Communication Dilemmas in Social Networks : A Game Theoretic Approach

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November 10, 2006

Keywords : social networks ; communication dilemma ; information.

Extended Abstract :

Social Dilemmas are situations in which individual rationality and collective rationality conflict. A *Communication Dilemma*, which is a special type of social dilemma, corresponds to a situation in which it is in the collective interest to share privately held information but in the individual interest to withhold it.

In the present study, we analyse a game-theoretical model of communication dilemma in which players are members of an exogenous communication network and strategically transmit their private information along its links. We wish to characterize the efficiency of different network structures in encouraging communication and link the centrality of players to their strategic behavior. The multi-stage game we consider is based on an experimental analysis of such situations in sociology by Bonacich [1990].

Bonacich's Experiments

Bonacich [1990] presents two experiments where subjects facing a communication dilemma are the members of a fixed communication network. Participants are initially given incomplete information in the form of non-overlapping subsets of letters from a quotation that the network, as a whole, has to identify. The communication network determines which pairs of agents can communicate. There are several communication rounds. A communication round corresponds to an opportunity for network members to communicate. On the one hand, in order to identify the quotation, the network's task is to centralize all the letters at one position, no matter which one. If the network reaches this collective goal, each member receives an identical reward, reduced by a penalty that increases in proportion with the time the network takes to succeed. On the other hand, the first network member who manages to

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identify the quotation receives an additional reward that he keeps for himself. This provides an incentive to hide information.

The game we build only differs from the setting of Bonacich's experiments in two ways. First, in the experiments, agents can guess the quotation even if they do not have all the letters whereas, in our model, the agents' task is to centralize all the pieces of information. Secondly, experimental subjects can give to their direct neighbors as many letters as they want, from 0 to 3, whereas, in our game, players either give or hide all the pieces of information they hold.

Bonacich's experiments support the hypothesis that the outcome of such conflicts of interests is affected by the structure of the communication network that agents are arranged in. More precisely, the author investigates whether occupants of different positions in a communication network and facing a communication dilemma have different incentives to behave cooperatively (by communicating freely) or competitively (by withholding information from other positions in the network).

Bonacich presents the *Centrality of a Position* in the network and *the Degree of Centralization* of the entire network as the key explanatory concepts of differences in behaviors. The fewer communication rounds an agent would need to centralize all the pieces of information, the more central is this agent. To create a measure that increases with position centrality, Bonacich uses the number of positions in the graph minus the number of rounds needed to centralize information at a position. The degree of centralization of a network is the one developed by Freeman [1979]. It corresponds to the sum of the differences in centrality measures between the more central agent and all other positions in the graph divided by the same sum for the most centralized graph (the star).

Bonacich studies four networks (two made up of 7 agents and two of 11) that present only three kinds of positions in term of centrality : central, middle and peripheral. The experimental results show significant differences in communicativeness by position. In the more centralized networks, peripheral and middle positions behave more cooperatively than central positions. In less centralized networks, middle and central positions communicate less than peripheral ones and the least cooperative behavior is found in the middle positions.

Economic Applications

Examples of situations in which network members face a communication dilemma include situations in which firms are arranged in R&D networks. In Goyal and Moraga [2001], horizontally related firms form a network of collaboration where ties are used to share R&D knowledge about a cost-reducing technology. A benefit arises when firms share knowledge but there is an incentive to hoard it as firms are rivals on the market.

Another example is a firm's internal organisation which can be viewed as a communication network whose members are employees [see Bolton and Dewatripont [1994] and Radner [1993]]. In some situations, there can be a collective interest for employees of a firm to communicate with each other. For instance, in Jehiel [1999], the whole organization is in charge of a decision d which may be either 1 or 0 (the status quo). The effect of the status quo is known to every one but each operating unit of the firm holds a partial and crucial information on the effect of $d = 1$. Jehiel characterizes the optimal communication structure for the firm assuming that communication in a group of agents may result in a loss of communicated pieces of information with a probability that solely depends on the group size. In Jehiel [1999], communication is not strategic but in a similar framework, a communication dilemma can emerge in the following sense : On the one hand, the decision has the same impact on each employee as a member of the same firm. On the other hand, some members are in competition with one another to get promoted, what is all the more relevant when considering employees belonging to the same hierarchical level. In this case, even if it is in the whole firm's interest to gather pieces of information individually held, each member of the firm could wish to be the one that manages to centralize the information useful to decision making.

