Ekeland Theorem is the following. It concerns the existence of a maximun of a linear perturbation of a function g with respect to distance (instead of a minimun), without continuity and compacity assumptions, when this function can have no maximun.

Let X a complete metric space with distance d. Suppose that $g : X \to R$ is bounded above, and upper semi continuous. For all $\theta > 0$, and every $x_0 \in X$, it exists $x^* \in X$ such that i) $g(x^*) - \theta d(x_0,y) \geq g(x_0)$, ii) $g(y) - \theta d(x^*,y) < g(x^*)$, for all $y \neq x^*$.

Then $x^* \in Arg\{g(x^*) - \theta d(x^*,y)\}$.

Moreover, if $\overline{g} - g(x_0) \leq \varepsilon$, for some $\varepsilon > 0$, then for any $k = \theta/\varepsilon > 0$, it exists $x^* \in X$, such that

a) $g(x^*) \geq g(x_0)$, b) $d(x_0,x^*) \leq 1/k = \varepsilon/\theta$ and c) $g(y) - \theta d(x^*,y) < g(x^*)$ for all $y \neq x^*$.

Existence of Stable Routines with Concavity and Compacity Assumptions

It is easy to show the existence of stable routines if one imposes concavity and compacity assumptions

to the "net advantage to move" function

$\Delta : (x,y) \in X \times X \to \Delta(x,y,\Omega) = A(x,y,\Omega) - C(x,y,\Omega)$.

The Ky Fan theorem works easily. But concavity or compacity hypothesis are not needed in our Cognitive process, the " Cognitive Version of the Ekeland Theorem". If one uses the Ky Fan theorem, one gets existence of stable routines. This is a Substantive approach of Rationality, but not a Procedural Approach as before. We cannot exhibit some convergence process towards a stable routine $x^* \in X$. There is no way to know how to reach such a stable routine. If we are there, at x^*, we prefer to stay there. But why are we there? We are only able to say that no move away from x^* is worthwhile.In our view this the great weakness of Substantive Rationality (for more on this point and a new variant of the Saddle Point Theorem, see Soubeyran A-Soubeyran B, 2005).

Sharing Incremental Advantages and Costs To Move Within a Group

Definition of a Feasible Sub Group Structure

Let $\Omega = \{\omega_1,\omega_2,....,\omega_n\}$ a set of agents. Let $\mathcal{F}$ a family of feasible subgroups $M \subset \Omega$. It is a σ ring ,or a σ additive family over Ω iff, i) $\Omega \in \mathcal{F}$, ii) $M \in \mathcal{F} \Rightarrow M^c = \Omega - M \in \mathcal{F}$,

iii) $M_j \in \mathcal{F}, j = 1,2,..., \Rightarrow \cup_{j=1}^{\infty} M_j \in \mathcal{F}$.

Let $m : M \in \mathcal{F} \to m(M) \in R$.It is a σ additive measure on Ω iff $m\left(\cup_{j=1}^{\infty} M_j\right) = \sum_{j=1}^{\infty} m(M_j)$

for any disjoint sequence $\{M_j\}$ of subsets of $\mathcal{F}$. It is a non negative σ additive measure if $m(M) \geq 0$, for all $M \in \mathcal{F}$.A measure is finite if $m(M) < +\infty$ for every $M \in \mathcal{F}$.

Advantages and Costs to move for the whole group Ω are

$A(x,y,\Omega) = t(y)\,(g(y,\Omega) - g(x,\Omega))$ and $C(x,y,\Omega) \geq 0$.

Sharing Advantages and Costs to Move within the group .

Let $A(x,y,M) = \lambda(x,y,M)\,(g(y,\Omega) - g(x,\Omega))$ the incremental advantage to move for a feasible sub group $M \in \mathcal{F}$.

Let $C(x,y,M) = \mu(x,y,M)\,C(x,y,\Omega)$ the incremental costs to move for this subgroup $M \in \mathcal{F}$.

The non negative real numbers $\lambda(x,y,M) \geq 0$, and $\mu(x,y,M) \geq 0$ represent shares of the incremental advantages and costs to move from x to y.Then, (the same is true for $\mu(x,y,M)$),

i) $\lambda(x,y,M) \geq 0$.

ii) $\lambda(x,y,M \cup N) = \lambda(x,y,M) + \lambda(x,y,N)$ for two disjoints feasible subgroups $M,N \in \mathcal{F}, M \cap N = \varphi$.

iii) $\lambda(x,y,\Omega) = 1$.

For any given pair $(x,y) \in X \times X$, the mapping $\lambda(x,y) : M \in \mathcal{F} \to \lambda(x,y,M) \geq 0$ is a non negative probability measure.

Then $\lambda(x,y)$ is sub additive: $\lambda(x,y,M \cup N) \leq \lambda(x,y,M) + \lambda(x,y,N)$ for any $M,N \in \mathcal{F}$

It is worthwhile to move for any sub group $M \in \mathcal{F}$ iff advantages to move are greater than costs to move :$A(x,y,M) \geq C(x,y,M)$ for all $M \in \mathcal{F}$. In other words, it is worthwhile to move for any sub group $M \in \mathcal{F}$ iff $\Delta(x,y,M) = A(x,y,M) - C(x,y,M) \geq 0$ for all $M \in \mathcal{F}$.

Let $\Delta(x,y) : M \in \mathcal{F} \to \Delta(x,y,M) \in R$ the signed measure of incremental net gain.

We define the "component order" over the σ algebra $\mathcal{F}$ of subsets $M \subset \Omega$.

$\Delta(x,y) \geq 0 \iff \Delta(x,y,M) \geq 0$, for all $M \in \mathcal{F}$.

ie $A(x,y) \geq C(x,y) \iff A(x,y,M) \geq C(x,y,M)$ for all $M \in \mathcal{F}$.

Towards Stable Routines: The Multi-Dimensional" Worthwhile To Move" Relation

Let the relation "It is Worthwhile to Move from x to y" for the group and its subgroups be defined by the relation $y \in W(x) \iff y \in W(x,M)$ for all $M \in \mathcal{F}$, where

$W(x,M) = \{y \in X, A(x,y,M) \geq C(x,y,M)\} = \{y \in X, \Delta(x,y,M) \geq 0\}$.

$W(x) = \{y \in X, A(x,y) \geq C(x,y)\} = \{y \in X, \Delta(x,y) \geq 0\}$.Then $W(x) = \cap_{M \in \mathcal{F}} W(x,M)$.

Resistance to Change as Maximal (or Pareto) Positions

A sub-group $M \in \mathcal{F}$ exhibits resistance to change from x to y if $A(x,y,M) < C(x,y,M)$.

There is **resistance to change** from x to y if it exists a subgroup $M \in \mathcal{F}$ such that $A(x,y,M) < C(x,y,M)$, ie if $y \notin W(x)$.

If $W(x^*) = \cap_{M \in \mathcal{F}} W(x^*,M) = \{x^*\}$, the performance or position $x^* \in X$ is a maximal element of the relation W. It exhibits resistance to change. For any intention to move from x^* to $y \neq x^*$, it exists a subgroup $M(x^*,y) \in \mathcal{F}$ which offers resistance to change.

$W(x)$ is the Worthwhile to Move Set, starting from x (anchored to x), ie

$W(x) = \{y \in X, t(y)\lambda(x,y,M)(g(y,\Omega) - g(x,\Omega)) \geq t(x,y)\mu(x,y,M)c(x,y,\Omega), M \in \mathcal{F}\}$.

We have $y \in_W x \iff t(y)\lambda(x,y,M)(g(y,\Omega) - g(x,\Omega)) \geq t(x,y)\mu(x,y,\Omega)c(x,y,\Omega)$

Definition of Stable Organizational Routines: it is a Position which exhibits Resistance to Change for any subgroup, and any new position:

$x^* \in X$ is a stable organizational Routine iff for all $y \neq x^*$, it exists a feasible subgroup $M^* = M(x^*,y) \in \mathcal{F}$ such that $A(x^*,y,M^*) < C(x^*,y,M^*)$. Then a stable routine x^* is such that for all move from x^* to $y \neq x^*$, there is resistance to change of at least one subgroup.

