How (Not) to Choose Peers in Studying Groups*

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Abstract

This paper looks into social group formation where groups acquire human capital. Within groups agents are subject to productive peer effects that increase human capital and consumptive peer effects that increase instantaneous utility. Agents are heterogeneous along two dimensions, ability and social skills. Building on established empirical facts, our model predicts segregation at the top and at the bottom of the attribute space as well as bunching for heterogenous intermediate types in a very general setting. We show that when agents value social activity and monetary payments are not used, there is little hope for the market allocation to yield cost-efficient human capital accumulation, in particular among elites. The theoretical results are robust to a number of different specifications of peer effect production.

1 Introduction

Important aspects of the economic allocation are crucially determined by the allocation of social ties between economic agents. For instance, in many economies an individual's success in encountering suitable employment depends largely on the social ties this individual has. Studies pioneered by Myers and Shultz (1951) find that individuals frequently encounter new jobs on the base of personal relationships. An individual with a greater number of loose acquaintances stands a better chance of encountering new employment (Granovetter, 1973). Other examples involve trading networks where

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trade opportunities are open only to network members, or implicit risk-sharing agreements in social groups (for a recent survey see Jackson, 2005).

Therefore we argue that economic efficiency of an allocation cannot be evaluated independently of the formation of such social groups. When social groups are formed by decentralized decision-making of economic agents, market prices for group membership or individual ties are seldomly observed. This is not to say that there does not exist a market and corresponding price for group membership, but prices may take the form of in kind payments or the exchange of favors. A natural although by no means exclusive example of economic activity conducted jointly in social groups is that of human capital acquisition, especially in education.

In education, the particular allocation of students into social groups can be expected to affect acquisition of human capital due to the widely observed presence of peer effects. Peer effects in education are commonly found to be positive, thereby introducing a local public good in human capital acquisition within social groups. This paper aims to analyze in its theoretical part the decentralized allocation of social groups. The model we employ is built on empirical regularities observed in the context of human capital acquisition in education.

In particular, we study groups whose members jointly produce human capital subject to peer effects. The key idea is to separate peer effects within groups into those that affect primarily instantaneous utility and those that have a lasting effect on agents' utility, for instance via an agent's attribute such as human capital. We call the former *consumption peer effects* which may be thought of as favors provided by group members to their group mates that have a predominantly consumptive character in the sense that these favors generate utility among group members but have no permanent effect. On the other hand, if peer effects have an effect on the future utility stream, most notably by changing characteristics of an agent such as productivity, we use the term *productive peer effects.*

Of course, in the process of social group formation individuals will be able to exploit both types of peer effects to generate utility. Peer effects are assumed to be subject to decreasing differences (see e.g. Hanushek et al., 2001, Vigdor and Nechyba, 2004, for some empirical support of this hypothesis). We argue that agents may be heterogenous with respect to their ability to produce consumption and production peer effects. This introduces a possibility for agents to "trade" across the two dimensions of peer effects. In the baseline model agents are restricted to their types, that is their attractiveness for a potential match is determined exclusively by their type. Intuitively, a charming person can potentially make up for a certain deficit in skill – and vice versa. However, agents with extreme types on both dimension will fail to find any agents either acceptable or willing to accept and segregate. This favors a decentralized allocation exhibiting bunching, that is heterogenous groups for intermediate types and homogeneous groups for extreme types. In an extension we show that this intuition carries over to the case of effort investments as long as agents cannot to commit to pre-specified investment levels or the transferability of utility generated by the possibility to try harder for a better match is sufficiently low.

Formally, the paper employs a one-sided matching model with endogenous group size and strictly nontransferable utility. It has been previously shown that in the literature that the matching pattern depends on agents' ability to transfer utility (e.g. Legros and Newman, 2004). Becker (1973) shows that positive assortative matching may occur although valuation functions have decreasing differences under strictly nontransferable utility. In contrast to this we find segregation and bunching in different regions of the type space. The reason for this is that agents may compensate each other across dimensions. The decentralized allocation fails to maximize aggregate surplus or human capital production since although heterogenous coalitions are formed, matching tends to be negative assortative for types on the same level set only. That is, the model predicts social segregation of types with similar attractiveness taking into account both dimensions, and heterogenous coalitions with respect to attributes on each dimension. Of course, the present model can also be interpreted as a pure exchange economy with indivisible goods. The segregation result implies then that a comparative advantage does not generally suffice for trade.

Finally, it stands to reason to take advantage of the model's predictions on the nature of equilibrium matching patterns with respect to human capital formation. We project to gather data among students at various German universities.

The paper proceeds by establishing empirical facts, such as the presence of positive peer effects, decreasing differences and congestion effects in peer effects. Section 3 introduces the baseline model without effort investments, where agents generate peer effects according to their two-dimensional type. In section 4 we analyze properties of the decentralized allocation of the baseline model. Introducing effort investments, section 5 provides some robustness results and section 6 concludes.

2 Empirical Findings

Let us first review the empirical facts that have been established about peer effects in education.

2.1 Peer effects exist and are positive.

There has emerged a quite large body of evidence stating that less able students consistently tend to benefit from more able peer groups. One strand of the literature observes positive peer effects on students' grades at school or classroom level for a number of countries.[1] A second group of empirical studies finds positive peer effects between randomly assigned roommates at college level.[2]

2.2 Less able student benefit more from more able peers than more able students.

Hanushek et al. (2001) find that math test scores of bad students are affected more by the presence of good peers at classroom level than the test scores of good students, a finding that is affirmed by Schneeweis and Winter-Ebmer (2005) for Austrian students with an emphasis on reading skills. Also a number of studies of roommate peer effects (Winston and Zimmerman, 2003, Zimmerman, 2003, Hoel et al., 2004) find that low ability students are affected to a greater extent by sharing rooms with high ability students than high ability students. This points to decreasing returns to scale for peer effects.

2.3 Peer effects eventually decrease in group size.

In the study of Hoel et al. (2004) peer effects decrease as social group size decreases. Maasoumi et al. (2004) find that whereas weak students benefit more from good peers in small classes, strong students benefit more in large classes. This lends some support to the view that peer effects are subject to congestion, and, in particular, that congestion decreases in average ability of the social group.

[1] See Hanushek et al. (2001), Hoxby (2000), Vigdor and Nechyba (2004) for the US, Robertson and Symons (2003) for the UK, McEwan (2003) for Chile, and Schneeweis and Winter-Ebmer (2005) for Austria among others.

[2] See e.g. Sacerdote (2001), Winston and Zimmerman (2003), Zimmerman (2003), Hoel et al. (2004).

3 A Simple Model

3.1 Agents

In the model economy there live a continuum of agents. An agent is characterized by his ability to generate consumption and production peer effects, that is a tuple $(\gamma_i, \theta_i) \in \Gamma \times \Theta$ where Γ where Θ are compact subsets of $\mathbb{R}$. Denote such a tuple by i. Only finitely many elements of $\Gamma \times \Theta$ are endowed with positive Lebesgue measure of agents. The set of tuples endowed with positive measure is denoted the agents set I. Agents have the following utility function u_i which is separable in consumption and production peer effects.

$$u_i = c_i + h_i,$$

where c_i denotes consumption peer effects agent i is exposed to and h_i the human capital accumulated by an agent i.[3]

3.2 Groups

Agents may form social groups to profit from positive peer effects. Membership in a group is exclusive. A group in this model can be interpreted as an organizational unit fully encompassing any direct utility benefits emerging from social interaction between agents. A group N is the set of its members $N = \{(\gamma_1, \theta_1), ..., (\gamma_i, \theta_i), ..., (\gamma_n, \theta_n)\}$ or, equivalently, the set of its members' types. The size of group N is given by n. We assume that each member has the possibility to unilaterally withdraw from a group and all members of a group must consent to the entry of a new member.

A group member i's benefit from consumption peer effect within the group is a function $c_i(.)$ of all the group members' types to produce consumption peer effects.

$$c_i = c_i(N).$$

That is, consumption peer effects are a social activity generating utility for all group members. A group member i's benefit from human capital production is a function $h_i(.)$ depending on all group members' types as well.

$$h_i = h_i(N).$$

This means an agent not only raises her own human capital by her presence, but also provides human capital to the remainder of the group. This gives automatically the outside option of an agent i who chooses to remain unmatched:

[3]For instance, productive peer effects may determine an agent's human capital which in turn determines the wage and thus lifetime consumption.

$c_i(\{i\})$ and $h_i(\{i\})$. Consistent with the empirical observations cited above, the assumptions on the peer effects production functions are stated as follows.

Assumption 1 (Peer effect production) *The peer effect production functions $c_i(.)$ and $h_i(.)$ have the following properties.*

(i) Regularity: For all $i \in I$, $c_i(.)$ and $h_i(.)$ are continuously differentiable, strictly increasing functions and symmetric in the arguments γ_j, respectively θ_j, with $j \neq i$.

