Job market paper

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Information Trading in Social Networks[2]

ABSTRACT. This paper considers information trading in fixed networks of economic agents who can only observe and trade with other agents with whom they are directly connected. We study the nature of price competition for information in this environment. The linear network, when the agents are located at the integer points of the real line, is a specific example I completely characterize. For the linear network there always exists a stationary equilibrium, where the strategies do not depend on time. I show that there is an equilibrium where any agent has a nonzero probability of staying uninformed forever. Under certain initial conditions this equilibrium is a limit of equilibria of finite-horizon games. The role of a transversality condition is emphasized, namely that the price in the transaction should not exceed the expected utility of all the agents who will get the information due to the transaction. I show that the price offered does not converge to zero with time.

Key Words and Phrases: networks, information trading, information diffusion.
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1. Introduction

In this paper, we consider agents located at points in a given social network, with each player only able to observe and communicate with his direct neighbors. Initially, each agent becomes informed with a probability p, independently of other agents. The informed agents then offer to sell the information to their uninformed neighbors who decide to accept or wait. The uninformed agents who buy the information can in turn sell it to their neighbors, if these neighbors are uninformed. We analyze the equilibria of this game.

"Neighbors" and "networks" need not be interpreted spatially. One can think of firms in similar markets as "neighbors" and the discovery of how to solve the problem of miniaturizing electronics, as in the 1970s, as the "information". Firms in similar industries become aware that their neighbors have solved a problem and might want to buy the solution, which might be of use in a wide variety of other industries. Similarly, prices need not be in terms of money but could be reciprocal exchange. Eric von Hipel [15] discusses a network of steel mini-mills, whose managers exchanged information on how to solve common problems, with the implicit contract being that each member would tell the others of relevant information. Exchange of gossip also falls into the category of such reciprocal exchange.

There have been many recent studies of learning through observing the actions or strategies of one's neighbors. Boyd and Richerson [5] consider this a fundamental means by which behavior spreads and call it cultural evolution. Empirical studies such as those of Banerjee and Munshi [2] show that the structure of the social network is especially important when the markets function imperfectly. These authors consider the effect of the social network on lending. In particular, it was found that migrants prefer to be in places close to their community's lending resources. This serves as evidence that there are benefits to being in proximity to the social network. The authors show that those who migrate to places with no access to the lending network are characterized by higher production ability. The relative independence of these migrants emphasizes the importance of the network for all the others who are less productive and therefore rely more on the lending network's benefits.

Foster and Rosenzweig [12] show that the structure of a social network plays an important role in spreading information about new technologies. They demonstrate that for farmers in India, imperfect knowledge about the management of high-yielding seed varieties is a significant barrier to their adoption, and neighbors' familiarity with these seed varieties significantly increases profitability. This neighbor effect indicates that farmers rely not only on official directions provided by the producers of the seed varieties but also on the experience of the people they know, which serves as a good example of the importance of a social network structure in information diffusion. Conley and Udry [7] also argue that the learning process about new technology in agriculture (they consider pineapple growing in Ghana) is rather social, and depends on one's neighbors' experience. The social nature of adopting new technology is explained by different conditions (soil, temperature, and so on) for different regions. The people paying someone they know to do research on financial markets (which stock to invest in) is one more example of information diffusion in a social environment.

This "network effect" — when people learn from their direct neighbors in accordance with the established connections — is evidence of market failure because the agents do not communicate with the rest of the group, and therefore information diffusion among the population is not socially optimal. This failure can be corrected only through the involvement of the government or related organizations; as Belli [3] observes, the agents themselves can not achieve a good level of communication.

In many of these models, the best interpretation of the network is either as a communication network or as a spatial structure. These models, however, do not explicitly consider the sale of information or exchange with the promise of reciprocity as in the earlier works mentioned.

This work combines three main theoretical strains in the literature: information diffusion, exogenously given network structure, and rational agents who trade the information. Muto [17] discusses the sale of information but does not model a network structure. He addresses the question of diffusion of an information good from a monopolistic owner to a finite number of demanders, and a seller being allowed to charge a price for the information. Although this problem was considered within some community, the structure of the connections was not taken into account. The assumption is that everyone is connected to everyone. Muto stresses the role of information resale, and analyzes the monopolist's and resellers' behaviors. If the resales are prohibited, then the outcome is always Pareto optimal (and therefore the society reaches maximum welfare), but if resales are allowed, then the outcome is not Pareto optimal. The author finds the number of final possessors of the information good.

Irrational agents whose response to the neighbors' actions is predetermined are studied in numerous papers. Chatterjee and Xu [6] consider myopic agents and place them at the integer points of the real line, i.e., everyone has exactly two neighbors. There are two types of technology, R(ed) and B(lue). Technology R is better than B because it provides a higher probability of success. Every period the agents decide on which technology to use. If there was a success in the technology the agent used during the last period, then he continues to use it. If there was a failure, then the agent chooses better technology based on his own and his neighbors' experience during the current period. The important finding of the paper is that sooner or later all the agents will switch to using the best technology.

Bala and Goyal [1] advance beyond the simple networks by taking into account an arbitrary structure, and investigate the process of social learning from neighbors. The agents are rational; however, the information about the right technology is not traded: the results of using different actions immediately become known to the neighbors. There is a finite connected social network of myopic agents who, without knowing actual payoffs, try to figure it out from their own and their neighbors' current and past experience. It was shown that the agents' beliefs converge to some limit with probability one, and this is used to show that at infinity, the utilities of all the agents are the same. As the network is finite, there is a chance that all the agents would not choose the right action (what would not happen in an infinite network).

This paper investigates information trading and information diffusion in the social network. The focus is on how the people trade, the equilibrium strategies and prices, and the final information distribution across the agents.

By "*information*" we mean a good that has the following properties (see Muto [17]):

- It delivers some level of utility to a person who has it (commodity);

- It is possible to duplicate it without any loss in the utility (free replication);
- Once a person knows the information, it is impossible to prohibit him from knowing it (irreversibility); and
- It is impossible to get utility from some part of the information (indivisibility).

For example, some financial information, technology, political news, or even gossip might be considered as the information.

The important property of information is everyone's ability to trade it. It can be paid for by barter or money — we do not distinguish between the two. Again, one may argue that it is difficult to trade gossip for money. It is true, however, by price here we mean obligation to provide another story in return next time — we can hardly imagine a person with whom other people want to share gossip and who never gives anything back. Usually people are eager to hear hot gossip; therefore, we might suggest that this gossip has a higher value, which is expressed in the amount of regular gossip.

"*Social network*" ("*social environment*"), in which the information diffusion is considered, is a set of agents with the following properties:

- Some agents are connected to each other (these agents are called "*neighbors*");
- The agents are able to trade only with their neighbors.

This social network is conveniently represented by a graph, where the agents are located at the nodes, and the connections of the agents are represented by the edges.

Only one sort of information is considered in the model. At the beginning, every agent independently with the same probability learns this information. At each period (time is discrete) the informed agents make offers to their uninformed agents by setting the price in exchange for providing the information. If the buyer accepts the offer, he becomes informed and can resell the information in the following periods of time.

The main assumptions made in the paper are the following:

1. **Everlasting offers.** Once made, the offer stays forever and the seller can not change it later. This assumption is made for the sake of simplicity of proofs to avoid dealing with evolving prices.

2. **Limited observability.** Any agent is always aware if his neighbor has the information or not, and all the offers made to the agent during previous periods of time. The

agents, however, have only general knowledge about the rest of the network and the game — who is whose neighbor, and what are the strategies, probabilities, distributions, and so on. No agent knows who, besides his neighbors, has the information, and the offers made to other agents.

The linear network is considered in the paper. Our main results are the following: it is shown that for any initial parameters there is a stationary equilibrium where the strategies do not depend on time, although the fraction of informed agents increases every period. This equilibrium is possible because the agent's beliefs about the distribution of uninformed agents prior to the next informed agent do not change with time. The price in the stationary equilibria does not converge to zero as it does in the random network.

The research demonstrates that for a small probability of learning the information at the beginning, the sellers' strategy always includes a mass point above the value of the information. The existence of this mass point above the personal valuation of the information leads to the possibility of a "low probability trap", when some agents may never get the information because both their neighbors make high enough offers at the same time, and each of these offers requires reselling in order to get a non-negative payoff. Moreover, the probability for the agent with two uninformed neighbors to stay uninformed forever does not change over time. For some initial parameters, this equilibrium is a limit of equilibria of finite-horizon games.

For the high probability of learning the information at the beginning, the strategy of the stationary equilibrium has continuous distribution below the personal valuation of the information, which means that every agent will get the information.

The rest of the paper is organized as follows: in Section 2 the model is described and the equilibrium concept is defined. The importance of *the transversality condition* in an equilibrium is emphasized: the price the agent pays for the information does not exceed the expected discounted utility of all the agents who will get the information due to the transaction. The linear model where the agents are placed in the integer points of the real line is considered in Section 4. The random matching example is considered in Section 5, where in every period the agents randomly form a new network.

The General Model

This section considers a general symmetric network without cycles. For this symmetric network the game is described and the equilibrium is defined.

The Model

Consider a network of agents without cycles, where every agent has exactly M neighbors. Because of the same number of neighbors for each of the agents, this network has the following symmetry: it looks the same way, whatever agent is located at the center and whatever order we choose for all the neighbors. An example of such a network for $M = 4$ is given in Figure 1.

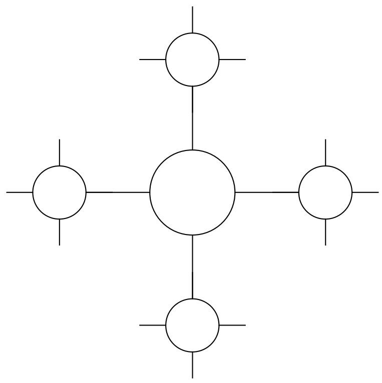

FIGURE 1. Example of a network for $M = 4$.

There is one sort of information (for example, some particular technology) every agent can use to extract a one-time utility u. Time is discrete, $t = 0, 1, \ldots$ The agents initially obtain i.i.d. realizations of a $\{0, 1\}$ random variable. If an agent gets a realization of 1 (this happens with probability p), he is "informed" (has the information), otherwise "uninformed". Once an agent has the information, he remembers it forever. Every agent always knows who of his neighbors is informed; however, no one knows anything about his neighbors' neighbors.

At the beginning of each period, the informed agents (sellers) decide on making offers to their uninformed neighbors (buyers). The offers are made separately to each of the neighbors. A seller can make an offer immediately, or wait till the next period. The seller offers the buyer the information in exchange for some price (the seller sets the price). Once made, the offer stays forever and can not be changed.

Then, the buyers make the decision to accept one of the offers, or to wait. At the end of the period the buyers with accepted offers become informed and can make offers during the next period.