Remark:

If it is worthwhile to move for all agents, it is worthwhile to move for the group.

If $y \quad W\ x,\omega_j$ for all agents $j \quad J$, then $y \quad W\ x,\Omega$.

Note that $y \quad _W\ x \qquad g\ y,\Omega \ - g\ x,\Omega \quad \geq 0$, ie the process is improving, because if $y \quad W\ x \qquad y \quad W\ x,\Omega \qquad t\ y \ \ g\ y,\Omega \ - g\ x,\Omega \quad \geq t\ x,y\ c\ x,y,\Omega \ \geq 0$, with $\lambda\ x,y,\Omega \ = \mu\ x,y,\Omega \ = 1$.Then $g\ y,\Omega \ - g\ x,\Omega \quad \geq 0$.

Existence of Stable Routines of A Worthwhile To Move Process:Potential Games.

In this section we will consider both Non Cooperative and Cooperative Potential Games.

Non Cooperative Potential Games

A Non Cooperative Game with Exploration Sets is the list $J,X, \ W_j,j \quad J \ , \ \Gamma_j,j \quad J$, ie a list of agents J, a set of decisions X, a list of semi preference relations $W_j,j \quad J$ and a list $\Gamma_j,j \quad J$ of feasible subsets to move for each agent.

A semi preference relation $x \quad X \qquad W_j\ x \qquad X$ is such that W_j is reflexive and transitive.

A strategy profile is $x = \ x_1,x_2,...x_n \qquad X = \Pi_{j\ J}X_j$.

The agent j prefers to move from x to y iff $y \quad W_j\ x$.

An "Feasible to Move Map" supposes that each player $j \quad J$ is only allowed to move from $x \quad X$ to the subset $\Gamma_j\ x \qquad X$. An exploration process represents such a map.

A position $x \qquad X$ is maximal for agent $j \quad J$ iff $W_j^\Gamma\ x \quad = W_j\ x \quad \cap \Gamma_j\ x \quad = \ x$. This means that this agent does not prefer to move from x to any feasible move $y \quad \Gamma_j\ x$, where $y \neq x$.

"**Resistance to Change**"**Positions as Non Cooperative Equilibrium**:

Suppose that any move are allowed, within the exploration subset of the group $E\ x,K\ x$ (to simplify).

.Each player $j \quad J$ is allowed to move from $x \quad X$ to $\Gamma_j\ x \ = E\ x,K\ x$.

If $x \qquad X$ is a maximal position for all players, ie $W_j^\Gamma\ x \quad = \ x$ for all $j \quad J$, all agents prefer to stay at x than to move to a different position $y \neq x$,$y \quad E\ x,K\ x$. We will say that $x \qquad X$ is a "**Resistance to Change**" **Non Cooperative Equilibrium**. No players want to move (deviate), within their local exploration set.

Be careful: This definition is different from $W^\Gamma\ x \quad = \cap_{j\ J}\ W_j^\Gamma\ x \quad = \ x$.

Non Cooperative Potential Games :

Suppose that it exists a reflexive and transitive relation Π (a semi-order) such that $y \quad W_j^\Gamma\ x \qquad y \quad \Pi\ x$ for all $j \quad J$, ie $\ _{j\ J}\ W_j^\Gamma\ x \qquad \Pi\ x$.

Then Π will be said a Potential for this Non cooperative game.

Nash Equilibrium :**Unilateral Moves**

Consider moves restricted to unilateral moves $x = \ x_j,x_{-j} \qquad y^j = \ y_j,x_{-j}$ where only one player is allowed to move from x_j to $y_j \neq x_j$, while others players stay there and repeat what they have done before, ($y_{-j} = x_{-j}$, ie do not move from x_{-j} to any different $y_{-j} \neq x_{-j}$. Only unilateral moves are allowed.

Player $j \quad J$ is allowed to move from $x \quad X$ to $\Gamma_j\ x \ = \left\{y = \ y_j,y_{-j} \qquad X, \text{ such that } y_{-j} = x_{-j}\ ,\right\}$

Then $x \qquad X$ is a Nash Equilibrium iff $W_j\ x \quad \cap \Gamma_j\ x \quad = \ x$ for all $j \quad J$.

Note that if $N_j(x) = W_j^{\Gamma}(x) = W_j(x) \cap \Gamma_j(x)$ is the Nash Worthwhile to Move set for agent $j \in J$, then $N_j(x) = W_j(x) \cap \Gamma_j(x) \subset W_j(x)$.

Suppose that it exists a Potential W for the initial game.

The inclusion $\cup_{j \in J} W_j(x) \subset \Pi(x)$ implies $\cup_{j \in J} N_j(x) \subset \Pi(x)$

Then Π will be a Potential for this Nash Non cooperative game.

Existence of Non Cooperative Stable Nash Routines follows immediately, applying our Cognitive Version of the Ekeland Theorem to the Potential Function Π.

In our context of a "Worthwhile to Move Process" the potential Π will appear just below in a natural way. It is related to the constraints of a "**Not Too Unfair Sharing Rule**".

Cooperative Potential Games

We repeat the same definitions, using as an index set the family of feasible subgroups $M \in \mathcal{F}$ instead of an index for agents $j \in J$.

A Non Cooperative Game is the triple $(J, \mathcal{F}, X, (W_M, M \in \mathcal{F}))$:a list of agents J, a list of feasible subgroups $\mathcal{F}$, a set of decision X, a list of preference relations $(W_M, M \in \mathcal{F})$, and a list $(\Gamma_M, M \in \mathcal{F})$ of feasible moves for each subgroup.

A preference relation $x \in X \mapsto W_M(x) \subset X$ where W_M is reflexive and transitive (pre-order).

A strategy profile $X = \Pi_{M \in \mathcal{F}} X_M$,

The subgroup M prefers to move from x to y iff $y \in W_M(x)$.

An "Allowed to Move Map" :each subgroup $M \in \mathcal{F}$ is only allowed to move from $x \in X$ to $\Gamma_M(x) \subset X$.

A position $x \in X$ is maximal for subgroup $M \in \mathcal{F}$ iff $W_M(x) \cap \Gamma_M(x) = \{x\}$ .This means that this subgroup does not prefer to move from x to any allowed to move $y \neq x$.

Existence of Stable Organizational Routines : **A Not Too Unfair Sharing Rule**

Let us show the following Result:

Result: The"Cognitive Ekeland Conditions" and a " Not Too Unfair Sharing Rule" within the Group imply the convergence of the "Worthwhile to Move Process" towards some stable routines which exist.

Suppose that the shares of the incremental Advantages and Costs $\lambda(x,y,M)$ and $\mu(x,y,M)$ are > 0, for all $M \in \mathcal{F}$, ie that each agent $j \in J$ have a strictly positive share $\lambda(x,y,\{j\})$ and $\mu(x,y,\{j\}) > 0$ of the incremental advantages and costs to move.

Suppose that $y \in W(x,M)$ for some $M \in \mathcal{F}$. Then $A(x,y,M) \geq C(x,y,M)$ is equivalent to

$t(y)\lambda(x,y,M)[g(y,\Omega) - g(x,\Omega)] \geq \mu(x,y,M)C(x,y,\Omega)$

Let $0 < v(x,y,M) = \mu(x,y,M)/\lambda(x,y,M) < +\infty$ be the shares of incremental costs to move relative to incremental advantages to move. Then $A(x,y,M) \geq C(x,y,M)$ implies

$t(y)[g(y,\Omega) - g(x,\Omega)] \geq v(x,y,M)C(x,y,\Omega)$.

Suppose that all the relative shares $v(x,y)$ are bounded from below. $v(x,y,M) \geq \underline{v} > 0$.