(ii) Decreasing differences for all $i \in I$:

$$\frac{\partial^2 h_i(N)}{\partial\theta_i\theta_j} < 0 \text{ and } \frac{\partial^2 c_i(N)}{\partial\gamma_i\gamma_j} < 0 \text{ for all } i, j \in N,$$
$$\frac{\partial^2 h_i(N)}{\partial\theta_i\gamma_j} < 0 \text{ and } \frac{\partial^2 c_i(N)}{\partial\gamma_i\theta_j} \leq 0 \text{ for all } i, j \in N,$$

(iii) Decreasing returns to size: For all $(\theta, \gamma) = (\theta_i, \gamma_i)$ with $i \in I$

$$\begin{aligned} h_i(N \cup \{i_{n+1}\}) - h_i(N) &< h_i(N) - h_i(N \setminus \{i_n\}), \\ c_i(N \cup \{i_{n+1}\}) - c_i(N) &< c_i(N) - c_i(N \setminus \{i_n\}) \end{aligned}$$

for all $i \in N$ with $N = \{i_1, i_2, ..., i_n\}$, $n > 1$ and $\theta_i = \theta$ and $\gamma_i = \gamma$.

(iv) Boundedness: For all $i \in I$ there exists $\bar{n}(\theta, \gamma) < \infty$ such that $h_i(\bar{N}) + c_i(\bar{N}) - (h_i(\bar{N} \setminus \{i_{\bar{n}}\}) + c_i(\bar{N} \setminus \{i_{\bar{n}}\})) \leq 0$ where $\bar{N} = \{i_1, i_2, ..., i_{\bar{n}}\}$ with $\theta_{i_j} = \theta_i$ and $\gamma_{i_j} = \gamma_i$ with $j = 1, ..., \bar{n}$.

Part (i) ensures tractability of the model by assuming sufficient regularity on the production functions. $h_i()$ and $(c_i(.)$ are defined on $\mathbb{R}^{2n}$ rather than on the type space to ensure compatability with section 5 where effort is introduced. (ii) reflects the empirical finding of decreasing returns to scale of peer effects in human capital production. Consumption peer effects have decreasing returns as well and there may be substitutability between consumption and production peer effects, for instance due to a concave utility function.[4] Part (iii) reflects a congestion effect on peer effects as the group size increases and the group gets more crowded and (iv) ensures that group sizes are finite as congestion precludes additional benefits from peer effects at some finite group size. Congestion is assumed to affect both consumption and production peer effect production similarly as both effects work primarily through the same channel,

[4]For increasing differences in production peer effects our results for decentralized version carry over if production peer effects dominate in terms of absolute size of the differences. Positive spillovers between production and consumption peer effects tend to reinforce our findings.

communication between agents. This is, for instance due to an agents' limited span of attention, severely impeded by congestion. Whether more or less able groups have greater sizes depends on whether differences in peer effects are decreasing or increasing in input levels.

3.3 Agents' valuations

An agent i's valuation from being matched into a particular social group N depends on the types of the other group members and on his own type and is denoted by v_i.

$$v_i(N) = c_i(N) + h_i(N).$$

This means an agent's valuation for a coalition N is the utility from being exposed to the peer effects in the coalition. If an agent remains solitary his valuation is given by $\underline{v}_i = v_i(\{i\}) \geq 0$. Note that $v_i(.)$ is submodular on $\Theta \times \Gamma$ due to decreasing returns to scale in Assumption 1.

3.4 Group formation

Group formation occurs on a matching market under complete information. Agents decide on whom to match with into a group and can commit to their decision. Group membership is strictly voluntarily. As equilibrium concept we choose a static one-sided matching equilibrium social groups. Let $\mathcal{F}(I)$ denote the set of all finite subsets of the agent space I. Formally, a matching equilibrium in this model is defined as follows.

Definition 1 *A matching equilibrium denoted by P^* is a partition of the agent space I into finite coalitions such that*

- *P^* is measure consistent, and*
- *$\nexists P'_i \in \mathcal{F}(I)$ such that*
  $v_i(P'_i) > v_i(P^*_i)\ \forall i \in P'_i$ (stability).

Measure consistency requires loosely speaking that the measure of first members of a certain group equals the measure of second members etc. Stability means that there exist no deviating coalition such that all members of the deviating coalition are better off than in the equilibrium allocation. Note that this equilibrium is in fact equivalent to the f-core.[5]

[5]See Kaneko and Wooders (1986, 1996) on measure consistency and the f-core.

3.5 Benchmark

Pareto optimality has not much bite in this framework, as any Pareto optimal allocation is a matching equilibrium. Therefore we introduce two concepts of optimality that may be of interest from a policy perspective. Consider first the objective to maximize the aggregate sum of utility in the economy, i.e. to finding a partition P of the type space into finite groups such that $\sum_{i \in I} v_i(P_i)$ with $P_i \in P$ is maximized. By decreasing differences of $h_i()$ and $c_i()$ under Assumption 1, for all $i \in I$ it is straightforward that matching must be component-wise negative assortative, that is $\theta_i > \theta_j$ implies $\theta_l \leq \theta_l$ for some $l \in P_i$ and some $k \in P_j$. Matching across dimensions must be negative assortative as well for negative spillovers between dimension and there is no indication for separability of both dimensions.[6]

4 Properties of equilibrium

As a first step we verify that an equilibrium indeed exists. This is fairly straightforward using a result by Kaneko and Wooders (1986). Thus it suffices to show that a characteristic function can be constructed from v_i meeting some regularity conditions and that per capita payoffs are bounded in all coalitions.

Proposition 1 *A matching equilibrium P^* exists.*

Proof: In Appendix.

Of primary interest is, of course, the matching pattern that emerges in equilibrium. Start by noting that an agent i can never persuade a social group where every member's type is at least as high as i's type on both dimensions to let him join. This is a direct consequence of strictly nontransferable utility but holds more generally, see section 5. The agent i may, however, be able to persuade a group to let him join if i replaces a member whose type is strictly lower at least on one dimension.

Agent i in turn must prefer to join the group as well. Hence, homogeneous groups are not stable if there exists an agent i whose type is (i) lower on one dimension and higher on the other than the group members' type, and (ii) i has an absolute advantage sufficiently great in producing different peer effects than the group. However, all agents can secure themselves at least the payoff

[6]If $c_i()$ has increasing differences for all $i \in I$, the utility maximizing matching must be negative assortative on Θ and positive assortative on Γ. If there are positive spillovers between dimensions, the benchmark allocation must have positive assortative matching between dimensions.

from being member of a homogenous group thus giving a lower bound on an agent's equilibrium payoff. We use this to derive a sufficient condition for the emergence of homogenous groups in equilibrium.

Proposition 2 *A homogeneous group of type (θ, γ) agents cannot be blocked if there does not exist a type (θ', γ') with $\text{sgn}(\gamma - \gamma') \neq \text{sgn}(\theta - \theta')$ and both $|\gamma - \gamma'|$ and $|\theta - \theta'|$ sufficiently great.*

Proof: Let N denote a homogenous group of agents of type (θ_N, γ_N) of size $n^*(\theta_N, \gamma_N)$. Let $n^* = \arg\max_n v_i(N)$. N cannot be blocked in an equilibrium allocation P if there is no N' with $(\theta_N, \gamma_N) \in N'$ such that for all $i \in N'$ it holds that $v_i(N') > v_i(N)$ if $i \in N$ and $v_j(N') > v_j(P_j)$ if $j \notin N$. For an agent j it must hold that $v_j(P_j) \geq v_j(M)$ where M is a homogenous group of type (θ_j, γ_j) agents of optimal size m^* as this gives the minimal payoff any agent j can obtain in equilibrium. That is, N cannot be blocked if there does not exist N' with $v_i(N') > v_i(N)$ if $i \in N$ and $v_j(N') > v_j(M)$ if $j \notin N$.

