Consumer Networks and Search Equilibria

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Abstract

Two identical firms compete in prices. A finite number of consumers is linked in a social network. Consumers obtain information about price quotations through costly non-sequential search. The price quotations each consumer privately observes are freely shared with his acquaintances. We show that social interactions, as a channel to exchange information, make the equilibrium outcome be closer to the monopolistic outcome. The reason is that connections create incentives for consumers to free-ride on each other. Overall, competition between firms decreases leading to higher expected prices and lower consumer surplus. This paper proposes a general research question, which is the study of how social interactions and markets jointly shape economic outcomes.

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1 Introduction

A persuasive body of empirical work shows that social interactions have a profound effect on market outcomes. Feick and Price (1986, 1987) show that consumers obtain much of their information about products via their social contacts. Consumer reviews on the Web contain information on qualities and prices of a variety of products, which is accessible by potential customers at no cost. Social networks are also crucial in labor markets. A prevalent use of social contacts in finding jobs is robust across professions, race, gender and countries, see e.g. Granovetter (1974), Pellizari (2004) and Rees and Shultz (1970). These examples identify the fundamental features of connections. Firstly, connections are a conduit of information. Secondly, they transform the information costly obtained by consumers into a public good. Thirdly, they facilitate the matching between buyers and sellers. The study of these effects and their implications on market outcomes is the object of this paper. Does equilibrium prediction change when social interactions and market jointly shape economic outcomes? Do firms benefit from consumers' information sharing? Are there network properties which have a profound and systematic effect on equilibrium outcomes?

We study a non-sequential search model. There are two identical firms producing a homogeneous good. Firms set prices so as to maximize profits. Consumers have a common willingness to pay for the good and buy at most a single unit. For a transaction to take place a consumer must know some price quotations. Each consumer searches a subset of the set of firms to obtain price quotations before receiving offers. This search is costly.[1] In addition, consumers are connected in a network of relations. The price quotations each consumer obtains from his own search are shared freely with his acquaintances. The game is a one-shot simultaneous move game. Firms set prices and consumers decide how many searches to make. Consumers share information and each consumer buys at the lowest price he observes, if he finds it profitable.

We show that in the presence of a social network consumers search less and firms' competition is less severe. Therefore in the presence of social interactions the equilibrium is closer to the monopolistic outcome. We also compare outcomes across networks. We show that when we add links to consumer networks the equilibrium expected price may increase and consumer surplus may be lower. These results have a very clear intuition. Social interactions, as a channel to share information, provide consumers with *wrong* incentives to engage in costly information acquisition. That is, connections increase consumers' incentives to free-ride on each other. This may be beneficial for firms, while it may make consumers worse-off.

In particular, the equilibrium expected price is higher in more connected networks when search costs are low. Low search costs imply that in equilibrium consumers search with high intensity. In this case, an increase in the connectivity of the network

[1] Morgan and Manning (1985) show that non-sequential search is appealing when consumers find it optimal to gather price information quickly. See Janssen and Moraga-Gonzalez (2004) for a variety of oligopolistic markets where consumers search non-sequentially.

generates a *strong* free-riding effect because connections are a reliable conduit of information. Similarly, an increase in the connectivity of the network so that the network becomes more clustered also increases the expected price. The reason being that in equilibrium a consumer does not have an incentive to search if one of his direct neighbors already searches.

Even if our model is quite stylized it provides new interesting insights on the overall functioning of the economy when networks and markets are incorporated in a single strategic framework. In particular, it shows that connections and consumers' incentives to search are substitute channels for information acquisition in the economy. These results provide a plausible explanation to why price dispersion is still persistent in the Internet and price levels are not lower than offline prices, see e.g. Bakos (1997), Brynjolfsson and Smith (2000). In the initial years of Internet, it was widely predicted that Internet would lead to a frictionless economy in which prices continually decrease and converge to perfect competition levels. The reason for such predictions was that in the Internet consumers face very low search costs and therefore firms would confront a world of increasing competition and transparency. However, a feature of the Internet is that it facilitates the information sharing between consumers, an effect which decreases competition and that is opposite to the effects provoked by a decrease in search costs.

We now discuss the related literature. The discussion shows that looking at markets or at social interactions alone may be misleading. The consumer search literature is well established in economics, e.g. Anderson and Renault (2000), Bester (1994), Braverman (1980), Burdett and Coles (1997), McAfee (1995), Morgan and Manning (1985), Stahl (1989,1996), Varian (1980). When consumers do not have connections our model degenerates to a duopolistic version of Burdett and Judd (1983). In this case, in equilibrium consumers always search with some probability. We show that when consumers have connections there are situations in which in equilibrium some consumers do not search at all. In these cases consumers are on average less informed so that firms' competition is lower.

Another paper which comes close to ours is Janssen and Moraga-Gonzalez (2004). They study a version of Burdett and Judd (1983) where consumers are ex-ante heterogeneous. One fraction of consumers searches at no cost and therefore they are fully informed. The remaining fraction has positive search cost. One may well think that an increase in the number of fully informed consumers has an effect which is equivalent to an increase in the connectivity of the consumer network. In Janssen and Moraga-Gonzalez (2004) average prices are (weakly) decreasing in the fraction of fully informed consumers. This contrasts with our findings.

We finally discuss the relation of our paper to the theory of networks. There is an increasing theoretical attention to the effect of decentralized interactions on a variety of settings, e.g. Bala and Goyal (1998), Ballester et al. (2004), Calvo and Jackson (2004), Goyal and Moraga-Gonzalez (2001) and Kranton and Minehart (2001). All these papers belong to a new general class of games in which the economic activity of players is embedded in a network, which affects players' incentives. Bramoulle and Kranton (2005) examine a model of social experimentation where individuals search for new information and the results of their search are non-excludable along links. In their model the benefit of searching is given by some exogenous function, while in our model it is endogenously determined by the optimal firms' price behavior. We can then study not only how social interactions influence search incentives but also how they influence market outcomes.

Section 2 presents the model. Section 3 focuses on the case where each consumer has the same number of links. Section 4 extends the analysis to arbitrary consumer networks. Section 5 concludes. Proofs are relegated to the appendix.

2 The Model

There are 2 firms producing a homogeneous good at constant returns to scale. We normalize their identical unit production cost to zero, without loss of generality. Firms compete in prices. We denote firm i's strategy by the price distribution $F_i(p)$ defined on a support $S_i \subset [\underline{p}_i, \bar{p}_i]$, $i = 1, 2$.

There are M identical consumers who want to buy a single unit of the product, $\mathbf{M} = \{1, ..., M\}$. The maximum willingness to pay for each consumer is $\tilde{p} > 0$. A consumer can buy from firm i only if he observes a price quotation of that firm. Each consumer searches a subset of the set of firms to obtain price quotations before receiving offers; the cost of each price quotation observed is $c > 0$, where $c < \tilde{p}$. Consumer m's strategy is then a probability distribution over the set $\{0, 1, 2\}$. We denote by q_x^m the probability that consumer $m \in \mathbf{M}$ searches x firms; thus m's strategy is $\mathbf{q}^m = \{q_0^m, q_1^m, q_2^m\}$, $q_x^m \in [0, 1]$, $\sum_{x=0}^{2} q_x^m = 1$. We also assume that when a consumer searches for one price quotation he evenly samples one of the two firms.

In addition, each consumer obtains price quotations by sharing information with other consumers. To model this we assume that consumers are connected in a network of relationships. The network therefore represents the topology of social interaction. The consumer network is exogenously given and it is described by the symmetric matrix $g \in \{0, 1\}^{M \times M}$, with $g_{m,l} = 1$ denoting that m and l are connected, while $g_{m,l} = 0$ meaning that the two consumers are not connected. For convenience it is assumed that $g_{m,m} = 0$ for all $m \in \mathbf{M}$. The set of acquaintances of consumer m is denoted by $\mathbf{M}_m(g) = \{l \in \mathbf{M} : g_{m,l} = 1\}$; let $\eta^m(g) = |\mathbf{M}_m(g)|$. We assume that the price quotations each consumer obtains by his private search are freely shared with his acquaintances. We remark that when the network is empty (there are no links) the model degenerates to a duopolistic version of Burdett and Judd (1983).

Firms and consumers know the architecture of the network and play a simultaneous-move game. We study Nash equilibria. An individual firm chooses its optimal price strategy, taking the price choices of its rivals, as well as consumers' search behavior as given. Consumers form conjectures about the firms' price behavior and optimally decide how many price observations to search for. Consumers share the information they have obtained and transactions take place. One of the main objective of the paper is to study the impact of adding links to the network on consumers' search intensity, firms' pricing, social welfare and consumer surplus.

To perform this analysis we will need to refine the Nash equilibrium set. We will consider a simple notion of stability based on Nash tâtonnement, see e.g. Fudenberg and Tirole (1996), Bramoulle and Kranton (2005), Fershtman and Fishman (1992). We say that a Nash equilibrium is stable if we can perturb it slightly and by letting agents play their best responses we converge to the original Nash equilibrium.

In the first part of the analysis we focus on a particular class of networks: regular networks. A regular network is a network g in which each consumer has the same number of links, $\eta^m(g) = k$ for all $m \in \mathbf{M}$. We denote by k the degree of a regular network and it takes value $k = 0, ..., M-1$. A regular network of degree k is denoted by g^k. Note that a regular network may not exist when M is odd. Hence, in the paper we assume that M is even. When we consider regular networks we focus on symmetric equilibria. Note that the characterization of symmetric equilibria in regular networks is equivalent to the characterization of equilibria in a model where each consumer shares information with a fixed number k of other consumers randomly sampled from the whole population. We provide this interpretation in Subsection 3.3. In the second part of the paper we provide a characterization of equilibria for arbitrary networks where consumers use a pure strategy.