Then $y \in W(x,M) \Rightarrow y \in \Pi(x)$, ie $W(x,M) \subset \Pi(x)$ where

$\Pi(x) = \{y \in X, t(y)[g(y,\Omega) - g(x,\Omega)] \geq \underline{v}C(x,y,\Omega)\}$.

Then $W(x) = \cap_{M \in \mathcal{F}} W(x,M) \subset \Pi(x)$.

This shows that Π is a Potential for this game.Existence of stable routines follows.

Bounded Rationality:Exploration-Exploitation Process to Know if it is Worthwhile to Move

Let us make more precise the exploitation -exploration process which drives our Cognitive Worthwhile to Move Process.

To be able to decide, in the near future, to move from $x \in X$ to a new state $y \in X$ of performances, and before choosing to move or stay there, the whole group and all its sub group have to be able to compare the incremental advantages $A(x,y) = (A(x,y,M) = t(y)(g(y,M) - g(x,M)), M \in \mathcal{F})$ and the incremental costs to move $C(x,y) = (C(x,y,M), M \in \mathcal{F})$. To be able to make such an estimation, the group and all its sub groups have to choose how much to explore around performance x. But the group and all its subgroups cannot stop exploitation. They must, at the same time, both spend time efforts and money, to use "explorers" (researchers,engineers) to explore new opportunities and to use "producers" (workers and engineers) devoted to exploitation, ie to production to enjoy the benefits (utility of profits) of the present ability to succeed to make performance $x \in X$.

A Dynamic Version of Bounded Rationality.

We define Bounded Rationality as the ability to be able to compare, each step, incremental advantages and costs to move. The way to hope to succeed to do it is to explore around. Then we cannot define Bounded Rationality without defining an Exploration Process. But we must also make precise the Adjoint Exploitation Process which is coupled with the Exploration Process in an intricated way (for example via Learning by Doing).

It is time now to make more explicit the underlying model of Exploration-Exploitation which drives our Cognitive model of Group Behavior (the "Worthwhile to Move Principle").

The first paradox is that the group (or one agent) have to decide how much to explore. To answer this question the group must know in advance if this will be rewarding (worthwhile). But without any preliminary exploration, in order to explore if it will be rewarding to explore (!), how to know that exploration will be rewarding ? There is a regression paradox to decide how much to explore. This is where individual and group psychology enters into the picture: the ability to take risk, or some strong resilience, which is the ability to accept to make errors and to make some sacrifices (to accept , ex ante, to loose exploration costs ex post, because of too much exploration, choosing wrong directions of search, making essays and errors).

The decision to explore more around some given performance is linked to the Group Motivation to move, ie the entwined contentment and unsatisfaction feelings of all agents.

This decision of "how much to explore" is also linked to the fact that exploration can be time consuming, and if the group has time constraints, (without the ability to hire new workers), the more it explores the less it exploits. This trade off (the Exploration-Exploitation trade offf) is even more crucial for an agent alone which cannot escape to time constraints.

A Present Exploitation-Exploration Phase

A recurrent period of Exploitation-Exploration works as follows. Suppose it is the present period at work. Let $t(x)$ its lenght of time, which is a choice variable for the Group (or the agent). This present period starts with a given known performance $x \in X$ and a given valence $g(x) \in L$.

i) **Exploitation Net Gain**

The Exploitation net gain per unit of time for the group is $f(x,\Omega) = \phi(x,\Omega) - m(x,\Omega)$

First $\phi(x,\Omega)$ is a per unit of time gross utility of the group Ω flowing from the ability to succceed to do and repeat performance $x \in X$. Second $m(x,\Omega)$ is a per unit of time maintenance cost of the group to be able to repeat performance $x \in X$. Then performance

$x \quad X$ is a temporary routine.

Revenues over the whole period of lenght $t\ x$ are $R\ x,\Omega\ = t\ x\ f\ x,\Omega$

ii) **Exploration Costs** $K\ x$ (as **Knowledge Costs**)

Exploration costs per unit of time spend by the group Ω are $k\ x,\Omega\ \geq 0$

Exploration costs over the whole period of lenght $t\ x$ are $K\ x,\Omega\ = t\ x\ k\ x,\Omega$

If the group and all its subgroups choose to spend the Costs of Exploration $K\ x,\Omega$ during the present period of choosen lenght $t\ x$, these costs allow them to explore within an Exploration Set $E\ x,K\ x,\Omega$ which includes $x \quad X$.

Let $G\ x,\Omega\ = R\ x,\Omega\ - K\ x,\Omega\ = t\ x\ \ \phi\ x,\Omega\ - m\ x,\Omega\ - k\ x,\Omega\ \ = t\ x\ g\ x,\Omega$

The net valence per unit of time is, for the whole group,

$g\ x,\Omega\ = \phi\ x,\Omega\ - m\ x,\Omega\ - k\ x,\Omega$

This exploration process around x allows the Group and its subgroups to discover both

i) the a priori unknown valence $g\ y,\Omega\ = \phi\ y,\Omega\ - m\ y,\Omega\ - k\ y,\Omega$ for all $y \quad E\ x,K\ x,\Omega$.

ii) the unknown physical Costs to Move $C\ x,y,\Omega$ for all $y \quad E\ x,K\ x,\Omega$.

Costs of exploration are used to discover both $g\ y,\Omega$ and $C\ x,y,\Omega$ (ie the landscape) around $x \quad X$.

An exploration phase can be divided into two succesive sub-phases of estimation :

A) **Improving** :A sub-phase where the agent or the group tries to estimate if it can improve its goal from $g\ x,\Omega$ to a greater level $g\ y,\Omega\ > g\ x,\Omega$. This is an "Improving Sub Phase" where the group builts its motivation to move which is strongly related to the estimation of the possibility of get some advantages to move per unit of time $\delta\ x,y,\Omega\ = g\ y,\Omega\ - g\ x,\Omega\ > 0$.

B) **Worthwhile To Move**: A subphase where the agent or the group estimates the costs to move $C\ x,y,\Omega$ only for those performances y which can improve, ie such that $g\ y,\Omega\ > g\ x,\Omega$.

C) **Decision To Move**: a final subphase of decision to move, comparing advantages and costs to move within the local exploration set.

A Moving (Swithching) Phase x,y

Suppose that the Group has decided to move from x to y, leaving a temporary routine phase, or static period of exploitation-exploration x , to enter a new temporary routine phase y . He is now able to decide to move or not (to choose between moving or not) because he has estimated by exploration during this initial period x , the balance between advantages and costs to move. More precisely he has estimated both the per unit of time advantage to move $g\ y,\Omega\ - g\ x,\Omega\ \ > 0$ (be careful,not the total advantage to move to be defined later) and the total cost to move $C\ x,y,\Omega$ from x to y. Let $t\ x,y\ = h > 0$ the estimated time spend to physically move from x to $y \neq x$, and $c\ x,y,\Omega\ \geq 0$ the per unit of time spend for that purpose. Then $C\ x,y,\Omega\ = c\ x,y,\Omega\ t\ x,y\ = c\ x,y,\Omega\ h$. Let $d\ x,y\ \geq 0$ be the distance between performance x and y, and $e\ x,y,\Omega\ \geq 0$ the effort or cost per unit of distance. Then the physical cost of moving is $C\ x,y,\Omega\ = e\ x,y,\Omega\ d\ x,y$. It depends both of the effort, or cost per unit of distance $e\ x,y,\Omega$, and the distance between performances x and y. The modulus of the speed of moving from the present exploitation-exploration phase x to a future one y is $\rho\ x,y\ = d\ x,y\ /t\ x,y$.

The relation between the per unit of time cost to move $c\ x,y,\Omega\ = C\ x,y,\Omega\ /t\ x,y$ and the per unit of distance cost to move $e\ x,y,\Omega\ = C\ x,y,\Omega\ /d\ x,y$ is $t\ x,y\ c\ x,y,\Omega\ = d\ x,y\ e\ x,y,\Omega \qquad c\ x,y,\Omega\ = \rho\ x,y\ e\ x,y,\Omega$.