Note that if N' exists, it must contain at least one agent of type (θ_M, γ_M) with $\theta_N > \theta_M$ and $\gamma_N < \gamma_M$ or vice versa by monotonicity of $h_i(.)$ and $c_i(.)$. Suppose without loss of generality that $\theta_N > \theta_M$ and $\gamma_N < \gamma_M$. Then the existence of N' implies that there exists x on the lattice $X = \{(\theta_N, \gamma_N); (\theta_M, \gamma_N)\}^{\max\{n^*;m^*\}}$ such that $h_i(N) - h_i(x) < c_i(x) - c_i(N)$ and $h_j(x) - h_j(M) > c_j(M) - c_j(x)$. This implies $c_i(M) - c_i(N) > h_i(N) - h_i(x)$ and $c_j(M) - c_j(x) < h_j(N) - h_j(M)$, that is $h_i(x) > h_i(N) - (c_i(M) - c_i(N))$ and $c_j(x) > c_j(M) - (h_j(N) - h_j(M))$ for all $i \in N'$ and $N' = x$. Since $c_j(x) > c_j(x')$ implies $h_i(x) < h_i(x')$, $x, x' \in X$, and the lattice X is a discrete space, there exist $\hat{c} > 0$ and $\hat{h} > 0$ such that $c_i(M) - c_i(N) > \hat{c}$ and $h_i(N) - h_i(M) > \hat{h}$ for $i \in N$, $j \in M$. Because of monotonicity of $h_i()$ and $c_i()$ there exist $\hat{\theta} > 0$ and $\hat{\gamma} > 0$ such that $\gamma_M - \gamma_N > \hat{\gamma}$ and $\theta_N - \theta_M > \hat{\theta}$.

That is, existence of N' implies existence of a type (θ_M, γ_M) with $\text{sgn}(\gamma_N - \gamma_M) \neq \text{sgn}(\theta_N - \theta_M)$ and $|\gamma_N - \gamma_M| > \hat{\gamma}$ and $|\theta_N - \theta_M| > \hat{\theta}$. The proposition follows by contraposition. ■

This means agents close to the border of the agent space on both dimensions segregate. Agents in the interior of the type space tend to form heterogenous groups if they are able to exploit returns to specialization. To capture this idea let us introduce the concept of *symmetric* types of agents.

Definition 2 (Symmetric Types) *Types $J = \{(\theta_1, \gamma_1), (\theta_2, \gamma_2), ...\}$ are*

- *symmetric at n if there exists $N^*(n)$ such that*
  $N^*(n) = \arg\max_{N \in J^n} v_j(N)$ for all $j \in J$, $J \subseteq N^*$ with
  $\frac{\partial v_j(N^*)}{\partial \theta_l} = -\frac{\partial v_j(N^*)}{\partial \gamma_l} \; \forall \; l \neq j \in N^*$ for all $j \in J$,

- *symmetric if they are symmetric at* n^* *and there exists* $N^* = \arg\max_{N \in \{N^*(n): n \in \mathbb{N}\}} v_j(N)$ *for all* $j \in J$.

For instance, in the most simple version of the model where $v_i = h(\theta)+c(\gamma)$ and $h(.) = c(.)$, i and j are symmetric types at even group sizes if $\theta_i = \gamma_j$ and $\gamma_i = \theta_j$ and i, j, and k are symmetric types at odd group sizes if i and j as above and $\theta_k = \frac{1}{2}(\theta_i + \theta_j) = \gamma_k$. Choosing a functional form $h(\theta) = (\sum_{l=1}^{n} \theta_l)^{1/n}$ and likewise for $c(.)$, types i and j are symmetric if $\theta_i = \gamma_i$ and $\theta_i + \theta_j \geq \frac{9}{4}$.[7] Note that heterogenous groups of agents with symmetric types fully exploit the returns to specialization under strictly nontransferable utility. Hence, coalitions of symmetric agents cannot be blocked.

Proposition 3 *An allocation with only heterogenous groups of symmetric agents is stable.*

Proof: Let J be symmetric agents. We first show that in a coalition of symmetric agents at n, $N^*(n) \in J^n$ defined as in Definition 2, an agent $l \notin N^*$ is preferred by $i \in J$ to $j \neq i \in J$, iff $\theta_l+\gamma_l > \theta_j+\gamma_j$. Then we extend this result to coalitions of symmetric agents with respect to deviations by any finite set of agents $L \notin N^*$. Finally, we argue that under an allocation of only groups of symmetric agents there exists no blocking coalition.

Let us look at agent i without loss of generality. For i to prefer l to j it must hold that $v_i(\{N', l\}) > v_i(\{N', j\})$ where $N' = N^* \setminus \{j\}$. This implies

$$h_i(\{N', l\}) - h_i(\{N', j\}) \quad > \quad c_i(\{N', j\}) - c_i(\{N', l\}). \tag{1}$$

Symmetry of i and j, monotonicity and decreasing differences of h_i and c_i in all arguments imply that $v_i(N)$ attains a maximum at $N = N^*(n)$ in the space $A = N' \cup \{(\theta, \gamma) \in \Theta \times \Gamma : \theta + \gamma \leq \theta_i + \gamma_i\}$. Hence, it is not possible to find $l \neq j \in A$ such that (1) holds with $\gamma_l + \theta_l \leq \gamma_i + \theta_i$. The extension of this argument to potential blocking coalitions with sets of agents $L \nsubseteq N^*(n)$ is straightforward as by Definition 2 $v_i(N)$ attains a maximum at $N = N^*(n)$ in the space $A^{n-1} = \{i\} \cup \{L \in (\Theta\times\Gamma)^{n'} : \theta_l+\gamma_l \leq \theta_i+\gamma_i \forall l \in N\}$ as well. Hence, for (1) to hold, there must be $l \in L$ with $\theta_l+\gamma_l > \theta_i+\gamma_i$. That is, a group $N^*(n)$ of agents symmetric at n is stable with respect to deviating coalitions of the same size. Hence, a group N^* such that $N^* = \arg\max_{N \in \{N^*(n), n \in \mathbb{N}\}} v_j(N)$ for all $j \in J$, that is J are symmetric, is stable with respect to potential blocking coalitions L containing only types l with $\theta_l + \gamma_l \leq \theta_i + \gamma_i$.

[7] For $\frac{2}{81} \leq \theta_i + \theta_j \leq \frac{9}{4}$ the preferred group size increases to 3 and i and j are symmetric if and only if $\theta_j = \frac{3}{4}\theta_i + \frac{1}{4}\gamma_i$ in which case $N^* = \{i, j, j\}$ or $\theta_j = -\frac{1}{2}\theta_i + \frac{3}{2}\gamma_i$ with $N^* = \{i, i, j\}$. Additionally there are symmetric types J with $|J| = 3$ and $\sum_{j \in J} \theta_j = \sum_{j \in J} \gamma_j$ and $\theta_j + \gamma_j = c$ for all $j \in J$.

This implies that a collection P of finite sets of agents containing only groups of symmetric types is stable. Under P an agent $i \in P_i \in P$ has no deviating coalition. For any L such that $v_i(\{i, L\}) > v_i(P_i)$ there must be $l \in L$ with $\theta_l + \gamma_l > \theta_i + \gamma_i$. By the above argument then $v_l(\{l, L\}) < v_l(P_l)$, $P_L \in P$. Hence, an allocation P with only symmetric groups of agents is stable. ■

However, whether an allocation of symmetric agents will indeed emerge in equilibrium will depend heavily on the distribution of types in the economy. Hence, Proposition 3 primarily aims to highlight the mechanism at work in the matching market. Heterogenous coalitions form if there exist types such that gains from specialization outweigh potentially differing levels of inputs.

5 Effort Investment

This section looks at a simple extension of the baseline model by letting agents choose effort levels on each dimension. This may introduce limited transferability of utility when effort levels proposed explicitly or implicitly at the matching stage are credible. Therefore both cases, with and without commitment, are considered. The results prove to be reasonably robust, as in the case of no commitment power, there is a unique Nash equilibrium within a group and this essentially permits the above analysis to hold true. In case of commitment power Proposition 2 continues to hold if the inputs are sufficiently complementary in the classical sense as to induce sufficient productivity loss when moving away from the optimal factor input ratio.

5.1 Framework

In this section agents have the opportunity to exert effort in order to provide consumption and production peer effects. Denote agent i's effort levels in consumption peer effects by $g_i \in \mathbb{R}$ and by $f_i \in \mathbb{R}$ in production peer effects. Exerting effort comes at a utility cost of $k_c(g_i, \gamma_i)$ and $k_h(f_i, \theta_i)$, respectively. An agent's cost of effort is affected by his type. The cost enters an agent's utility function additively, $u_i = c_i + h_i - k_c - k_h$. Cost functions are strictly convex in investments g_i, f_i and differ across agents only by the parameters γ_i and θ_i. In particular, we assume that higher types always face higher marginal costs all else equal.

Assumption 2 (Single crossing) *For all $i, j \in I$:*

$$\gamma_i > \gamma_j \quad \Leftrightarrow \quad \frac{\partial k_c(g,\gamma_i)}{\partial g} < \frac{\partial k_c(g,\gamma_j)}{\partial g} \; \forall\, g \geq 0 \textit{ and}$$
$$\theta_i > \theta_j \quad \Leftrightarrow \quad \frac{\partial k_h(f,\theta_i)}{\partial f} < \frac{\partial k_h(f,\theta_j)}{\partial f} \; \forall\, f \geq 0.$$

This means the higher an agent's type the lower the marginal cost for producing peer effects at any given level of peer effects. Production of peer effects is adapted to the introduction of effort. Consumption peer effects in a group N are generated according to the function $c_i((f_j, g_j)_{j\in N}$. Production peer effects are generated according to the function $h_i((f_j, g_j)_{j\in N})$.

