In Search of Stars:
Network Formation among Heterogeneous Agents

Jacob K. Goeree, Arno Riedl, and Aljaž Ule*

July 24, 2006

(first version: June, 2003)

Abstract

This paper reports results from a laboratory experiment on network formation among heterogeneous agents. The experimental design extends the Bala-Goyal (2000) model of network formation with decay and two-way flow of benefits by allowing for agents with lower linking costs or higher benefits to others. Furthermore, agents' types may be common knowledge or private information. In all treatments, the (efficient) equilibrium network has a "star" structure. With homogeneous agents, equilibrium predictions fail completely. In contrast, with heterogeneous agents stars frequently occur, often with the high-value or low-cost agent in the center. Stars are not born but rather develop: with a high-value agent, the network's centrality, stability, and efficiency all increase over time. Probit estimations based on best-response behavior and other-regarding preferences are used to analyze individual linking behavior. Our results suggest that heterogeneity is a major determinant for the predominance of star-like structures in real-life social networks.

JEL classification: C72; C92; D82; D85

Keywords: Network formation; Experiment; Heterogeneity; Private information

*Ule (corresponding author): CREED, Faculty of Economics and Business, University of Amsterdam, Roetersstraat 11, 1018 WB, Amsterdam, the Netherlands, a.ule@uva.nl, tel +31205254205, fax +31205255283. Goeree: Division of the Humanities and Social Sciences, California Institute of Technology, Mail code 228-77, Pasadena, CA 91125, USA, jkg@hss.caltech.edu. Riedl: Department of Economics, Faculty of Economics and Business Administration, University of Maastricht, P.O. Box 616, 6200 MD, Maastricht, the Netherlands, a.riedl@algec.unimaas.nl. We gratefully acknowledge financial support from the National Science Foundation (0551014), the Alfred P. Sloan Foundation, and the Dutch National Science Foundation (VICI 453.03.606).

1 Introduction

Many social, economic, and information networks exhibit uneven, hierarchical structures. The Internet, for example, consists of relatively few web-sites with a huge number of links and a preponderance of web-pages with only a few links.[1] Co-author networks typically display a few well-connected researchers collaborating with many others.[2] Finally, casual observation suggests that ongoing social relations require the presence of relatively few active organizers who are central to a large network of friends or colleagues.

Several explanations have been put forth to explain the emergence of such hierarchical structures. Barabási and Albert (1999) demonstrate that hub-like structures can result from a simple dynamic process where the probability that a new agent entering the network connects with an existing agent is proportional to the number of links the existing agent has. They do not, however, give a rationale for such linking behavior. Jackson and Rogers (2005) consider a model where entering agents perform local searches in the neighborhoods of agents they randomly meet. They show that the resulting model is capable of reproducing many characteristics of socially generated networks. Both papers focus on large networks with agents that have imperfect knowledge about the prevailing network structure.[3]

In contrast, Bala and Goyal (2000) study a game-theoretic model where agents possess complete information about the network, an assumption that is probably more realistic in small networks with relatively few agents. Their approach is based on the idea that a link between two agents is established whenever at least one of them is willing to pay the cost.[4] Bala and Goyal show that extremely uneven or hierarchical networks can arise in equilibrium if benefits can be accessed through the network regardless of who sponsors its links (two-way flow of benefits). Indeed, for a wide range of parameter values the predicted equilibrium network has a "star"-like structure with a single agent (the center) being directly connected to all other (peripheral) agents.[5] Stars are predicted to occur even though agents are symmetric with identical incentives and opportunities.

Network formation is hard to investigate in the field because of many potentially confounding factors, e.g., possible asymmetries in agents' information or motivations, imperfect knowledge about linking opportunities or existing network structures, etc. A valuable alternative is provided by controlled laboratory experiments, an approach used by Falk and Kosfeld (2005) to study network formation among homogeneous agents. They find that linking behavior does not converge in settings where the unique strict Nash equilibrium

[1] See, e.g., Barabási (2002).

[2] See, e.g., Newman (2004) and Goyal et al. (2004).