Notice that the advantage to move per unit of time $g\ y,\Omega\ - g\ x,\Omega\ \ > 0$ and the cost to move $C\ x,y$ from x to y are known by the agent within his exploration set, ie for all $y \quad E\ x,K\ x,\Omega$.

The agent has a Bounded Rational behavior because he has choosen to pay (via exploration costs) for the ability to be able to compare locally advantages and costs to move. This definition of Bounded Rationality is more precise and more operational that the one pionnered by Simon (1975).

The Future Exploitation-Exploration Phase y

If we consider the temporary routine phase x where the agent both exploits and explores, at some time the agent has to decide when to leave this temporary routine phase x to move and enter a new temporary routine phase y . This means that the lenght $t\ x$ of the present temporary routine phase x is a choice variable for the Group. The longuer $t\ x$ the more the group both exploits and explores. The more the Group can hope a large future improving gap $g\ y,\Omega\ - g\ x,\Omega\ > 0$. But the lower the speed of improving , spending too much time to explore and exploit.

The agent continues the temporary routine phase x , as long as he has not founded (estimated), by exploration, some new performance y whose estimated gain ”improves enough”, ie $g\ y,\Omega\ - g\ x,\Omega\ > 0$, in a sense to be made more precise later.

Then, the agent has to estimate the cost of moving $C\ x,y,\Omega$ to be able to compare advantages and costs of moving. We turn now to the estimated total incremental advantages to move $A\ x,y,\Omega$ .They depend of the lenght $t\ y$, choosen ex ante, for the new temporary routine phase y and of the incremental advantages to move, per unit of time,$g\ y,\Omega\ - g\ x,\Omega\ > 0$.Notice again that the lenght $t\ y$ of the new temporary routine phase y is a choice variable for the agent, as well as the lenght $t\ x$ of the previous temporary routine phase x .

The choosen lenghts $t\ x$ and $t\ y$ of the successive exploitation-exploration periods and the more constraint lenght $t\ x,y$ of the moving period determine the speed of the Process of incremental improvement. As we will see in the proof of our main result the speed of improvement matters much.

The estimated total incremental advantages to move are
$A\ x,y,\Omega\ = t\ y\ \ g\ y,\Omega\ - g\ x,\Omega\ \ > 0$.

Remark:

Our model describes a Punctuated Dynamic between static exploration -exploitation periods of temporary routines and dynamic moving periods, ie periods where nothing change and periods where a lot of things change.

PART II

Non Incremental Sharing Rule

The Non Incremental Landscape of a Group and its Subgroups

We move now from the whole group to the individual agents and all the feasible subgroups they can form to define their utility, advantages and costs to move, via a non incremental sharing rule for the gains (utility, valence).

The Non Incremental Landscape of the Group and all its feasible Subgroups is defined by the pair $g\ y,M\ = \lambda\ y,M\ g\ y,\Omega$ and $C\ x,y,M\ = \mu\ x,y,M\ C\ x,y,\Omega$ for all feasible subgroups $M\quad \mathcal{F}$.

The real numbers $\lambda\ y,M\ \geq 0$ and $\mu\ x,y,M\ \geq 0$ represent shares of the whole cake $g\ x,\Omega$ and of the Total Costs to Move $C\ x,y,\Omega$ .We have $\lambda\ x,\Omega\ = 1$ and $\mu\ x,y,\Omega\ = 1$, and $\lambda\ x,M\ = \sum_{j\ M} \lambda\ x,\omega_j$, and $\mu\ x,y,M\ = \sum_{j\ M} \mu\ x,y,\omega_j$ .Shares are Finite Non Negative Probabilistic Measures (non negative, and σ additive ,ie additive on disjoints infinite sequences of subgroups of the σ ring $\mathcal{F}$).

We note these measures as vectors with components $\lambda(x,M)$ and $\mu(x,y,M)$, where the set of indexes are the σ ring of feasible subgroups $M \in \mathcal{F}$. They represent the sharing rules, $\lambda(x) = (\lambda(x,M), M \in \mathcal{F}) \geq 0, \mu(x,y) = (\mu(x,y,M), M \in \mathcal{F}) \geq 0$.

Remark:

Previously we have defined shares over incremental advantages to move, $\lambda(x,y,M)(g(y,\Omega) - g(x,\Omega))$.

Now we define shares over the net gain $\lambda(y,M)g(y,\Omega)$. The two cases are admissible.

Worthwhile Process For a Group and its SubGroups

A feasible sub-group $M \in \mathcal{F}$ can benefit of the share $A(x,y,M) = t(y)(g(y,M) - g(x,M)) > 0$ of the incremental gains and must support the share $C(x,y,M) = t(x,y)c(x,y,M) > 0$ of the whole costs of moving. The shares will be:

$g(y,M) = \lambda(y,M)g(y,\Omega)$ and $g(x,M) = \lambda(x,M)g(x,\Omega)$,

$A(x,y,M) = t(y)(g(y,M) - g(x,M))$, ie

$A(x,y,M) = t(y)\lambda(y,M)(g(y,\Omega) - g(x,\Omega)) + t(y)(\lambda(y,M) - \lambda(x,M))g(x,\Omega)$.

Sub Groups Resistance to Change

A sub-group $M \in \mathcal{F}$ will feel some incentives to change from x to y if his shares of the whole group advantages to move are higher than his share of the costs to move of the whole group, ie $y \geq_M x \Leftrightarrow A(x,y,M) \geq C(x,y,M)$.

A contrario a sub-group $M \in \mathcal{F}$ exhibits resistance to change from x to y if $A(x,y,M) < C(x,y,M)$.

The Banach Lattice (and Kakutani) Space of Rewarding Rules

In this paragraph we give general abstract definitions to be able to state our more general "Cognitive Worthwhile To Move Process" Theorem which will give, as a by product, an extension of the celebrated Ekeland Theorem in a multidimensional setting.

A Banach Lattice Space is also named a Riesz space. A Kakutani -Riesz space adds one more condition over the norm : the condition $\phi \geq 0, \psi \geq 0 \Rightarrow \|\phi + \psi\| = \|\phi\| + \|\psi\|$..

We modelize Dynamic Rewarding and Sharing Rules Process for a group and all its subgroups, moving from one performance to a new one, ie from x to y.

Rewarding and Sharing Rules for a Group and its Subgroups:**Non Negative Measures**

Family of feasible subgroups of a group: **a σ ring**.

Let $\mathcal{F}$ a family of subgroups (subsets) $M \subset \Omega$ of a group (set) Ω. This family of subsets $\mathcal{F}$ is a σ ring if

i) $\Omega \in \mathcal{F}$, ii) If $M \in \mathcal{F}$, then $M^c = (\Omega - M) \in \mathcal{F}$, and $(M_j \in \mathcal{F}, j = 1,2,...,\infty) \in \mathcal{F}$ implies

$\cup_{j=1}^{\infty} M_j \in \mathcal{F}$.

Family of Weights (Rewarding Rules) attached to a Family of Subgroups of a Group:

Let $\phi : M \in \mathcal{F} \to \phi(M) \in R$ be a real valued weight (a signed weight: positive, zero, or negative) attached to each subgroup M, for example its net gain or reward.

Rewarding Rules or Signed Measure

A weight function ϕ is a signed measure or a Rewarding Rule, or a list of net gains of each subgroup, for a group and its subgroups if ϕ is such that

i) $\phi(M)$ is a real number $,\geq,=\leq 0$, for all $M \in \mathcal{F}$, and

ii) ϕ is σ additive, ie $\phi(\cup_{j=1}^{\infty} M_j) = \sum_{j=1}^{\infty} \phi(M_j)$ for any sequence $(M_j \in \mathcal{F}, j = 1,2,...,)$ of disjoint subsets of $\mathcal{F}$.

Let L the set of all these signed measures (weights) attached to each subgroup.