3 Regular Networks

In this section we study symmetric Nash equilibria in regular consumer networks. The first result characterizes equilibria in which consumers use symmetric pure strategies.

Proposition 3.1 *For every g^k the only generic equilibria in which consumers use a symmetric pure strategy take the following form: consumers never search, $q_0 = 1$, and firms charge a price $p \in [\tilde{p} - c, \tilde{p}]$.*[2]

If consumers search surely for one price quotation, then firms price in such a way that consumers will either prefer to search less, if search costs are sufficiently high, or to search more otherwise. If consumers search surely for two price quotations, firms' competition will drive prices to marginal cost. This creates incentives for consumers to search less. Thus, we investigate equilibria in which consumers use a mixed strategy. The next lemma shows the possible candidates for an equilibrium.

[2] In the proof of this Proposition we show that there exists a non-generic equilibrium in which consumers search for one price quotation and firms price randomly. This equilibrium is not generic because it exists for a specific search cost.

Lemma 3.1 *For every g^k in every equilibrium in which consumers use a symmetric mixed strategy firms price according to an atomless price distribution, $F(p)$, defined on a convex support S. Moreover, if $k = 0$ then $q_1 + q_2 = 1$, $q_1, q_2 \in (0, 1)$, while if $k > 0$, then either $q_1 + q_2 = 1$, $q_1, q_2 \in (0, 1)$ or $q_0 + q_1 = 1$, $q_0, q_1 \in (0, 1)$.*

We first remark that equilibria must involve random pricing. Since consumers search randomly, some consumers in the market are ex-post more informed than others; a fact which allows firms to extract profits by randomizing their prices. More interesting is that a new possible equilibrium candidate emerges when networks interplay with markets, i.e. $k > 0$. This has the property that consumers randomize between searching once and not searching at all. The intuition for this is that social interactions imply that, with some probability, even consumers who do not search at all observe both firms' prices. This allows firms to extract profits by randomizing in prices.

We shall now characterize equilibria and provide some comparative statics results. We say that an equilibrium is a high search intensity equilibrium (HSIE) if $q_1 + q_2 = 1$, $q_1, q_2 > 0$. Similarly, a low search intensity equilibrium (LSIE) is an equilibrium where $q_0 + q_1 = 1$ $q_0, q_1 > 0$. For expositional reasons we first analyze the HSIE and then we turn to the LSIE.

3.1 High search intensity equilibria

We denote by $E[p]$ the expected price associated to a price distribution $F(p)$. Similarly, we denote by $E[\min\{p_i, p_j\}]$ the expected minimum price obtained by randomly sampling two prices from the support of $F(p)$. The next result characterizes high search intensity equilibria.

Theorem 3.1 *For every g^k a HSIE takes the following form:*

(i) Firms price according to an atomless price distribution $F(p) = 1 - \frac{q_1^{*k+1}}{2(2^k - q_1^{*k+1})} \frac{\tilde{p}-p}{p}$, $S = [\frac{q_1^{*k+1}}{2^{k+1} - q_1^{*k+1}} \tilde{p}, \tilde{p}]$;

(ii) consumers search for a price quotation with probability q_1^*, where q_1^* is a solution of $\frac{q_1^k}{2^k}[E(p) - E[\min\{p_i, p_j\}]] = c$; with the remaining probability consumers search for two price quotations.

For every k there exists $\bar{c}_k > 0$ such that a HSIE exists if and only if $c \in (0, \bar{c}_k)$. For every k and $c \in (0, \bar{c}_k)$ there exists a unique stable HSIE.

Figure 1 provides a graphical representation of the equilibrium condition of an arbitrary consumer. Figure 1a plots the consumer's marginal gain from sampling two prices instead of one price when $k = 0$ and when $k = 1$. Given the equilibrium price distribution this is a function of q_1. In the figure we denote by q_1^{*k} the stable HSIE

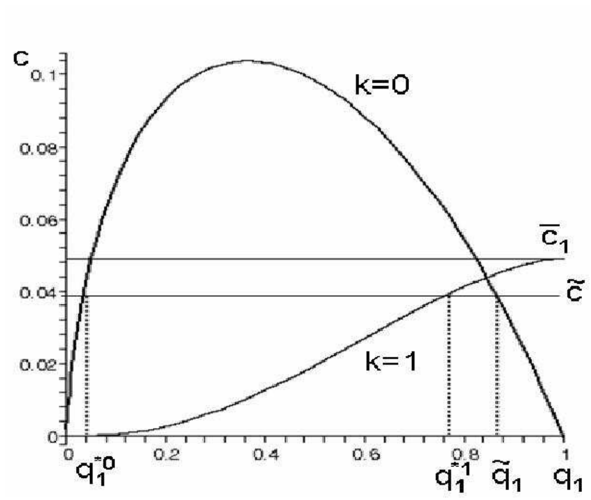


Figure 1a

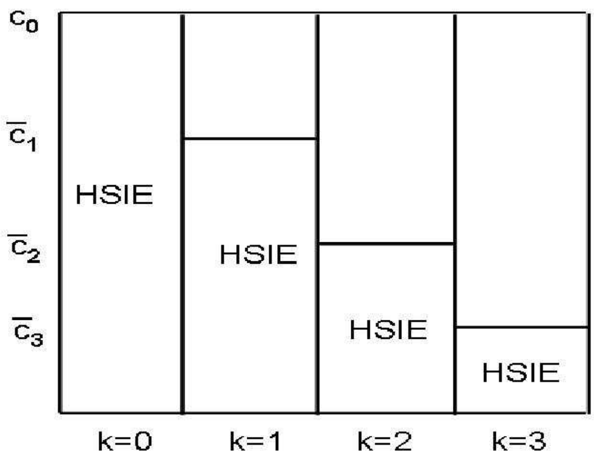


Figure 1b

Figure 1: High Search Intensity Equilibrium

for search costs $\tilde{c}$.[3] Figure 1b shows for which search costs the stable HSIE exists for different values of k.

Figure 1a and Figure 1b illustrate the impact that networks have on equilibria. Essentially, consumer networks reduce the incentives that each consumer has to search for price quotations. Thus, for sufficiently high search costs a HSIE does not exist when $k > 0$, while it exists when $k = 0$. The intuition behind this result is crucial and we now elaborate on it. The equilibrium condition of a consumer is the following:

$$\frac{q_1^k}{2^k}[E(p) - E[\min\{p_i, p_j\}]] = c \tag{1}$$

Each consumer trades-off the marginal cost from sampling two prices instead of one price with its marginal gain. The marginal gain is the difference between buying at the expected price and at the expected minimum price, weighted by the probability with which a consumer who searches for one price will indeed observe only one price quotation. When consumers are linked in a network, each consumer is more likely to observe two price quotations even when he samples only one price. This reduces

[3] We elaborate on stability. Consider Figure 1a and suppose $k = 0$. Note that the equilibrium defined by $\tilde{q}_1$ is unstable. Indeed, for $\tilde{q}_1 + \epsilon$, where ϵ is small and positive, the marginal cost $\tilde{c}$ is higher than the associated marginal gain. This implies that a consumer has an incentive to search once with higher probability so that we move away from $\tilde{q}_1$.

consumers' incentives to search. Thus, social interactions lead to a severe free-riding problem.

Ultimately, we are interested in how different patterns of social interactions alter market outcomes. This amounts to comparing market outcomes across networks with different connectivity. Do more connected consumer networks increase firms' competition, or quite the opposite? The next proposition establishes a clear relationship between the degree of the network and players' search intensity, expected utilities, consumer surplus and social welfare. We focus on the stable HSIE.

Proposition 3.2 *Consider g^k and g^{k+1}, $k \in [1, ..., M-2]$ and assume that $c < \bar{c}_{k+1}$. In the stable HSIE the following holds: (a) consumers search less frequently in g^{k+1} than in g^k; (b) expected price is higher in g^{k+1} than in g^k; (c) social welfare is higher in g^{k+1} than in g^k; (d) consumer surplus is lower in g^{k+1} than in g^k.*

Proposition 3.2 shows that when we add links to a regular consumer network the expected price becomes closer to the monopolistic price. An increase in the network degree induces two effects. The first effect is that keeping constant consumers' behavior, the expected number of consumers who observe two price quotations increases. Hence, firms compete more intensively and therefore they price more aggressively. Indeed, it is easy to show that for each $q_1 \in (0, 1)$, the equilibrium price distribution defined in Theorem 3.1 can be ranked with respect to k in the first-order stochastic sense, i.e. $F(p)$ associated to g^k first order stochastically dominates $F(p)$ associated to g^{k+1}.[4] The second effect is the free-riding effect which we have discussed above.

Overall the expected price increases because the free-riding effect offsets the former effect. This crucially depends on the fact that consumers are searching with high intensity. Indeed, in this case, a consumer will heavily rely on connections to observe price quotations. This makes free-riding strong enough to increase the number of consumers that, in expected terms, observe only one price quotation. The implications for consumer surplus and social welfare are also interesting. Consumer surplus decreases because prices at which consumers buy are on average higher. Social welfare increases because of the saving on search costs.