[3] The resulting network typically contains many different sub-architectures and the distribution of the number of agents' links is approximately "scale-free" (i.e. follows a power law), see, e.g., Barabási and Albert (1999) and Barabási (2002).

[4] See Jackson and Wolinsky (1996) for a model where links require both agents' consent. Jackson (2005) provides a recent comprehensive survey of the literature on networks.

[5] A similar prediction is true for the symmetric connections model studied by Jackson and Wolinsky (1996).

network has a star-like structure[6] and argue that *strategic asymmetry* and *payoff asymmetry* account for the failure of Nash predictions.[7] These negative results are interesting in that they highlight the difficulties in forming star-like networks even in small groups of (four) agents who possess complete information about others' types and the prevailing network structure.

As the aforementioned examples indicate, however, star-like networks do emerge in the real-world. One obvious difference between the experimental setup and the real-life examples is the assumed symmetry across agents. Casual observation suggests that in practice, individual differences may play an important role in network formation. For instance, some people are better "networkers" in that they have a taste for linking to others or lower opportunity costs of maintaining their connections. Likewise, some people possess skills that are more scarce, making them more valuable to others. Individual differences in linking costs or benefits-to-others may resolve some of the strategic asymmetry faced by the agents, e.g. a high-value co-author may be more easily targeted to become the center of a periphery-sponsored star. Furthermore, individual heterogeneity may alleviate payoff asymmetries or make such asymmetries more acceptable. Indeed, the Greek proverb "success has many friends" suggests people have a preference for being connected to highly-rewarded individuals despite the resulting payoff inequalities.

The impact of individual heterogeneity on actual network formation is difficult to address using field data since "linking costs" and "benefits to others" are hard to measure and even harder to vary in a systematic way. The experiments reported in this paper provide a careful evaluation of the effects of asymmetries on linking behavior in a controlled laboratory setting. Besides an homogeneous environment where all agents have identical linking costs and are of equal value to others, we consider heterogeneous environments where one of the agents has lower linking costs, or a higher value to others, or a combination of two such agents. We consider cases where agents' types are common knowledge and those where their types are private information. For each of the resulting treatments, we provide a complete characterization of the (Bayesian) Nash equilibrium networks and study (i) whether uneven structures emerge, (ii) what network positions different types of agents occupy, (iii) how efficient and stable the observed networks are, and (iv) what factors determine individual linking behavior.

We find that the introduction of different types of agents has a dramatic impact on linking behavior and observed networks. While almost no stars are formed among symmetric agents, they are prevalent in some of the heterogeneous treatments. These findings are far from obvious: in Bala and Goyals (2000) model of network formation, heterogeneity is not only not necessary for star networks to form but, in fact, stars are *most likely* among

[6]Falk and Kosfeld (2005) include treatments where only the agent that sponsors the link receives the benefits from the link (one-way flow model), see also Callander and Plott (2005). In this case, observed linking behavior does converge to the Nash equilibrium network, which has a "wheel"-type structure. See Corbae and Duffy (2003) for an earlier network experiment and Kosfeld (2004) for a recent survey.

[7]Strategic asymmetry reflects the idea that in a completely symmetric setting it may be hard for subjects to coordinate on a very asymmetric outcome such as a star. Likewise, inequality averse subjects may find it hard to accept the payoff asymmetries that occur in star-like networks.

the networks formed by homogenate agents under simple best-response dynamics. This theoretical prediction was not confirmed by earlier experimental studies (Falk and Kosfeld, 2005), and we provide insights for why it was not.

Stars, however, are not born but develop over time: none of the treatments show a significant number of stars in the first five rounds of the experiments, but in several treatments stars are the most prevalent architecture in the final five rounds. Network centrality displays a strong tendency to rise over time in treatments with a high-value agent. A similar trend is observed for the network's efficiency and its stability. In summary, while star formation is absent initially (and remains absent with symmetric agents), repetition and experience enable subjects to coordinate on these hierarchical structures in some of the heterogeneous treatments.