Unfortunately, it is necessary to put some more regularity on the functions c_i and h_i to ensure tractability of the model when allowing for effort investments.

Assumption 3 (Peer effect production) *In addition to Assumption 1 the functions c_i and h_i have the following properties.*

(i) $c_i(N, f, g) = c_i((g_j)_{j\in N})$ *and* $h_i(N, f, g) = h_i((f_j)_{j\in N}$*,*

(ii) for all $i, j \in I$, c_i and h_i are linear transformations of c_j and h_j, respectively.

Part (i) of this assumption assumes separability of consumption and production peer effects and (ii) is needed for comparative static analysis.

5.2 Agents' valuations

An agent i's valuation from being matched into group N depends on the investments in peer effects of the other group members and on his own investment net of effort cost and is denoted by v_i.

$$v_i(N, f, g) = c_i((f_j, g_j)_{j\in N}) + h_i((f_j, g_j)_{j\in N}) - k_c(g_i, \gamma_i) - k_h(f_i, \theta_i).$$

If an agent i remains solitary his valuation is given by $v_i(i, f_i^*, g_i^*) \geq 0$ where (f_i^*, g_i^*) maximizes $c_i + h_i - k_c(g_i, \gamma_i) - k_h(f_i, \theta_i)$. Given the assumptions on regularity this program has always a unique solution. Note that $v_i(.)$ is submodular in (f, g) due to decreasing returns to scale in Assumption 1.

5.3 Equilibrium Concept

The equilibrium concept employed needs some modification as well in order to account for effort decisions. Let a matching equilibrium with effort investments under commitment be defined as follows.

Definition 3 *A matching equilibrium denoted by* (P^*, f^*, g^*) *is a partition* P^* *of the agent space* I *into finite coalitions and individual investment plans in human capital* f^* *and social activities* g^* *such that*

- P^* *is measure consistent, and*
- $\nexists P_i' \in \mathcal{F}(I)$ *such that*
  $v_i(P_i', f_{P_i'}, g_{P_i'}) > v_i(P_i^*, f_{P_i^*}, g_{P_i^*}) \, \forall i \in P_i'$ *(stability).*

The key difference to section 3 is that with effort investment and commitment there arises a limited opportunity for transferring utility between group members. Existence is straightforward by a version of the proof of Proposition 1.

5.4 Constrained efficient investments

Let an agent i be member of social group N of size n. Let $V(N, f, g) = \sum_{i \in N} v_i(N, f, g)$ denote the value of a group N given vectors of investments (f, g). Surplus maximizing investments within this social group solve the following optimization problem of the coalition N.

$$\max_{(f_i, g_i), i \in N} \sum_{i \in N} v_i(N, g, f). \tag{2}$$

First order necessary conditions are then for all $i \in N$

$$\begin{aligned} \sum_{i \in N} \frac{\partial c(g_1, ..., g_n)}{\partial g_i} &= \frac{\partial k_c(g_i, \gamma_i)}{\partial g_i} \text{ and} \\ \sum_{i \in N} \frac{\partial h(f_1, ..., f_n)}{\partial f_i} &= \frac{\partial k_h(f_i, \theta_i)}{\partial f_i}. \end{aligned} \tag{3}$$

Denote the investment vectors defined by these equations by f^* and g^*.

Then define the potential value of a coalition N by

$$V^*(N) = \arg \max_{(f_i, g_i), i \in N} \sum_{i \in N} v_i(N, g, f).$$

$V^*(N)$ is the highest attainable sum of utilities in social group N. Note, however, that $(f_i^*, g_i^*)_{i \in N}$ pins down uniquely the allocation of the surplus among group members. The allocation of surplus within the group can be changed by varying investment levels of agents. This induces a fundamental nontransferability of utility. Define now the utility possibility frontier $\phi_i(u_{-i})$ of agent $i \in N$ in group N by the maximum amount of utility i can secure for himself provided all agents $j \neq i \in N$ obtain at least u_j each. u_{-i} is shorthand for the vector of the remaining agents' minimum utility.

$$\phi_i(N, u_{-i}) = \max_{(f_j, g_j), j \in N} v_i(N, g, f) \text{ s.t. } \forall\, j \neq i \in N, v_j(N, g, f) = u_j \geq 0. \tag{4}$$

Note that for sufficiently high u_j this maximization problem has no solution. Set in this case $\phi_i(u_{-i}) = 0$. Then the utility possibility frontier of coalition N, $\Phi(N)$ can be written as

$$\Phi(N) = \{u \in \mathbb{R}^n : u_i = \phi_i(u_{-i}) \, \forall \, i \in N\}.$$

$\Phi(N)$ gives all distributions of surplus within group N on the efficient frontier. The optimization problem (4) requires agents to choose investments as to minimize utility cost when providing a given utility level to the other group members. That means for all points of $\Phi(N)$ all members of N have to equate their marginal costs of investment in human capital and social activities.

For all points $u \in \Phi(N)$ it must hold that the associated investment levels are constrained optimal, that is

$$\begin{aligned}
\left(1 + \sum_{j \neq i \in N} \lambda_j\right) \frac{\partial h(.)}{\partial f_i} &= \frac{\partial k_h(f_i, \theta_i)}{\partial f_i} \text{ and} \\
\left(1 + \sum_{j \neq i \in N} \lambda_j\right) \frac{\partial h(.)}{\partial f_j} &= \lambda_j \frac{\partial k_h(f_j, \theta_j)}{\partial f_j} \, \forall \, j \neq i \in N, \\
h(g, f) + c(g, f) - k_h(f_j) - k_c(g_j) &= u_j \, \forall \, j \neq i \in N.
\end{aligned}$$

where λ_j denote the Lagrange multipliers. Necessary conditions for investment in g_i are derived analogously. Let us state now a number of technical preliminaries that will become useful later on.

Lemma 1 *(Monotonicity) For any $N \in \mathcal{F}(I)$, it holds for all $i \in N$ that $\gamma_i > \gamma_j \Rightarrow g_i^* > g_j^*$ and $\theta_i > \theta_j \Rightarrow f_i^* > f_j^*$ for g^* and f^* that solve (2).*

Proof: f^* and g^* solve problem (2). Hence, the necessary conditions (3) hold. Let us focus on investments in productive peer effects first. Symmetry and strict concavity of $h(.)$ imply that

$$\sum_{j \in N} \frac{\partial h(.)}{\partial f_i} < \sum_{j \in N} \frac{\partial h(.)}{\partial f_j} \Leftrightarrow f_i^* > f_j^*. \tag{5}$$

By convexity of the cost function and single crossing in Assumption 2 it holds that

$$\begin{aligned}
\theta_i > \theta_j \quad \Rightarrow \quad & \left(\frac{\partial k_h(f_i^*, \theta_i)}{\partial f_i} \geq \frac{\partial k_h(f_j^*, \theta_j)}{\partial f_j} \wedge f_i^* \geq \underline{f}(\theta_i, \theta_j, f_j^*)\right) \\
& \vee \left(\frac{\partial k_h(f_i^*, \theta_i)}{\partial f_i} < \frac{\partial k_h(f_j^*, \theta_j)}{\partial f_j} \wedge f_i^* < \underline{f}(\theta_i, \theta_j, f_j^*)\right),
\end{aligned} \tag{6}$$

for some well-defined $\underline{f}(\theta_i, \theta_j, f_j^*) > f_j^*$. Using the identities (3) and equivalence (5) the first case yields a contradiction. Hence, using (3) and (5),

$$\theta_i > \theta_j \Rightarrow \underline{f}(\theta_i, \theta_j, f_j^*) > f_i^* > f_j^*.$$

An analogous argument hold for g^*. ■

Define now the surplus maximizing group size of a homogenous group of type (γ, θ) agents, $n^*(\gamma, \theta)$.

$$n^*(\gamma, \theta) = \arg\max_n V^*(N) \text{ with } n = |N| \text{ and } (\gamma_i, \theta_i) = (\gamma, \theta) \, \forall \, i \in N. \tag{7}$$

Denote such a group by $N^*(\gamma, \theta)$. Then $V^*(N^*(\gamma, \theta))$ gives the maximal value of a homogenous coalition. The following lemma shows that this determines an agent i's minimal payoff in a matching equilibrium, $\underline{v}(\gamma, \theta)$.