We emphasize that the results presented in Proposition 3.2 could not be obtained in the absence of social interactions. To illustrate this we compare our result with the results provided by Janssen and Moraga-Gonzalez (2004). They study an oligopolistic version of Burdett and Judd (1983) where some consumers observe all price quotations for free. An increase in the number of these consumers create externalities to all other players and therefore alters the equilibrium. This exercise is somewhat analogous to adding links in the network; yet, the implications on how equilibria change are sharply

[4]Note that if a consumer could choose whether to share his information with his neighbors he would do so. Indeed, given $q_1 \in (0, 1)$ and $F(p)$, when a consumer shares information with his neighbors, the probability that an acquaintance observes two prices instead of one price quotation increases. This creates positive externalities to all consumers since firms compete more often so that they price more aggressively.

different. Indeed, in Janssen and Moraga-Gonzalez (2004) an increase in the number of fully informed consumers always produce positive externalities in the sense that average price becomes always closer to the competitive price and consumer surplus never decreases.

3.2 Low search intensity equilibria

We now characterize low search intensity equilibria.

Theorem 3.2 *For every g^k, $k = 1, ..., M-1$, a LSIE takes the following form:*

(i) Firms price according to an atomless price distribution

$$F(p) = 1 - \frac{(1+q_0^*)^{k+1} - 2^{k+1} q_0^{*k+1}}{2(2^k(1+q_0^{*k+1}) - (1+q_0^*)^{k+1})} \frac{\tilde{p} - p}{p}, S = \left[\frac{(1+q_0^*)^{k+1} - 2^{k+1} q_0^{*k+1}}{2^{k+1} - (1+q_0^*)^{k+1}} \tilde{p}, \tilde{p} \right] \tag{2}$$

(ii) consumers do not search with probability q_0^*, where q_0^* is a solution of

$$\frac{(1+q_0)^k - 2^k q_0^k}{2^k} [E[p] - E[\min\{p_i, p_j\}]] + q_0^k (\tilde{p} - E[p]) = c;$$

and with the remaining probability consumers search for one price quotation.

For every k, there exists a $c_k^* > 0$ such that a LSIE exists if and only if $c \in (0, c_k^*)$. Furthermore, there exists a $\bar{c}_k > 0$, where $c_k^* > \bar{c}_k$, such that for all $c \in (\bar{c}_k, c_k^*)$ a stable LSIE exists.

Figure 2 plots the marginal gain from sampling one price quotation instead of not searching when $k = 1$ and when $k = 2$; q_0^{*k} represents the probability that a consumer does not search in the stable LSIE (for search cost $\tilde{c}$).

For moderate search costs there exists a stable LSIE. The proof in the appendix also shows that there always exists at least another equilibrium, which however is unstable. Numerical simulations reveal that these are the only two possible LSIE. To validate our comparative static analysis, it is important to note that a stable LSIE exists for search costs higher than $\bar{c}_k$. When search costs are exactly $\bar{c}_k$ the stable HSIE described in Theorem 3.1 and the stable LSIE described in Theorem 3.2 coincide. Specifically, consumers sample one price quotation with probability one. So the stable HSIE and the stable LSIE do not generally overlap.

We now study the impact of adding links on the stable LSIE. The intractability of the equilibrium conditions forces us to rely on simulations. The findings are summarized in the following remark.[5]

[5] We have run simulations for $k = 1, ..., M-1$, for different value of M. For any (k, M) we determine the range of the search costs for which the stable equilibrium exists, say $[\bar{c}_1(k, M), c^*(k, M)]$. For each c in this interval we derive the stable solution $q_0^*(k, c)$. Using this value we compute the expected price, social welfare and consumer surplus.

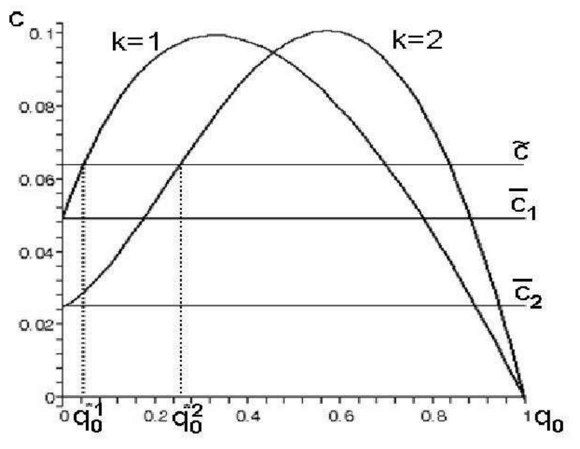


Figure 2: Low Search Intensity Equilibrium

Remark 3.1 *Consider g^k and g^{k+1}, $k \in [1, ..., M-2]$, and assume that $c \in (\bar{c}_k, c_k^*)$. In the stable LSIE the following holds: (a) consumers search less frequently in g^{k+1} than g^k; (b) expected price is lower in g^{k+1} than g^k; (c) social welfare is lower in g^{k+1} than g^k; (d) consumer surplus is higher in g^{k+1} than g^k.*

Figure 3 illustrates the results obtained from the numerical simulation. There, I plot, for example, the equilibrium average price for different search costs and different network degrees. Note that the two effects described in the HSIE are still at work in this new equilibrium. However, in sharp contrast with Proposition 3.2, the overall effect of an increase of the network degree is to boost firms' competition and therefore expected prices become closer to the competitive price.

The reason is that in the LSIE consumers have less incentives to free-ride on others than in the HSIE. To see this note that the equilibrium condition for a consumer is as follows:

$$\frac{(1+q_0)^k - 2^k q_0^k}{2^k}[E[p] - E[\min\{p_i, p_j\}]] + q_0^k(\tilde{p} - E[p]) = c \qquad (3)$$

The marginal gain for a consumer from sampling one price instead of not searching depends on two factors. Similarly to the HSIE, when a consumer searches one firm he is more likely to observe two price quotations. However, in the LSIE while a consumer who samples one price will always have the necessary information to buy, this is not the case for a consumer who does not search. The incentives a consumer has to free-ride on other consumers is therefore reduced. The fact that social welfare decreases also illustrates the risk inherent in free-riding. Indeed, the losses generated by the transactions which do not take place offset the savings on search costs. Finally, consumer surplus increases because prices are on average lower and consumers are more likely to compare prices.