The effects of incomplete information vary across treatments. In treatments with a high-value agent the periphery-sponsored star with this agent in the center is a frequent outcome in both information conditions. In contrast, in treatments with a low-cost agent, incomplete information has the surprising effect that it raises the occurrence of stars with the low-cost agent at the center. When both a low-cost and a high-value agent are present, incomplete information clearly aggravates the coordination problem subjects face: fewer stars are formed and not all of them are periphery-sponsored stars with the high-value agent at the center. In contrast, the abundance of stars observed for the complete information case are all of this type. This suggests that "networking" increases one's centrality when individual abilities are hidden but that it is typically the most "talented" individual that becomes (the center of) a star.

We also test how heterogeneity affects individual behavior. We consider several possible determinants of agents' linking decisions, including best-response behavior and "other regarding" preferences such as a taste for efficiency, envy, or guilt.[8] In most treatments, all these factors play a significant role in individual decision making. In particular, envy (more so than guilt) and a taste for efficiency have a significant effect on linking behavior. However, in treatments with a high-value agent, envy is less important and a taste for efficiency dominates. This seems in line with our earlier observation that in real life, people do choose to connect to highly-rewarded individuals.

This paper is organized as follows. In the next section we outline the key theoretical concepts of network formation with heterogeneous agents and incomplete information (sections 2.1 and 2.2). We present our experimental design, the experimental parameters and procedures (sections 2.3 and 2.4), and discuss theoretical predictions (section 2.5). Section 3 presents results on the empirical frequency of Nash networks (section 3.1), stars (section 3.2), the efficiency of observed networks (section 3.3), their stability (section 3.4), and the determinants of individual linking behavior (section 3.5). Section 4 concludes. Appendix A contains proofs and Appendix B contains the instructions.

[8] An agent experiences envy (guilt) when others' net payoffs from the formed network are higher (lower). See Fehr and Schmidt (1999) for an analysis of envy and guilt in general games, and Falk and Kosfeld (2005) for an application to networks.

2 Model, experimental design, and theoretical predictions

We extend the two-way flow model with decay of Bala and Goyal (2000) to allow for agents that differ in costs or benefits of linking. For alternative approaches to heterogeneity in networks see the models of Johnson and Gilles (2000), and Haller and Sarangi (2005), in which links rather than agents vary in costs or reliability. A related model of Galeotti et al. (2005) studies network formation among heterogeneous agents without decay.

2.1 Basic network concepts

Let $N = \{1, ..., n\}$ denote the set of agents, with distinct generic members i and j. Agents make links to other agents, e.g. to access their skills or information. Any agent can make a link to any other agent. Agent i's links can be represented by the *linking vector* $g_i = (g_{i1}, ..., g_{in})$ such that $g_{ii} = 0$ and $g_{ij} \in \{0, 1\}$ for each $j \in N \backslash \{i\}$ where $g_{ij} = 1$ if and only if i made a link with j. The collection of all agents N and all their links $\{(i, j); g_{ij} = 1\}$ constitute a *directed graph*, or a *network*, which can be represented by the matrix $g = (g_1, ..., g_n)$.

Graphically we represent agents by small circles and their links by arrows. A link made by agent i to agent j is represented by a line starting at i with the arrowhead pointing towards j. Figure 1 shows an example of a network, where agents 4 and 6 made no links, agent 1 made a link to agents 2 and 4, and agents 2, 3 and 5 made a link to agent 4.

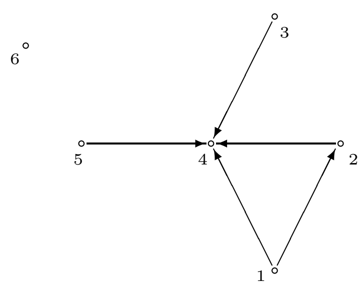


Figure 1: Example of a network of directed links among 6 agents.