Objective and Related Literature

In situations where members of a communication network face a communication dilemma, incentives to cooperate or to compete depend on the position in the network, as suggested by Bonacich's experiments. We especially aim at identifying, for a given network, which positions are at the origin of a behavior of strategic retention of information and which positions facilitate the gathering of all the dispersed pieces of information. We also examine which network structures are favorable for the whole network to centralize the information as quickly as possible. To address these questions, this paper builds a multi-stage network game in which players strategically choose either to *Hide* or to *Pass On* information they hold to their neighbors in the network.

Galeotti et al. [2006] present a very general framework for games played by members of a fixed network. The authors allow for a general class of payoffs assuming that the payoff of a player depends on his own action as well as on the actions that his direct neighbors take. In the game described in Galeotti et al. [2006], individual incentives depend on neighbor's actions and the nature of the neighborhood effect depends on whether the game exhibits strategic complements or substitutes. The main difference between our paper and Galeotti and al.'s is that our game is dynamic. In our game, payoffs depend on the history of the game, namely on the actions taken by every player at every period of play. Besides, the payoff function has a different form for each position in a network.

Model

The structure of the model is as follows. We consider n players arranged in a fixed and connected network that determines which pairs of agents can communicate with each other. The network is represented as a graph g which is assumed to be undirected so that the communication of information can take place along the links in both ways. The set of agents directly linked to player i in g is called i's neighborhood.

We assume that there are n different pieces of information numbered from 1 to n. Initially, each player is given a unique piece of information, which he is the only player to hold. Player i is given the piece of information numbered i.

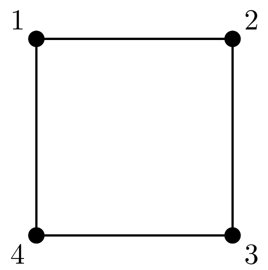
The game is played over discrete time periods denoted t. The length of the game is finite ($t \leq T$). At each date t, each player i chooses an action : either, player i *Passes On* all the pieces of information he holds at time t to every agents in his neighborhood, or player i *Hides* all the pieces of information he holds at time t to every player. Players perfectly observe the actions of every other players at each date and have perfect recall.

The state of players' information at each date t is described by a $n \times n$ matrix of information denoted V^t. The component v_{ij}^t of V^t is equal to 1 if, at date t, player i holds the piece of information numbered j and 0 otherwise. Initially, $V^0 = Id$ since each player i only holds the piece of information numbered i. The state of players' information evolves in time as players *Pass On* or *Hide* their pieces of information. If i belongs to k's neighborhood and player k *Passes On* at date t, player i receives every piece of information held by player k at the beginning of period t. Once i has received j's piece of information, directly from j or indirectly from another player, the jth component of the ith line of the matrix of information becomes equal to 1 until the end of the game. Each final history of the game defines uniquely a sequence of matrices of players' information $(V^0, ..., V^T)$. Payoffs are written as a function of the sequence of information matrices.

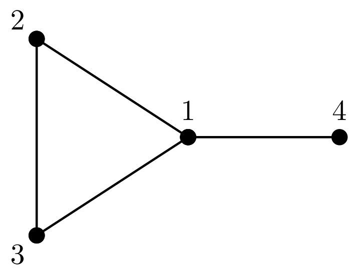
The payoff structure is as follows. If there is no player that has managed to gather the n pieces of information before the end of the game, then players earn nothing. On the contrary, if there is one player or more that have managed to gather the n pieces of information before the end of the game, then players are rewarded. In this case, we denote τ the first period in which the n pieces of information are gathered by a player. The game ends at τ. At this date, a *Collective Reward* is equally shared between all the players and the first player that has gathered the n pieces of information, called the *winner*, receives an *Individual Reward* that he keeps for himself. In the case where there are several winners, the one who is given the *Individual Reward* is randomly selected among them. Payoffs are discounted according to some common discount factor $\delta \in [0, 1]$.