The discount factor is equal to $\delta \in (0, 1)$. All the agents are risk neutral. The agent's utility is equal to

$$U = \begin{cases} 0, & \text{the agent is never informed;} \\ \delta^t(u - v) + W, & \text{the agent gets the information at period } t, \end{cases}$$

where v is the price the agent pays for the information, and W is the total discounted revenue from selling the information to the neighbors. The agents maximize their expected utility.

The Strategies and the Equilibrium Concept

At every period t agent α has a personal history

$$h_t^\alpha = (\{s_{tn}^\alpha\}_{n=1}^M, \{(s_{tn}^{\alpha B}, v_{tn}^{\alpha B})\}_{n=1}^M, \{(s_{tn}^{\alpha S}, v_{tn}^{\alpha S}, \tilde{s}_{tn}^{\alpha S})\}_{n=1}^M, (s_t^\alpha, m_t^\alpha)),$$

where

s_{tn}^α — the time when neighbor n got informed;

$s_{tn}^{\alpha B}, v_{tn}^{\alpha B}$ — the time when neighbor n made an offer, and the price offered;

$s_{tn}^{\alpha S}, v_{tn}^{\alpha S}, \tilde{s}_{tn}^{\alpha S}$ — the time of the offer to neighbor n, the price and the time of acceptance, if any;

s_t^α, m_t^α — the time when the agent got the information, and the neighbor from whom he got it.

All the histories are consistent across the agents and across time. Denote H_t — the set of all possible histories at time t. The state of the world at time t is the set of all histories for all agents $\{h_t^\alpha\}_\alpha$. The buyer is an agent for whom $s_t^\alpha = \emptyset$ (he has not got the information yet), and the seller is an agent for whom $s_t^\alpha \in \mathbb{N}$.

The buyers' pure strategy is the decision to buy the information from one of the neighbors, or to wait (0 corresponds to waiting):

$$R_t^{\alpha B} : H_t \to \{0, 1, \ldots, M\}.$$

The sellers' pure strategy for each of the uninformed neighbors is the decision to wait (represented by $\emptyset$, which is also played for the informed neighbors) or a price:

$$R_t^{\alpha S} : H_t \to \{\mathbb{R}_+, \emptyset\}^M.$$

Denote the sets of pure strategies by $\mathbf{R}_t^B$ and $\mathbf{R}_t^S$ correspondingly. We allow mixed strategies, i.e. some measures $\mu_{Bt}(\cdot)$ and $\mu_{St}(\cdot)$ over $\mathbf{R}_t^B$ and $\mathbf{R}_t^S$.

We consider a symmetric case, i.e. all the agents use the same strategies. The network does not contain cycles; therefore, we can assume that all the agents act independently.

Beliefs

The following parameters and functions constitute the agents' beliefs:

1. Probability p_t that an uninformed neighbor of an uninformed agent will get the information during the current period t, conditionally that this neighbor is uninformed at the beginning of the period;

2. Probability $p_t(l)$ that an uninformed neighbor of an uninformed agent has exactly l informed neighbors at the beginning of period t;

3. Distribution $G_{tl}(v)$ of the lowest offer at the beginning of period t, conditional on the fact that this agent has exactly l informed neighbors and is still uninformed;

4. Expected payoff $\pi_{St}(v)$ at time t from selling the information to the specific uninformed neighbor at all the consequent periods by offering price v;

5. Expected payoff $\pi_{Bt}(l, v)$ of the buyer with the lowest price v and with l informed neighbors at time t;

6. Expected payoff π_{St} at time t from selling the information to the specific uninformed neighbor at all the consequent periods;

7. Probability $g_{tk}(v)$ that the offer v made at time t to an uninformed neighbor will be accepted at time $t + k$.

As in Fudenberg, Levine, and Maskin [13], all the beliefs are calculated based on the strategies played.

Equilibrium Definition and Its Characterization

An analogue of the Perfect Bayesian Equilibrium is used in the model. In particular, this means that any agent plays the best response to any history in accordance with his beliefs, even if this history is not achievable under the given equilibrium strategies. This equilibrium concept shares with the Perfect Bayesian Equilibrium the idea that every agent maximizes the expected utility in every state, given the system of beliefs consistent with all the other players' strategies. Since there are infinitely many agents in the game, we can not directly apply the PBE concept, but need to generalize it in order to apply it to our context. This is similar in spirit to the local perfect equilibrium in Fudenberg, Levine, and Maskin [13].

Definition. An equilibrium of the game is a sequence of strategies $\{\mu_{Bt}(\cdot), \mu_{St}(\cdot)\}_{t\geq 0}$ and a sequence of beliefs

$$\{p_t, p_t(l), G_{tl}(x)\pi_{St}(v), \pi_{Bt}(l,v), \pi_{St}, g_{tk}(v)\}_{t\geq 0},$$

such that the agents' beliefs are consistent with the strategies and no agent with any history (on or off the equilibrium path) can get extra payoff from deviating given that all the other agents play equilibrium strategies.

Proposition 1. For any symmetric equilibrium there is an equivalent one (in terms of buyers' strategy, beliefs and expected payoffs), in which all the informed agents make their offers immediately.

From the equilibrium definition, the buyers' strategy is a function

$$K_t : \{1, 2, \ldots, M\} \to \mathbb{R}_+,$$

which determines the threshold for the best price offered (reservation price), given the number of informed neighbors. The sellers' strategy is a distribution function $F_t(\cdot)$ of the offers made to the neighbors who are still uninformed.

If all of the buyer's neighbors are informed, then his strategy is simple:

Proposition 2.

$$K_t(M) = u \quad \forall t \in \mathbb{N}_+.$$

We deal with an infinite-horizon game with infinitely many agents, where equilibria may have the property of the Ponzi game, and the prices are not supported by the utility the agents get from the information. Indeed, from any equilibrium one might get a new one by increasing $F_t(\cdot)$ and $K_t(l)$. To avoid this kind of situation, we will use a modified *transversality condition*. We will require that

$$K_t(l) \leq u * \mathbf{E}\, A_{tl} \qquad \forall l, t, \tag{1}$$

where A_{tl} stands for the random variable representing the discounted number of uninformed agents who will get the information due to the transaction between the seller and his neighbor, if the neighbor himself has exactly l informed neighbors at period t. This condition means that the price can not exceed the expected discounted utility of all the agents who will get the information due to the transaction.

An example of an equilibrium when the transversality condition fails and the prices grow to infinity is given on page 17.

Infinite Linear Network

This section considers the special case of an infinite symmetric network without cycles — the infinite linear network (see figure 2), where every agent is connected to exactly two other agents.

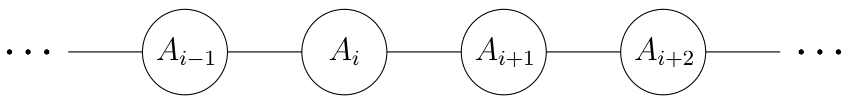

Figure 2. Infinite linear network.

General Results for the Linear Network

As in the general case, the sellers' strategy is the distribution function of offers $F_t(\cdot)$, and the buyers' strategy is the threshold $K_t(l)$ for the prices when exactly l neighbors are informed.

Denote event "agent i is informed at time t" as $\overline{A_i^t}$, and event "agent i is uninformed at time t" as A_i^t.

The next proposition describes the agent's beliefs about the distance till the first informed agent. Although the fraction of informed agents increases over time (the agents do not forget the information, and some of them buy it), this belief does not change over time if the agent himself and his neighbor are uninformed.

Proposition 3. The number of uninformed agents preceding the next informed one, conditional on the fact that the original agent himself and his neighbor are uninformed, has geometric distribution with parameter p:

$$\mathbf{P}(A_{i-1}^t A_i^t \dots A_{i+k}^t \overline{A_{i+k+1}^t} | A_{i-1}^t A_i^t) = p(1-p)^k.$$

The proposition claims that from the point of view of any uninformed agent with an uninformed neighbor, the distribution of the number of uninformed agents preceding the first informed one is geometric and does not depend on time. If the agent knows that his right neighbor is uninformed, then he has a subjective probability $(1-p)^k$ that the next k agents to the right are uninformed.

This counterintuitive result holds for the following reasons. First, we calculate this belief conditionally on the fact that the agent and his neighbor are uninformed. Second, the initial distribution of the number of uninformed agents preceding the first informed one is geometric because at the beginning everyone independently learns the information. And third, geometric distribution has the property that is a discrete analogue of constant hazard rate property of exponential distribution.

Denote $V_t = \sup(\operatorname{supp} F_t(v))$ — the maximal price that can be offered at period t. The transversality condition limits this value. As the distribution of the number of uninformed agents is known, the following proposition finds this bound.

Proposition 4. In any equilibrium satisfying the transversality condition, the price at any period of time does not exceed

$$V_t \leq u + u \sum_{k=0}^{\infty} \delta^k k (1-p)^k p = u \left(1 + \frac{1}{p}\right).$$

This proposition bounds the price by the expected utility of all the agents who are located from one side of the informed agent. To go further, the bound of the price at every period can be found more precisely.

Proposition 5. For any period of time t value $V_t \leq \frac{u}{1-\delta(1-p^2)}$ for any equilibrium which satisfies the transversality condition.

The upper bound $\frac{u}{1-\delta(1-p^2)}$ we found is achievable for some values p, and this is exactly the price for the pure strategy equilibrium.

Proposition 6. For any value $p \in \left(0, \frac{2\delta+1-\sqrt{4\delta+1}}{2\delta}\right]$ there is the unique pure strategy equilibrium which satisfies the transversality condition, and

$$K_t(1) = \frac{u}{1-\delta(1-p)^2}.$$

There is no pure strategy equilibrium for $p > \frac{2\delta+1-\sqrt{4\delta+1}}{2\delta}$.

As the buyers use the same strategy every period of time in this pure strategy equilibrium, the information always diffuses from an informed agent to his uninformed neighbor if this neighbor has only one offer.

The equilibria when an uninformed agent with one offer always buys the information has the property that is related to the distribution of uninformed agents. Namely, if the probability that the uninformed neighbor's neighbor has the information is equal to p (from proposition 3), then the probability that during the next period the uninformed neighbor of an uninformed agent will acquire the information is also equal to p (the product of p and the probability that the information will be transferred, which is equal to one). This is an informal proof for the following proposition.

Proposition 7. If in equilibrium an agent with one uninformed neighbor always buys the information, then the probability that an agent with an uninformed neighbor will become informed is equal to p and does not depend on time.