Two agents are *linked* whenever at least one of them made (maintains) a link to the other. It is thus useful to define the *closure* of g: this is a non-directed network $\overline{g} = cl(g)$, defined by $\overline{g}_{ij} = \max\{g_{ij}, g_{ji}\}$ for each $i, j \in N$. A *path of length* k between i and j is a sequence of distinct agents $(i, j_1, ..., j_{k-1}, j)$, such that $\overline{g}_{ij_1} = \overline{g}_{j_1 j_2} = ... = \overline{g}_{j_{k-1} j} = 1$. If at least one path exists between i and j then j is *accessible* for i (and vice versa), otherwise j is *inaccessible* for i (and vice versa). A path $(j_1, ..., j_k)$ is a *cycle of length* k if $k \geq 3$ and $\overline{g}_{j_k j_1} = 1$. The *distance* between i and j, $i \neq j$, denoted $d(i, j; g)$, is the length of the shortest path between i and j. If j is inaccessible for i then the distance is $d(i, j; g) = \infty$. If i and j are (directly) linked, that is, if $\overline{g}_{ij} = 1$, then $d(i, j; g) = 1$. For completeness we set $d(i, i; g) = 0$. The *degree* of an agent i, $\deg_i(g) = \sum_{j \neq i} \overline{g}_{ji}$, is the number of other agents with whom she is linked. Figure 1 shows a network with, among else, two paths between agents 1 and 5, $(1, 4, 5)$ and $(1, 2, 4, 5)$, and a cycle $(1, 2, 4, 1)$. The distance between agents 1 and 5, $d(1,5) = 2$, is implied by the length of the shortest path between them, $(1,4,5)$. Agent 2 is linked with agents 1 and 4, thus her degree is 2.

A non-empty subset of agents $M \subset N$ is a *component* of network g if every two distinct members of M access each other but no agent in M accesses any agent in $N \backslash M$. An agent having no links in the closure $\overline{g}$ is *isolated*, and forms a component consisting of one member. A network is *connected* if each agent accesses all other agents. In a connected network there are no isolated agents and the unique component consists of the whole set of agents N. A network is *minimally sponsored* if $g_{ij} = 1$ implies $g_{ji} = 0$ for any i and j, that is, if every link is maintained by exactly one agent. A network is *minimally connected* if it is minimally sponsored, connected and contains no cycles. In such a network there is a unique path between every pair of distinct agents. For example, all networks shown in Figures 3-5 below are minimally connected. The network in Figure 1 is not minimally connected as it contains a cycle and an isolated agent.

An agent that has exactly one link in the closure $\overline{g}$ is called *periphery*. We say that agent i *sponsors* the link with agent j when $g_{ij} = 1$ and $g_{ji} = 0$. It is convenient to identify some prominent classes of networks. For a graphical representation of some of the following networks, see Figures 2-5.

Empty network (EN): No links are made, $g_{ij} = 0$ for all i and j.

Complete network (CN): All links are made, $\overline{g}_{ij} = 1$ for all i and j.

Wheel network (WN): The links of the network form a cycle of length n spanning all agents.

Star (S): There is one central agent i. Each other agent $j \neq i$ is periphery, linked only with the central agent i. The direction of links is arbitrary. Formally, $\overline{g}_{ik} = 1$ and $\overline{g}_{jk} = 0$ for $j, k \neq i$.

Minimally-sponsored star (MSS): A star where each link is sponsored either by the center agent i or by the periphery agent. Formally, $g_{ki} = 1 \Longleftrightarrow g_{ik} = 0$ and $g_{kj} = 0$ for all $j, k \neq i$.

Periphery-sponsored star (PSS): A minimally-sponsored star where all links with the central agent i are sponsored by the periphery agents. Formally, $g_{ki} = 1$, $g_{ik} = 0$ and $g_{kj} = 0$ for all $j, k \neq i$.

Center-sponsored star (CSS): A minimally-sponsored star where the central agent i sponsors all the links. Formally, $g_{ki} = 0$, $g_{ik} = 1$ and $g_{kj} = 0$ for all $j, k \neq i$.

Linked star (LSS): There are two linked non-periphery agents i and j while all other agents are periphery, linked with either i or j. Formally, $\overline{g}_{ij} = 1$, $\overline{g}_{ki} = 1$ or $\overline{g}_{kj} = 1$ but not both, and $\overline{g}_{kl} = 0$ for $k, l \notin \{i, j\}$.

