Sign, then Ratify: Negotiating about Thresholds

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Abstract

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1 Introduction

Contrary to popular belief, international treaties are not negotiated, then signed and then enter into force. The situation is, in fact, more complicated, particularly in the case of multilateral agreements. In this paper we will look at the procedures through which multilateral agreements are arrived at and put into effect. In accordance with the 1969 Vienna Convention on treaty law, the procedure for implementing any international treaty must involve two steps. The first or negotiation phase culminates in the signature of the treaty by the different parties. The second or ratification phase requires the ratification by the appropriate national bodies who thus give the official consent of each country to the agreement. When the agreement is multilateral, there may be special requirements specifying what stage in the ratification process has to be reached before the agreement enters into force. In the particular case of the Kyoto protocol, the parties had first to agree on the amount by which total greenhouse gas emissions should be reduced. Then they had to decide on how to split the efforts necessary to achieve this amount Lastly they had to determine how far the ratification process had to advance before the treaty could come into effect.

Justifications for the basic two step procedure can be found in different parts of the literature particularly in political science. The usual reason given there is that "once an agreement is found at the international level, its prescriptions must be translated into domestic laws through a ratification process" Currarini and Tulkens (2004). National delegates negotiate at the international level but the acceptability of this negotiation's outcome needs to be checked at the national level.

However, what concerns us in this paper, is not so much the justification for the existence of the ratification process itself as the reasons for the rules which govern it. The Kyoto protocol, like all treaties, specifies that it can only be implemented if it has been ratified. However it adds two further conditions which must be met before the agreement comes into effect. The first rule is that at least 55% of the countries which originally signed must also have ratified. The second rule requires that the countries which have ratified must have accounted for at least 55% of the pollution produced by signatory countries in the reference year (1990). From these thesholds it is clear that the ratification process does not ever have to be fully completed. Ratification by a subset of the signatories is sufficient.

What sort of theoretical tools can we use to analyse this sort of procedure? The double rule governing the implementation of the treaty is nothing other than the imposition of thresholds which must be met before the treaty comes into effect. Such thresholds are often invoked in the literature on contributions to a public good, for example. It is argued there that they are incentive compatible (cf. Bagnoli and Lipman (1989), Palfrey and Rosenthal (1984)). However, in that case the situation is rather different from the one we are considering here. In that literature, thresholds occur for two reasons. Either they are necessary because of the indivisible nature of the public good, (a certain minimum level of contribution is necessary to construct it for example), or they are imposed by a benevolent planner. Yet, in the case of multilateral agreements, the choice of the thresholds is made by the parties themselves and this choice forms an integral part of the negotiation process.

The aim of this paper is to explain the ratification procedure in the framework of the literature on coalition formation. As a first approach we propose a model in which the only strategic choice is that involving the choice of a threshold for ratification before the agreement comes into force. We consider a group of countries who have agreed upon a sharing rule, the Shapley Value. However, they have to agree on the rule for the ratification threshold. The idea is simple, the threshold will consist of a list of those coalitions such that if any of these ratifies the agreement will be implemented. A strategy for a player is simply a list of those coalitions which she considers sufficient for implementation. After the specification of the strategies of each of the players, the outcome is determined by a variant of a rule proposed by Hart et Kurz (1983). The result will be a list of coalitions such that if one of them forms, the Shapley Value can be implemented between its members. We will call these coalitions, threshold-coalitions.

We shall assume that, during the negotiation process each delegate has little or no information about the strategic behaviour of the political institutions of the country that she represents. This will lead to us considering the ratification process, not as a strategic game but rather as determining a value. We shall further assume that the order in which the signatories ratify the treaty is random but that it must be consistent with the set of threshold coalitions determined in the prevoius stage. By this, we mean that, given the order specified, the players who ratify first make up one of the threshold coalitions. Hart and Kurz (1983) defined a Shapley Value for a given coalition structure. They considered that players had the possibility to form coalitions during the negotiation process to protect their own interests. Here however, the set of possible ratification orders is not constrained by a partition of the players into coalitions but rather by a collection of coalitions, the threshold coalitions, the elements of which may well have a non-empty intersection.