Rewarding Rules as Sharing Rules or Non Negative Measures

A weight function ϕ is a non negative measure, or a Sharing Rule, ie a Non Negative Rewarding Rule, for a group and its subgroups if

i) ϕ is non negative, $\phi(M) \geq 0$, for all $M \in F$, and

ii) ϕ is σ additive, ie $\phi(\cup_{j=1}^{\infty} M_j) = \sum_{j=1}^{\infty} \phi(M_j)$ for any sequence $M_j \in F, j = 1,2,...,$ of disjoint subsets of F.

Let L^+ be the set of all non negative measures.

Then a non negative measure is subadditive : $\phi(M \cup N) \leq \phi(M) + \phi(N)$ for any $M,N \in F$.

Then the triple (Ω, F, ϕ) is called a measurable space.(Yosida 1968, p 15).

If $\phi(\Omega) = 1$, then $\phi \in L^+$ is a probabilistic measure.

Banach Lattice and Kakutani Banach Lattice

The Linear Space L of Signed Measures

Let $\phi, \psi \in L$.We define the sum and scalar product of non negative measures,

$(\phi + \psi)(M) = \phi(M) + \psi(M)$ and $(\alpha\phi)(M) = \alpha\phi(M)$, for $\alpha \in R_+$, and all $M \in F$.

The Lattice of Non Negative Measures

Suppose that each $\phi \in L$ is finite: $|\phi(M)| < +\infty$ for all $M \in F$.

The "component partial order" $\phi \leq \psi \Leftrightarrow \phi(M) \leq \psi(M)$ for all $M \in F$, and $\phi, \psi \in L^+$ makes L^+ a Lattice, where $\sup(\phi, \psi) = \sup\{\phi(M), \psi(M), M \in F\}$, and $\inf(\phi, \psi) = \inf\{\phi(M), \psi(M), M \in F\}$.

The Vector Lattice of Non Negative Measures

A real vector space V is said to be a vector lattice if it is a lattice by a partial order relation $\geq$ satisfying

the conditions $\phi \leq \psi$ implies i) $\phi + \xi \leq \psi + \xi$, and ii) $\alpha\phi \leq \alpha\psi$ for every $\alpha \in R_+$ (or $\alpha\phi \geq \alpha\psi$ for every $\alpha \in R_-$).

In a vector lattice V we define the positive and negative value of an element $\phi \in V$,

$\phi^+ = \sup(\phi, 0)$, $\phi^- = \inf(\phi, 0)$, and its absolute value $|\phi| = \phi^+ - \phi^-$. Then $\phi = \phi^+ + \phi^-$.

The vector space of signed measures $V = L$ is a vector lattice.

Let $\phi^+(M) = \sup\{\phi(N), N \in F, N \subseteq M\}$ the positive variation of ϕ on M.

Let $\phi^-(M) = \inf\{\phi(N), N \in F, N \subseteq M\}$ the negative variation of ϕ on M.

The Total Variation of ϕ over all $M \in F$ is $|\phi| = \sup(\phi, -\phi) = \phi^+ - \phi^-$.

Order complete Vector Lattice

The definition of a σ complete (ie order complete) vector lattice V is equivalent to: V is a σ complete vector lattice iff every monotone increasing and bounded sequence $(\phi_n) \subseteq V$ have a supremun $\sup(\phi_n) \in V$. A **Normed Linear Space** is a vector space where a norm have been defined. A **Banach space** is a (metric) complete normed linear space. A **Banach Lattice** is a vector lattice V which is a Banach space such that $|\phi| \leq |\psi| \Rightarrow \|\phi\| \leq \|\psi\|$, for $\phi, \psi \in V$. A **Kakutani Banach Lattice** is such that $\phi \geq 0, \psi \geq 0$, imply $\|\phi + \psi\| = \|\phi\| + \|\psi\|$.

Proposition: A σ complete F lattice satisfying $|\phi| \leq |\psi| \Rightarrow \|\phi\| \leq \|\psi\|$ and $\phi \geq 0, \psi \geq 0$, imply $\|\phi + \psi\| = \|\phi\| + \|\psi\|$.

Theorem:The space of signed measures L is a complete lattice.

Main Example

The space of finite signed measures L is a Banach space, ie a complete normed vector space, where

$\phi = \phi^+ + \phi^-$, the modulus of ϕ is $|\phi| = \phi^+ - \phi^-$ and the norm of $\phi = \phi^+ + \phi^-$ is the total variation of ϕ over the whole space Ω, ie (Yosida, p 369),

$\|\phi\| = |\phi| \ \Omega \ = \phi^{+} \ \Omega \ - \phi^{-} \ \Omega$.

In the space $L^{+} \quad L$ of non negative finite measures

$\|\phi\| = \phi^{+} \ \Omega \ = |\phi| \ \Omega \ = \phi \ \Omega \ \geq 0.$

Banach Lattice- Kakutani Space

To sum up: a Banach Lattice-Kakutani space is such that :

L is a lattice, and a complete normed linear vector space (ie a Banach space), with a partial order ≥ such that

if $|\phi| = \sup \ \phi, -\phi$ is the modulus of ϕ,

1) $\phi, \psi, \xi \quad L$ and $\phi \geq \psi \qquad \phi + \xi \geq \psi + \xi$, and $\alpha\phi \geq \alpha\psi$ for $\alpha \geq 0$.

2) $|\phi| \leq |\psi| \qquad \|\phi\| \leq \|\psi\|$

3) For all $\phi, \psi \quad L$, $\sup \ \phi, \psi$ and $\inf \ \phi, \psi$ exist, and belong to L.

4) $\phi \geq 0, \psi \geq 0 \qquad \|\phi + \psi\| = \|\phi\| + \|\psi\|$.

5) If $\phi_{n \ \ n \ N}$ is a sequence bounded above and increasing, then $\sup \ \phi_n, n \quad N$ exists.

Rewarding Rules

In our model the measures represent Rewarding Rules for the Group Ω and the family of all its feasible subgroups F, for different states of performances, moving from $x \quad X$ to $y \quad X$ (where performances represent the degrees of resolution of all the problems of the group). These measures define the Landscape of the group and its Subgroups: they are $\phi = g \ x \quad L^{+}, \psi = g \ y \quad L^{+}$, or $\eta = A \ x,y \quad L$, and $\xi = C \ x,y \quad L^{+}$. The Landcape of the group Ω and the familiy of its feasible Subgroups F define first ,

for any given pair of performances $x,y \quad X \times X$, the Rewarding Rules linked to these pair of performance x,y :

$\phi \ \cdot \quad L : M \quad F \quad \phi \ \cdot, M \quad R$, where $\phi \ \cdot, M \ = g \ x, M \quad R_{+}$, or $g \ y, M \quad R_{+}$, or $A \ x, y, M \quad R$ or $C \ x, y, M \quad R_{+}$.

The Landscape of the group Ω and the familiy of its feasible Subgroups F includes also an Exploitation-Exploration Process. This have been made explicit previously.

Upper Semi Continuity of Sub Group Landscapes Functions

Let X be a complete metric space. A function $g : X \quad L$ is sais to be sequentially upper semicontinuous if for all sequence $x_n \quad X$, with

$x_n \quad x \quad X$, and $g \ x_n$ not decreasing, and bounded above, ie $g \ x_{n+1} \ \geq g \ x_n$, and $g \ x_n \ \leq \overline{g}$, for all $n \quad N$, then $\sup \ g \ x_n \ \leq g \ x$, ie

$\limsup \ g \ x_n \ \leq g \ x$ .Remember that $g \ x_{n+1} \ \geq g \ x_n$ is equivalent to $g \ x_{n+1}, M \ \geq g \ x_n, M$ for all $M \quad F$.

Remark: if g_1, nor g_2 are not upper semi continuous, nor $g_1 + g_2$, however the pair g_1, g_2 can be upper semi continuous.This remark makes the following theorem not so obvious.