Stationary Equilibria

Proposition 7 showed that if an agent with just one informed neighbor always buys the information, then the probability of an uninformed agent's uninformed neighbor getting the information equals p and does not change over time. This allows us to assume that there might be stationary equilibria, where the strategies $F_t(\cdot)$ and $K_t(1)$ do not change over time: $F_t(\cdot) = F(\cdot)$, $K_t(1) = K$. In this subsection all the stationary equilibria will be found and characterized.

The property "if the information always diffuses from an informed agent to his uninformed neighbor with just one offer, then there might be stationary equilibrium" can be reversed.

Proposition 8. In any stationary equilibrium the following equality holds:

$$K(1) = V \equiv \sup(\operatorname{supp} F(v)),$$

i.e., an agent with only one offer will accept this offer immediately.

All the strategies possible in a stationary equilibrium can be characterized using the following proposition.

Proposition 9. $F(v)$ can have a continuous part

$$F(v) = \frac{1}{p} - \frac{C}{v}$$

for $v \in [pC, \min(V, u)]$, where the constant $C > 0$ is such that $pC < u$ and $F(pC) = 0$. If $K(1) > u$, then the strategy $F(v)$ has a mass point at $V = K(1)$. If $F(\cdot)$ contains both a continuous part and a mass point at $V > u$, then

$$pC = (1 - p)V.$$

Based on proposition 9, there are three possibilities for $F(v)$:

1. $F(u) \in (1, \infty)$. The distribution function of the offers is

$$F_1^m(v) = \begin{cases} 0, & v < pC; \\ \frac{1}{p} - \frac{C}{v}, & pc \le v \le \frac{pC}{1-p}; \\ 1, & v > \frac{pC}{1-p}. \end{cases}$$

2. $F(u) \in (0, 1]$. The distribution function of the offers is

$$F_2^m(v) = \begin{cases} 0, & v < pC; \\ \frac{1}{p} - \frac{C}{v}, & pc \le v \le u; \\ \frac{1}{p} - \frac{C}{u}, & u < v < V; \\ 1, & v \ge V. \end{cases}$$

3. $F(u) \in (-\infty, 0]$. The distribution function of the offers is

$$F^p(v) = \begin{cases} 0, & v < V; \\ 1, & v \ge V. \end{cases}$$

All three possibilities for $F(v)$ are depicted in Figure 3.

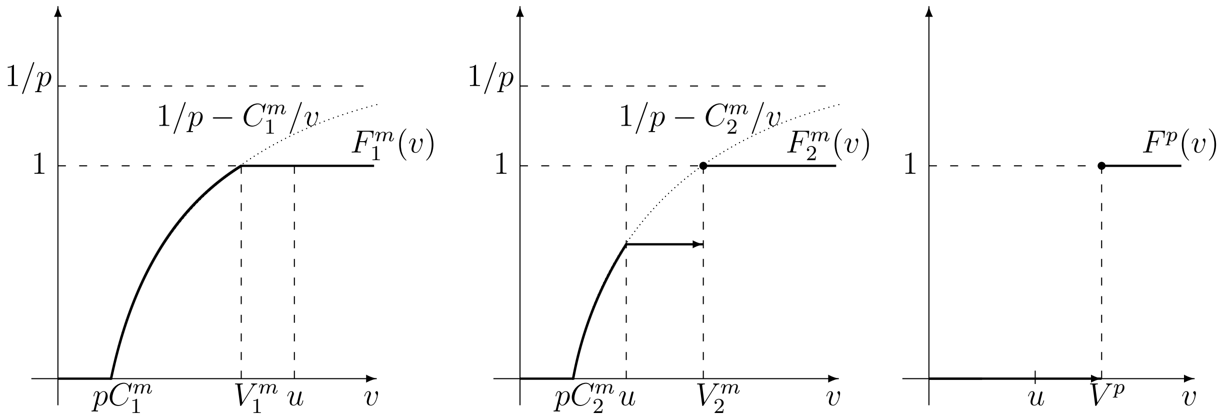

FIGURE 3. Stationary equilibrium strategies in the infinite linear network. Left graph: strategy $F_1^m(v)$; Center graph: strategy $F_2^m(v)$; Right graph: pure strategy $F^p(v)$.

Now our goal is to find out under which values of the initial parameter p each of equilibria $F_1^m(v)$, $F_2^m(v)$, and $F^p(v)$ exists.

All these strategies will exist for different values of p. To define the interval for each strategy, we need the following technical lemma.

Lemma 1. For any $\delta \in (0,1)$ equation

$$p - (1-\delta(1-p))(1-p)^2 + (1-p)\ln(1-p) = 0 \tag{2}$$

has unique solutions p^* which belongs to the interval $(0,1)$. Also,

$$p^* > \frac{2\delta+1-\sqrt{4\delta+1}}{2\delta}.$$

For any δ two values p^* and $\frac{2\delta+1-\sqrt{4\delta+1}}{2\delta}$ from lemma 1 divide the interval $(0,1)$ into three parts.

Proposition 10. The strategy $F_2^m(\cdot)$ exists if and only if

$$p \in \left(\frac{2\delta+1-\sqrt{4\delta+1}}{2\delta}, p^*\right).$$

The constant C is uniquely determined by the equation

$$\frac{u}{pC} + \delta(1-p) - \frac{1}{1-p} = \frac{\delta}{1-\delta(1-p)}\left(\frac{u}{pC} - 1 - \ln\left(\frac{u}{pC}\right)\right). \tag{3}$$

Equation 3 is homogeneous in $\frac{u}{C}$, which corresponds to the intuition that for the given probability p ratio

$$\frac{u}{C} \equiv \frac{p}{1-p}\frac{u}{V}$$

is fixed.

The pure strategy $F^p(v)$ is completely described by the constant $V > u$. In accordance with proposition 6, there is the unique pure strategy equilibrium for any $p \in \left(0, \frac{2\delta+1-\sqrt{4\delta+1}}{2\delta}\right]$, and the constant V is equal to

$$V = \frac{u}{1-\delta(1-p)^2}.$$

Proposition 11. The strategy $F_1^m(\cdot)$ exists if and only if $p \geq p^*$. The constant C is equal to $C = \frac{V(1-p)}{p}$, where V is given by

$$V = \frac{u(1-\delta)}{(1-\delta(1-p))(1-\delta(1-p)^2) + \delta(1-p)\ln(1-p)}, \tag{4}$$

and $V(p)$ is a decreasing function of p for $p \geq p^*$.

From formula 4, the value V converges to $u(1-\delta)$ as $p \to 1$. Also, C converges to 0 and therefore $F(v) = \frac{1}{p} - \frac{C}{v}$ weakly converges to zero, which means that the price converges to zero with time.

For any p there is the unique stationary equilibrium. For small p, this is a pure strategy equilibrium, for medium p this is an equilibrium with both a continuous part and a mass point in $F(v)$, and for high p this is a completely mixed equilibrium.

A strategy $F^p(\cdot)$ exists for $p \leq \frac{2\delta+1-\sqrt{4\delta+1}}{2\delta}$, and a strategy $F_1^m(\cdot)$ exists for $p \in \left(\frac{2\delta+1-\sqrt{4\delta+1}}{2\delta}, p^*\right)$. Both of these strategies have a mass point $V > u$. Because this mass point is above the agent's personal valuation of the information, there is a non-zero probability that an agent will get two offers V at the same time, and therefore will stay uninformed forever.

Proposition 12. In a stationary equilibrium with $g \equiv 1 - F(u) > 0$ every agent has probability $\frac{pg^2}{2-p}$ to stay uninformed forever.

For any $p < p^*$ there is a non-zero probability that an agent will stay uninformed forever. For $p \geq p^*$ every agent will get the information. This threshold p^* divides interval (0,1) into two areas of efficient and non-efficient equilibria. The analysis shows, that the the answer to the question if everyone will get the information, depends not on the value of the information, but on how well known this information is at the beginning.

Although those equilibria with a mass point above u might seem unusual, they are the limit of finite-horizon games for small p. Namely, for any T consider a game which stops at time T, i.e., $t = 0, 1, \ldots, T$.

Proposition 13. For small enough p the equilibria for the finite-horizon games converge to the pure strategy equilibrium for the infinite-horizon game.

Therefore, the equilibria, when not all the agents get the information, are not only impossible, but naturally appear as the limit of the finite horizon games equilibria.

Equilibria with Unbounded Price

The transversality condition restricts the values for the price (proposition 4). The question is if an equilibrium with an unbounded price does exist. An example of the family of such equilibria is given in this subsection.

For simplicity, we restrict our attention to the equilibria with pure strategies only. Consequently, the following equation should hold for any t:

$$u - V_t + \delta(1-p)^2 V_{t+1} = 0.$$

After rearranging the terms, one can get

$$V_{t+1} - \frac{u}{1-\delta(1-p)^2} = \frac{V_t - \frac{u}{1-\delta(1-p)^2}}{\delta(1-p)^2}.$$

Taking into account that in the stationary pure strategy equilibrium maximal offer $\bar{V} = \frac{u}{1-\delta(1-p)^2}$, we get

$$V_{t+1} - \bar{V} = \frac{V_t - \bar{V}}{\delta(1-p)^2}. \tag{5}$$

As $\delta(1-p)^2 < 1$, the difference $V - \bar{V}$ grows exponentially if the initial V_0 is greater than $\bar{V}$:

$$V_t = \bar{V} + (V_0 - \bar{V}) \left(\frac{1}{\delta(1-p)^2} \right)^t.$$

The only additional requirement for V_0 is that the seller does not deviate to offering u at $t = 0$, i.e. $V_0(1-p) > u$.

Random Networks

This section deals with the random network model, when a new network is formed every period of time. For this model, I find the equilibrium which satisfies the transversality condition.

Every period of time the agents are randomly matched with M other agents, and the network formed does not contain cycles. It means that at every period there is a new M-network or several of them, and no past history influences an agent's current or future decisions. (The probability of being matched with the same partner twice is assumed to be equal to zero.) Therefore, the agents' actions are independent across time and across neighbors, within the network(s).

As before, all the informed agents simultaneously make their offers and the uninformed agents choose the best offer they have (if any). As we deal with a random network, the informed agents make offers to uninformed ones every period of time, and the offers made expire at the end of each period with the abortion of the connections. The agents use the same strategies, i.e., the equilibria are symmetric.

A seller's strategy is a distribution function of offers $F_t(v)$. A buyer's strategy is K_t — the maximal price at which he is ready to buy the information. As a new network is randomly formed each period of time, K_t does not depend on the number of informed neighbors. As a part of equilibrium, there is a parameter p_t — a probability that a randomly chosen agent has the information.

Proposition 14. The sellers' strategy is the distribution function

$$F_t(v) = \frac{1}{p_t} - C_t v^{-\frac{1}{M-1}}. \tag{6}$$

with the support $[\tilde{V}_t, K_t]$, where the the constant $C_t > 0$,

$$V_t = \left(\frac{p_t C_t}{1-p_t}\right)^{M-1}, \tag{7}$$

$$\tilde{V}_t = V_t(1-p_t)^{M-1}.$$

As K_t coincides with V_t, later in this section V_t will represent both constants.