We will show that there is always an equilibrium in which the grand coalition forms. Furthermore, when the game is defined by a characteristic function and there are not externalities between coalitions, this equilibrium is unique. However, when the game can be specified in partition form which specifies the externalities between coalitions, there may be equilibria in which there are threshold coalitions strictly smaller than the grand coalition.

2 T-Value

Consider a set of players $N = \{1, ..., n\}$. A coalition of players is a subset $S \subset N$. A coalition structure $B = (S_1, ..., S_n)$ is a partition of the set of players, $\cup_{k=1,...,m} S_k = N$ and $S_k \cap S_{k'} = \varnothing, \forall k \neq k'$. We denote by $\langle N \rangle$ the trivial coalition structure in which each player is independent and $\langle N \backslash S \rangle$ the partition of the set of players who do not belong to coalition S such that each of them is independant. We denote by B_S the coalition structure formed by a unique non trivial coalition S and $N \backslash S$ independent players. A partition function game (N, v) is defined for a given partition function v, which associates a value $v_B(S)$ for each coalition $S \subset N$ and for each given coalition structure B, to which S is an element. We consider superadditive games. In the case of partition functions, the property of superadditivity is defined as follows:

Definition 1 *A partition function game (N, v) is superadditive if, for each coalition structure B and two elements S and $T \in B$:*

$$v_B(S) + v_B(T) < v_{B'}(S \cup T)$$

in which B' is the structure formed taking out of B the elements S and T and adding the element $S \cup T$.

Note that, in our specific framework,

$$v_{B_S}(S) + \sum_{i \in N \backslash S} v_{B_S}(\{i\}) < v(N)$$

We give a simple extention of the Shapley value for a game in partition function form (N, v) drawn from Pham Do and Norde (2002). For a given coalition S, the incremental value of player i is $_{B_S}(S) - v_{B_{S \backslash i}}(S \backslash i)$ We could have considered more complex incremental values representing the player's contribution to a coalition when other coalitions are also already formed. However, if this could be an important point in a general framework with externalities represented by the partition function, it is not needed in our specific model in which the only objective of players is to sign a unique agreement. Therefore, in this simple framework the Shapley value for player i is his expected incremental value on all possible orders:

$$\varphi_i(N, v) = \frac{\sum_{S \subset N, i \in S} (s-1)!\,(n-s)! \left(v_{B_S}(S) - v_{B_{S \backslash i}}(S \backslash i) \right)}{n!}$$

In the same way, we can define a Shapley value for a subset of players set:

$$\varphi_i(M \subset N, v) = \frac{\sum_{S \subset M, i \in S} (s-1)!\,(m-s)! \left(v_{B_S}(S) - v_{B_{S \backslash i}}(S \backslash i) \right)}{m!}$$

This is the value obtained by each member of coalition M when the different members share $v_{B_M}(M)$, given that players in $\backslash M$ remain independent.

Now, consider the following situation. A player is a pair (C_i, R_i) in which C_i is a country, $i = 1, ..., n$ and R_i is its representative at the negotiation table. A set N of representatives for n countries would like to sign an agreement whose objective is the sharing of a value $v(N)$. They meet around the table to negotiate this agreement, but each representatives knows that she is not in the position to impose this negotiation's result to her country. In other words, they know that the negotiation, if it succeeds, will be followed by a ratification process. We assume that, during the negotiation, they do not have any idea about the order in which the different country will ratify the agreement. In other words, they anticipate that the order in which the countries will ratify is random. They agree on two points:

1) First, the implementation of the agreement when a coalition M of countries have ratified means that they will share $v(M)$ applying the Shapley value.

2) Indeed, the agreement will be only implemented by the signatories who have also ratified.

In this framework, an order designates the order in which the players ratify the agreement. If all the players have ratified, the agreement can be implemented, which means that each player receives her Shapley value $\varphi_i(N, v)$.

We consider as given, a set of $T-$coalitions $T(\sigma)$ which will be generated by the first stage of our model described in the following section. At this point, a formal definition of this set is needed.