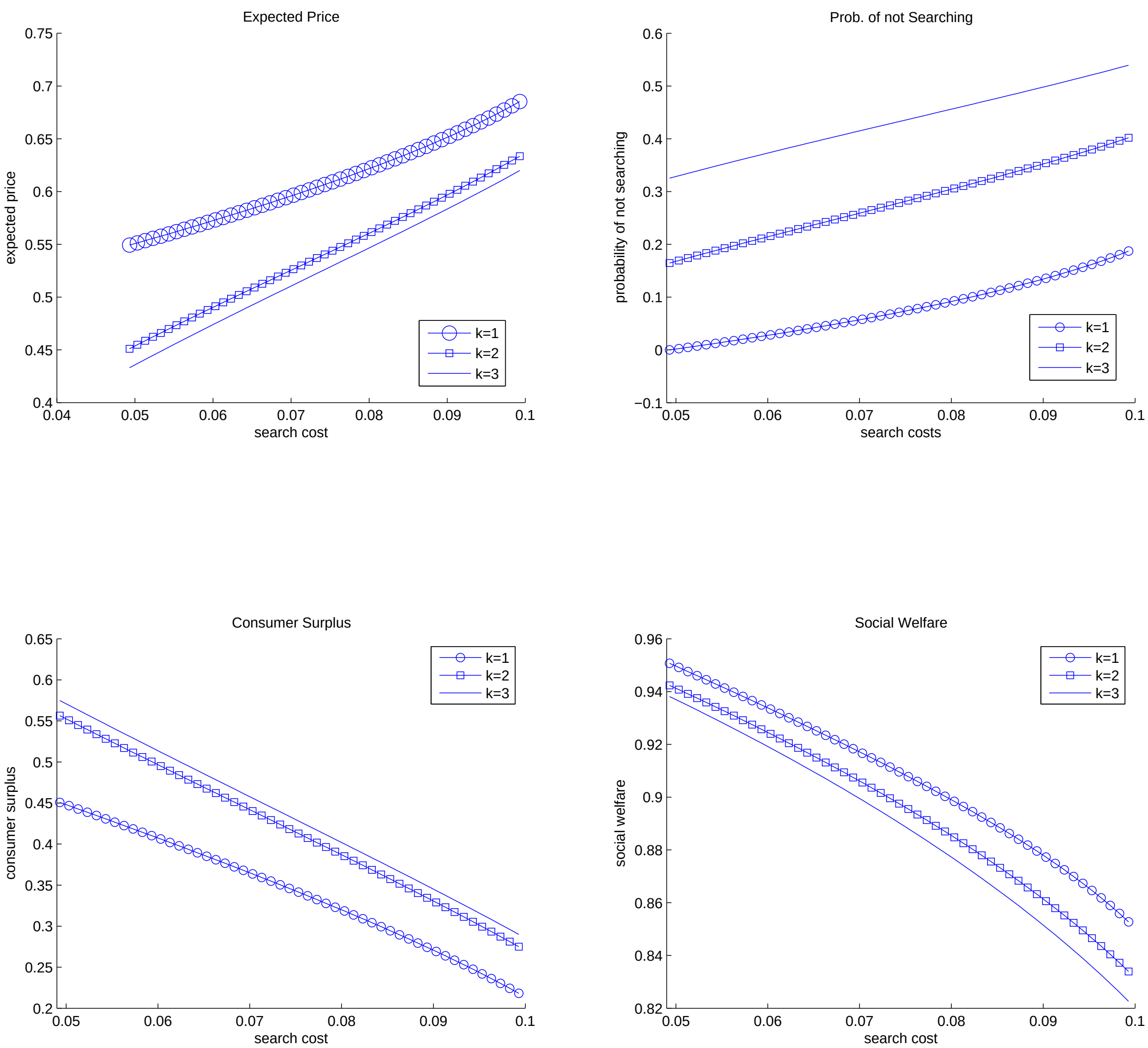


Figure 3: Comparative statics of the Low Search Intensity Equilibrium

3.3 Interpretation: A Model of Internet Competition

The analysis developed so far has focused on regular consumer networks and symmetric strategies. This analysis is useful because it clearly extends to a situation where the population of consumers is large and the specific pattern of interactions is not known beforehand but each consumer anticipates that he will be interacting with some fixed number of other consumers randomly sampled from the whole population.[6] To fix ideas, consider the following model of internet competition. Firms are e-retailers which are selling a homogeneous good on the Internet. Consumers on the Internet collect information via costly search. Each consumer anticipates that he will be able to read k consumer reviews. The search strategy employed by consumers determines the probability that a consumer review provides relevant information (in this specific example price information) about a particular product. The analysis developed in this section provides a complete equilibrium characterization of this model.

At the beginning of the Internet most economists and managers predicted the so called *frictionless e-commerce hypothesis*: price dispersion and price levels in Internet should be much lower than in traditional markets. The prediction relied on the fact that Internet would lower search costs and that firms would confront a more transparent and therefore more competitive market. However online price dispersion is persistent and online prices are no lower than offline prices, see e.g. Bakos (1997), Brynjolfsson and Smith (2000). Our paper illustrates a possible "anti"-competitive feature of Internet. Internet reduces search costs, but it also facilitates the exchange of information among consumers. Actually, firms promote consumer reviews, a fact which, at first glance, may seem peculiar. The point is that while lower search costs increase competition by boosting the incentives of consumers to search for product information, consumer reviews, or more generally every method which facilitates information sharing, may balance (or offset) this effect by creating free-riding problems among consumers, thus hindering the acquisition of valuable information.

4 Arbitrary Networks

In this section we turn our attention to arbitrary networks. To get a handle on this problem we focus on equilibria where consumers use pure strategy. We note that it is easy to extend the characterization provided by Proposition 3.1 to arbitrary networks. We therefore analyze equilibria in which consumers use asymmetric pure strategies (hereafter asymmetric equilibria). While in the previous section we have shown that the degree of a regular network is sufficient to characterize symmetric equilibria, now

[6] This scenario is present in much of the theory of social learning and word of mouth communication (Ellison and Fudenberg (1995)), and the study of how social norms arise in large population (Kandori (1992)). Recently this framework has been used to study how local externalities shape behaviors when agents interact in a network of relationships, whose architecture is not common knowledge (Galeotti and Vega Redondo (2005)).

it is plausible that even if two consumers have the same degree their search activity in equilibrium will be different. Indeed, even if two consumers have the same degree, the neighbor of one consumer may search more than the neighbor of another consumer. This suggests that the nature of asymmetric equilibria will depend on the overall network architecture. In what follows we characterize asymmetric equilibria and we will also illustrate the effect of adding links using some examples.

We first introduce some additional notation. An independent set $\mathbf{I}_g$ of a network g is a set of consumers such that no two consumers who belong to $\mathbf{I}_g$ are linked. A maximal independent set is an independent set which is not a proper subset of any other independent set. We denote by $\mathcal{I}_g$ a maximal independent set of g and $\phi^{\mathcal{I}_g} = |\mathcal{I}_g|$. We also denote by $\mu_x^{\mathcal{I}_g}$ the number of consumers which do no belong to $\mathcal{I}_g$ and that are linked with x consumers belonging to $\mathcal{I}_g$. Clearly, for every maximal independent set $\mathcal{I}_g$ we have that $M = \phi^{\mathcal{I}_g} + \sum_{x=1}^{\phi^{\mathcal{I}_g}} \mu_x^{\mathcal{I}_g}$. Note that for a network g there may be several maximal independent sets. It will be useful to denote by $\mathcal{I}_g^*$ a maximal independent set which satisfies the following two properties: (a) at least two consumers belong to the maximal independent set, i.e. $\phi^{\mathcal{I}_g^*} \geq 2$ and (b) at least one consumer who does not belong to the maximal independent set is linked with two consumers who belong to the maximal independent set, i.e. $\mu_1^{\mathcal{I}_g^*} \neq M - \phi^{\mathcal{I}_g^*}$. Figure 4 and 5 depicts some network architectures. There, for each network the set of 1-nodes form a maximal independent set which satisfies those properties.

We say that a consumer network is connected if and only if there is a sequence of links for each pair of consumers. A complete network, g^C, is a regular nework of degree $k = M - 1$. The following proposition provides a characterization of asymmetric equilibria for every connected consumer network.[7]

Theorem 4.1 *Let g be a connected consumer network. An asymmetric equilibrium exists for some search costs if and only if $g \neq g^C$. Let $g \neq g^C$ an asymmetric equilibrium takes the following form:*

I. For every consumer $m \in \mathbf{M}$ either $q_1^m = 1$ or $q_0^m = 1$, and $q_1^m = 1$ if and only if $m \in \mathcal{I}_g^*$.

II. Each firm prices randomly according to an atomless distribution function

$$F(p) = 1 - \frac{\frac{\phi^{\mathcal{I}_g^*}}{2} + \sum_{x=1}^{\phi^{\mathcal{I}_g^*}} \frac{\mu_x^{\mathcal{I}_g^*}}{2^x}}{\sum_{x=1}^{\phi^{\mathcal{I}_g^*}} \mu_x^{\mathcal{I}_g^*} \left(1 - \frac{1}{2^{x-1}}\right)} \frac{\tilde{p} - p}{p}, S = \left[\frac{\frac{\phi^{\mathcal{I}_g^*}}{2} + \sum_{x=1}^{\phi^{\mathcal{I}_g^*}} \frac{\mu_x^{\mathcal{I}_g^*}}{2^x}}{\frac{1}{2}\left(2m - \phi^{\mathcal{I}_g^*} - \sum_{x=1}^{\phi^{\mathcal{I}_g^*}} \frac{\mu_x^{\mathcal{I}_g^*}}{2^{x-1}}\right)} \tilde{p}, \tilde{p} \right]$$

[7]This proposition may be easily extended to networks which are not connected. Essentially, the conditions provided in part (I) of Theorem 4.1 must hold at least in one network component. This implies that in the empty network asymmetric equilibria do not exist. The complete characterization is available upon request to the author.

Part I requires that each consumer either does not search or samples one price and that consumers who sample one price quotation form a maximal independent set. The maximal independent set must satisfy property (a) and (b), which we have introduced above. The reason for these conditions is that for every other search profile firms will charge either the competitive price or the monopolistic price. In both situations some consumers gain by deviating. In contrast, these conditions together are necessary and sufficient to assure that in expectation some consumers observe only one price and the others observe both firms' prices. This implies that firms will price randomly.

As a final remark note that we do not allow consumers to coordinate in their search. However, it is quite easy to see that consumers cannot coordinate their search activity in equilibrium. Suppose that there are only two consumers and suppose that they coordinate in the following way: one consumer searches one firm and the other consumer searches the other firm. In this case the two consumers observe both firms' prices and therefore firms must charge the competitive price. However, this is not a Nash equilibrium because a consumer has an incentive to free-ride on the other consumer.

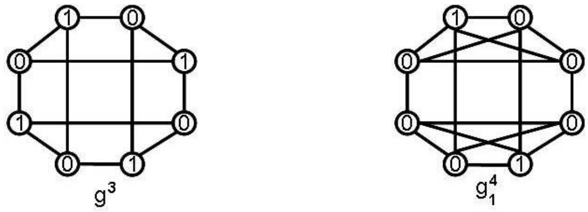
The following two examples illustrate the effect of adding links on market outcomes when consumers use asymmetric strategies. In the first example the consumer network is a regular graph. In the second example the consumer network is asymmetric.

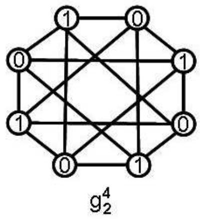
Example 4.1 *Asymmetric Equilibria and Regular Consumer Networks*

Figure 4 depicts three consumer regular networks. A 1-node represents a consumer who samples one price, while a 0-node is a consumer who does not search. The network g^3 is a regular network of degree 3. Starting from g^3 and adding a link to each consumer we obtain regular networks of degree 4. The network g_1^4 is obtained by adding to each consumer in g^3 a link with one of his neighbor's neighbors. The network g_2^4 is obtained by adding to each consumer in g^3 a link with a neighbor of his neighbors' neighbors. For each of these networks there exists a unique asymmetric equilibrium (up to permutation of consumers) which is summarized in the figure. We also summarize expected prices, consumer surplus and social welfare in the three different networks for given search costs.[8].