2.2 Heterogeneous agents, two-way flow of benefits, and decay

Suppose each agent i provides some service (the quality of which depends on personal skill or information) that has positive value v_i to others. Agents can materialize these values by (simultaneously) forming links with other agents. Each agent i incurs positive costs c_i for every link she forms (capturing the time, effort, or money invested to form and maintain the link). When there is a link between agents i and j, made either by i or j (or by both), both enjoy the other's value. Establishing a link therefore does not require mutual consent. Agents benefit also from any other agent they access through the network although the benefit decreases with distance (reflecting the idea that information may become less accurate, or less valuable as it propagates through the network).

Following previous literature on network formation games we consider only pure strategies.[9] The set of pure strategies of agent i, $\mathcal{G}_i$, is the set of all her possible linking vectors g_i. The strategy space of all agents is given by $\mathcal{G} = \mathcal{G}_1 \times \cdots \times \mathcal{G}_n$. Any profile of strategies $g = (g_1, ..., g_n)$ constitutes a (directed) network. Agents' benefits depend on $\overline{g}$, the undirected closure of g.

The benefit that agent i extracts from agent j depends on the value v_j as well as on the distance $d(i, j; g)$ as given by the *decay function* $\Phi(v_j, d(i, j; g))$. We assume that $\Phi : \mathbb{R}_+ \times \{\{0, 1, 2, ..., n-1\} \cup \{\infty\}\} \rightarrow \mathbb{R}$ is strictly increasing in value and decreasing in distance, with $\Phi(v_j, 0) = \Phi(v_j, \infty) = 0$, that is, an agent receives no benefit from herself nor from those other agents she cannot access.

We assume a linear payoff function, in line with the models of Bala and Goyal (2000). The payoff to agent i is the sum of the benefits accessed through the network minus the costs of links she maintains. Let $\mu_i(g) = |\{j \in N; g_{ij} = 1\}|$ be the number of links that agent i maintains. The payoff to agent i in the network g is given by

$$\pi_i(g) = \sum_{j \in N} \Phi(v_j, d(i, j; g)) - \mu_i(g) c_i. \quad (1)$$

If agents' values and linking costs and the decay function are common knowledge then the network formation game is played under complete information. In this case we define *Nash networks* to be the networks that are established in the (pure-strategy) Nash equilibria of the game $\langle N, \mathcal{G}, (\pi_i)_{i=1}^n \rangle$. Given a network g let g_{-i} denote the network obtained when all links maintained by agent i are removed. The network g can thus be written as $g = g_i \oplus g_{-i}$, where the symbol '$\oplus$' indicates that g is formed by the union of links in g_i and g_{-i}. A strategy g_i is a best response of agent i to network g if

$$\pi_i(g_i \oplus g_{-i}) \geq \pi_i(g_i' \oplus g_{-i}) \text{ for all } g_i' \in \mathcal{G}_i.$$

The set of all best responses of agent i to g is denoted by $BR_i(g)$. A network $g = (g_1, ..., g_n)$ is a Nash network if $g_i \in BR_i(g)$ for each i. A Nash network is *strict Nash* if $|BR_i(g)| = 1$ for each i, i.e. each agent is playing her *unique* best response to the network established by the other agents.

[9] This literature is reviewed in Jackson (2004).