Definition 2 *A set of T-coalitions $T(\sigma)$ is a set of coalitions in which no pair of coalitions is such that one is included in the other:*

$$\forall S_k, S_{k'} \in T(\sigma), \text{ we do not have } S_k \subset S_{k'} \text{ nor } S_{k'} \subset S_k$$

Remark that a set of T-coalitions is not a partition of the set of players. We may have that players belong to different T-coalitions in the same set. Now, we have the ingredients to define a value for a given partition function game (N, v) and a given set of T-coalitions $T(\sigma)$. An *order* $\rho_N = (r_i)_{i \in N}$ is a ranking of the set of players N. By extension, we define a ranking for each subset of the players set $\subset N$ and we will denote it by $\rho_S = (r_i)_{i \in S}$.

In the previous definition of the Shapley value for partition functions, as in the regular definition for characteristic functions, the assumption is made such that each possible order defined on the set of players N appears with the same probability $\frac{1}{n!}$. Here, we want to define a new value such that the set of T-coalitions restricts the set of possible orders. Hart and Kurz (1983) proved that the Owen value can be obtained when the possible orders are restricted to the set of orders "consistent" with a coalition structure. We adopt the same approach in the sense that we want to restrict the set of possible orders, but here, the restriction is imposed by a set of coalitions which does not necessarily form a partition. However, the main difference is in the way the consistency is defined. We restrict the orders to those in which the members of a $T-$oalition in the set $T(\sigma)$, occupy the first places. Formally, we keep the order $\rho = (r_i)_{i \in N}$ if there exists a T-coalition $S \in T(\sigma)$ such that $foralli \in S, r_i \leq s$. For each T- coalition of size t, there are $t!(n-t)!$ orders like this. However, we only consider the part of the order defined on the T-coalition. We will speak about *incomplete orders*. Therefore, for each given set of $T-$coalitions $T(\sigma)$, we will only consider $\sum\limits_{K \in T(\sigma)} k!$ orders. We assume that each one of these incomplete orders appears with the same probability $\frac{1}{\sum\limits_{K \in T(\sigma)} k!}$.

Therefore, if we consider all the possible incomplete orders, each coalition $M \in T(\sigma)$ appears with probability $\frac{m!}{\sum\limits_{K \in T(\sigma)} k!}$. For a given set $T(\sigma)$, each player gets her expected value on the different (incomplete) orders consistent with $T(\sigma)$. Two configurations can occur for the $i-$player in a given (incomplete) order associated with a coalition $M \in T(\sigma)$. When the $i-$layer belongs to the coalition $M \in T(\sigma)$ which is associated with the incomplete order, this player gets her Shapley value in this coalition. When she does not appear in the incomplete order, she gets her value defined by the partition function when the coalition $M \in T(\sigma)$ is formed. We will denote by $T_i(\sigma) \subset T(\sigma)$ the set of coalitions to which player i belongs. Therefore, her $T-$value, $\phi_i(v, T(\sigma))$ is defined by the expected payoff:

$$\phi_i(v, T(\sigma)) =$$

$$\frac{\sum\limits_{M \in T_i(\sigma)} \sum\limits_{S \subset M} (s-1)!(m-s)! \left(v_{B_S}(S) - v_{B_{S \setminus \{i\}}}(S \setminus \{i\})\right) + \sum\limits_{C \in T(\sigma) \setminus Ti(\sigma)} c! v_{B_M}(\{i\})}{\sum\limits_{K \in T(\sigma)} k!}$$

We make two remarks about this $T-$value. First, when the grand coalition $\in T(\sigma)$, the value is the Shapley value for partition games:

$$\phi_i(v, T(\sigma) = \{N\}) = \varphi_i(v)$$

The second remark is that nul players may have a non-zero $T-$value. Indeed, when a nul player belongs to a $T-$coalition, for the orders in which this coalition appears, the members get their Shapley value and the nul player gets nothing but for the orders such that the nul player does not belong to the coalition which appears, she does not participate to any value, but she can benefit from the value generated by the others throught the externalities. Therefore, if i is a nul player, her $T-$value is:

$$\phi_i(v, T(\sigma)) = \frac{\sum\limits_{C \in T(\sigma) \setminus Ti(\sigma)} c! v_{B_M}(\{i\})}{\sum\limits_{K \in T(\sigma)} k!}$$