Lemma 2 *(Segregation Payoffs) In any matching equilibrium, an agent i of type (γ, θ) obtains at least payoff*

$$\underline{v}(\gamma, \theta) = h(g_1^*, ..., g_{n^*(\gamma,\theta)}^*) - k_c(g^*) + h(f_1^*, ..., f_{n^*(\gamma,\theta)}^*) - k_h(f^*),$$

where $g_i^* = g^*$ and $f_i^* = f^*$, with $i = 1, ..., n^*(\gamma, \theta)$ solve (3) for $N^*(\gamma, \theta)$ and $n^*(\gamma, \theta)$ is defined as in (7).

Proof: All types are endowed with positive measure. Hence, an agent i of type (γ, θ) cannot receive less payoff in an equilibrium group than by matching into a homogeneous group. The homogeneous group generating maximum surplus is given by $N^*(\gamma, \theta)$. Constrained efficient investments f_i^* and g_i^* are given by (3). By Lemma 1, for a homogeneous group it must hold that $g_i^* = g_j^*$ and $f_i^* = f_j^*$ with $i \neq j \in N^*(\gamma, \theta)$. Therefore any agent i of type (γ, θ) can secure himself at least payoff $\underline{v}(\gamma, \theta) = h(g^*) - k_c(g^*) + h(f^*) - k_h(f^*)$ in an efficient homogeneous coalition $N^*(\gamma, \theta)$.

For any equilibrium with positive measure of a homogeneous coalition N, each $i \in N$ must receive the same payoff and (n, f, g) must maximize group surplus under equal sharing. Hence, the only stable homogeneous coalition is $N^*(\gamma, \theta)$ with investments $f^*(N^*(\gamma, \theta))$ and $g^*(N^*(\gamma, \theta))$. Otherwise, there is a positive measure of agents with lower payoff who are able to form this group and thus obtain strictly higher payoffs. ■

Lemma 2 gives agents' segregation payoffs which coincide with payoffs of agents matched into homogenous coalitions with positive measure in equilibrium. Moreover, the regularity assumptions on the production function and cost allow us to state the following important preliminary that will become useful for determing the efficiency benchmark.

Lemma 3 $V^*(N)$ *is strictly increasing and strictly submodular in members' types* (γ, θ).

Proof: The system of equations (3) defines $f^*(N, \theta)$ and $g^*(N, \gamma)$ because of separability of utility. Differentiating $V^*(N) = V(N, g^*(N, \gamma), f^*(N, \theta))$ with respect to γ_i and θ_i yields by the envelope theorem

$$\begin{aligned} \frac{\partial V^*(N, \gamma, \theta)}{\partial \gamma_i} &= -\frac{\partial k_c(g_i^*(\gamma), \gamma_i)}{\partial \gamma_i} > 0 \text{ and} \\ \frac{\partial V^*(N, \gamma, \theta)}{\partial \theta_i} &= -\frac{\partial k_h(f_i^*(\theta), \theta_i)}{\partial \theta_i} > 0. \end{aligned}$$

The cross partial derivatives are then

$$\begin{aligned} \frac{\partial^2 V^*(N, \gamma, \theta)}{\partial \gamma_i \partial \gamma_j} &= -\frac{\partial k_c(g_i^*(\gamma), \gamma_i)}{\partial \gamma_i} \frac{\partial g_i^*(\gamma)}{\partial \gamma_j} \text{ and} \\ \frac{\partial^2 V^*(N, \gamma, \theta)}{\partial \theta_i \partial \theta_j} &= -\frac{\partial k_h(f_i^*(\theta), \theta_i)}{\partial \theta_i} \frac{\partial f_i^*(\theta)}{\partial \theta_j}. \end{aligned}$$

Because of monotonicity of investments in types from Lemma 1 and decreasing returns to scale from Assumption 1, indeed it must hold that $\frac{\partial g_i^*(\gamma)}{\partial \gamma_j} < 0$, and likewise for θ. Hence, the cross partial derivatives are strictly negative and $V^*(N)$ is strictly submodular in the vectors of types, γ and θ.[8] ■

5.5 Benchmark

We are interested in a benchmark allocation to compare with matching outcomes. To reflect potentially diverging interests in human capital formation and utility we provide two benchmarks. The first maximizes the sum of utilities in the economy and can be derived by the allocation (P^U, f^U, g^U). The second benchmark is the allocation maximizing human capital formation, (P^H, f^H). Submodularity of $V^*(N)$ in types and separability of the utility function then imply almost immediately the following proposition.

Proposition 4 *Both benchmark allocations,* (P^U, f^U, g^U) *and* (P^H, f^H)*, are characterized by negative assortative matching on each type dimension, that is groups must be heterogeneous along each dimension.*

Proof: Let $P \in P^U$ with $P = \{(\gamma_1, \theta_1), ..., (\gamma_n, \theta_n)\}$. The value of group P is then accordingly $V^*(P, \gamma, \theta)$. If $P \in P^U$ with positive measure, it must hold that

$$|P| V^*(P, \gamma, \theta) \geq \sum_{P_i' \in P'} V^*(P_i', \gamma, \theta)$$

[8] There is another way of proof by preservation of supermodularity under maximization, see Theorems 2.6.2 and 2.7.6 in Topkis (1998); however, it involves some more notation.

for all partitions P' of $\bigcup_{i=1}^{|P|} P$. That is, partitioning a subset $\bigcup_{i=1}^{|P|} P$ of the agent space into $|P|$ groups P must induce at least the same social surplus as any partition of that subset of the agent space. Strict submodularity of $V^*(N)$ in γ and θ from Lemma 3 implies strictly decreasing differences (Theorem 2.6.1 in Topkis, 1998). That is,

$$\sum_{i \in P} V^*(N'(i)) \leq |P| V^*(P) \text{ with } N'(i) = \bigcup_{j=1}^{|P|} \{(\gamma_i, \theta_i)\},$$

with strict inequality for $\gamma_i \neq \gamma_j$ or $\theta_i \neq \theta_j$ for at least one pair $i \neq j \in P$. Thus allocations that maximize aggregate surplus on each dimension of peer effects must have heterogeneous groups along both type dimensions. This extends to the allocation (P^H, f^H) as well because of separable utility. A surplus maximizing allocation indeed exists, because both optimization problems are equivalent to matching problems with fully transferable utility. Analogous to the proof of Proposition 1 the matching problems can be rewritten to yield convex games for which a solution exists (see Kaneko and Wooders, 1996).■

Socially optimal outcomes always have heterogeneous groups with respect to human capital acquisition. This result obviously depends on the assumption of decreasing differences in peer effects. For increasing differences of $c(.)$ the proof can be reversed yielding a positive assortative matching pattern of the benchmark allocation with respect to cost of consumption peer effects. It is noteworthy though that negative assortative matching with respect to θ in both benchmark allocations does not depend on the separability of utility. The critical assumption is monotonicity, that is differences of $h(.)$ are increasing globally and independently from $c(.)$.

5.6 No Commitment

Let us first assume that agents do not possess the power to commit to effort levels announced at the matching stage but play a noncooperative investment game within groups. Start by analyzing the within group game in some coalition $N \in \mathcal{F}(I)$. An agent $i \in N$'s strategy space is given by $(f_i, g_i) \in F \times G$, his payoff by $v_i(N, f, g)$ where g and f are $(1 \times n)$ vectors of investment levels.

Proposition 5 (Noncooperative investments)

(i) The game $(G \times F, v_i, i \in N)$ has a nonempty, Pareto-rankable set of Nash equilibria (f^*, g^*).

(ii) For any equilibrium agent i's investment f_i^* (g_i^*) strictly increases in own type θ_i (γ_i) and the remaining players' investment f_j^* (g_j^*) strictly decreases in i's type θ_i (γ_i), $j \in N \setminus \{i\}$.

(iii) $v_i(N, f^*, g^*)$ strictly decreases in own type θ_i (γ_i) and strictly increases in another player's type θ_j (γ_j), $j \in N \setminus \{i\}$.

(iv) $v_i(N, f^*, g^*)$ has decreasing differences in $(\theta_j)_{j \in N}$, $v_i(N, f^*, g^*)$ has decreasing differences in $(\gamma_j)_{j \in N}$.

Proof: (i) Existence is immediate as $G \times F$ is compact, f_i and g_i are continuous and v_i is continuously differentiable in (f, g). Note that $f(.)$ is submodular in (f, g) and $k_i(.)$ is supermodular in (f_i, g_i). Hence, $(G \times F, v_i, i \in N)$ is a submodular game (see Topkis, 1998, for details). That means that the set of equilibria of this game has a partial order and the lowest element of the set is Pareto best.