We first observe that market outcomes are remarkably different in g_1^4 and g_2^4. Note that in g_1^4 almost every pair of connected consumers have a link with a common consumer. This is not the case in g_2^4. That is, the network g_1^4 is more clustered than the network g_2^4. Theorem 4.1 and in particular the notion of maximal independent set implies that if three consumers are all linked to each other, i.e. they form a cycle, at most one of them may search for prices. Thus, in g_1^4 more consumers free-ride than in g_2^4, leading to higher expected prices and lower consumer surplus.

[8]Clearly, we consider search costs where equilibria exist in the three networks depicted in the figure. Also, the comparison provided in the figure holds for all search costs where this happens. The computation of these equilibria is provided upon request to the author





Search cost: c=0.2	g^3	g_1^4	g_2^4
Expected Price	0.66	0.87	0.60
Consumer Surplus	2.2	0.6	2.7
Social Welfare	7.2	7.6	7.2

Figure 4: Adding links to regular networks

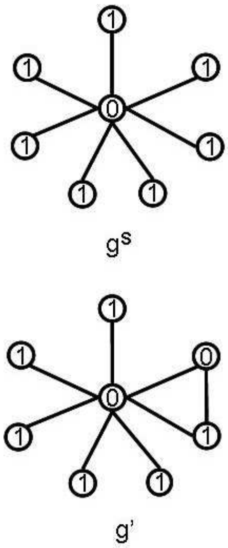
The second remark is that the effect of adding links on firms' pricing and consumer surplus depends on how we allocate the new links. If we add links to g^3 so that we end-up in g_1^4, expected prices increase and consumer surplus decreases. The reverse is true if we move from g^3 to g_1^4. Thus, the impact of social interaction on market outcomes is inherently linked on how networks evolve.

Example 4.2 *Asymmetric Equilibria and Asymmetric Networks*

We now consider the star network, say g^S, and a network g' obtained by adding to g^S a link to two spoke consumers. These two network architectures are illustrated in Figure 5. It is easy to show that in each of these networks there exists a unique asymmetric equilibrium (up to permutation of consumers). The equilibrium market outcome in each of the two networks is also summarized in the figure.

In the star network there exists only one consumer, the center, who free-rides on his neighbors. When we add a link in the star a new consumer starts free-riding on his neighbors. Again this decreases competition between firms and leads to higher expected prices. The additional link increases consumer surplus because the savings on search costs outweigh the increase in prices. However, note that this does not mean that in g' all consumers are better-off than in g^S. On the contrary, only one consumer gains by the additional link. This is the consumer who free-rides on his neighbors in the new network, while he was searching in the star network.

We conclude this Section by emphasizing that Example 4.1 and 4.2 show that the insights we gained in Section 3 are robust to arbitrary networks. What is specific to the analysis of Section 3 is that the degree of the network is sufficient to describe how market outcomes differ across regular graphs. In arbitrary networks, however, we must expect that market outcomes depend also on other architectural properties



Search Costs: c=0.1	g^s	g’
Expected Prices	0.883	0.885
Consumer Surplus	0.27	0.33
Social Welfare	7.3	7.4

Figure 5: Adding a link to Star Networks

of networks which reflect how links are allocated across consumers. Above all, the examples suggest that clustering has a profound effect on market outcomes.

5 Conclusion

We have introduced a simple theoretical model to study how markets and social interactions shape outcomes. We have shown that more connected networks do not necessarily increase firms’ competition. On the contrary, it is often the case that by adding connections in a network market outcomes become closer to the monopolistic outcome. The research question formulated in this paper goes beyond our analysis. However, our model provides interesting new insights and it is simple enough to be extended in a variety of ways. In what follows we discuss some new questions which are inspired by this work.

First, how does entry of firms affect market outcomes in different networks? That is, instead of comparing market outcomes across different networks for a fixed number of competitors, we could compare market outcomes across different oligopolistic markets within the same pattern of consumers’ interactions. A natural question is whether there are networks in which systematic relations between the number of competitors and market outcomes emerge. Second, in this paper we have assumed that consumers search non-sequentially. The effect of social interactions on search behavior depend on how consumers search. An alternative search protocol is the sequential search, e.g. Stahl (1989).

Finally, networks influence consumers’ incentives to search and they also influence firms’ incentive to invest in advertising. An example is firms’ use of network-based marketing strategies. The term *viral* marketing denotes every marketing strategy that induces Web sites or users to pass on a marketing message to other sites or users, creating a potentially exponential growth in the message's visibility and effect. How social interactions influence the firms' strategy to market their product and the implications on market outcomes is an interesting and open research question.

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Appendix

Proof of Proposition 3.1.

It is immediate to verify that $\{p, q_0 = 1\}$, with $p \in [\tilde{p} - c, \tilde{p}]$ is a Nash equilibrium. I now prove that these are the only (generic) equilibria in which consumers employ pure strategies. First, suppose $q_1 = 1$. If $k = 0$, each consumer observes only one price; hence firms charge $p = \tilde{p}$. Since $c > 0$, a consumer strictly gains by not searching at all. Suppose $k > 0$. In equilibrium firms price according to an atomless price distribution $F(p)$ defined on a convex support S. Indeed, if $k > 0$ and $q_1 = 1$ some consumers observe both firms' prices and others only one price. Thus, if firms charge a price p with a mass point, they will tight at that price with strictly positive probability. Then a firm strictly gain by undercutting the atom. Next, note that, given $k > 0$, an equilibrium where $q_1 = 1$ exists for a unique value of search costs, i.e. it is not generic. Indeed, the utility to a consumer is $Eu\left(q_1 = 1\right) = \tilde{p} - \frac{1}{2^k} E\left(p\right) - \left(1 - \frac{1}{2^k}\right) E[\min\{p_i, p_j\}]\left(p\right) - c$. For an equilibrium $Eu\left(q_1 = 1\right) \geq Eu^d\left(q_x = 1\right),\ x = 0, 2,$ where $Eu^d\left(q_0 = 1\right) = \tilde{p} - \frac{1}{2^{k-1}} E\left(p\right) - \left(1 - \frac{1}{2^{k-1}}\right) E[\min\{p_i, p_j\}]$ and $Eu^d\left(q_2 = 1\right) = \tilde{p} - E[\min\{p_i, p_j\}]\left(p\right) - 2c$. Solving the two inequalities we obtain that:

$$c = \frac{1}{2^k}\left(E\left(p\right) - E[\min\{p_i, p_j\}]\right)$$

Finally, suppose $q_2 = 1$ and $k \geq 0$. Then each consumer observes always two prices. Thus, firms charge the competitive price, i.e. $p = 0$. However, a consumer strictly gains by searching only once. This completes the proof of the Proposition.■

Proof Lemma 3.1.

First, we claim that $F\left(p\right)$ is atomless. If $k = 0$ this follows from Burdett and Judd (1983). Suppose $k > 0$ and suppose p^* is an atom. Since consumers search at least once with positive probability and $k > 0$, some consumers observe two prices. Therefore, firms tie at p^* with positive probability; however a firm gains by undercutting p^*. Next, for any $k \geq 0$, S is convex. Suppose not, i.e. $\exists \tilde{S} \subsetneq S : F\left(p\right) = \eta$ $\forall p \in \tilde{S}$, $\eta \in [0, \tilde{p}]$. Let $p^* = \inf \tilde{S}$, then a firm charging p^* gains by increasing such price.

Suppose $q_0 + q_2 = 1$ is part of equilibrium. Since a consumer either observe no price or both prices, in equilibrium $p = 0$, but then a consumer strictly gain by searching only once. Next, suppose $q_0 + q_1 + q_2 = 1$ is part of equilibrium. We know that $F(p)$ is atomless and σ is convex. Let $\alpha(k) = \sum_{x=1}^{k} \binom{k}{x} \frac{q_0^{k-x} q_1^x}{2^x}$. We have that:

$$\begin{aligned}
Eu(q_0 = 1) &= \tilde{p}(1 - q_0^k) - \alpha(k)E(p) - (1 - q_0^k - \alpha(k))E_{\min}(p) \\
Eu(q_1 = 1) &= \tilde{p} - \left(q_0^k + \frac{\alpha(k)}{2}\right)E(p) - \left(1 - q_0^k - \frac{\alpha(k)}{2}\right)E[\min\{p_i, p_j\}] - c \\
Eu(q_2 = 1) &= \tilde{p} - E[\min\{p_i, p_j\}] - 2c
\end{aligned}$$

For equilibrium: $Eu(q_x = 1) = Eu(q_y = 1)$, $x, y = 0, 1, 2$. Solving:

$$[\tilde{p} - E(p)] = [E(p) - E[\min\{p_i, p_j\}]]$$

This is impossible. Indeed,

$$E(p) - E[\min\{p_i, p_j\}] = \int_{\underline{p}}^{\tilde{p}} 2pf(p)F(p)dp - E(p)$$

Integrating by parts we have that:

$$\int_{\underline{p}}^{\tilde{p}} 2pf(p)F(p)dp = \tilde{p} - \int_{\underline{p}}^{\tilde{p}} [F(p)]^2 dp$$

Thus,

$$E(p) - E[\min\{p_i, p_j\}] = [\tilde{p} - E(p)] - \int_{\underline{p}}^{\tilde{p}} [F(p)]^2 dp < [\tilde{p} - E(p)] \tag{4}$$