The definition of equilibrium networks is more intricate for network formation games with incomplete information.[10] For convenience, the definition we give here is limited to the setup employed in our experiments. Let the decay function Φ be common knowledge. Let the profile of agents' value-cost pairs $\theta = ((v_1, c_1), ..., (v_n, c_n))$ be randomly drawn from some finite space of profiles Θ and let the ex-ante probability distribution p over Θ be common knowledge. We denote by $v_i(\theta)$ and $c_i(\theta)$ the value and linking cost of agent i given a profile θ. For any profile $\theta \in \Theta$ and any network g the payoff to agent i is given by

$$u_i(g; \theta) = \sum_{j \in N} \Phi(v_j(\theta), d(i, j; g)) - \mu_i(g) c_i(\theta). \tag{2}$$

Once a profile $\theta \in \Theta$ is drawn each agent i learns her own value and linking cost, $\theta_i = (v_i(\theta), c_i(\theta))$, which permits her to calculate her (subjective) beliefs $p_i^{\theta_i}$ over Θ and her expected payoff $E_i u_i(g; \theta_i) = \sum_{\theta' \in \Theta} u_i(g; \theta') p_i^{\theta_i}[\theta']$ about her payoff in the network g. Formally, this defines the Bayesian game $\left\langle N, \mathcal{G}, \Theta, ((p_i^{\theta_i})_{\theta \in \Theta})_{i=1}^n, (u_i)_{i=1}^n \right\rangle$.

The set of equilibrium networks may depend on the realized value-cost profile. A network g is a *Bayesian Nash network given profile* $\theta \in \Theta$ if, for each agent i,

$$E_i u_i(g_i \oplus g_{-i}; \theta_i) \geq E_i u_i(g_i' \oplus g_{-i}; \theta_i) \text{ for all } g_i' \in \mathcal{G}_i \backslash \{g_i\}. \tag{3}$$

In a Bayesian Nash network no agent can increase her expected payoff given her beliefs about the profile of values and linking costs among the agents.[11] Network g is *strict* Bayesian Nash, given θ, if all inequalities (3) are strict.

2.3 Experimental parameters and design

The set of (Bayesian) Nash networks depends on the parameters of the model such as the linking costs c_i and values v_i of individual agents. Theoretical literature suggests that it is difficult to analytically characterize equilibrium networks for arbitrary profiles of costs and values and arbitrary decay functions. For instance, Bala and Goyal (2000) only partially characterize the Nash networks for the model with homogeneous costs and values with decay while Galeotti et al. (2005) partially characterize Nash networks for a particular case of cost heterogeneity without decay.[12] General characterization is beyond the scope of this paper. Instead we motivate below our choice of cost and value parameters for the experiments and provide a complete characterization of the set of (Bayesian) Nash networks for these parameter values.

[10] To the best of our knowledge, network formation under incomplete information has been studied only by McBride (2005).

[11] The relation between ex-ante defined Bayesian equilibria and the "interim" defined Bayesian Nash networks is as follows. A (pure) strategy profile $s : \Theta \to \mathcal{G}$ is a Bayesian equilibrium of our network formation game if and only if $s(\theta)$ is a Bayesian Nash network given any allocation $\theta \in \Theta$. See e.g. Fudenberg and Tirole (1991) for discussion on the equivalence between ex-ante and "interim" formulations of equilibria in Bayesian games.

[12] In both papers the set of Nash networks is characterized completely only for certain ranges of parameters. For other parameter values some prominent Nash networks are identified. See also Haller et al. (2006).

To separate the effects of heterogeneity in linking costs and heterogeneity in values we consider four situations. In the "homogeneous" case, all agents have identical costs and values. In the "low-cost" case, one agent has lower linking costs than others while all other parameters are identical to the homogeneous case. In the "high-value" case, the value of one agent exceeds that of others while all other parameters are identical to the homogeneous case. Finally, we study a situation with one low-cost and one high-value agent while keeping all other parameters identical to the homogeneous case. In this section we focus on network formation games with complete information.