For a given partition function game (N, v) and a set of $T-$oalitions $T(\sigma)$, the value for player i can be written:

$$\phi_i(v, T(\sigma)) = \frac{\sum\limits_{M \in T_i(\sigma)} m! \varphi_i(M, v) + \sum\limits_{C \in T(\sigma) \setminus Ti(\sigma)} c! v_{B_M}(\{i\})}{\sum\limits_{K \in T(\sigma)} k!}$$

We should make several remarks in order to compare this value with famous values we can find in the litterature. It is an extension of the Shapley value for games in partition form. In this aspect it differs from Aumann and Dreze (1964) and Owen's (1974) or Hart and Kurz (1983) values. On the opposite it is clearly related to a more recent value proposed by Phan Do and Norde (2002). In a second aspect it differs from this three values because it is defined for a set of $T-$coalitions. Aumann and Dreze's extension of the Shapley value and Hart and Kurz's value are defined for a given coalition structure. Here, coalitions in a $T-$set do not constitute a partition of the set of players. In these authors framework the coalition structure restricts the set of possible orders and the same coalition structure emerges in the different orders. On the opposite, here, the coalition structure to be formed depends on the order considered and the restriction on the set of possible orders works in a different way.

3 The negotiation process

The negotiation process is modeled as a normal form game. The players are the different countries' representatives. They have to choose the ratification process' rule. This rule concerns the conditions under which the ratification process will be considered as sucessfull and the agreement implemented. For each player $i \in N$, consider the set of non trivial coalitions to which she could belong: $E_i = \{S \mid S \subset N, i \in S, S \neq \{i\}\}$. Her strategy is then to choose among this set, which coalitions she wants to propose: it is a subset of the set of coalitions to which she could belong: $\sigma_i \subset E_i$. For a given strategy profile σ, some coalitions are said to be *feasible*, others are not. A coalition is feasible if it appears in the strategy of each of its members.

Definition 3 *Coalition S is feasible if and only if* $\forall i \in S, S \in \sigma_i$.

When players are seating around the table to negotiate, maybe they do not have the same preferences on the agreement which they want to sign, but at least each player has to show her willingness to negotiate. In the formal framework, this is reflected in two assumptions. First, we assume that each player makes at least one proposal, in other words $\sigma_i \neq \{\emptyset\}$. The second assumption which will be called assumption A is stronger and will be added to the model in a second analysis.

Assumption A says that each player has to make at least one *feasible* proposal.

Said in another way, the negotiation fails if one player proposes only non-feasible coalitions. Then, no coalition can be fixed, no agreement will be reached and each player will get $v_{\langle N\rangle}(\{i\})$. This assumption is equivalent to say that one part of the negotiation is already done. Subsets of players agree to propose feasible coalitions. This could be compared to the Γ-game proposed by Hart and Kurz (1983). In their game, each player proposes a unique coalition to which she wants to belong but the feasible coalitions are the only coalitions to be realized. Our game is different in two aspects. First, a proposal may be composed of several coalitions. The second point is that, Under assumption A, if one player proposes only non feasible coalition, all the negotiation fails. However, this is not in contradiction with their model since, here, the objective is to form a unique agreement. As we will see, this strong assumption does *not* facilitate the full cooperation.

We call these proposals, *threshold coalitions (*$T-$coalitions). They are feasible coalitions which satisfy an additional characteristic, which is that there is no other threshold coalition in which it is included.

Definition 4 *Let $T(\sigma)$ be the set of threshold coalitions. It is the largest set such that each coalition $S \in T(\sigma)$ satisfies two characteristics: (i) it is feasible, (ii) there is no other $K \in T(\sigma)$ such that $\subset K$.*

We denote by $T_i(\sigma)$ the subset of $T-$coalitions to which player i belongs. Payoffs are determined by the value defined in the previous section. For a given strategy profile σ, an i-player's payoff is: $\phi_i(v, T(\sigma))$. We use a concept of strong Nash equilibrium.