(ii) Agent i's reaction correspondence $(g_i, f_i)(g_{-i}, f_{-i})$ to other players' strategies (f_{-i}, g_{-i}) can be written as $f_i(f_{-i})$ and $g_i(g_{-i})$ due to separability of consumption and production peer effects. The proof focusses on production peer effects, extending it to consumption peer effects is straightforward. $f_i(f_{-i})$ is implicitly given by the first order necessary conditions of his optimization problem.

$$\frac{\partial h_i(f_i, f_{-i})}{\partial f_i} = \frac{\partial k_h(f_i, \theta_i)}{\partial f_i}. \tag{8}$$

It follows immediately from submodularity of $h_i(.)$ and convexity of $k_h(.)$ that f_i decreases in f_{-i}. The next step is to establish that f_i increases and f_{-i} decrease in θ_i. Suppose the first order conditions (8) hold for all $i \in N$. The proof proceeds by contradiction. Let θ_i decrease to $\theta_i' < \theta_i$. Suppose f_i increases to $f_i' \geq f_i$ in response and therefore by (8)

$$\frac{\partial h_i(f_i', f_{-i}')}{\partial f_i} > \frac{\partial h_i(f_i, f_{-i})}{\partial f_i}. \tag{9}$$

Then by submodularity of $h_i(.)$ for some $j \in N \setminus \{i\}$ it holds that $f_j' < f_j$. But this means $\frac{\partial k_h(f_j', \theta_j)}{\partial f_j} < \frac{\partial k_h(f_j, \theta_i)}{\partial f_j}$ and consequently $\frac{\partial h_j(f_j', f_{-j}')}{\partial f_j} < \frac{\partial h_j(f_j, f_{-j})}{\partial f_j}$ by j's first order condition. This is a contradiction to (9) due to part (ii) of Assumption 3. Hence, $f_i' < f_i$.

Suppose now there is some $j \in N \setminus \{i\}$ such that $f_j' \leq f_j$. This means

$$\frac{\partial h_j(f_j', f_{-j}')}{\partial f_j} \leq \frac{\partial h_j(f_j, f_{-j})}{\partial f_j}, \tag{10}$$

as argued above. From above we know that $f_i' < f_i$ and thus by submodularity of $h_i(.)$ there must be an $l \in N \setminus \{i\}$ with $f_l' > f_l$. For $n = 2$, $j = l$ which already contradicts (10). For $n > 2$, $l \neq j$ and $\frac{\partial h_l(f_l', f_{-l}')}{\partial f_l} > \frac{\partial h_l(f_l, f_{-l})}{\partial f_l}$ by l's first order condition. By part (ii) of Assumption 3 this contradicts (10) and $f_j' > f_j$ for all $j \in N \setminus \{i\}$.

(iii) Differentiating $v_i(N, f^*, g^*)$ with respect to θ_i ceteris paribus and applying the envelope theorem yields

$$\frac{\partial v_i(N, f^*, g^*)}{\partial \theta_i} = \sum_{j \neq i \in N} \frac{\partial h_i(f^*)}{\partial f_j^*} \frac{\partial f_j^*}{\partial \theta_i}. \tag{11}$$

By part (ii) of this proof this expression must be negative. Now let θ_j decrease to $\theta_j' < \theta_j$. As shown above, $f_i' > f_i$ and therefore $\frac{\partial h_i(f_i', f_{-i}')}{\partial f_i} > \frac{\partial h_i(f_i, f_{-i})}{\partial f_i}$. By strict monotonicity of both $h_i(.)$ and $k_h(.)$, $h_i(f') < h_i(f)$ and $k_h(f_i') > k_h(f_i)$ and thus $v_i(N, f', g) < v_i(N, f, g)$.

(iv) It suffices to establish that expression (11) is decreasing in θ_j. Let θ_j increase to $\theta_j' > \theta_j$. By symmetry of h_i in all f_j, (11) can be rewritten as $\frac{\partial h_i(f^*)}{\partial f_j} \sum_{j \neq i \in N} \frac{\partial f_j^*}{\partial \theta_i}$. Let θ_j increase to $\theta_j' > \theta_j$. By part (ii) we know that $\frac{\partial h_i(f^{*\prime})}{\partial f_i} < \frac{\partial h_i(f^*)}{\partial f_i}$ and therefore $\frac{\partial h_i(f^{*\prime})}{\partial f_l} < \frac{\partial h_i(f^*)}{\partial f_l}$ for all $l \neq i \in N$.

Hence, we only need to show that $\sum_{j \neq i \in N} \frac{\partial f_j^*}{\partial \theta_i} \geq \sum_{j \neq i \in N} \frac{\partial f_j^{*\prime}}{\partial \theta_i}$. That is, in terms of differences, $\sum_{j \neq i \in N} (f_j^*(\theta_i', \theta_j) - f_j^*(\theta_i, \theta_j)) \geq \sum_{j \neq i \in N} (f_j^*(\theta_i', \theta_j') - f_j^*(\theta_i, \theta_j')$. Note that from part (ii) of this proof that $\sum_{j \neq i \in N} (f_j^*(\theta_i', \theta_j) - f_j^*(\theta_i, \theta_j))$ must strictly increase in the difference $\frac{\partial h_i(f^*(\theta_i, \theta_j))}{\partial f_j} - \frac{\partial k_h(f_i^*(\theta_i, \theta_j), \theta_i')}{\partial f_i} = \frac{\partial k_h(f_i^*(\theta_i, \theta_j), \theta_i)}{\partial f_i} - \frac{\partial k_h(f_i^*(\theta_i, \theta_j), \theta_i')}{\partial f_i}$. The latter is weakly increasing in f_i^* due to decreasing differences and strict convexity of $k_h(.)$ from Assumption 2. Since $f_i^*(\theta_i, \theta_j) > f_i^*(\theta_i, \theta_j')$, we have that indeed for the differential operator D_{θ_j}, $D_{\theta_j} \sum_{j \neq i \in N} (f_j^*(\theta_i', \theta_j) - f_j^*(\theta_i, \theta_j)) \leq 0$. This means, by separability of $h_i(.)$ and $c_i(.)$, $v_i(f^*)$ has decreasing differences in θ. ■

Part (i) of the proposition implies that applying the Pareto criterion for equilibrium selection the equilibrium effort levels are unique. Hence, there exist functions $f_i^* : \mathcal{F}(I) \mapsto F$ and $g_i^* : \mathcal{F}(I) \mapsto G$ determining effort levels of all $i \in N$ for any $N \in \mathcal{F}(I)$. That is, an agent i's valuation $v_i(N)$ depends only on N and can be written as part

$$\begin{aligned} v_i(N) \quad = \quad & h_i((f_j^*(N), g_j^*(N))_{j \in N}) + c_i((f_j^*(N), g_j^*(N))_{j \in N}) \\ & - k_h(f_i^*(N), \theta_i) - k_c(g_i^*(N), \gamma_i). \end{aligned}$$

Combined with part (iv) this implies that $v_i^*(N) = v_i(N, f^*, g^*)$ is submodular on $\Theta \times \Gamma$. This is quite useful as Propositions 2 and 3 can be applied.

Part (ii) and (iii) give some intuition on the mechanics under noncooperative investments. Effort investments are determined by an agent's relative cost of effort. Hence, a high cost type on both dimensions will free ride on effort investments and therefore will not be accepted by any type with lower cost type on both dimensions. Moreover, equilibrium investment levels are monotone in type and do not fully internalize submodularity of the effort provision technology. Hence, there are gains from specialization at the matching stage and heterogenous groups of agents from the interior of the type space may emerge.

5.7 Commitment

In this section agents forming a coalition have the possibility to choose levels of individual investments in peer effects when forming the group. We assume that agents can indeed commit to these investment levels when group production of human capital takes place. Several natural motivations come to mind. First agents' behavior in social groups are a possibly infinitely repeated game by nature. Hence, there exists the possibility of supporting equilibrium allocations Pareto-dominating an equilibrium where every group member free rides by adequate use of punishments in players' strategies. Secondly, social groups can be interpreted to be formed on a daily basis, so that the assumption of commitment power reduces really to assuming commitment power over a very limited amount of time.[9] Moreover, our results remain unchanged for limited commitment as long as all groups have access to the same commitment technology. However, individual heterogeneity of the agents with respect to commitment ability could be accommodated by this model if it is possible to interpret the ability to commit as part of an individual's social skills.

Given that agents are now able to transfer utility within groups by allocating effort levels among members it can no longer be expected that the baseline results, notably Proposition 2, continue to hold. It has been shown previously (e.g. in Legros and Newman, 2004) that as transferability improves sufficiently sorting will inevitably match the pattern maximizing total surplus, that is negative assortative in our case. Intuitively, under full transferability of utility bad types can compensate better types fully for their presence. When the benefit of good type to a bad type exceeds the benefit of a good type to a good type, the bad type will be able to outbid the good type.