Finally consider $k = 0$. Previous arguments rule out $q_0 + q_2 = 1$ and $q_0 + q_1 + q_2 = 1$. Suppose $q_0 + q_1 = 1$ is part of equilibrium. Then firms charge $\tilde{p}$; but then $q_0 = 1$. The proof of the Proposition is completed.■

Proof Theorem 3.1

Assume each consumer randomizes between searching for one price and for two prices, i.e. $q_1 + q_2 = 1$, $q_x \in (0, 1), x = 1, 2$. We define by D_i the expected number of consumers who observe only the price of firm i. Similarly, we define by $D_{i,j}$ the expected number of consumers who observe both prices, $D_{i,j}$. These are functions of k and q_1 and take the following form:

$$D_i(k, q_1) = \frac{mq_1^{k+1}}{2^{k+1}} \tag{5}$$

$$D_{i,j}(k, q_1) = m\left(1 - \frac{q_1^{k+1}}{2^k}\right) \tag{6}$$

Given the conditions in Lemma 3.1, for an asymmetric equilibrium $F(p)$ and S must be such that the expected profit of each firm is constant for all $p_i, p_j \in S$. Formally,

$$E\pi_i(p_i, p_j) = D_i(k, q_1)p_i + D_{i,j}(k, q_1)p_i[1 - F(p_i)] = E\pi^* \tag{7}$$

We derive $F(p)$ and S. Note that $\overline{p} = \tilde{p}$; for otherwise a firm charging $\overline{p} < \tilde{p}$ strictly gains by increasing it. Thus, $E\pi^* = E\pi_i^*(\tilde{p}, p_j) = \frac{mq_1^{k+1}}{2^{k+1}}\tilde{p}$. Solving condition (7) we obtain $F(p)$ defined in Theorem 3.1. Furthermore, by solving $E\pi_i(\underline{p}, p_j) = E\pi_i^*$ we obtain $\underline{p}$ defined in Theorem 3.1.

Consider now the problem of an arbitrary consumer who searches once with probability q_1 and twice with the remaining probability. The expected utilities to a consumer from the two distinct searching alternatives are:

$$Eu(q_1 = 1) = \tilde{p} - \frac{q_1^k}{2^k}E[p] - \left(1 - \frac{q_1^k}{2^k}\right) E[\{\min[p_i, p_j]\}] - c \tag{8}$$

$$Eu(q_2 = 1) = \tilde{p} - E[\{\min[p_i, p_j]\}] - 2c \tag{9}$$

In equilibrium a consumer should be indifferent between the two different alternatives, i.e. $Eu\,(q_1 = 1) = Eu\,(q_2 = 1)$. Solving we obtain the following equilibrium condition:

$$\frac{q_1^k}{2^k}[E(p) - E[\{\min[p_i, p_j]\}]] = c \tag{10}$$

We now show existence. Assume, without loss of generality that $\tilde{p} = 1$. The proof for the case $k = 0$ follows from Burdett and Judd (1983). Next, assume $k \geq 0$. Let the RHS of (10) be $\phi(p; k, q_1) = \frac{q_1^k}{2^k}[E[p] - E[\{\min[p_i, p_j]\}]]$. We show that $\frac{\partial \phi(p;k,q_1)}{\partial q_1} > 0$. To do this we first invert $F(p)$ derived in Theorem 3.1 and we obtain:

$$p(z; k, q) = \frac{1}{g(z; k, q_1)}, \tag{11}$$

where

$$g(z; k, q_1) = 1 + \frac{2(2^k - q^{k+1})}{q^{k+1}}(1 - z) \tag{12}$$

Next, note that

$$\frac{2^k}{q_1^k}\phi(p; k, q_1) = 2\int_{\underline{p}(k,q_1)}^{1} pf(p)(1 - F(p)dp - \int_{\underline{p}(k,q_1)}^{1} pf(p)dp,$$

where f(p) is the density function of F(p). Integrating by parts yields,

$$\frac{2^k}{q_1^k}\phi(p; k, q_1) = \int_{\underline{p}(k,q_1)}^{1} [F(p)(1 - F(p))]dp$$

Using the inverse function $p\,(z; k, q_1)$, we rewrite this expression as:

$$\frac{2^k}{q_1^k}\phi(z; k, q_1) = \int_0^1 [p(\sqrt{z}; k, q_1) - p(z; k, q_1)]dz$$

Or,

$$\frac{2^k}{q_1^k}\phi(z; k, q_1) = \int_0^1 p(z; k, q_1)(2z - 1)dz$$

Let $a = q^{k+1}$, $b = 2(2^k - q^{k+1})$ and $c = 2^{k+1}(2k+1)$. Then,

$$\begin{aligned}\frac{2^k}{q_1^{2k}}\frac{\partial\phi(z;k,q_1)}{\partial q_1} &= -\int_0^{1\backslash 2}\frac{[ka(2z-1)+c(1-z)](1-2z)}{[a+b(1-z)]^2}dz + \\ &+\int_{1\backslash 2}^1\frac{[ka(2z-1)+c(1-z)](2z-1)}{[a+b(1-z)]^2}dz\end{aligned}$$

Note that $\frac{[ka(2z-1)+c(1-z)]}{[a+b(1-z)]^2}$ is positive and increasing in z for all $z \in (0, 1/2)$ and that $[a+b(1-z)]^2$ is decreasing in z. Therefore:

$$\frac{2^k}{q_1^{2k}}\frac{\partial\phi\left(z;k,q_1\right)}{\partial q_1} > -\int_0^{1\backslash 2}\frac{2^k\left(2k+1\right)\left(1-2z\right)}{2^{2k}}dz + \int_{1\backslash 2}^1\frac{\left[ka\left(2z-1\right)+c\left(1-z\right)\right]\left(2z-1\right)}{2^{2k}}dz$$

Since $[ka(2z-1)+c(1-z)]$ is positive and decreasing in z,, we have that:

$$\begin{aligned}\frac{2^k}{q_1^{2k}}\frac{\partial\phi(z;k,q_1)}{\partial q_1} &> -\int_0^{1\backslash 2}\frac{2^k(2k+1)(1-2z)}{2^{2k}}dz + \int_{1\backslash 2}^1\frac{q_1^{k+1}k(2z-1)}{2^{2k}}dz \\ &> \left(\frac{2^k(2k+1)+q_1^{k+1}k}{2^{2k}}\right)\int_0^1(2z-1)dz \\ &> \left(\frac{2^k(2k+1)+q_1^{k+1}k}{2^{2k}}\right)\left(\frac{(2z-1)^2}{4}\right)_0^1 = 0\end{aligned}$$

Next, note that $\lim_{q\to 0}\phi(p;k,q_1) = 0$ and that

$$\lim_{q\to 1}\phi(p;k,q_1) = \bar{c}_k = \frac{1}{2^k(2^{k+1}-2)}\left(\frac{2^{k+1}}{2^{k+1}-2}\ln(2^{k+1}-1)-2\right) > 0.$$

These three facts just proved show that there exists a unique solution, say $q_1^* \in (0,1)$, of the equilibrium condition (10).

We conclude the proof by showing that consumers do not want to deviate. Note that:

$$Eu^d(q_0 = 1) = \tilde{p} - \frac{q_1^k}{2^{k-1}}E[p] - \left(1 - \frac{q_1^k}{2^{k-1}}\right)E[\{\min[p_i, p_j]\}]$$

For an equilibrium $Eu^d(q_0 = 1) \leq Eu(q_1 = 1)$, which is satisfied if and only if:

$$c \geq \frac{q_1^k}{2^k}[E[p] - E[\{\min[p_i, p_j]\}]]$$

This completes the proof of the Theorem. ■

Proof Proposition 3.2

We first show that $\frac{\partial \phi(z;k,q_1)}{\partial k} < 0$. We observe that:

$$\begin{aligned}\frac{\partial \phi(z;k,q_1)}{\partial k} &= -\frac{q_1^{2k+1}}{2^k}\ln\frac{2}{q_1}\int_0^1 \left(\frac{q_1^{k+1}+2(2^{k+1}-q_1^{k+1})(1-z)}{[q_1^{k+1}+2(2^k-q_1^{k+1})(1-z)]^2}\right)(2z-1)dz + \\ &+\frac{q_1^{2k+1}}{2^k}\ln q_1 \int_0^1 \frac{q_1^{k+1}(2z-1)^2}{[q_1^{k+1}+2(2^k-q_1^{k+1})(1-z)]^2}dz\end{aligned}$$

The second term is weakly negative and also $-\frac{q_1^{2k+1}}{2^k}\ln\frac{2}{q_1}$ is weakly negative. Then, it is sufficient to show that:

$$\xi = \int_0^1 \left(\frac{q_1^{k+1}+2(2^{k+1}-q_1^{k+1})(1-z)}{[q_1^{k+1}+2(2^k-q_1^{k+1})(1-z)]^2}\right)(2z-1)dz > 0$$

To see this note that:

$$\begin{aligned}\xi &= -\int_0^{1/2} \left(\frac{q_1^{k+1}+2(2^{k+1}-q_1^{k+1})(1-z)}{[q_1^{k+1}+2(2^k-q_1^{k+1})(1-z)]^2}\right)(1-2z) + \\ &+\int_{1/2}^1 \left(\frac{q_1^{k+1}+2(2^{k+1}-q_1^{k+1})(1-z)}{[q_1^{k+1}+2(2^k-q_1^{k+1})(1-z)]^2}\right)(1-2z)dz\end{aligned}$$