Notice that an agent becomes informed once he has at least one informed neighbor. As the probability of being informed is equal to p_t, then

$$p_{t+1} = p_t + (1-p_t)(1-(1-p_t)^M) = 1-(1-p_t)^{M+1};$$
$$1-p_{t+1} = (1-p_t)^{M+1};$$
$$p_t = 1-(1-p_1)^{(M+1)^{t-1}}, \tag{8}$$

and p_t approaches 1 exponentially.

If the equilibrium satisfies the transversality condition, then

$$\sup_t V_t \leq \sup_t \left(u + \frac{u(1-p_{t+1})}{p_{t+1}}\right) < \sup_t \left(u + \frac{u}{p_t}\right) = u + \frac{u}{p_1} < \infty, \tag{9}$$

i.e., the supports of the distributions are bounded by some constant. For all the equilibria in the rest of the section it will be assumed that the transversality condition holds.

Proposition 15. In an equilibrium, the following law of motion for V_t holds:

$$\frac{1}{\delta}(V_{t-1} - u) = V_t(1-p_t)^{M-1}(M + (1-p_t)) - u. \tag{10}$$

The proposition 15 helps to restore the whole equilibrium, using only one value V_1. For any given p_1 formula 8 gives all values $\{p_t\}_{t=1}^{\infty}$. Equation 10 defines the sequence $\{V_t\}_{t=1}^{\infty}$. Based on it, values $\{C_t\}_{t=1}^{\infty}$ (equation 7) can be calculated. Therefore, any equilibrium is determined by one value V_1.

At the same time, there is no freedom in choosing value V_1. In particular, the following proposition states how u and $\lim_{t\to\infty} V_t$ are related.

Proposition 16. In an equilibrium the following equations hold:

$$\lim_{t\to\infty} V_t = u(1-\delta);$$
$$\lim_{t\to\infty} \tilde{V}_t = 0;$$
$$\lim_{t\to\infty} \mathbf{E}_{F_t} v = 0.$$

Also, proposition 16 says that the average price the agents charge in a period t converges to zero. Given that the price is non-negative and the support for this price is limited by $u(1+1/p_t)$ (formula 9), distributions $F_t(v)$ weakly converge to the degenerated distribution with all the mass concentrated at zero.

Denote

$$g_t = (1 - p_t)^{M-1}(M + (1 - p_t)).$$

Value V_1 not only is determined by u, but it also has a simple representation. Moreover, it is unique, which means that there is only one equilibrium that satisfies the transversality condition. The buyer's thresholds V_t converge to $u(1-\delta)$ monotonically.

Proposition 17. There is only one equilibrium that satisfies the transversality condition. For this equilibrium

$$V_1 = u(1-\delta)\left(1 + \sum_{i=3}^{\infty}\prod_{j=2}^{i-1}\delta g_i\right);$$

$$V_t = V_1 \prod_{i=2}^{t}\frac{1}{\delta g_i} - u(1-\delta)\sum_{i=2}^{t}\prod_{j=i}^{t}\frac{1}{\delta g_j}, \tag{11}$$

and V_t is a decreasing sequence.

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Appendix

Lemma 2. The differential equation

$$af'(x)x = 1 - bf(x)$$

for $b \neq 0$, $a \neq 0$ has solution

$$f(x) = \frac{1}{b} - Cx^{-b/a}. \tag{12}$$

Proof of lemma 2.

The solution is verified by substituting formula (12) for $f(x)$ into the original equation and the fact that the first-order differential equation has only one undetermined constant.

□

Proof of proposition 1. Consider one informed agent A and his uninformed neighbor B. By waiting agent A can observe only the fact that B got the information from his another neighbor (what makes impossible selling information to B). At the same time, agent A makes an offer which maximizes his expected payoff. Therefore, by waiting agent A looses expected revenue, and he is not worse off by making his offer as soon as he gets the information.

There are 2 cases:

Case 1. There is a chance that agent B will accept the offer before the time when agent A would normally make the offer. Then because of discounting agent A is strictly better off by making an offer immediately, and therefore this is not an equilibrium to delay with the offer.

Case 2. Agent B does not accept the offer earlier. Then by making the offer earlier agent A does not change anything and we have new equivalent equilibrium.

□

Proof of proposition 2.

The buyer with all informed neighbors will buy the information for any price not exceeding u. At the same time, buying the information for price above u will result in negative payoff, therefore $K_t(M) = u$.

□ **Proof of proposition 3.**

The idea of the proof is the following. Fix one agent and consider the part of the network from one side of this agent, as if another half of the network does not exist, although it is assumed that the agents use the same strategies as with the whole network. We will define random variables ξ_t which will represent the distance from the agent till the first informed agent at time t. Since the question of interest is how the information reaches the agent, we do not need to take into account all the informed agents in the network, but rather the one closest to the original agent. We will define random variable η_t, which will represent the waiting time of the agent before transferring the information to the next agent, for the agent who got the information at time t. These variables capture the fact that the seller might charge different prices and, based on these prices, and the buyer waits before accepting the offer. Note that the distribution of η_t may depend on time.

Then I will show that if ξ_0 has a geometric distribution, then random variables ξ_t will also have the same distribution, conditional on $\xi_t \geq 0$.

Consider two sequences of random variables $\xi_t \in \mathbb{N}_+$ and $\eta_t \in \mathbb{N}_+$, $t \in \mathbb{N}$. Assume that the following properties hold:

1. $\mathbf{P}\{\xi = k\} = p(1-p)^k, \qquad k = 0, 1, 2, \ldots;$
2. $\xi_t - \xi_{t+1} \in \{0, 1\}$;
3. η_t are independent of each other and ξ_0;
4. Denote $t_0 = 0$, $t_1 = \eta_0, \ldots$, $t_{i+1} = t_i + \eta_i$. Then for any $i \geq 0$

$$\xi_{t_i} = \xi_{t_i+1} = \ldots = \xi_{t_{i+1}-1} = \xi_{t_{i+1}} + 1.$$

Variables t_i have the following meaning. Those are the times when the information is transferred to the next agent. Correspondingly, η_{t_i} is the delay in transferring to the next person.

The proposition will follow if the following result will be proven for every $t \geq 0$ and $k \geq 0$:

$$\mathbf{P}\{\xi_t = k | \xi_t \geq 0\} = p(1-p)^k,$$

which holds for $t = 0$ because of the initial conditions.

Define $\tilde{\eta}_t$ — the period which the agent waits before transferring the information at time t,

$$\xi_t = \xi_{t+1} = \ldots = \xi_{t+\eta_t-1} = \xi_{t+\eta_t} + 1.$$

Obviously, $\tilde{\eta}_0 = \eta_0$, and

$$\tilde{\eta}_t = \begin{cases} \tilde{\eta}_{t-1} - 1 & , \quad \tilde{\eta}_{t-1} > 1; \\ \eta_t & , \quad \tilde{\eta}_{t-1} = 1. \end{cases}$$

By initial independence random variables $\xi_0, \tilde{\eta}_0$ and $\{\eta_t\}_{t>0}$ are independent.

The proposition will be proven by induction. The base of the induction holds because $\mathbf{P}\{\xi_0 = k | \xi_t \geq 0\} = p(1-p)^k$. The step of the induction will be proven into tow steps: in the first step we will show that if ξ_t has specified geometric distribution and $\xi_t, \tilde{\eta}_t$ are independent, then ξ_{t+1} has the same geometric distribution, conditional on $\xi_{t+1} \geq 0$. In the second step we will show that if $\xi_t, \tilde{\eta}_t$ and $\{\eta_{t'}\}_{t'>t}$ are independent, then $\xi_{t+1}, \tilde{\eta}_{t+1}$ and $\{\eta_{t'}\}_{t'>t+1}$ are also independent, conditional on $\xi_{t+1} \geq 0$.

Step 1. Suppose that ξ_t has geometric distribution and is independent of $\tilde{\eta}_t$. We want to show that ξ_{t+1} has the same distribution, conditional on $\xi_{t+1} \geq 0$.

Denote $\bar{\eta} = \begin{cases} 0 & , \quad \tilde{\eta}_t > 1; \\ 1 & , \quad \tilde{\eta}_t = 1. \end{cases}$

Then $\xi_{t+1} = \xi_t - \bar{\eta}$ and

$$\begin{aligned}
\mathbf{P}(\xi_{t+1} = k | \xi_{t+1} \geq 0) &= \sum_{l \in \{0,1\}} \mathbf{P}(\xi_{t+1} = k, \bar{\eta} = l | \xi_{t+1} \geq 0) = \sum_{l \in \{0,1\}} \frac{\mathbf{P}(\xi_t = k + l, \bar{\eta} = l)}{\mathbf{P}(\xi_{t+1} \geq 0)} \\
&= \frac{\sum\limits_{l \in \{0,1\}} \mathbf{P}(\xi_t = k + l)\, \mathbf{P}(\bar{\eta} = l)}{\sum\limits_{l \in \{0,1\}} \mathbf{P}(\xi_t \geq l, \bar{\eta} = l)} = \frac{\sum\limits_{l \in \{0,1\}} \mathbf{P}(\xi_t = k + l)\, \mathbf{P}(\bar{\eta} = l)}{\sum\limits_{l \in \{0,1\}} \mathbf{P}(\xi_t \geq l)\, \mathbf{P}(\bar{\eta} = l)} \\
&= \frac{\sum\limits_{l \in \{0,1\}} p(1-p)^{k+l}\, \mathbf{P}(\bar{\eta} = l)}{\sum\limits_{l \in \{0,1\}} (1-p)^l\, \mathbf{P}(\bar{\eta} = l)} = p(1-p)^k,
\end{aligned}$$

and the last formula means that random variable ξ_{t+1} (defined on $\xi_{t+1} \geq 0$) has the same geometric distribution as random variable ξ_t.

Step 2. In this step we want to show that if $\xi_t, \tilde{\eta}_t$ and $\{\eta_{t'}\}_{t'>t}$ are independent, then ξ_{t+1}, $\tilde{\eta}_{t+1}$ and $\{\eta_{t'}\}_{t'>t+1}$ are also independent conditional on $\xi_{t+1} \geq 0$.

To prove this we have to show that ξ_{t+1} and $\tilde{\eta}_{t+1}$ are independent conditional on $\xi_{t+1} \geq 0$, or $\sigma(\xi_{t+1} | \xi_{t+1} \geq 0) \perp \sigma(\tilde{\eta}_{t+1} | \xi_{t+1} \geq 0)$. By definition, $\{\eta_t'\}_{t'>t+1}$ are independent of $\tilde{\eta}_{t=1}$ and ξ_{t+1} because $\tilde{\eta}_{t+1}$ and ξ_{t+1} are some functions of ξ_0 and $\{\eta_{t'}\}_{t' \leq t+1}$.