Due to monotonicity of constrained efficient effort investments in type we

[9]Note that much of the literature on social loafing in social psychology lends some support to this argument, see for instance Karau and Williams (1993, 1997) and Shepperd (1993) among others.

know that Proposition 2 holds point-wise for constrained efficient effort levels. That is, under constrained efficient effort levels an agent of worse type and strictly worse on at least one dimension cannot compensate an agent of higher type. However, under effort investments transferability is limited. In order to transfer utility among peers a group of agents will have to choose inefficient effort levels. For Proposition 2 to carry over the efficiency loss incurred by a departure from constrained efficient effort investments must outweigh the gain by exploiting submodularity of $c(.)$ and $h(.)$. Putting the condition in terms of the utility possibility frontier yields that for all $i, j \in I$ with $\theta_j \geq \theta_i$ and $\gamma_j \geq \gamma_i$ with one strict inequality it must hold that

$$\phi_j(N, (\underline{v}(i))_{i \in N' \setminus \{j\}}) < \underline{v}(j).$$

However, the properties of the utility possibility frontier are notoriously hard to ascertain. Therefore a simple example may be in order.

5.7.1 An example

Let $\Gamma \times \Theta = \{1; 2\} \times \{1; 2\}$, that is agents' types are binary in each dimension. Assume that each type (γ, θ) occurs with equal probability in the population of agents. Cost of effort is assumed to be $k_c(g_i, \gamma_i) = \frac{1}{\gamma_i} g_i^2$ and $k_h(f_i, \theta_i) = \frac{1}{\theta_i} f_i^2$. Specify the peer effect production technology by assuming $c_i(g_1, ..., g_n) = \sqrt{\sum_{i=1}^n g_i}$ if $n \leq 2$ and 0 otherwise; and $h_i(f_1, ..., f_n) = \sqrt{\sum_{i=1}^n f_i}$ if $n \leq 2$ and 0 otherwise. That is, groups have at most size $n = 2$. This parametric specification is in line with Assumptions 2 and 1, except for strictly decreasing returns to size which are only weakly decreasing in this example. As this already suffices to prevent groups of size $n > 2$, this deviation is of no further consequence. This means an agent i's valuation for coalition N is given by

$$v_i(N, f, g) = \left(\sum_{i=1}^{2} g_i \right)^{-\frac{1}{2}} + \left(\sum_{i=1}^{2} f_i \right)^{-\frac{1}{2}} - \frac{g_i^2}{\gamma_i} - \frac{f_i^2}{\theta_i}.$$

As we have shown above, the efficient allocation is then given by negative assortative matching on each dimension, that is for all groups N it must hold that $\theta_i \neq \theta_j$ and $\gamma_i \neq \gamma_j$ for $i \neq j \in N$. Combined with constrained efficient investments given by equations (3) this pins down the efficient allocation P^*: for all $P_i^* \in P^*$, $|P_i^*| = 2$, $\theta_1 \neq \theta_2$ and $\gamma_1 \neq \gamma_2$ and $f^* = \left(\left(\frac{2}{3}\right)^{\frac{1}{3}}, \frac{1}{2} \left(\frac{2}{3}\right)^{\frac{1}{3}} \right) = g^*$ with higher investment by the lower cost agent.

Now we turn to the utility possibility frontier of a social group N, $\Phi(N)$ which is of interest only for $n = 2$. Although the problem is quite simple, deriving the utility possibility frontier is not trivial. Hence, we provide a numerical solution in Figure 1 depicting the relevant utility possibility frontiers in different coalitions. The first tuple in the coalition denotes agent i' type and the second one agent j's type.

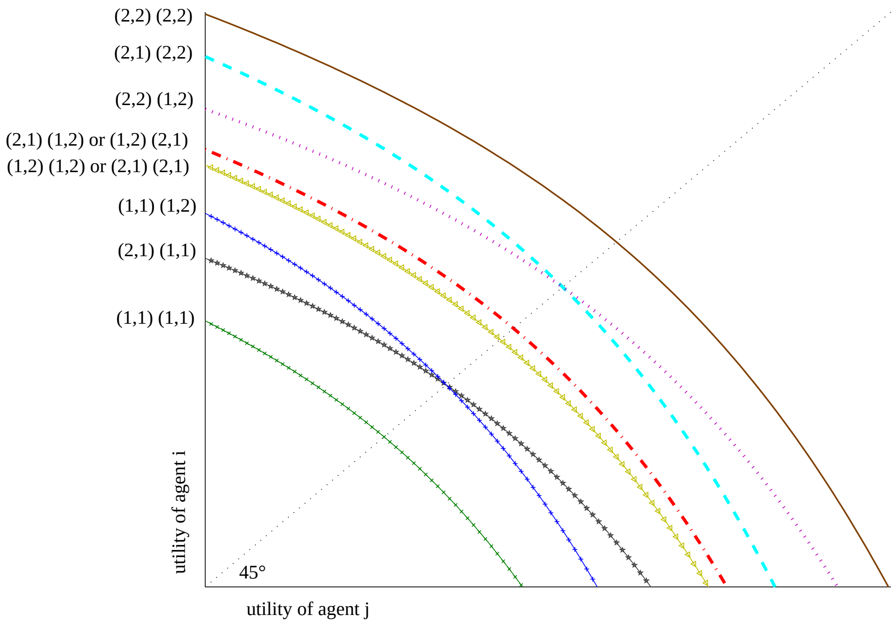

Figure 1: Utility possibility frontier for possible social groups

To solve for market allocation start by checking homogeneous groups of low cost $(1,1)$ agents. For stability group members have to split the surplus which coincides with constrained efficient investments, $f = \left(2^{-\frac{1}{3}}, 2^{-\frac{1}{3}}\right) = g$. That is, $\underline{v}(1,1) = \frac{3}{2}2^{\frac{1}{3}}$. Moreover, $\underline{v}(1,2) = \frac{3}{4}(1+2^{\frac{1}{3}})$. Numerical analysis shows that the maximum payoff of an agent i with type $(1,1)$ when matching with a $(1,2)$ agent is given by $\phi_{(1,1)}((1,1),(1,2),\underline{v}(1,2)) < \underline{v}(1,1)$. Hence, groups consisting of both $(1,2)$ and $(1,1)$ agents are not stable. One can show by solving problem (4) numerically, that the same argument goes through for $(2,1)$ and $(2,2)$ agents. Hence, homogeneous coalitions of $(1,1)$ agents are stable.

Now we turn to homogeneous groups of $(2,1)$ and $(1,2)$ agents, respectively. It is immediate from decreasing returns to scale that there exist payoffs for agents in $((2,1),(1,2))$ or $((1,2),(2,1))$ groups that dominate homogeneous coalitions, that is $\phi_{(2,1)}((2,1),(1,2),\underline{v}(1,2)) > \underline{v}(2,1)$ and vice versa. Note that $\underline{v}(2,1) = \underline{v}(1,2)$ due to the symmetry of the valuation function. Numerical analysis can be used to show that also $\underline{v}(2,1) > \phi_{(2,1)}((2,1),(2,2),\underline{v}(2,2))$ and $\underline{v}(1,2) > \phi_{(1,2)}((1,2),(2,2),\underline{v}(2,2))$. Hence, $((2,1),(1,2))$ or $((1,2),(2,1))$ are stable. This also implies that homogeneous $(2,2)$ groups are stable as well.

The decentralized allocation in this little example is summarized in the following table.

	$\gamma_i = 1$	$\gamma_i = 2$
$\theta_i = 1$	segregates	matches with $(1, 2)$
$\theta_i = 2$	matches with $(2, 1)$	segregates

That is, the matching has segregation at the (component-wise) top and the (component-wise) bottom of the agent space. Agents in between form heterogeneous groups. Note that heterogeneous coalitions of intermediate types (e.g. $(1, 2), (2, 1)$) dominate homogenous coalitions (e.g. $(1, 2), (2, 1)$). This shifts the relevant utility possibility frontier in Figure 1 to the right which induces homogenous coalitions of $(1, 1)$ and $(2, 2)$ agents to be stable. This intuition lies at the heart of Proposition 2.

The matching pattern does not depend on the distribution of types among agents although the distribution of payoffs does. Investment in peer effects are constrained efficient in homogeneous groups which depends on equal weighting of peer effects but these groups are subject to mismatch. Sorting for intermediate agents is efficient, but investments may not be efficient depending on payoffs and thus on the type distribution.

6 Conclusion

To be written yet...