Since $\left(\frac{q_1^{k+1}+2(2^{k+1}-q_1^{k+1})(1-z)}{[q_1^{k+1}+2(2^k-q_1^{k+1})(1-z)]^2}\right)$ is increasing in z for all $z \in (0,1/2)$ and $[q_1^{k+1}+2(2^k-q_1^{k+1})(1-z)]$ is decreasing in z, then:

$$\xi > -\int_0^{1/2}\frac{2^{k+1}}{2^{2k}}(1-2z)dz + \int_{1/2}^1 \left(\frac{q_1^{k+1}+2(2^{k+1}-q_1^{k+1})(1-z)}{2^{2k}}\right)(1-2z)dz$$

Furthermore, $q_1^{k+1}+2(2^{k+1}-q_1^{k+1})(1-z)$ is also decreasing in z; thus

$$\begin{aligned}\xi &> -\int_0^{1/2}\frac{2^{k+1}}{2^{2k}}(1-2z) + \int_{1/2}^1 \frac{q_1^{k+1}}{2^{2k}}(1-2z) = \\ &= \frac{2^{k+1}+q_1^{k+1}}{2^{2k}}\int_0^1(2z-1)dz \\ &= \frac{2^{k+1}+q_1^{k+1}}{2^{2k}}\left(\frac{(2z-1)^2}{4}\right)_0^1 = 0\end{aligned}$$

The fact that $\frac{\partial \phi(z;k,q_1)}{\partial k} < 0$ and that $\frac{\partial \phi(z;k,q_1)}{\partial q_1} > 0$ implies that q_1^* is increasing in k.

Second, we show that expected prices are increasing in k. Let $\psi(k,q_1) = \frac{q_1^{k+1}}{2(2^k-q_1^{k+1})}$, then $F(p)$ defined in Theorem 3.1 can be rewritten as $F(p) = 1-\psi\frac{\tilde{p}-p}{p}$. It is enough to show that:

$$\frac{d\psi}{dk} = \frac{\partial\psi}{\partial k} + \psi\frac{\partial q_1}{\partial k} > 0$$

Denote $\phi_k(k,q_1) = \partial\phi(k,q_1)/\partial k$ and $\phi_{q_1}(k,q_1) = \partial\phi(k,q_1)/\partial q_1$. Using the equilibrium condition $\phi(k,q_1) - c = 0$ and applying the implicit function theorem we derive $\frac{\partial q}{\partial k} = -\frac{\phi_k(\cdot)}{\phi_{q_1}(\cdot)}$, where

$$\begin{aligned}
\phi_k(k,q_1) &= -\frac{q_1^k}{2^k}\ln\frac{2}{q_1}\psi[(1+2\psi)\ln\frac{1+\psi}{\psi} - 2] + \frac{q_1^k}{2^k}\psi_k[(1+4\psi)\ln\frac{1+\psi}{\psi} - \frac{3+4\psi}{1+\psi}] \\
\phi_{q_1}(q_1,k) &= \frac{kq_1^{k-1}}{2^k}\psi[(1+2\psi)\ln\frac{1+\psi}{\psi} - 2] + \frac{q_1^k}{2^k}\psi_{q_1}[(1+4\psi)\ln\frac{1+\psi}{\psi} - \frac{3+4\psi}{1+\psi}]
\end{aligned}$$

Plugging the expressions for $\frac{\partial q_1}{\partial k}$ in $\frac{d\psi}{dk}$, we obtain

$$\frac{d\psi}{dk} = \frac{\psi}{\phi_{q_1}}\left(\left((1+2\psi)\ln\frac{1+\psi}{\psi} - 2\right)\left(\psi_k\frac{kq^{k-1}}{2^k} + \psi_{q_1}\frac{q_1^k}{2^k}\ln\frac{2}{q_1}\right)\right)$$

Since ϕ_{q_1} and $\psi(q_1,k)$ are strictly positive, it follows that $\frac{d\psi}{dk} > 0$ if and only if

$$((1+2\psi)\ln\frac{1+\psi}{\psi} - 2)(\psi_k\frac{kq_1^{k-1}}{2^k} + \psi_{q_1}\frac{q_1^k}{2^k}\ln\frac{2}{q_1}) > 0$$

Computing $\psi_k = -\frac{q_1^{k+1}2^k}{2(2^k - q_1^{k+1})^2}\ln\frac{2}{q_1}$ and $\psi_{q_1} = \frac{(k+1)q_1^k 2^k}{2(2^k - q_1^{k+1})^2}$ it follows that:

$$\psi_k\frac{kq_1^{k-1}}{2^k} + \psi_{q_1}\frac{q_1^k}{2^k}\ln\frac{2}{q_1} = \frac{q_1^{2k}}{2(2^k - q_1^{k+1})^2}\ln\frac{2}{q_1} > 0$$

and using $\psi(k,q_1)$ it also follows that:

$$\begin{aligned}
(1+2\psi)\ln\frac{1+\psi}{\psi} - 2 &= \frac{2^k}{2^k - q_1^{k+1}}\ln\left(\frac{(2^{k+1} - q_1^{k+1})}{q^{k+1}}\right) - 2 > \\
&> \frac{2^k}{2^k - 1}\ln(2^{k+1} - 1) - 2 > \\
&> -2 + 2\ln 3 > 0
\end{aligned}$$

This proves the claim.

Third, social welfare is increasing in k. The social welfare is $SW(k,q_1,c) = \tilde{p} - q_1 c - (1-q_1)2c = \tilde{p} - 2c + q_1 c$; since q_1 is increasing in k, the claim follows. Finally, consumer surplus decreases in k. Consumer surplus is $CS = Eu(q_1 = 1) = Eu(q_2 = 1) = \tilde{p} - E_{\min}(p) - 2c$. Using $F(p;k,q_1)$ in Theorem 3.1, we obtain that the distribution function of the second order statistic is $F(\min\{p_i,p_j\}) = F(p)(2 - F(p))$. Therefore $\frac{\partial F(\min\{p_i,p_j\})}{\partial k} < 0$ iff $2\frac{\partial F(p)}{\partial k}\left[1 - \frac{\partial F(p)}{\partial k}\right] < 0$ iff $\frac{d\psi}{dk} > 0$, which follows from above. This completes the proof.■

Proof of Theorem 3.2

Assume consumers randomize between searching from one price and not searching, i.e. $q_0 + q_1 = 1, q_x \in (0,1), x = 0,1$. We denote by $f(q_0,k,x) = \binom{k}{x}\frac{q_0^{k-x}(1-q_0)^x}{2^x}$. Note that:

$$D_i(k,q_0) = \frac{m(1-q_0)}{2}\sum_{x=0}^{k} f(q_0,k,x) + mq_0\sum_{x=1}^{k} f(q_0,k,x) \tag{13}$$

$$D_{i,j}(k,q_0) = m(1-q_0)\left(1-\sum_{x=0}^{k} f(q_0,k,x)\right) + mq_0\left(1-q_0^k - 2\sum_{x=1}^{k} f(q_0,k,x)\right) \tag{14}$$

These expressions can be rewritten as follows:

$$D_i(k,q_0) = \frac{m[(1+q_0)^{k+1} - 2^{k+1}q_0^{k+1}]}{2^{k+1}} \tag{15}$$

$$D_{i,j}(k,q_0) = \frac{m[2^k(1+q_0^{k+1}) - (1+q_0)^{k+1}]}{2^k} \tag{16}$$

In a symmetric equilibrium $F(p)$ and S must be such that the profits to each firm must be constant for all $p_i, p_j \in S$. Formally:

$$E\pi(p_i,p_j) = D_i(k,q_0)p_i + D_i(k,q_0)p_i[1-F(p_i)] = E\pi^* \tag{17}$$

Next, note that for an equilibrium $\overline{p} = \tilde{p}$; for otherwise a firm charging $\overline{p} < \tilde{p}$ strictly gains by increasing such price. Thus, the equilibrium profit is $E\pi^* = (m((1+q)^{k+1} - 2^{k+1}q^{k+1})/2^{k+1})\tilde{p}$. We can now solve condition (17) and obtain the distribution function $F(p)$ defined in Theorem 3.2. Furthermore, by solving $E\pi(\underline{p}) = E\pi^*$ we obtain $\underline{p}$ as defined in Theorem 3.2.