Note that $\xi_{t+1} = \xi_t - \bar{\eta}$, and

$$\begin{aligned}
\mathbf{P}(\tilde{\eta}_{t+1} = k, \xi_{t+1} = s) &= \mathbf{P}(\tilde{\eta}_{t+1} = k, \xi_{t+1} = s, \bar{\eta} = 1) + \mathbf{P}(\tilde{\eta}_{t+1} = k, \xi_{t+1} = s, \bar{\eta} = 0) \\
&= \mathbf{P}(\eta_{t+1} = k, \xi_t = s + 1, \tilde{\eta}_t = 1) + \mathbf{P}(\tilde{\eta}_t = k + 1, \xi_t = s) \\
&= \mathbf{P}(\eta_{t+1} = k, \tilde{\eta}_t = 1)\, \mathbf{P}(\xi_t = s + 1) + \mathbf{P}(\tilde{\eta}_t = k + 1)\, \mathbf{P}(\xi_t = s) \\
&= \mathbf{P}(\eta_{t+1} = k, \tilde{\eta}_t = 1) p(1-p)^{s+1} + \mathbf{P}(\tilde{\eta}_t = k + 1) p(1-p)^s.
\end{aligned}$$

Therefore for $s \geq 0$

$$\begin{aligned}
\mathbf{P}(\tilde{\eta}_{t+1} = k, \xi_{t+1} = s | \xi_{t+1} \geq 0) &= \frac{\mathbf{P}(\tilde{\eta}_{t+1} = k, \xi_{t+1} = s)}{\mathbf{P}(\xi_{t+1} \geq 0)} \\
&= \frac{\mathbf{P}(\eta_{t+1} = k, \tilde{\eta}_t = 1) p(1-p)^{s+1} + \mathbf{P}(\tilde{\eta}_t = k + 1) p(1-p)^s}{\mathbf{P}(\xi_{t+1} \geq 0)}.
\end{aligned}$$

Also,

$$
\begin{aligned}
\mathbf{P}(\tilde{\eta}_{t+1} = k|\xi_{t+1} \geq 0) &= \frac{\mathbf{P}(\tilde{\eta}_{t+1} = k, \xi_{t+1} \geq 0)}{\mathbf{P}(\xi_{t+1} \geq 0)} = \frac{\sum\limits_{l \geq 0} \mathbf{P}(\tilde{\eta}_{t+1} = k, \xi_{t+1} = l)}{\mathbf{P}(\xi_{t+1} \geq 0)} \\
&= \frac{\sum\limits_{l \geq 0} \mathbf{P}(\eta_{t+1} = k)\, \mathbf{P}(\tilde{\eta}_t = 1) p(1-p)^{l+1} + \mathbf{P}(\tilde{\eta}_t = k+1) p(1-p)^l}{\mathbf{P}(\xi_{t+1} \geq 0)} \\
&= \mathbf{P}(\tilde{\eta}_{t+1} = k, \xi_{t+1} = s|\xi_{t+1} \geq 0) \frac{\sum\limits_{l \geq 0} (1-p)^l}{(1-p)^s} \\
&= \mathbf{P}(\tilde{\eta}_{t+1} = k, \xi_{t+1} = s|\xi_{t+1} \geq 0) \frac{1}{p(1-p)^s} \\
&= \mathbf{P}(\tilde{\eta}_{t+1} = k, \xi_{t+1} = s|\xi_{t+1} \geq 0) \frac{1}{\mathbf{P}(\xi_{t+1} = s|\xi_{t+1} \geq 0)}.
\end{aligned}
$$

From the last equation,

$$\mathbf{P}(\tilde{\eta}_{t+1} = k|\xi_{t+1} \geq 0)\, \mathbf{P}(\xi_{t+1} = s|\xi_{t+1} \geq 0) = \mathbf{P}(\tilde{\eta}_{t+1} = k, \xi_{t+1} = s|\xi_{t+1} \geq 0),$$

which says that random variables $\tilde{\eta}_{t+1}$ and ξ_{t+1} are independent conditional on event $\xi_{t+1} \geq 0$.

Now all the ξ_t conditional on $\xi_t \geq 0$ have the same geometric distribution because of the induction:

$$\mathbf{P}\{\xi_t = k|\xi_t \geq 0\} = p(1-p)^k, \quad k = 0, 1, \ldots.$$

□

Proof of proposition 4.

The proposition follows from the requirement by the transversality condition that the price does not exceed the expected utility of all the agents who will get the information due to the transaction.

□

Proof of proposition 5.

From proposition 2 $K_t(2) = u$. Therefore, we have to prove that

$$K_t(1) \leq \frac{u}{1 - \delta(1-p)^2}$$

for any t because $\sup V_t \leq \sup(\max(K_t(1), K_t(2)))$ as any price offered should have some chance to be accepted.

Suppose that the proposition does not hold, i.e. that that

$$(1 - \delta(1-p)^2) \sup_t K_t(1) > u. \tag{13}$$

Choose τ such that $K_\tau(1) > \sup_t K_t(1) - \epsilon$ (we can do this for any arbitrary small ϵ). Because the buyer should be indifferent between buying (in the case he decides to resell it next period) and waiting, the following inequality should hold:

$$-K_\tau(1) + u + \delta(1-p)^2 V^1_{\tau+1} \geq 0,$$

where $V^1_{\tau+1}$ — first of the mass points in the distribution of offers above u (and the probability that this offer is accepted is not greater than $(1-p)^2$). Therefore we have:

$$\begin{aligned} V^1_{\tau+1} &\geq \frac{K_\tau(1) - u}{\delta(1-p)^2} > \frac{\sup_t K_t(1) - \epsilon - u}{\delta(1-p)^2} \\ &= \frac{\sup_t K_t(1) - \sup_t K_t(1)(1-\delta(1-p)^2)}{\delta(1-p)^2} \\ &\quad + \frac{\sup_t K_t(1)(1-\delta(1-p)^2) - \epsilon - u}{\delta(1-p)^2} \\ &= \sup_t K_t(1) + \frac{\sup_t K_t(1)(1-\delta(1-p)^2) - \epsilon - u}{\delta(1-p)^2}. \end{aligned}$$

Because of inequality (13) we can always choose ϵ so small enough that

$$\sup_t K_t(1)(1-\delta(1-p)^2) - \epsilon - u > 0,$$

and therefore

$$V^1_{\tau+1} > \sup_t K_t(1),$$

which, along with $K_t(1) \leq \frac{u}{1-\delta(1-p)^2}$, says that this value $V^1_{\tau+1}$ can never be accepted. Therefore no agent will ever offer price $V^1_{\tau+1}$. Contradiction.

□

Proof of proposition 6.

Taking into account proposition 5, it is enough to show that $K_t(1) \geq \frac{u}{1-\delta(1-p)^2}$.

Assume opposite, i.e. $\inf_t K_t(1) < \frac{u}{1-\delta(1-p)^2}$. Choose such minimal τ that $K_\tau(1) < \inf_t K_t(1) + \epsilon$ for some arbitrary small ϵ. Because we chose minimal τ we have $V^i_t \geq K_\tau(1)$ for any i and $t \leq \tau$.

Because we have pure strategy equilibrium, no offer below u is made. Therefore, the buyer's expected utility should always be zero (if he waits, he will not get offer below u, but his opportunity to resell will expire), and this is a sufficient and necessary condition. So, we should have

$$-K_\tau(1) + u + \delta(1-p)^2 V^1_{\tau+1} \geq 0.$$

However

$$\begin{aligned} -K_\tau(1) + u + \delta(1-p)^2 V^1_{\tau+1} &\geq -\inf_t K_t(1) - \epsilon + u + \delta(1-p)^2 \inf_t K_t(1) \\ &= -\epsilon + \left(u - (1-\delta(1-p)^2)\inf_t K_t(1)\right), \end{aligned}$$

and the last expression is greater then zero for small ϵ because $\inf_t K_t(1) < \frac{u}{1-\delta(1-p)^2}$. Therefore this is not an equilibrium because the value $K_\tau(1)$ can be increased and the offer above $K_\tau(1)$ will still be accepted.

The sellers will dot deviate to u for $p \in (0,1)$ if and only if

$$\frac{u}{1-\delta(1-p)^2}(1-p) \geq u;$$

$$p \leq \delta(1-p)^2;$$

$$p \in \left(0, \frac{2\delta + 1 - \sqrt{4\delta+1}}{2\delta}\right].$$

□

Proof of proposition 7. This proposition follows from proposition 3. Using the same notation,

$$\mathbf{P}\{A^t_{i-1}, A^t_i, \overline{A^{t+1}_i} | A^t_{i-1}, A^t_i\} = \mathbf{P}\{A^t_{i-1}, A^t_i, \overline{A^t_{i+1}} | A^t_{i-1}, A^t_i\} = p.$$

□

Proof of proposition 8.

As $K(2) = u$, then for offer $V > u$ to have a chance to be accepted, the proposition should hold.

For $V \leq u$, if $K(1) > V$ then $\pi(K(1)) > \pi(V)$ as V is the maximal possible offer and the probability of accepting $K(1)$ and V by the buyer is the same, which contradicts the equilibrium.

The case when $V \leq u$ and $K(1) < V$ is the only situation left to consider. The main idea here is the following. If $K(1) < V$, then in equilibrium an uninformed agent with one high offer will wait for the offer from another neighbor. This possible waiting for the second offer will make the beliefs about the number of uninformed agents till the next informed one change over time, i.e. the equilibrium can not be stationary.

The buyer will wait if he gets offer $v \in (K(1), V] \subset (0, u]$, which happens with probability $(1 - F(K(1))) \equiv 1 - h > 0$. For $v = K(1)$ the buyer should be indifferent between waiting and accepting the offer. Suppose that agent A_0 has an offer $\tilde{v} \in (K(1), V]$ at time t (the time when the agent got the offer is not important).

Denote event "the agent A_i is uninformed at time t" as A_{it}, and event "there are i uninformed agents behind agent A_1 and the price offered to the last uninformed agent is not higher than $K(1)$" as S_{it}. Then event S_{it} implies that agents $A_1, A_2, \dots, A_{i+1}$ are uninformed, agent A_{i+2} is informed, and during all the transactions when the information was transferred to A_{i+2} from A_{i+t+2} the price did not exceed $K(1)$. Therefore,

$$\mathbf{P}\{S_{it}|A_1\} = (1-p)^{i+t}ph^t.$$

The probability that agent A_0 will get offer from A_1 is not greater than

$$g_t = \sum_{i\geq 0}(1-h)^i \, \mathbf{P}\{S_t(i)|A_1\} \leq \sum_{i\geq 0}(1-p)^{i+t}ph^t \leq h^t$$

and converges to zero as $t \to \infty$. The expected utility of an agent with one offer $\tilde{v}$, who follows the equilibrium strategy and waits for another offer, is

$$\pi_{Bt}(\tilde{v}) \leq g_t u,$$

and converges to zero as $t \to \infty$. Hence, an agent with one offer $\tilde{v} > K(1)$ will be better off by accepting this offer at time t such that $u - \tilde{v} > g_t u$. Therefore, in stationary equilibrium $K(1) = V$, which concludes the proof.