A Appendix: Proofs

A.1 Proof of Proposition 1

The proof of existence closely follows Legros and Newman (1996). We need to use a modified version of v_i, v_i^M, to construct a super-additive characteristic function of the game along the lines of Shubik and Wooders (1983). That means it must be ensured that any union of disjoint coalitions must be able to reach the same allocation as the disjoint coalitions. Hence we define $v_i^M(P_i, N) = v_i(P_i)$, where $N \in \mathcal{F}(I)$ and $P_i \subset N$. v_i^M specifies agent i's valuation for being member of group P_i which may or may not coincide with coalition N. Now let $V(N)$ denote the characteristic function of the assignment game (I, v_i^M) where $N \in \mathcal{F}(I)$ and P_N is a partition of N.

$$V(N) = \{(v_i^M(P, P_i)_{i \in N} : \forall\, i \in N,\ P_i \in P_N)\}.$$

Then construct the comprehensive extension of $V(N)$ by defining

$$\hat{V}(N) = \{x \in \mathbb{R}^{|N|} : x \leq V(N)\}.$$

$\hat{V}(N)$ defines a set in $\mathbb{R}^{|N|}$ that is bounded above by $V(N)$ for any finite set of agents N. Then $\hat{V}(N)$ has the following properties:

$$\hat{V} \text{ is a non-empty, closed subset of } \mathbb{R}^p \,\forall\, P \in \mathcal{F}, \tag{12}$$

$$\hat{V}(P) \times \hat{V}(O) \subseteq \hat{V}(P \cup O) \,\forall\, O, P \in \mathcal{F}(I) \text{ with } O \cap F = \emptyset, \tag{13}$$

$$\inf \sup \hat{V}(\{i\}) > -\infty, \tag{14}$$

$$\text{for any } N \in \mathcal{F}(I), x \in \hat{V}(N) \text{ and } y \in \mathbb{R}^p \text{ with } y \leq x, y \in \hat{V}(N), \tag{15}$$

$$\forall\, N \in \mathcal{F}(I), \hat{V}(N) \setminus \bigcup_{i \in N} \left[\text{int} V(\{i\}) \times \mathbb{R}^{|N|-1}\right] \text{ is non-empty and bounded.} \tag{16}$$

Properties 12, 13 and 15 follow directly by definition of v_i, v_i^M and $\hat{V}$, respectively. Property 14 follows from the existence of an outside option, $V(\{i\}) \geq 0$. This and $v_i(P_i) > 0$ for all set of agents $1 < |P_i| < \bar{n}$ from Assumption 1 also imply property 16. Therefore $\hat{V}$ is a characteristic function in the sense of Kaneko and Wooders (1986).

Denote by $N_{(\theta,\gamma)}\{i \in N : (\theta_i, \gamma_i) = (\theta, \gamma)\}$ a subset of N of positive measure containing all agents of types (θ, γ). Then there exists a partition of N into subsets of equal types, $(N_{(\theta_k,\gamma_k)})_{k=1}^T$. Hence, the game $(I, \hat{V})$ has the *r-property* with respect to $(N_{(\theta_k,\gamma_k)})_{k=1}^T$ in the terminology of Kaneko and Wooders (1986).

The game $(I, \hat{V})$ is *per-capita bounded* with respect to $(N_{(\theta_k,\gamma_k)})_{k=1}^T$ iff there is a positive real number $0 < \delta < 1$ and a $K \in \mathbb{R}$ such that for any $S \in \mathcal{F}(I)$, where payoffs $x \in \hat{V}(S)$ are such that $x_i = x_j$ for all $i, j \in N_k \cap S$ for $k = 1, ..., T$, $(1 + \delta)\frac{\mu(N_k)}{\mu(N)} \geq \frac{|S \cap N_k|}{|S|} \geq (1 - \delta)\frac{\mu(N_k)}{\mu(N)}$ implies $x_i < K$ for all $i \in S$.

Any agent $i \in S$ obtains some payoff $x_i \in \hat{V}(S)$. By construction of $\hat{V}$ and strict non-transferability, $x_i \leq v_i(S_i)$ with $S_i \subset S$ such that $i \in S_i$. By Assumption 1 there exists $\bar{n}(\theta_i, \gamma_i) < \infty$ such that $v_i(\bar{N}) \geq v_i(\bar{N} \cup N')$ where $\bar{N} \subset N_i$ and $N' \subset N_i$. Hence, $v_i(S_i) < \infty$ for all $i \in N$ and S_i. An upper bound K can be obtained by setting $K = \max_{i \in I} \underline{v}_i$. Thus $(I, \hat{V})$ is per-capita bounded.

Now we are in a position to apply Theorem 1 from Kaneko and Wooders (1986) and existence of the f-core of the characteristic function game associated with $\hat{V}$ follows. What remains to be shown is that an allocation in the f-core of $\hat{V}$ is also an equilibrium as defined above. A vector of payoffs in the f-core of $\hat{V}$, $\hat{x} \in \hat{V}(N)$ cannot be improved upon, that is there exists no $S \in \mathcal{F}(I)$ with $x' \in \hat{V}(S)$ such that $x' > x$ for each $i \in S$. By construction of $\hat{V}$ there must exist $x \in V(N)$ with $x \geq \hat{x}$. Hence, neither can x be improved upon. Then the partition (P_N) of N such that $x_i = v_i(P_i)$ with $P_i \in P_N$ and $i \in P_i$ for all $i \in N$ defines the equilibrium coalitions in the sense of our equilibrium definition. ■

References

Becker, G. S.: 1973, 'A Theory of Marriage: Part I'. *Journal of Political Economics* **81**(4), 813–846.

Granovetter, M.: 1973, 'The Strength of Weak Ties'. *American Journal of Sociology* **78**, 1360–1380.

Hanushek, E. A., J. F. Kain, J. M. Markmann, and S. G. Rivkin: 2001, 'Does Peer Ability Affect Student Achievement?'. *NBER Working Paper Series* **8502**.

Hoel, J., J. Parker, and J. Rivenburg: 2004, 'Peer Effects: Do First Year Classmates, Roommates, and Dormmates Affect Students' Academic Sucess'. *mimeo, Reed College.*

Hoxby, C.: 2000, 'The Effects of Class Size on Student Achievement: New Evidence from Population Variation'. *Quarterly Journal of Economics* **115**(4), 1239–1285.

Jackson, M. O.: 2005, 'The Economics of Social Networks'. *mimeo California Institute of Technology.*

Kaneko, M. and M. H. Wooders: 1986, 'The Core of a Game with a Continuum of Players and Finite Coalitions: the Model and Some Results'. *Mathematical Social Sciences* **12**, 105–137.

Kaneko, M. and M. H. Wooders: 1996, 'The Nonemptiness of the f-Core of a Game Without Side Payments'. *International Journal of Game Theory* **25**, 245–258.

Karau, S. F. and K. D. Williams: 1993, 'Social loafing: A Meta-analytic Review and Theoretical Integration'. *Journal of Personality and Social Psychology* **65**, 681–706.

Karau, S. F. and K. D. Williams: 1997, 'The effects of group cohesiveness on social loafing and social compensation'. *Group Dynamics* **1**, 156–168.

Legros, P. and A. F. Newman: 1996, 'Wealth Effects, Distribution, and the Theory of Organization'. *Journal of Economic Theory* **70**, 312–341.

Legros, P. and A. F. Newman: 2004, 'Beauty is a Beast, Frog is a Prince'. *Working Paper.*

Maasoumi, E., D. L. Millimet, and V. Rangaprasad: 2004, 'Class Size and Educational Policy: Who Benefits from Smaller Classes?'. *mimeo, Southern Methodist University.*

McEwan, P.: 2003, 'Peer Effects on Student Achievement: Evidence from Chile'. *Economics of Education Review* **22**, 131–141.

Myers, C. A. and G. P. Shultz: 1951, *The Dynamics of a Labor Market.* Prentice-Hall, New York.

Robertson, D. and J. Symons: 2003, 'Do Peer Groups Matter? Peer Group versus Schooling Effects on Academic Attainment'. *Economica* **70**(277), 31–53.

Sacerdote, B.: 2001, 'Peer Effects with Random Assignment: Results for Dartmouth Roommates'. *Quarterly Journal of Economics* **116**(2), 681–704.

Schneeweis, N. and R. Winter-Ebmer: 2005, 'Peer Effects in Austrian Schools'. *CEPR Discussion Paper Series* **5018**.

Shepperd, J. A.: 1993, 'Productivity Loss in Performance Groups: A Motivation Analysis'. *Pyschological Bulletin* **113**, 67–81.

Shubik, M. and M. H. Wooders: 1983, 'Approximate Cores of Replica Games and Economies. Part II: Setup-Cost and Firm Formation in Coalition Production Economies'. *Mathematical Social Sciences* **6**, 285–306.

Topkis, D. M.: 1998, *Supermodularity and Complementarity.* Princeton University Press, Princeton.

Vigdor, J. and T. J. Nechyba: 2004, 'Peer Effect in North Carolina Public Schools'. *CesIfo Conference Paper.*

Winston, G. C. and D. J. Zimmerman: 2003, 'Peer Effects in Higher Education'. *NBER Working Paper Series* **9501**.

Zimmerman, D. J.: 2003, 'Peer Effects in Academic Outcomes: Evidence from a Natural Experiment'. *Review of Economics and Statistics* **85**(1), 9–23.