The Not Worthwhile To Move Theorem :Existence of Stable Organizational Routines

Main Theorem :

Let X a complete metric space and L a Kakutani Riesz Space.

Let $C \ x,y \ : X \times X \quad L^{+}$, $g : X \quad L, g$ bounded above.

i) For all $\ > 0$, it exists $\delta > 0$, such that $\|C \ x,y \ \| \leq \delta \qquad d \ x,y \ \leq$.

ii) $C \ x,x \ = 0$, for all $x \quad X$.

iii) For all $x \quad X, y \quad X \qquad C \ x,y$ is continuous

iv) $C \ x,z \ \leq C \ x,y \ + C \ y,z$ for all $x,y,z \quad X$

v) g is upper semi continuous.

Let $W\ x\ =\ y\quad X, g\ y\ - g\ x\ \geq \theta C\ x,y$ for some $\theta = 1/\ 1 + \overline{\gamma}\ > 0$.

Then it exists $x\quad X$ such that for all $y \neq x$,we have $W\ x\ =\ x$,ie for every $y \neq x\ , y\quad W\ x$.

Comments:

Such an $x\quad X$ is a maximal element of the relation $y\quad W\ x$, the analogue of a Pareto element of $\widetilde{X}$ when the decision space reduces from X to $\widetilde{X}\quad X$.

If the Kakutani Riesz space L is the set of non negative measure defined by a Sub Group Landscape Function $g:\ x,M\quad X \times F\quad g\ x,M\ \geq 0$,

L is the family of non negative measures. In this case $x\quad X$ is a stable routine for the group.

Ekeland Theorem appears to be a special case of our multidimensional sub group approach, when the sub group landscape function $g:\ x,M\quad X \times F\quad g\ x,M\ \geq 0$ and the subgroup cost to move function $C:\ x,y,M\quad X \times X \times F\quad C\ x,y,M\ \geq 0$, are separable, ie when

$g\ x,M\ = \lambda\ M\ g\ x,\Omega$ and $C\ x,y,M\ = \mu\ M\ C\ x,y,\Omega$, where λ and μ are non negative measures.

Then for all $x\quad X$, it exists $x\quad X$ such that for all $y \neq x$, it exists $M\ x\ ,y\ = M\quad F$ such that this sub group M will resist to change, because its share of the incermental gain $\lambda\ M\ \ g\ y,\Omega\ - gx,\Omega$ does not compensate for his share of the costs to move $\mu\ M\ C\ x,y,\Omega$,ie

$\lambda\ M\ \ g\ y,\Omega\ - gx,\Omega\ < \mu\ M\ \theta C\ x,y,\Omega$.

If the shares of the incremental gains are equal to the shares of the costs to move for all sub group, ie if the two non negative measures λ and μ are equal, then $g\ y,\Omega\ - gx,\Omega\ < \theta C\ x,y,\Omega$. This because $\lambda\ M\ = \mu\ M$ is necessarily > 0. If not we will have $0 < 0$.

If the cost to move for the whole group $C\ x,y,\Omega$ is proportional to the distance d of the metric space X, ie $C\ x,y,\Omega\ = \theta d\ x,y\ , \theta > 0$, the Ekeland Theorem follows.

The Takahashi Theorem is a direct consequence, as it is well known.

Routinezed Nash Equilibration :Worthwhile To Move Non Cooperative Sub-Group Games

Let X be a complete metric space. Let $T:\ x\quad X\quad T\ x\quad X$ be a response function or a routinized way of moving (not necessarily a best response function) for a set of agents in a non cooperative game.

The Multi dimensional Carisiti Theorem:

Consider the "Worthwhile To Move Relation" defined in the "Not Worthwhile To Move"Theorem.Let us adopt the hypothesis of this theorem.

Suppose also that for each $x\quad X$, we have $T\ x\quad W\ x$, ie $g\ T\ x\ - g\ x\ \geq C\ x, T\ x\ , x\quad X$.

Then it exists an $x\quad X$ such that $T\ x\ =\ x$, ie a fixed point of T.

Proof: The "Not Worthwhile to Move " theorem shows that it exists a position $x\quad X$ such that $W\ x\ =\ x$. But $T\ x\quad W\ x$. The result follows.

Comments:

1)"**Worthwhile To Move**" **Non Cooperative Games**

A response $T:\ x\quad X\quad T\ x\quad X$ such that $T\ x\quad W\ x$ is a worthwhile to move response of a σ

ring family F of subgroups of a group Ω.

It can represent a best response for a non cooperative game, or a routine behavior for a list of players, (for a feasible family F of subgroups of players) such that for each of them it is worthwhile to move. Such Worthwhile to Move Games" are Potential Games.

Our theorem represents a multi dimensional generalization of the famous Caristi Theorem (Caristi,1976).

2) **From Routinezed Actions to Routinezed Responses** (**Ways of Doing**).

Let Λ the set of actions of an agent or a group of agents. Let $\alpha = \ x, y \quad \Lambda$ one of them which helps him to move from x to y. A routinezed way of doing is a map $x \quad X \quad \alpha\ x \quad \Lambda$ which, starting from any given performance $x \quad X$ chooses a new performance $y = T\ x \quad X$ such that $\alpha = \ x, T\ x \quad \Lambda$.

In a Banach space we can write $y = x + hv$, where $v \quad F\ x, h$ is a feasible subset of speed of moving which depends both of the present position x, and the time spend to move $h > 0$.

The map $x, h \quad X \times R_{+} \quad F\ x, h \quad X$ is a difference-differential inclusion.(see Aubin,1984).

A routinezed action $\alpha \quad \Lambda$ is such that for every $x \quad X$, we have $y = T\ x \ = x + hv$.This uniquely determines at each position x the speed of movement $v = v\ x \ = \ T\ x \ - x\ /h$.

To simplify we take the travel time $h = t\ x, y$ as a given constant.

These kind of formulation will lead us to Dynamic Games with Differential Inclusions (see the Viability Theory, Aubin, 1984), which is out of our scope.Anchoring effects can be modelized in this way.

Cognitive Principles of the Proof of the Theorem

Sketch of the List of the Main Ten Principles and Concepts.

1) Building The Knowledge Space, the network of the space of Problems and Criteria, the state space of Performances and the underlying space of Actions (from one performance to an other one), a metric or a proximity relation between performances and between actions......the Riemanian geometry of the Difficulty to Solve problems, Costs to Move, Advantages to Move,...the Speed of Resolution of problems ....

2) The "Enclosing Search space" Principle : example $C\ x, y, \Omega \ \geq \theta d\ x, y$ .Dealing with some Lack of Precision.Using Inclusions.

3) A Worthwhile Process is Nested if .....

Definition of a Nested Process :$y \quad W\ x \quad A\ x, y, M \ \geq C\ x, y, M$, for all $M \quad F$, where W is a pre-order ,ie reflexive and transitive.

4) The Worthwhile process $x_{n+1} \quad W\ x_n \ , n \quad 0, 1, 2, ...$ converges if Costs to Move are continuous and the space of performances is a complete metric space (or a complete uniform space) .

5) W is inductive : if $x_{n+1} \quad W\ x_n \ , n \quad 0, 1, 2, ...$, it exists an $x \quad X$, such that $x \quad W\ x_n$ for all $n = 0, 1, 2, ..$

6) $\| g\ x \ \| : X \quad 0, \overline{g}$ is increasing, and bounded above.

7) The Brezis-Browder Principle applies to $\| g\ x \ \|$.

8) To Explore, Improve and Satisfice Enough.

9) No Surprise upward, starting from any position: Upper semi Continuity assumptions.

10) Shrinking Process.

and A Cognitive Approach of the Brezis -Browder Principle as a basic guide framework.

Conclusion

Sketch of the Conclusion: Our original ”Worthwhile to Move Process” have direct applications and strong connexions with a lot of important aspects, both in Economics and Psychology and in Mathematics :

Economics and Psychology:

Landscape Approach :Unknown Goal , Utility, Valence function.The Role of Stress in Decision making.