□

Proof of proposition 9.

From proposition 3, for $K(1) > u$ the distribution function $F(v)$ is continuous for $v \leq u$ and has no measure above u except for $K(2)$, where a mass point can be located because $K(1) = u$ from proposition 2.

A seller's expected payoff from setting the price to $v \in [0, \min(u, K(1))]$ is equal to

$$\pi(v) = (1-p)v + pv(1 - F(v)) \tag{14}$$

because with probability the neighbor has only one offer and therefore accepts it unconditionally, and with probability p the neighbor gets another offer distributed in accordance with $F(\cdot)$, which is accepted if it is better than v.

Differentiating equation 14 with respect to v gives us

$$(1-p) + p(1 - F(v)) - pvf(v) = 0;$$

$$1 - pF(v) - pvf(v) = 0.$$

Applying lemma 2, $F(v) = \frac{1}{p} - \frac{C}{v}$. Constant C is greater than zero because $F(v)$ is a non-decreasing function. Equation $F(pC) = 0$ follows from the expression for $F(v)$.

$F(v)$ has a mass point at $K(1)$ if $K(1) > u$ because otherwise an agent offering the highest possible price u will be better off by increasing it to $K(1)$.

Finally, $pC = (1-p)K(1)$ because offer $v = K(1) > 0$ is accepted with probability $(1-p)$ (if another neighbor is uninformed), and offer pC is always accepted (it is the minimal offer possible), therefore the equation holds because all the strategies in the support should deliver the same expected payoff.

□

Proof of lemma 1.

Note that $\frac{2\delta+1-\sqrt{4\delta+1}}{2\delta}$ is the only solution of equation

$$p - \delta(1-p)^2 = 0.$$

Rewrite equation 2:

$$p - \delta(1-p)^2 = (1+\delta p)(1-p)^2 + (1-p)\ln(1-p). \tag{15}$$

The left-hand side of equation 15 is continuous and changes from $-\delta$ to 1 as p increases from 0 to 1. The right-hand side is continuous and changes from 1 to 0. Therefore, there is at least one solution of equation 2.

Right-hand side of equation 15 is always positive for $p \in (0,1)$. The left-hand side of equation 15 is positive if and only if $p > \frac{2\delta+1-\sqrt{4\delta+1}}{2\delta}$. Therefore, $p^* > \frac{2\delta+1-\sqrt{4\delta+1}}{2\delta}$ for any solution of equation 2.

To prove uniqueness, divide left-hand side of equation 2 by $1-p$ and take derivative:

$$\begin{aligned}
&\left(\frac{p}{1-p} - (1-\delta(1-p))(1-p) + \ln(1-p)\right)' = \\
&\quad = \frac{1}{1-p} + \frac{p}{(1-p)^2} - \delta(1-p) + (1-\delta(1-p)) - \frac{1}{1-p} \\
&\quad = \frac{p}{(1-p)^2} - 2\delta(1-p) + 1 = \frac{p - 2\delta(1-p)^3 + (1-p)^2}{(1-p)^2} \\
&\quad = \frac{p - \delta(1-p)^3 + (1-p)^2 - \delta(1-p)^3}{(1-p)^2} \\
&\quad > \frac{p - \delta(1-p)^2 + (1-p)^2 - (1-p)^2}{(1-p)^2} > 0
\end{aligned}$$

as $p - \delta(1-p)^2 > 0$ because $p > \frac{2\delta+1-\sqrt{4\delta+1}}{2\delta}$.

As the derivative increases, there exists at most one solution.

□

Proof of proposition 10.

In order to find constant C we have to consider the maximal offer V. The agent with offer V and one uninformed neighbor should be indifferent between accepting the offer and waiting for another one. The expected payoff from buying the information immediately is equal to

$$u + \delta(1-p)pC - V.$$

If the agent waits for another offer, then he will get the information if and only if his other neighbor will offer this information for a price less then u. Then given that another price payoff is $u - v'$, expected payoff from waiting is

$$\begin{aligned}
\mathbf{E}\left(\sum_{t\geq 1}\delta^t p(1-p)^{t-1}(u-v')I_{\{v'\leq u\}}\right) &= \frac{p\delta}{1-\delta(1-p)}\left(F(u)u - \int\limits_{pC}^{u} v\,d\,F(v)\right) \\
&= \frac{p\delta}{1-\delta(1-p)}\left(F(u)u - \int\limits_{pC}^{u} v\frac{C}{v^2}d\,v\right) \\
&= \frac{p\delta}{1-\delta(1-p)}\left(F(u)u - C\ln\left(\frac{u}{pC}\right)\right).
\end{aligned}$$

Equating expected payoff of the agent from buying the information now and waiting till the offer from another neighbor, one may get

$$\begin{aligned}
u + \delta(1-p)pC - V &= \frac{p\delta}{1-\delta(1-p)}\left(F(u)u - C\ln\left(\frac{u}{pC}\right)\right); \\
\frac{u}{pC} + \delta(1-p) - \frac{1}{1-p} &= \frac{\delta}{1-\delta(1-p)}\left(\frac{u}{pC} - 1 - \ln\left(\frac{u}{pC}\right)\right).
\end{aligned}$$

This strategy is possible if and only if

$$F(u) > 0 \Leftrightarrow \frac{u}{pC} > 1.$$

Rewriting equation 3for $\frac{u}{pC} = 1$, we have

$$\begin{aligned}
1 + \delta(1-p) - \frac{1}{1-p} &= \frac{\delta}{1-\delta(1-p)}\left(1 - 1 - \ln 1\right); \\
1 + \delta(1-p) - \frac{1}{1-p} &= 0.
\end{aligned}$$

The last equation has two roots $p_{1,2} = \frac{2\delta+1\pm\sqrt{4\delta+1}}{2\delta}$, of which only one $p_1 = \frac{2\delta+1-\sqrt{4\delta+1}}{2\delta}$ belongs to the interval $[0,1]$.

Now I will show that for $p > p_1$ there exists unique C satisfying $F(u) > 0$ and equation 3, and there is no such C for $p < p_1$. Rewrite equation 3 in the following way:

$$\frac{u}{pC} - \frac{\delta}{1-\delta(1-p)}\left(\frac{u}{pC} - 1 - \ln\left(\frac{u}{pC}\right)\right) = \frac{1}{1-p} - \delta(1-p).$$

Replace $\frac{u}{pC}$ by some variable x:

$$x - \frac{\delta}{1-\delta(1-p)}(x - 1 - \ln x) = \frac{1}{1-p} - \delta(1-p). \tag{16}$$

Note that

$$x = \frac{u}{pC} = \frac{u}{(1-p)V} < \frac{u}{(1-p)u} = \frac{1}{1-p}.$$

The right-hand side of equation 16 is an increasing function of p:

$$\frac{1}{1-p} - \delta(1-p) = (1-\delta) + p(1+\delta) + p^2 + p^3 + \ldots.$$

Denote the left-hand side of equation 16 by $h(x, \delta, p)$. Derivative

$$\begin{aligned} \frac{\partial h(x,\delta,p)}{\partial x} &= 1 - \frac{\delta}{1-\delta(1-p)}(1 - 1/x) \\ &> 1 - \frac{\delta}{1-\delta(1-p)}(1-(1-p)) \\ &> \frac{1-\delta}{1-\delta(1-p)} > 0. \end{aligned} \tag{17}$$

Therefore, $h(x, \delta, p)$ increases with x for the allowable range of values. Also, $h(1, \delta, p) = h(1, \delta, p_1) \equiv x$.

$$h(1,\delta,p) = h(1,\delta,p_1) = \frac{1}{1-p_1} - \delta(1-p_1).$$

If $p > p_1$, then $\frac{1}{1-p} - \delta(1-p) > \frac{1}{1-p_1} - \delta(1-p_1)$, and for $h(x, \delta, p)$ to be equal to $\frac{1}{1-p} - \delta(1-p)$ variable x should be greater than one.

If $p < p_1$, then $\frac{1}{1-p} - \delta(1-p) < \frac{1}{1-p_1} - \delta(1-p_1)$, and for $h(x, \delta, p)$ to be equal to $\frac{1}{1-p} - \delta(1-p)$ variable x should be less than one. This completes the proof.

Strict monotonicity of $h(x, \delta, p)$ in x gives uniqueness of constant C. Also, we have that or pC decreases when p increases.

□

Proof of proposition 11.

Note that from $F_2^m(V) = 1$ follows $V = \frac{pC}{1-p}$.

As in proposition 10, the buyer is indifferent between buying at the maximal price and waiting for the best offer:

$$\begin{aligned}
u+\delta(1-p)pC-V &= \sum_{t\geq 1}\delta^t p(1-p)^{t-1}\int\limits_{pC}^{\frac{pC}{1-p}}(u-v)\,dF_2^m(v)\\
&= \frac{p\delta}{1-\delta(1-p)}\int\limits_{pC}^{\frac{pC}{1-p}}(u-v)\frac{C}{v^2}\,dv\\
&= \frac{p\delta C}{1-\delta(1-p)}\left(\left(-\frac{u}{\frac{pC}{1-p}}+\frac{u}{pC}\right)-\left(\ln\frac{pC}{1-p}-\ln pC\right)\right)\\
&= \frac{p\delta u}{1-\delta(1-p)}+\frac{p\delta C\ln(1-p)}{1-\delta(1-p)};
\end{aligned}$$

$$u\frac{1-\delta}{1-\delta(1-p)}=V\left(1-\delta(1-p)^2+\frac{\delta(1-p)\ln(1-p)}{1-\delta(1-p)}\right);$$

$$V(p)=\frac{u(1-\delta)}{(1-\delta(1-p))(1-\delta(1-p)^2)+\delta(1-p)\ln(1-p)}.$$

Derivative of the denominator is equal to

$$\left(\frac{u(1-\delta)}{V}\right)'=\delta(1-\delta(1-p)^2)+2\delta(1-p)(1-\delta(1-p))-\delta\ln(1-p)-\delta$$

All the terms in the right-hand side except for δ are strictly increasing, which makes the derivative to increase. Therefore, function $V(p)$ either always decreases or first increases and then decreases.