We begin with the homogeneous case where values and linking costs are the same across agents, that is, $v_i = v > 0$ and $c_i = c > 0$ for all i. Let $n \geq 4$ and assume an exponential decay function $\Phi(v, d) = \delta^d v$ with decay parameter $\delta \in (0, 1)$ (Jackson and Wolinsky (1996), Bala and Goyal (2000)). Using Propositions 5.3 and 5.4 in Bala and Goyal (2000) we obtain the following characterization: (a) if $0 < c < v - \delta v$ then minimally connected complete networks are the only Nash networks; (b) if $v - \delta v < c < v$ then any minimally-sponsored star is a Nash network, and minimally connected but non-star Nash networks may exist; (c) if $v < c < v+(n-2)\delta v$ and δ is sufficiently close to 1 then the empty network and all periphery-sponsored stars are the only Nash networks; (d) if $v + (n - 2)\delta v < c$ then the empty network is the unique Nash. Given our interest in star networks, we set $c = 24$, $v = 16$ and $\delta = 0.75$ in our experiments. The resulting setup corresponds to case (c), having a simple but interesting set of star networks that can be established in equilibrium. Employing an exponential decay function yields benefits that often cannot be expressed as integers. For convenience to the experimental subjects we truncate our chosen exponential decay function to integer values.

The second case we study is identical to the homogeneous case except that the linking cost of agent i_c is lower than the linking costs of other agents; $c_{i_c} = c^l < c = c_i$ for all $i \neq i_c$. In this case we call agent i_c the "low-cost" agent and all other agents the "normal" agents. If c^l is sufficiently close to c then the set of Nash networks coincides with that of the homogeneous case. On the other hand, if c^l is sufficiently close to 0 then agent i_c is willing to sponsor a link with any other agent regardless of the rest of the network, which implies that minimally-sponsored stars with agent i_c in the center are the only Nash networks. We selected an intermediate value $c^l = 7$ for which all minimally-sponsored stars with agent i_c in the center as well as periphery sponsored stars with any agent in the center are Nash networks, but the empty network is not. Intuitively, this makes it more likely (compared to the homogeneous case) that a star network is established and that agent i_c is in its center.

The third case we consider has parameters identical to the homogeneous case except that the value of agent i_v is higher than that of other agents: $v_{i_v} = v^h > v = v_i$ for all $i \neq i_v$. In this case we call agent i_v the "high-value" agent and all other agents the "normal" agents. Again, if v^h is sufficiently close to v then the set of Nash networks coincides with that of the homogeneous situation. On the other hand, if v^h is very high then any agent is willing to sponsor a link with agent i_v regardless of the rest of the network, which implies that the periphery-sponsored star with agent i_v in the center is the unique Nash network.

We selected an intermediate value $v^h = 32$ for which all periphery sponsored stars, with any agent in the center, are Nash networks, but the empty network is not. Intuitively, this makes it more likely (compared to the homogeneous case) that a star network is established but all agents are equally likely in its center.

We also study the more complex situation with one low-cost agent i_c, one high-value agent i_v, and $n-2$ normal agents, using the parameters from the cases above. In all our treatments we study groups of $n = 6$ agents. Table 1 summarizes the parameter values used in our experiments.

Table 1: Experimental parameters.

a) Linking costs and values of different agent types.

	cost per link made	value to other agents
normal agent	24	16
low cost agent	7	16
high value agent	24	32

b) Benefits (per agent accessed) from accessing different types of agents at different distances.

distance	1	2	3	4	5	∞
normal or low cost agent	16	12	9	7	5	0
high value agent	32	24	18	14	10	0

We investigate the network formation behavior in the four situations described above using the following seven treatments (where the 'I' stands for complete and the 'N' for incomplete information):

(BI) baseline treatment with groups of 6 normal agents,

(CI & CN) treatments with groups of 5 normal agents and 1 low-cost agent,

(VI & VN) treatments with groups of 5 normal agents and 1 high-value agent, and

(CVI & CVN) treatments with groups of 4 normal agents, 1 low-cost agent, and 1 high-value agent.

Since we are particularly interested in the dynamics of network formation the network formation game is repeated for 30 rounds within each group. In each round, subjects first simultaneously make their linking decisions and then each subject is informed about the established network, her total benefits, her costs, and her payoff. Groups and individual types are fixed throughout the experiment and in each group each subject keeps a unique identification tag. In treatments BI, VI, CI, and CVI, the type of each agent is publicly announced at the beginning of the first round of the experiment. In treatments CN, VN, and CVN, the *collection* of types in the group is publicly announced at the beginning of the first round of the experiment, but the type of each agent remains private information throughout the experiment. Treatments CN, VN, and CVN differ from treatments CI, VI, and CVI, respectively, only in terms of the quality of information. See Table 2 for an overview.