Instrumentalism versus Incrementalism : Endogenous Step by Step Goal Formation along the process .

Lack of Knowledge is modelized as Inclusions (point to set maps). This is different from Uncertainty, which is modelized via probabilities.

Modelization of Psychological Aspects :Anchoring Effects, Motivation-Deception-Contentment

-Frustration.

Bounded Rationality Aspects : Exploration by Rejection and Elimination.

Satisficing and Procedural Rational Process, the Dynamics of Heuristics and Costs Regression.

From Procedural to Substantive Rationality : Saddle Point Theorems.

Routines, Inertia, Reactivity. Punctuated Dynamics.

Coupled Landscapes in Management Sciences: a Rudge Landscape Approach, a coupled Exploration-Exploitation Process.How much to Explore? Coupled Dynamics

Dynamic Game Theory, Games with Costs to Move and Exploration Process.

N Agents Dynamic Bargaining Process with Contentment and Deception ”à la Kalai-Smorodinsky”.. Potential Games.

Mathematics

Dynamical Dissipative (Expansive) Process with an Entropy. Liapunov Functions.

Ekeland Theorem, Approximate Optimization, Optimun, when agents optimize in real life?

Fixed Point Theorem: The Caristic Theorem without compacity assumptions.

Variational Analysis.Marginalism.Variational Inequalities.

Viability Theory and Differential Inclusions.

Moutain Pass Theorems. Exploration in the Large, in Parallel, in Series.Critical Points

Second Order Gradient Process with Inertia. Proximal Algorithms.

Direct Search Methods in Applied Optimization: Improving Process without Knowledge of the Gradient. Simulated Annealing as Improving Process.

Dynamic Game Theory, Games with Costs to Move and Exploration Process.Coupled Dynamics.

N Agents Dynamic Bargaining Process with Contentment and Deception ”à la Kalai-Smorodinsky”.. Potential Games.

References

-Aghion, P. Bolton, C. Harris and B. Jullien (1991), Optimal Learning by Experimentation, *Rev. Econ. Stud.* 58, 621-654

-Attouch H-X Goudou and P Redont (2000) ”The Heavy Ball with Friction Method,(part1),Communication to Contemporary Mathematics,vol 2,no 1,1-34.

-Attouch H-Soubeyran A (2004) ”Towards Stable Routines :Improving and Satisficing Enough by Exploration-Exploitation on an Unknown Landscape”,Working paper ,GREQAM.

-Attouch H-Teboulle M (2001) ”A Regularized Lotka-Volterra Dynamical System as a Continuous Proximal-Like
Method in Optimization”,ACSIOM,Université de Montpellier 2.
-Aubin J.P (1977) ”Applied Abstract Analysis ”,JohnWiley.New York.
-Aubin J.P -Y Ekeland (1983). ”Applied Non Linear Analysis”,
Wiley-Interscience.
-Aubin J.P and A Cellina (1984) ”Differential Inclusions” Springer-Verlag.
-Aubin-J.P(1994)”Initiation à l’Analyse Appliquée”,Masson.
-Benabou R-Tirole J (2001)”Self Confidence and Personal Motivation”,IDEI, Toulouse.
-Berge C (1970) ”Graphes et Hypergraphes”,Dunod.
-Berthoz A (2003)”La Decision”,Odile Jacob.
-Berthoz A (1997) ”Le Sens du Mouvement”,Odile Jacob.
-Bolte J (2003) ”Sur Les Systèmes Dynamiques Dissipatifs de Type Gradient.Applications en Optimisation” Thèse.Université de Montpellier 2.
-Brezis H-Browder F.E (1976) ”A General Principle on Ordered Sets in Non Linear Functional Analysis”,Adv.Math,21,355-364.
-Boujon C (2002)”L’Inhibition”,Solal.
-Caristi J (1976)” Fixed Point Theorems for Mapping Satisfying Inwardness Conditions”,
Trans.Am.Math,215,241-251.
-Cyert-March (1963)”A Behavioral Theory of the Firm”,Prentice Hall,
Englewood Cliffs.
-Conlisk J (1996) ”Why Bounded Rationality ”,Journal of Economic Litterature,Vol XXXIV,669-700.
-Dasgupta S (2003)”Multidisciplinary Creativity:the Case of Herbert
A.Simon”,Cognitive Science,Vol 27,Issue 5,683-707.
- Dar-El,E (2000) “Human Learning: From Learning Curves to Learning Organizations”, Kluwer Academic Publishers, Boston, 2000.
-Dutta J (2002)”Optimality Conditions in Approximate Optimization”,
Universitat Autonoma de Barcelona.
-Ekeland I (1974) ”On the Variational Principle”,
J Math.Analysis Applications,47,325-353.
-Fan K(1972) ”A Minimax Inequality and Applications”, in Inequalities 3,O Sisha Ed,103,113,Academic Press.
-Gittins J.C (1989)” Multi Armed Bandit Allocations Indices”Wiley,New York.
-Holland JH (1973) ”Genetic Algorithms and the Optimal Allocations of Trials”,SIAM Journal of Computing,2,88-105
-Horst R and P Pardalos (1995) :”Handbook of Global Optimization”,Kluwer Academic Publishers.
-Kang BG-Park S (1990) ”On Generalized Ordering Principles in Non Linear Analysis” ,,Non Linear Analysis,Theory ,Methods and Applications,,vol 14,no 2,159-165
-Le Boterf G (2001) ”Construire les Competences Individuelles et Collectives”,
Editions L’Organisation.
-Leloup B(2002) ”L’Incertitude de Deuxieme Ordre en Economie:Le Compromis Exploration-Exploitation”,These Nouveau Régime,Ecole Normale Superieure de Cachan, GRID-CREA.
-Levy-Leboyer C (2003)”La Motivation dans L’Entreprise ”,
Editions d’Organisation.
-Leibenstein H (1989) ”The Collected Essays of H Leibenstein”,

K Button,Edward-Elgar,Vol.2.
-Levinthal D-Warglien M(1999)"Landscape Design:Designing for Local Action in Complex Worlds",Organization Science,Vol 10,no3,342-357.
-Lewin A.Y,and Alii (1999)"The Coevolution of New Organizational Forms", Organization Science,Vol 10,Issue 5,535-550.
-Locke EA-Lathan GP (1990)"A Theory of Goal Setting and Task Performance ,Englewood Cliffs Prentice Hall.
-Martinez-Legaz J-Soubeyran A (2002) "Learning from Errors". UAB.University of Barcelona.
-Nelson R-Winter S (1997) "An Evolutionary Theory of Economic Change",in "Resources Firms and Strategies",Oxford Management Readers, Oxford.
-Pingle M-Day R (1996) "Modes of Economizing Behavior:Experimental Evidence", Journal of Economic Behavior and Organization,vol 29,191-209.
-Postrel S-Rumelt R (1992) "Incentives,Routines and Self Command ",Oxford University Press.
-Rant M (2003)"Organizational Fit and Organization, Environment Co-Evolution",European Applied Business Conference,Venice.
-Robson J (2001) "The Biological Basis of Economic Behavior",Journal of Economic Litterature ,Vol XXXIV,11-33.
-Rockafellar R(1972) "Convex Analysis".Princeton University Press
-Rockafellar R(1984)"Network Flows and Monotropic Optimization",Wiley-Inter Sciences Series.
-Rojot J (2003), "Théorie des Organisations", Eska Ed.
-Rothschild M (1974) "A Two Armed Bandit Theory of Market Pricing " Journal of Economic Theory,9,185-202.
-Rumelt R (1990) "Inertia and Transformation",mimeo.
-Samose JP-Sarrazin P-Cury F (1998) "la Fixation de But : une Technique pour Surmonter l'Anxieté et Augmenter la Confiance en Soi",mimeo.
-Simon H (1987) "Satisficing",in the New Palgrave,:A Dictionary of Economics",Eds Eatwell J-Milgate M-Newman P,London,Mac Millan,243-245.
-Simon H (1955) "A Behavioral Model of Rational Choice",Quaterly Journal of Economics,69,99-118.²
-Sinclair B -Desgagné-Soubeyran A (2000) " A Theory of Routines as Mindsavers", CiranoWorking Paper, 2000s,52.
- Singh S-Watson B-Srivastava P (1997) "Fixed Point Theory and Best Approximation: The KKM Map Principle",Kluwer.
- Sobel J (2000) , Economists' Models of Learning, *J. Econom. Theory* 94 , 241-261.
-Takahashi W (1976)" Non Linear Inequalities and Fixed Point Theorems, J.Math.Soc.Japan,168-181.
-Turinici M (1984) "A Generalization of Altman's Ordering Principle", Proc.Am.Math,90, 128-132.
-Tversky A (1972) "Choice by Elimination", Journal of Mathematical Psychology,9,341-367.
- Vroom,VH (1964) "Work and Motivation",NewYork,Wiley.
-Zeidler E (1991):"Non Linear Functional Analysis and its Applications", part 1,"Fixed Points Theorems",Springer.