An Example

The dynamic game with simultaneous moves and perfectly observable actions that we describe has a very large set of Nash Equilibria. We restrict our attention to Subgame Perfect Nash Equilibria (SPNE). We denote $\Gamma(n, g, T)$ the game with n players arranged in a network g and lasting T periods. As an example, let's consider the 4 following games : $\Gamma(4, g_1, 2)$, $\Gamma(4, g_2, 2)$, $\Gamma(4, g_1, 3)$ and $\Gamma(4, g_2, 3)$.



g_1 : The Square



g_2 : The Kite

Games $\Gamma(4, g_1, 2)$ and $\Gamma(4, g_2, 2)$ can end without any *winner* as, in both games, there exists a SPNE whose outcome is "every player *Hide* at every date". On the contrary, in games $\Gamma(4, g_1, 3)$ and $\Gamma(4, g_2, 3)$ in which players have one more period of play, in every SPNE, the game ends with a *winner* with a strictly positive probability.

When players are arranged in the Square, no player is linked to every other player directly. Therefore, in g_1, 2 periods at least are necessary to centralize dispersed information at one position. On the contrary, when players are arranged in the Kite, it is possible for player 1 to centralize the four dispersed pieces of information in the first period. "Players 2,3 and 4 *Pass On* in the first period" is the outcome of a SPNE of the game $\Gamma(4, g_2, 2)$. Networks differ in the minimal time needed by their members to centralize the information and this has obviously an impact on the SPNE for which the end of the game is the fastest.

When players have 3 periods of play, there is a SPNE for which players play in mixed strategy in the last period. In $\Gamma(4, g_1, 3)$, 2 players at most use mixed strategy in the last period whereas in $\Gamma(4, g_2, 3)$ they can be 3 which is worse for the probability of getting a *winner*.

Results

We provide a necessary and sufficient condition for the existence of a *winner* with a strictly positive probability in every equilibrium of a game $\Gamma(n, g, T)$, namely $T \geq n - 1$. This condition is independent of the structure of the connected network g : if there is a *winner* with a strictly positive probability in every SPNE of a game $\Gamma(n, g, T)$, the same conclusion applies in every game $\Gamma(n, g', T)$, no matter how players are arranged in g'.

In the large set of SPNE of each game $\Gamma(n, g, T)$, we restrict our attention to two SPNE outcomes : the *best* equilibria corresponding to the one leading to the highest aggregate payoffs and the *worst* equilibria corresponding to the one leading to the lowest aggregate payoffs. First, we give a general characterization of the *best* and the *worst* equilibria for games $\Gamma(n, g, T)$. Then, for a given number of players n and a given deadline T, we arrange networks according to two criterion of network efficiency.

The first efficiency criteria is the following : a network g is said to be more efficient than a network g' if the *best* equilibrium in the set of Equilibria of $\Gamma(n, g, T)$ leads to higher aggregate payoffs than the *best* one in the set of Equilibria of $\Gamma(n, g', T)$. We show that the arrangement of networks under this first efficiency criteria is made with respect to a degree of centralization of the whole graph. The degree of network centralization that we consider differs slightly from the one used in Bonacich [1990].

In a second part, a network g is said to be more efficient than a network g' if the *worst* equilibrium in the set of Equilibria of $\Gamma(n, g, T)$ leads to higher aggregate payoffs than the *worst* one in the set of Equilibria of $\Gamma(n, T, g')$. We show that the arrangement of networks under this second efficiency criteria is made with respect to a very simple graphical measure, likely linked to the degree of network centralization.

Actually, for given n and T, the first efficiency criteria arranges networks with respect to the minimal time needed to have a *winner* and the second efficiency criteria arranges networks with respect to the probability to have a *winner* at the last period of play.

Eventually, we investigate whether some network structures can be the most efficient ones under the two criterion at the same time. We conjecture that star-networks are the best candidates.

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