Definition 5 *A strategy profile σ is a strong Nash equilibrium if and only if:*

$$\nexists M \subset N \text{ and } \sigma'_M \text{ such that } \phi_i(v, T(\sigma)) > \phi_i(v, T(\sigma'_M, \sigma_{-M})), \forall i \in M$$

in which σ_M and σ'_M are profile of strategies of M-members and σ_{-M} the profile of strategies chosen by players who do not belong to M.

Proposition 6 *If (N, v) is a supperadditive game, the strategy profile $sigma_i = \{N\}, \forall i \in N$, such that the grand coalition forms and each player gets her Shapley value is always an equilibrium.*

Proof: The proof is obvious since the only possible deviation for any coalition D is, for its members, to propose smaller coalitions and to cancel the grand coalition. Then, if we denote by σ' the new strategy profile after the deviation, $N \notin T(\sigma')$ and clearly $\phi_i(v, T(\sigma')) < \varphi_i(N, v), \forall i \in D$. Indeed:

$$\phi_i(v, T(\sigma')) = \frac{\sum\limits_{M \in T_i(\sigma')} m! \varphi_i(M, v)}{\sum\limits_{K \in T(\sigma')} k!} < \varphi_i(N, v), \forall i \in D$$

Proposition 7 *If (N, v) is a superadditive characteristic function game (no externalities), the grand coalition is the unique stable structure.*

Proof: First, note that we always have $\varphi_i(M, v) < \varphi_i(N, v)$ and that, if the game is superadditive but without externalities, $v(\{i\}) < \varphi_i(N, v)$. Now, consider a strategy profile σ such that $N \notin T(\sigma)$. Then, it is beneficial for all the players to deviate and propose N. Indeed:

$$\phi_i(v, T(\sigma')) =$$

$$\sum_{M \in T_i(\sigma')} m! \varphi_i(M, v) + \sum_{C \in T(\sigma') \setminus Ti(\sigma')} c! v(\{i\}) < \varphi_i(N, v) \sum_{K \in T(\sigma')} k!$$

Proposition 8 *If (N, v) is a partition function game, $\sigma_i = \{N\}\ \forall i \in N$ is always an equilibrium but other equilibria can exist in which the grand coalition does not form.*

Proof: cf. the examples presented in the following section.

4 Inefficiency

Example 9 *Three symmetric players, $N = \{1, 2, 3\}$,*

$$v_{B_{\langle N \rangle}}(\{i\}) = 0, v_{B_{(i,j)}}(i, j) = a, v_{B_{(i,j)}}(k) = b, v_{B_N}(1, 2, 3) = c$$

The game is superadditive and thus, $a > 0, c > a + b$. The Shapley value for each player i is $\varphi_i = \frac{c}{3}$. We consider that there are positive externalities such that player k free rides on the two other players when they form a coalition: $v_{B_{(i,j)}}(k) = b > \frac{a}{2} = \varphi_i(i, j) = \varphi_j(i, j)$. We consider first what happens without Assumption A. Two cases have to be considered. If $3b < c$, the only equilibrium is $\sigma_i^* = \{N\}\ \forall i \in N$, and the only threshold coalition is the grand coalition. If $3b > c$, there are three other equilibria:

$$\sigma_i^{**} = \{(i, j)\}, \sigma_j^{**} = \{(i, j)\}, \sigma_k^{**} = \{(j, k)\}.$$

and the outcome is a threshold coalition of size two. The T-value is $\phi_i = \phi_j = \frac{a}{2}$ and $\phi_k = b$. Now, we consider what happens under Assumption A. σ^* is still an equilibrium. But there is another equilibrium in which the grand coalition is not a threshold coalition. Consider the following strategy profile:

$$\sigma_i^{***} = \{(i, j)\}, \sigma_j^{***} = \{(i, j); (j, k)\}, \sigma_k^{***} = \{(j, k)\}.$$

There are two threshold coalitions: (i, j) and (j, k). The T-value is: $\phi_2 = \frac{a}{2}$ and $\phi_1 = \phi_3 = \frac{a}{4} + \frac{b}{2}$. Of course, if the condition $b > \frac{a}{2}$ is not verified there is no possibility to free ride and the unique equilibrium is associated with the grand coalition.