Appendix: Proof of the Main Theorem

TO BE WRITTEN: for an analogous proof in a simpler case (uni dimensional case), see the original model of Attouch-Soubeyran (2004), using our "Worthwhile to Move

Cognitive Principles".

Non Cooperative Short Term Dynamic Games With Anchoring Effects

Let us show how our formulation can be linked to more traditional presentations of a dynamic game. In our case, it is a "sliding one shot dynamic game" with an added exploration process. Our presentation puts more weights on "Anchoring Effects", due to exploration around at each step. The main aspect which enchores the game to the past is the explicit modelization of costs to move, which generates Inertia-Reactivity or Learning Effects.

One Shot Control Formulation of Dynamic Games :**Games with Costs to Move**.

Start from the State space of performances (degree of resolution of problems) and define an Action or Choice space over this state space. This will define a "Decision Network". Let the vector of degrees or states of resolution of the list of problems P be the nodes (or state variables) $x \quad X, y \quad X$. Let a feasible move from x to y an edge (arc), ie a feasible choice or action $\alpha = \quad x \quad\quad y \quad\quad \Lambda \; x \quad\quad \Lambda$. The map $x \quad X \quad\quad \Lambda \; x \quad\quad \Lambda$ defines the feasible moves starting from $x \quad X$.

Let $P = \quad p_j, j \quad J$ the list of problem to solve. Let $x = \quad x_j, j \quad J$ the vector of their degrees of resolution. To simplify let us suppose that agent j has only one problem to solve, let us say p_j, and that the degree of resolution of his problem is x_j.

An action for agent j is a move $\alpha_j = \quad x_j \quad\quad y_j$ from x_j to y_j. The vector of actions $\alpha = \quad \alpha_j, j \quad J$ for the set of all agents impulses a general move from $x = \quad x_j, j \quad J$ to $y = \quad y_j, j \quad J$, an action $\alpha = \quad x \quad\quad y$ ..

An unilateral move for the group of agents Ω (or a Nash move) is a move α_j of only one agent j from $x = \quad x_j, x_{-j}$ to $y = \quad y_j, y_{-j}$ where $\alpha_{-j} = \quad x_{-j} \quad\quad y_{-j} = x_{-j}$.

Let $g_j \; x$ the utility for agent j when all problems are partially solved, for a vector degree of resolution $x \quad X$.

Let us suppose a move of all agents from $x = \quad x_j, x_{-j}$ to $y = \quad y_j, y_{-j}$

The dynamic gain of agent j is $G_j \quad x \quad\quad y \; /x \; = G_j \; \alpha/x \; = t \; y \; g_j \; y \; - C_j \; x,y$ .This gain includes both the costs to move during the moving phase and the exploitation gains during the next exploitation -exploration phase of lenght $t \; y \; \geq 0$.

If agent j prefers to stay there (a routine), his dynamic gain will be

$G_j \quad x \quad\quad x \; /x \; = t \; y \; g_j \; x \; - C_j \; x,x$ for $x = \quad x_1, x_2, ..., x_l$ and $y = \quad y_1, y_2, ..., y_l$,with $y_j = x_j$.

These formulations can help to make the link between our network formulation and the more traditional formulation of a dynamic game (one shot dynamic game for us, each step). In a vector space we can define a "vector velocity" $v = \quad y - x \; /h$ where $h = t \; x,y \; \geq 0$ is the time spend to move from x to $y \quad\quad y = x + hv$.

A "difference inclusion" $v = \quad y - x \; /h = v \quad\quad F \; x,h \quad\quad X$ defines the feasible "velocities of resolution of the problems" at each position $x \quad X$.

These dynamic gains are the dual of energy functions in a continuous time formulation.

Games in Characteristic Form:The Static Case

As we will see now, our formulation have a strong connection with Games in Caracteristic Form.

Consider the incremental advantage $A \; x,y,M$ of the subgroup M as a share $A \; x,y,M \; = \xi \; x,y,M \; A \; x,y,\Omega$ of the incremental advantage to move of the whole group.

The sharing rule ξ of the advantages to move is defined by $:M \in \mathcal{F} \to \xi(x,y,M) \geq 0, \xi(x,y,\Omega) = 1$. Suppose that this sharing rule is state independant, ie that it remains the same for any move, from any $x \in X$ to any $y \in X$. In other words $\xi(x,y,M) = \xi(M)$ for all $x,y \in X$.

This sharing rule $\xi : M \in \mathcal{F} \to \xi(M) \geq 0$ is also a non negative probabilistic measure which gives to each feasible subgroup $M \in \mathcal{F}$ the share of the advantages to move $A(x,y,M) = \xi(M)A(x,y,\Omega)$. A more usual way is to define such a non negative measure as an imputation $\xi = (\xi(\omega_1) = \xi_1, \xi(\omega_2) = \xi_2, ..., \xi(\omega_n) = \xi_n) = (\xi_1, \xi_2, ..., \xi_n) \geq 0$ such that $\xi(M) = \sum_{j \in M} \xi(\omega_j) = \sum_{j \in M} \xi_j$. Note that $\xi(\Omega) = 1$.

The same can be said for Costs Sharing Rules $\mu(x,y) : M \in \mathcal{F} \to \mu(x,y,M) \geq 0$, when $\mu(x,y,M) = \mu(M)$, and $C(x,y,M) = \mu(x,y,M)C(x,y,\Omega)$.

The worthwhile to move relation simplifies :

$A(x,y,M) \geq C(x,y,M) \Leftrightarrow \xi(M)A(x,y,\Omega) \geq \mu(M)C(x,y,\Omega)$.

Suppose to simplify even more an equalitarian sharing rule for costs to move $\mu(M) = (1/n)\mu(\Omega)$

The worthwhile to move relation simplifies :

$A(x,y,M) \geq C(x,y,M) \Leftrightarrow \xi(M)A(x,y,\Omega) \geq (1/n)C(x,y,\Omega)$.

Let us take $x,y \in X$ as given. Suppose that each feasible sub group $M \in \mathcal{F}$ can guaranty for himself the share $v(M)A(x,y,\Omega) = v(M)A(x,y,\Omega) \geq 0$ of the incremental advantages to move of the whole group, where $v : M \in \mathcal{F} \to v(M) \geq 0$, $v(\Omega) = 1$ is a characteristic function of a Cooperative game (Owen, 1982, chapter VIII) such that $v(\varphi) = 0$, $v(M \cup N) \geq v(M) + v(N)$, for $M,N \in \mathcal{F}$, and $M \cap N = \varphi$ (super additivity). Then the caracteristic function v is a suradditive non negative measure..

If we impose $\xi(M) \geq v(M) \geq 0$ for all $M \in \mathcal{F}$, ie $\xi \geq v \geq 0$, we are in a situation analog to the static core of a game. In this case the core asociated to the caracteristic function v is the set of non negative probabilistic measures such that $\xi \geq v$.