Also,

$$V(0)=\frac{u(1-\delta)}{(1-\delta)(1-\delta)}=\frac{u(1-\delta)}{(1-\delta)}>u;$$
$$V(1-0)=u(1-\delta)<u$$

because $\lim_{x\to 0+}x\ln x=0$.

As $V(p)$ either always decreases or first increases and then decreases, there exists unique $p^*(\delta)$ such that $V(p^*(\delta))=u$, $V(p)<u$ for $p>p^*(\delta)$, and $V(p)>u$ for $p<p^*(\delta)$.

□

Proof of proposition 12.

The agent will get two offers simultaneously only if the informed agents on the opposite sides are located on the same distance. Therefore, the probability of staying uninformed forever is equal to

$$\sum_{t\geq 0}((1-p)^t p)^2 g^2 = p^2 g^2 \frac{1}{1-(1-p)^2} = \frac{pg^2}{2-p}.$$

□

Proof of proposition 13.

Denote all the strategies in the game with horizon T by upper-index T.

The finite horizon equilibria we will be looking for will have the same property as the infinite horizon stationary equilibria: the agent with one informed neighbor only is always buying the information. Therefore, at the last period of time the sellers' strategy will be

$$F_T^T(v) = \frac{1}{p} - \frac{C}{v},$$

where constant C is such that $F_t^T(u) = 1$, i.e.

$$F_t^T(v) = \frac{1}{p}\left(1 - \frac{u(1-p)}{v}\right).$$

The expected payoff of the agent with one offer at the last period is

$$\begin{aligned}\pi &= \int_{pC}^{u}(u-v)\,dF_T^T(v) = u - \int_{(1-p)u}^{u} v\frac{u(1-p)}{pv^2}\,dv = u - \int_{(1-p)u}^{u}\frac{u(1-p)}{pv}\,dv \\ &= u - \frac{u(1-p)}{p}\ln\left(\frac{1}{1-p}\right) < u - \frac{u(1-p)}{p}p = pu\end{aligned}$$

for small enough p because

$$\begin{aligned}\ln\frac{1}{1-p} &= \ln(1+p+p^2+\ldots) > \ln(1+p+p^2) \\ &= (p+p^2) - \frac{1}{2}(p+p^2)^2 + \frac{1}{3}(p+p^2)^3 - \ldots > p.\end{aligned}$$

Distribution functions $F_t^T(v)$ with mass points at V_t^T, $t = 0, 1, \ldots, T-1$, where V_t are found from formula

$$\delta^{T-t}(1-p)^{T-t-1}p\pi = -V_t^T + u + \delta(1-p)^2 V_{t+1}^T \tag{18}$$

because the left-hand side represents the buyers' expected payoff from waiting, and the right-hand side represents the expected payoff from buying the information for price V_t^T. The equation for V_{T-1}^T

$$\begin{aligned}\delta\pi &= -V_{T-1}^T + u + \delta(1-p)(1-p)u; \\ V_{T-1}^T &= -\delta\pi + u + \delta(1-p)(1-p)u; \\ V_{T-1}^T &> u(1+\delta(1-p)^2) - pu.\end{aligned}$$

By induction, all V_t^T are greater than $u(1+\delta(1-p)^2) - pu$ because from formula 18

$$\begin{aligned} V_t^T &= -\delta^{T-t}(1-p)^{T-t-1}p\pi + u + \delta(1-p)^2 V_{t+1}^T \\ &> -pu + u + \delta(1-p)^2(u(1+\delta(1-p)^2) - pu) \\ &= -pu + u(1+\delta(1-p)^2) + u\delta(1-p)^2(\delta(1-p)^2 - p) \\ &> -pu + u(1+\delta(1-p)^2). \end{aligned}$$

No seller will deviate from V_t^T because expected payoff from V_t^T is greater than expected payoff from u:

$$(1-p)(u(1+\delta(1-p)^2) - pu) > u.$$

All V_t^T do not exceed pure strategy equilibrium in infinite horizon game

$$\bar{V} = \frac{u}{1-\delta(1-p)^2}$$

because $V_{T-1}^T < \bar{V}$ and therefore by induction

$$\begin{aligned} V_t^T &= -\delta^{T-t}(1-p)^{T-t-1}p\pi + u + \delta(1-p)^2 V_{t+1}^T \\ &< u + \delta(1-p)^2 \frac{u}{1-\delta(1-p)^2} = \frac{u}{1-\delta(1-p)^2}. \end{aligned} \tag{19}$$

V_t^T as an increasing function of V_{t+1}^T and decreasing function of $\delta^{T-t}(1-p)^{T-t-1}p\pi$. As $\delta^{T-t}(1-p)^{T-t-1}p\pi$ decreases when t decreases and $V_{T-2}^T > V_{T-1}^T$, then V_t^T is a decreasing sequence, therefore no buyer will deviate from his strategy.

Finally, V_t^T converges as T increases to infinity because V_t^T is limited, decreasing and $V_{t+1}^{T+1} = V_t^T$. It converges to $\bar{V}$ because for any t

$$\begin{aligned} \bar{V} - V_t^T &= \delta(1-p)^2(\bar{V} - V_{t+1}^T) + \delta^{T-t}(1-p)^{T-t-1}p\pi \\ &\leq \delta^s(\bar{V} - V_{t+s}^T) + \delta^{T-s}\pi\sum_{i=0}^{s-t}\delta^i \to 0 \end{aligned}$$

if $T \to \infty$ and $s = T/2$.

□

Proof of proposition 14.

No offer above V_t will be accepted, therefore $F_t(V_t + 0) = 1$. The distribution function $F_t(\cdot)$ does not have mass points because otherwise the seller would prefer to decrease his offer by some small ϵ.

The set $\text{supp}(F_t(\cdot))$ is closed because of the continuity of the expected payoff function for $v < V_t$ and on the left from V_t. The set is connected because by increasing the offer in a gap a seller would increase his expected payoff as the acceptance probability of the offer stays the same. Because of the same argument, $V_t \in \text{supp}(F_t(\cdot))$, which, along with $F_t(V_t+0) = 1$, gives us

$$\sup(\text{supp}\, F_t(\cdot)) = V_t.$$

A seller's expected payoff from one uninformed neighbor is equal to

$$\begin{aligned}
\pi(v) &= v\,\mathbf{P}\{v \leq \text{other offers }\} \\
&= v \prod_{i=1}^{M-1} (\mathbf{P}\{v \leq \text{offer from neighbor } i, \text{ if any}\} + \\
&\qquad \mathbf{P}\{\text{there is no offer from neighbor } i\}) \\
&= v \prod_{i=1}^{M-1} (\mathbf{P}\{v \leq \text{offer } i | \text{there is an offer}\}\, \mathbf{P}\{\text{there is an offer}\} + (1 - p_t)) \\
&= v \prod_{i=1}^{M-1} ((1 - F_t(v))p_t + (1 - p_t)) \\
&= (1 - p_t F_t(v))^{M-1}\, v.
\end{aligned}$$

Solving maximization problem

$$\pi(v) \to \max_v$$

we have:

$$(1 - p_t F_t(v))^{M-1} - p_t f_t(v) v (1 - p_t F_t(v))^{M-2}(M-1) = 0;$$

$$p_t f_t(v) v (M-1) = 1 - p_t F_t(v).$$

Using Lemma 2, the solution is

$$F_t(v) = \frac{1}{p_t} - C_t v^{-\frac{1}{M-1}}.$$

Therefore, the distribution $F_t(v)$ has support $[\tilde{V}_t, V_t]$, where

$$\tilde{V}_t = (p_t C_t)^{M-1},$$

$$V_t = \left(\frac{p_t C_t}{1 - p_t}\right)^{M-1},$$

$$\tilde{V}_t = V_t (1 - p_t)^{M-1}.$$

□

Proof of proposition 15.

Let U_t^i be the expected payoff of an informed agent at the beginning of time period t, and let U_t^u be the expected payoff an uninformed agent at the beginning of time period t. Then

$$U_t^i = M(1-p_t)\tilde{V}_t + \delta U_{t+1}^i = \sum_{i=t}^{\infty} \delta^{i-t} M(1-p_i)\tilde{V}_i; \tag{20}$$

$$U_t^u = (u - \mathbf{E}\, v_t + \delta U_{t+1}^i)(1-(1-p_t)^M) + \delta U_{t+1}^u (1-p_t)^M, \tag{21}$$

where $\mathbf{E}\, v_t$ stands for the expected price an agent pays for acquiring the information at period t (conditional on the fact that there is at least one offer).

For the equilibria satisfying the transversality condition the informed agent's expected payoff is limited because

$$\begin{aligned} U_t^i &= \sum_{i=t}^{\infty} \delta^{i-t} M(1-p_i)\tilde{V}_i \leq \\ & \sum_{i=t}^{\infty} \delta^{i-t} M(1-p_i) V_i < \infty. \end{aligned}$$

In an equilibrium the buyer with the highest possible offer V_t is indifferent between accepting the offer and waiting, therefore

$$u - V_t + \delta U_{t+1}^i = \delta U_{t+1}^u. \tag{22}$$

We see that $U_t^u < U_t^i$, therefore the uninformed agent's expected payoff is also bounded. Using formulas 22, 20 and 21, one can get

$$\begin{aligned} \frac{1}{\delta}(V_{t-1} - u) &= U_t^i - U_t^u \\ &= M(1-p_t)\tilde{V}_t + \delta U_{t+1}^i \\ &\quad -(u - \mathbf{E}\, v_t + \delta U_{t+1}^i)(1-(1-p_t)^M) \\ &\quad -\delta U_{t+1}^u (1-p_t)^M \\ &= (V_t - u)(1-p_t)^M + M(1-p_t)^M V_t \\ &\quad +(\mathbf{E}\, v_t - u)(1-(1-p_t)^M) \\ &= (M+1)V_t(1-p_t)^M + \mathbf{E}\, v_t(1-(1-p_t)^M) - u. \end{aligned}$$

The distribution function of the minimal price from l independent offers $v, v_2, \ldots, v_l$ is equal to

$$\begin{aligned} F_l(v) &\equiv \mathbf{P}(\min(v_1, \ldots, v_l) \le v) \\ &= 1 - (1 - \mathbf{P}(v_1 \le v))^l \\ &= 1 - \left(C_t v^{-\frac{1}{M-1}} - \left(\frac{1}{p_t} - 1 \right) \right)^l . \end{aligned}$$

The average minimalprice with l independent offers is equal to

$$\begin{aligned} \mathbf{E}\, v_{F_l} &\equiv \int_{\tilde{V}_t}^{V_t} v \, dF_l(v) \\ &= \int_{\tilde{V}_t}^{V_t} vd \left(1 - \left(C_t v^{-\frac{1}{M-1}} - \left(\frac{1}{p_t} - 1 \right) \right)^l \right) \\ &= \int_{\tilde{V}_t}^{V_t} \frac{C_t l}{M-1} v^{-\frac{1}{M-1}} \left(C_t v^{-\frac{1}{M-1}} - \left(\frac{1}{p_t} - 1 \right) \right)^{l-1} dv. \end{aligned}$$