We now consider consumers' behavior. The expected utilities a consumer gets from the two distinct search alternatives are:

$$Eu(q_1 = 1) = \tilde{p} - \sum_{x=0}^{k} f(q_0,k,x)E[p] - \left(1-\sum_{x=0}^{k} f(q_0,k,x)\right)E[\min\{p_i,p_j\}] - c \tag{18}$$

$$Eu(q_0 = 1) = \tilde{p}(1-q_0^k) - \sum_{x=1}^{k}\frac{f(q_0,k,x)}{2}E[p] - \left(1-q_0^k-\sum_{x=1}^{k}\frac{f(q_0,k,x)}{2}\right)E[\min\{p_i,p_j\}] \tag{19}$$

In equilibrium each consumer must be indifferent between searching once and not searching at all, i.e $Eu\left(q_1 = 1\right) = Eu\left(q_0 = 1\right)$. This condition is satisfied if and only if:

$$\frac{(1+q_0)^k - 2^k q_0^k}{2^k}[E[p] - E[\min\{p_i,p_j\}]] + q_0^k(\tilde{p} - E[p]) = c \tag{20}$$

We now prove existence. Without loss of generality, let $\tilde{p} = 1$. We invert $F(p)$ defined in Theorem 3.2, to obtain:

$$p(z,k,q_0) = \frac{1}{g(z;k,q_0)} \tag{21}$$

where

$$g(z;k,q_0) = 1 + \frac{2^{k+1}(1+q_0^{k+1}) - 2(1+q_0)^{k+1}}{(1+q_0)^{k+1} - 2^{k+1}q_0^{k+1}}(1-z) \tag{22}$$

Using (21), the equilibrium condition (20) can be rewritten as follows:

$$\frac{(1+q_0)^k - 2^{k+1}q_0^k}{2^k}\int_0^1 p(z,k,q_0)(2z-1)dz + q_0^k\left(1 - 2\int_0^1 p(z,k,q_0)(1-z)dz\right) = c \tag{23}$$

Denote by $\rho(z;k,q_0)$ the LHS of (23). Note that $\lim_{q_0\to 0}\rho(z;k,q_0) = \bar{c}_k$ and $\lim_{q_0\to 1}\rho(z;k,q_0) = 0$. Furthermore, the limit when q_0 goes to zero of the derivative of $\rho(z,k,q)$ is positive:[9]

$$\lim_{q_0\to 0}\frac{\partial\rho(z;,k,q_0)}{\partial q_0} = 1$$

Since $\rho(q_0,k)$ is positive at $q_0 = 0$ and it is zero at $q_0 = 1$, it follows that for every $k > 0$ there exists a $c^* > 0$ such that for every $c \in [0, c^*]$ there exists a solution of the equilibrium condition (20). Furthermore, since $\rho(q_0,k)$ is increasing in the neighbor of $q_0 = 0$ we also have that that if $c \in [\bar{c}_k, c^*]$ there exist a solution which is stable.

Finally, a consumer does not have an incentive to deviate. To see this note that $Eu^d(q_2 = 1) = \tilde{p} - E[\min\{p_i,p_j\}] - 2c \leq Eu(q_1 = 1)$ if and only if

$$c \geq \frac{(1+q_0)^k}{2^k}[E[p] - E[\min\{p_i,p_j\}]]$$

Using the equilibrium condition (20) we can rewrite this inequality as

$$E[p] - E[\min\{p_i,p_j\}] \leq \tilde{p} - E[p]$$

which is always true because:

$$E[p] - E[\min\{p_i,p_j\}] = [\tilde{p} - E[p]] - \int_{\underline{p}}^{\tilde{p}}[F(p)]^2dp < [\tilde{p} - E[p]]$$

This completes the proof.■

Proof. Theorem 4.1

The following Lemma is useful to prove Theorem 4.1.

Lemma 5.1 *Suppose firms price randomly. For a generic equilibrium it must be the case that for all* $m \in \mathbf{M}$ *such that* $q_1^m = 1$ *there exists either one* $l \in \mathbf{M}_m(g)$ *such that* $q_2^l = 1$ *or* $q_0^l = 1$ *for all* $l \in \mathbf{M}_m(g)$.

[9] I develop the result using the program Mathematica. To do this I compute the following transformation. Let $\rho_{q_0}(q_0,k) = \frac{\partial\rho(q_0,k)}{q_0}$ then $\lim_{q_0\to 0}\rho_{q_0}(q_0,k) = e^{\lim_{q_0\to 0}\ln(\rho_{q_0}(q_0,k))^{q_0}}$. The computation is avaialable upon request of the author.

Proof. Proof Lemma 5.1 Suppose there exists a $m \in \mathbf{M}$ such that $q_1^m = 1$ and $q_2^l = 0$ for all $l \in \mathbf{M}_m(g)$ and $q_1^l = 0$ for some $l \in \mathbf{M}_m(g)$. Denote the number of consumers linked with m who searches once by k.The utility of consumer m is $Eu(q_1^m = 1) = \tilde{p} - \frac{1}{2^k}E[p] - \left(1 - \frac{1}{2^k}\right) E[\min\{p_i, p_j\}] - c$. For an equilibrium $Eu(q_1^m = 1) \geq Eu^d(q_x^m = 1)$, $x = 0, 2$, where $Eu^d(q_0^m = 1) = \tilde{p} - \frac{1}{2^k}E[p] - \left(1 - \frac{1}{2^k}\right) E[\min\{p_i, p_j\}]$ and $Eu^d(q_2^m = 1) = \tilde{p} - E[\min\{p_i, p_j\}] - 2c$. Solving the two inequalities we obtain that:

$$c = \frac{1}{2^k}E[p] - E[\min\{p_i, p_j\}] \tag{24}$$

■

We first show Part I. We claim that in equilibrium $q_0^m = 1$ for at least one $m \in \mathbf{M}$, $q_1^m = 1$ for at least one $m \in \mathbf{M}$ and $q_2^m = 0$ for all $m \in \mathbf{M}$. We must rule out three possibilities. One, suppose $q_0^m = 1$ for at least one $m \in \mathbf{M}$, $q_2^m = 1$ for at least one $m \in \mathbf{M}$ and $q_1^m = 0$ for all $m \in \mathbf{M}$. In this case each consumer m either observes two prices or he does not observe any price. Thus, firms must charge the competitive price. However, each m such that $q_2^m = 1$ strictly gains by searching only once. This is a contradiction.

Two, suppose $q_1^m = 1$ for at least one $m \in \mathbf{M}$, $q_2^m = 1$ for at least one $m \in \mathbf{M}$ and $q_0^m = 0$ for all $m \in \mathbf{M}$. The same argument we have just used implies that it must exist a consumer m such that $q_1^m = 1$ and $q_1^l = 1$ for all $l \in \mathbf{M}_m(g)$. This implies that firms will price randomly. Lemma 5.1 leads to a contradiction. Three, suppose $q_0^m = 1$ for at least one $m \in \mathbf{M}$, $q_1^m = 1$ for at least one $m \in \mathbf{M}$ and $q_0^m = 1$ for at least one $m \in \mathbf{M}$. Here, note that it must be the case that if $g_{m,l} = 1$ and $q_2^m = 1$ then $q_0^l = 1$. This implies that if $g_{m,l} = 1$ and $q_1^m = 1$ then either $q_1^l = 1$ or $q_0^l = 1$. Lemma 5.1 leads to a contradiction.

Lemma 5.1. now also implies that consumers who search once form a independent set. Suppose now that the independent set is not maximal. This implies that there exists m such that $q_0^m = 1$ and $q_0^l = 1$ for all $l \in \mathbf{M}_m(g)$. Since we are assuming equilibrium it must be the case consumer m strictly gains be searching for one price.

Third, we claim that $\phi^{\mathcal{I}_g} \geq 2$. Suppose $\phi^{\mathcal{I}_g} = 1$. Then each consumer will observe only one price. For an equilibrium firms will charge $\tilde{p}$. Then the consumer who searches will deviate. Fourth, $\mu_1^{\mathcal{I}_g} \neq m - \phi^{\mathcal{I}_g}$. Suppose $\mu_1^{\mathcal{I}_g} m - \eta^{\mathcal{I}_g}$ then again each consumer observes only one price, firms must charge $\tilde{p}$, which implies that a consumer who searches strictly gains by not searching.

We now prove part (II). Let consumers' behavior specified as in Part I. The expected profit of a firm is

$$E\pi_i(p, p_j) = \left(\frac{\phi^{\mathcal{I}(g)}}{2} + \sum_{x=1}^{\phi^{\mathcal{I}_g}} \frac{\mu_x^{\mathcal{I}_g}}{2^x}\right) p + \sum_{x=1}^{\phi^{\mathcal{I}_g}} \mu_x^{\mathcal{I}(g)} \left(1 - \frac{1}{2^{x-1}}\right) p\,[1 - F(p)] \tag{25}$$

Standard arguments imply that firms must price randomly and the support should be convex with upper bound $\tilde{p}$. Thus the expected profit in equilibrium is

$$E\pi^* = \left(\frac{\phi^{\mathcal{I}_g}}{2} + \sum_{x=1}^{\phi^{\mathcal{I}_g}} \frac{\mu_x^{\mathcal{I}_g}}{2^x} \right) \tilde{p} \tag{26}$$

The distribution function provided in the proposition and the lower bound $\underline{p}$ of the support are obtained by solving $E\pi_i(p, p_j) = E\pi^*$ and $E\pi_i(\underline{p}, p_j) = E\pi^*$, respectively.

Finally, we prove existence. Here we note that if $g = g^C$ then every $|\mathcal{I}_g| = 1$, which contradicts Part I of the Theorem. It is readily seen that for every connected network $g \neq g^C$ a maximal independent set satisfying the desired properties exists. We then need to check only incentives to deviate of consumers. Two observations are important. The first is that each $m \in \mathcal{I}(g)$ has the same incentive to deviate. The second is that if $m \in \mathcal{I}(g)$ does not gain by deviating then every $l \notin \mathcal{I}(g)$ does not deviate as well. Select an arbitrary consumer $m \in \mathcal{I}(g)$. For an equilibrium it must be the case that $Eu_m(q_1^m = 1) \geq Eu_m(q_x^m = 1), x = 0, 2$. This is satisfied if and only if

$$E[p] - E[\min\{p_i, p_j\}] \leq c \geq \tilde{p} - E[p] \tag{27}$$

The derivation in Lemma 3.1 shows that there always exist search costs for which these two inequalities hold with strictly. The proof is now complete. ■