Example 10 *Three asymmetric players:* $N = \{1,2,3\}, v_{B_{\langle N\rangle}}(\{i\}) = 0$, $v_{B_{(1,2)}}(1,2) = 1$, $v_{B_{(1,2)}}(\{3\}) = 1$, $v_{B_{(1,3)}}(1,3) = 1$, $v_{B_{(1,3)}}(\{2\}) = \frac{3}{4}$ $v_{B_{(2,3)}}(2,3) = \frac{1}{2}$, $v_{B_{(2,3)}}(\{1\}) = \frac{1}{2}$, ${}_{B_N}(1,2,3) = 2$. *In this game, the Shapley value is:* $\varphi_1 = \frac{10}{12}$, $\varphi_2 = \varphi_3 = \frac{7}{12}$ *and corresponds to the payoffs at the equilibrium* $\sigma_i^* = \{N\}\ \forall i \in N$. *Player 1 has no advantage to free ride. Her participation to the coalition is more effective than the other's participation. The value of coalitions* $(1,2)$ *or* $(1,3)$ *is twice the value of coalition* $(2,3)$. *Her payoff as a free rider on coalition* $(2,3)$, *which is* $\frac{1}{2}$, *is the same as her Shapley value in the coalition* $(1,2)$ *or* $(1,3)$ *and it is less than its Shapley value* $\left(\frac{1}{2} < \frac{10}{12}\right)$. *She is the participant who is the most anxious to reach the full agreement. On the opposite, players 2 and 3 have a lot to win free riding:*

$$v_{B_{(1,2)}}(\{3\}) = 1 > \frac{7}{12} = \varphi_3 \text{ and } v_{B_{(1,3)}}(\{2\}) = \frac{3}{4} > \frac{7}{12} = \varphi_2$$

Without Assumption A, there are three equilibria:

$$\sigma_i^* = \{N\}\ \forall i \in N$$

the grand coalition forms and payoffs are $\varphi_1 = \frac{10}{12}$, $\varphi_2 = \varphi_3 = \frac{7}{12}$. *Player 1 gets more because her participation is more effective: her incremental value to the grand coalition is bigger.*

$$\sigma_1^{**} = \{(1,2);(1,3),(1,2,3)\}, \sigma_2^{**} = \{(1,2)\}, \sigma_3^{**} = \{(2,3)\}.$$

The unique threshold coalition is $(1,2)$. *Payoffs are* $\phi_1 = \phi_2 = \frac{1}{2}, \phi_3 = 1$

$$\sigma_1^{***} = \{(1,2);(1,3),(1,2,3)\}, \sigma_2^{***} = \{(2,3)\}, \sigma_3^{***} = \{(1,3)\}.$$

The unique threshold coalition is $(1,3)$. *Payoffs are* $\phi_1 = \phi_3 = \frac{1}{2}, \phi_2 = \frac{3}{4}$. *Now we assume that players have to propose at least one feasible coalition. The grand coalition is still stable, the two others are not. However, there is another equilibrium. Consider the following strategy profile:*

$$\sigma_1^{****} = \{(1,2);(1,3)\}, \sigma_2^{****} = \{(1,2)\}, \sigma_3^{***}* = \{(1,3)\}$$

The associated threshold set is $T(\sigma^{****}) = \{(1,2),(1,3)\}$ *and the value:*

$$\phi_1(v, T(\sigma^{**})) = \frac{1}{2} < \varphi_1; \phi_2(v, T(\sigma^{**})) = \phi_3(v, T(\sigma^{**})) = \frac{5}{8} > \varphi_2 = \varphi_3.$$

Clearly this is an equilibrium. Players 2 and 3 are better in comparison with σ^* *because they free ride on player 1.*

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