Note that

$$\begin{aligned} \sum_{l=1}^{M} l g^{l-1} C_M^l p_t^l (1-p_t)^{M-l} &= \sum_{l=1}^{M} l g^{l-1} \frac{M!}{l!(M-l)!} p_t^l (1-p_t)^{M-l} \\ &= \sum_{l=1}^{M} M p_t g^{l-1} C_{M-1}^{l-1} p_t^{l-1} (1-p_t)^{M-l} \\ &= M p_t \sum_{l=0}^{M-1} C_{M-1}^l g^l p_t^l (1-p_t)^{(M-1)-l} \\ &= M p_t \left(g p_t + (1-p_t) \right)^{M-1} . \end{aligned}$$

Taking into account that the probability of exactly l informed neighbors is equal to $C_M^l p_t^l (1-p_t)^{M-l}$, we have

$$
\begin{aligned}
\mathbf{E}\, v_t(1-(1-p_t)^M) &= \sum_{l=1}^{M} \mathbf{E}\, v_{F_l} C_M^l p_t^l (1-p_t)^{M-l} \\
&= \sum_{l=1}^{M} C_M^l p_t^l (1-p_t)^{M-l} \\
&\qquad \int\limits_{\tilde{V}_t}^{V_t} \frac{C_t l}{M-1} v^{-\frac{1}{M-1}} \left(C_t v^{-\frac{1}{M-1}} - \left(\frac{1}{p_t} - 1 \right) \right)^{l-1} dv \\
&= \int\limits_{\tilde{V}_t}^{V_t} \frac{C_t}{M-1} v^{-\frac{1}{M-1}} \\
&\qquad \sum_{l=1}^{M} l C_M^l p_t^l (1-p_t)^{M-l} \left(C_t v^{-\frac{1}{M-1}} - \left(\frac{1}{p_t} - 1 \right) \right)^{l-1} dv \\
&= \int\limits_{\tilde{V}_t}^{V_t} \frac{C_t}{M-1} v^{-\frac{1}{M-1}} M p_t \\
&\qquad \left(p_t \left(C_t v^{-\frac{1}{M-1}} - \left(\frac{1}{p_t} - 1 \right) \right) + (1-p_t) \right)^{M-1} dv \\
&= \int\limits_{\tilde{V}_t}^{V_t} \frac{C_t}{M-1} v^{-\frac{1}{M-1}} M p_t \left(p_t C_t v^{-\frac{1}{M-1}} \right)^{M-1} dv \\
&= \int\limits_{\tilde{V}_t}^{V_t} \frac{M (p_t C_t)^M}{M-1} v^{-\frac{1}{M-1}-1} dv = - M (p_t C_t)^M v^{-\frac{1}{M-1}} \Big|_{\tilde{V}_t}^{V_t} \\
&= M (p_t C_t)^M \left(\left((p_t C_t)^{M-1} \right)^{-\frac{1}{M-1}} - \left(\left(\frac{p_t C_t}{1-p_t} \right)^{M-1} \right)^{-\frac{1}{M-1}} \right) \\
&= M p_t^M C_t^{M-1} = V_t M p_t (1-p_t)^{M-1}. \qquad (23)
\end{aligned}
$$

Substituting $\mathbf{E}\, v_t(1-(1-p_t)^M)$ (formula 23) into formula 10 we have

$$
\frac{1}{\delta}(V_{t-1} - u) = V_t (1-p_t)^{M-1} (M + (1-p_t)) - u,
$$

w.r.t.p.

□

Proof of proposition 16.

By inequality 9, values V_t are limited and as $\lim_{t\to\infty} p_t = 1$ (formula 8) we have

$$\lim_{t\to\infty} V_t(1-p_t)^{M-1}(M+(1-p_t)) = 0.$$

Then, using formula 23, one can get

$$\lim_{t\to\infty} \frac{1}{\delta}(V_{t-1}-u) = \lim_{t\to\infty} \big(V_t(1-p_t)^{M-1}(M+(1-p_t)) - u\big) = -u;$$

$$\lim_{t\to\infty} V_t = u(1-\delta).$$

The lower bound of the support of $F_t(v)$ converges to zero because

$$\lim_{t\to\infty} \tilde{V}_t = \lim_{t\to\infty} V_t(1-p_t)^{M-1} \le \sup_t V_t \cdot \lim_{t\to\infty}(1-p_t)^{M-1} = 0.$$

The average price at period t for $M > 2$

$$\begin{aligned}
\mathbf{E}_{F_t} v &= \int\limits_{\tilde{V}_t}^{V_t} v\, d\left(\frac{1}{p_t} - C_t v^{-\frac{1}{M-1}}\right) = \int\limits_{\tilde{V}_t}^{V_t} \frac{C_t}{M-1} v^{-\frac{1}{M-1}}\, dv = \frac{C_t}{M-2} v^{\frac{M-2}{M-1}}\Bigg|_{\tilde{V}_t}^{V_t} \\
&= \frac{V_t^{\frac{1}{M-1}}}{M-2}\frac{1-p_t}{p_t}\left(1-(1-p_t)^{M-2}\right) V_t^{\frac{M-2}{M-1}} \overset{t\to\infty}{\longrightarrow} 0;
\end{aligned}$$

and the average price for $M = 2$

$$\begin{aligned}
\mathbf{E}_{F_t} v &= \int\limits_{\tilde{V}_t}^{V_t} \frac{C_t}{M-1} v^{-\frac{1}{M-1}}\, dv = \frac{C_t}{M-1} \ln v\Bigg|_{\tilde{V}_t}^{V_t} = \\
&= \frac{V_t^{\frac{1}{M-1}}}{M-2}\frac{1-p_t}{p_t}\left(1-(M-1)\ln(1-p_t)\right)\ln V_t \overset{t\to\infty}{\longrightarrow} 0.
\end{aligned}$$

□

Proof of proposition 17.

From proposition 15

$$\frac{1}{\delta}(V_{t-1}-u) = V_t g_t - u;$$

$$\frac{1}{\delta}(V_{t-1}-u(1-\delta)) = V_t g_t;$$

$$V_t = \frac{V_{t-1}}{\delta g_t} - \frac{u(1-\delta)}{\delta g_t}.$$

Therefore, formula 11 holds for $t = 2$. Suppose that formula 11 holds for some t. Then

$$\begin{aligned}
V_{t+1} &= \frac{V_t}{\delta g_{t+1}} - \frac{u(1-\delta)}{\delta g_{t+1}} \\
&= \frac{V_1 \prod_{i=2}^{t} \frac{1}{\delta g_i} - u(1-\delta) \sum_{i=2}^{t} \prod_{j=i}^{t} \frac{1}{\delta g_j}}{\delta g_{t+1}} - \frac{u(1-\delta)}{\delta g_{t+1}} \\
&= V_1 \prod_{i=2}^{t+1} \frac{1}{\delta g_i} - u(1-\delta) \sum_{i=2}^{t} \prod_{j=i}^{t+1} \frac{1}{\delta g_j} - \frac{u(1-\delta)}{\delta g_{t+1}} \\
&= V_1 \prod_{i=2}^{t+1} \frac{1}{\delta g_i} - u(1-\delta) \sum_{i=2}^{t+1} \prod_{j=i}^{t+1} \frac{1}{\delta g_j},
\end{aligned}$$

which proves formula 11 for any $t > 2$.

Expressing V_1 through V_t, we get

$$\begin{aligned}
V_1 &= \left(V_t + u(1-\delta) \sum_{i=2}^{t} \prod_{j=i}^{t} \frac{1}{\delta g_j} \right) \prod_{i=2}^{t} \delta g_i \\
&= \left(V_t \prod_{i=2}^{t} \delta g_i + u(1-\delta) \left(1 + \sum_{i=3}^{t} \prod_{j=2}^{i-1} \delta g_j \right) \right).
\end{aligned}$$

From proposition 16,

$$\lim_{t\to\infty} V_t = u(1-\delta)$$

and therefore

$$\lim_{t\to\infty} V_t \prod_{i=2}^{t} \delta g_i = 0$$

as $\lim_{t\to\infty} g_t = 0$.

Therefore, there exists a unique V_1 satisfying inequality 9, and

$$\begin{aligned}
V_1 &= u(1-\delta) \left(1 + \sum_{i=3}^{\infty} \prod_{j=2}^{i-1} \delta g_j \right) \\
&= u(1-\delta)(1 + \delta g_2 + \delta^2 g_2 g_3 + \delta^3 g_2 g_3 g_4 + \ldots) \\
&= u(1-\delta)(1 + \delta g_2(1 + \delta g_3(1 + \delta g_4(1 + \ldots)))).
\end{aligned}$$

Starting from some period of time t, values $g_t = (1-p_t)^{M-1}(M + (1-p_t)) < 1$, therefore $\sum_{i=3}^{\infty} \prod_{j=2}^{i-1} \delta g_j$ converges and V_1 is finite.

To prove that V_t is a decreasing sequence, express V_t in terms of p_t and g_t:

$$\begin{aligned}
V_t &= V_1 \prod_{i=2}^{t} \frac{1}{\delta g_i} - u(1-\delta) \sum_{i=2}^{t} \prod_{j=i}^{t} \frac{1}{\delta g_j} \\
&= u(1-\delta)\left(1 + \sum_{i=3}^{\infty} \prod_{j=2}^{i-1} \delta g_j\right) \prod_{i=2}^{t} \frac{1}{\delta g_i} - u(1-\delta) \sum_{i=2}^{t} \prod_{j=i}^{t} \frac{1}{\delta g_i} \\
&= u(1-\delta)\left(1 + \sum_{i=t+2}^{\infty} \prod_{j=t+1}^{i-1} \delta g_j\right).
\end{aligned}$$

The values

$$g_t = (1-p_t)^{M-1}(M + (1-p_t)) = (1-p_1)^{M^{t-1}(M-1)}(M + (1-p_1)^{M^{t-1}})$$

decrease with p_t, p_t increases with t, and therefore $\sum_{i=t+2}^{\infty} \prod_{j=t+1}^{i-1} \delta g_j$ decreases with t because

$$\sum_{i=t+2}^{\infty} \prod_{j=t+1}^{i-1} \delta g_j < \sum_{i=t+2}^{\infty} \prod_{j=t+1}^{i-1} \delta g_{j-1} = \sum_{i=(t-1)+2}^{\infty} \prod_{j=(t-1)+1}^{i-1} \delta g_j.$$

□