Table 2: Group composition and information condition in each of our treatments.

	complete information	incomplete information
6o	BI (7)	-
5o + 1c	CI (6)	CN (6)
5o + 1v	VI (4)	VN (6)
4o + 1c + 1v	CVI (4)	CVN (6)

Note: Numbers of independent observations (i.e., groups) in parenthesis.
Abbreviations: o - normal agent, c - low-cost agent, v - high-value agent.

2.4 Experimental procedures

The experimental sessions were conducted in Spring of 2003 at the CREED laboratory at the University of Amsterdam and at the Social Science Experimental Laboratory at the California Institute of Technology. In total 234 subjects participated. Each experimental session lasted between 45 and 90 minutes. Subjects' total earnings were determined by the sum of the points earned over all the rounds, using a conversion rate of 70 points per Euro or US dollar (the benefits and costs listed in Table 1 are all in points). The average earnings were 22.8 Euros or (roughly) USD 25 at the time the experiments were conducted. Most subjects were economics or business administration undergraduates. They were recruited through notices on bulletin boards and through email announcements. Each subject participated in only one session and none had previously participated in a similar experiment. To ensure anonymity, at least twelve subjects were recruited for every session and were randomly divided into at least two independent groups. They were seated in separated cubicles, which eliminated most possibilities for communication other than via the computer network. We consider each group as one independent observation, see Table 2 for numbers of groups in different treatments.

At the beginning of a session, subjects were told the rules of conduct and provided with detailed instructions. The instructions were read aloud and shown on the computer screen.[13] After having finished the instructions, subjects received a printed summary and were asked to complete a questionnaire designed to test their understanding of the network formation game, the payoff calculations, and the computer interface. Once all subjects had correctly answered the questionnaire, we conducted a single practice round (without providing feedback about others' choices). The experiment started after all subjects confirmed they had no further questions.

[13]The experiment and the questionnaire were computerized using software developed at CREED by Jos Theelen.

Care was taken to minimize differences in the instructions for different treatments.[14] In all treatments, subjects knew the size of their group, the numbers of normal, low-cost, and high-value types in their group, the type-conditional payoff functions, the information condition, and the number of rounds. To increase anonymity and avoid suggestive framing, each subject's screen displayed herself as "Me" and the other five members in her group as "A", "B", "C", "D", and "E". Each letter corresponded to the same member in the group throughout the session, and this was common knowledge. The game was neutrally framed: each subject was either "green" (normal), "purple" (low-cost), or "blue" (high-value) and was forming "links" with other subjects. In the complete information treatments, each subject observed the color of all other subjects in the group. In the incomplete information treatments, each subject only observed her own color.

2.5 Equilibrium and efficiency analysis

For reference, subscripts c, v, and o, respectively, indicate a low-cost agent, a high-value, or a (normal) other agent in the center of a minimally-sponsored star MSS_x, a periphery-sponsored star PSS_x, or a center-sponsored star CSS_x, where x can be c, v or o. LS^p_{cx} denotes a linked star with a low-cost agent in one center sponsoring links with p periphery agents, and an agent of type x in the other periphery-sponsored center linked with $4-p$ periphery agents (LS^1_{co}, LS^2_{cv} and LS^1_{cv} are shown in Figures 3d, 5c, and 5d, respectively). PSS^{-v} denotes a star with a normal agent in the center that is periphery-sponsored except for the center sponsoring one link to the high-value agent (see Figure 4c).

Propositions 1 and 2 give complete characterizations of the (Bayesian) Nash networks for the one-shot network formation games in each of our treatments. The proofs are provided in the Appendix. Illustrations of the equilibrium networks are given in Figures 2-5.

Proposition 1 *The following are all Nash networks for the complete information treatments:*

BI: all PSS and the EN.
VI: all PSS and all PSS^{-v}.
CI: all PSS, all MSS_c including the CSS_c, and all LS^1_{co}.
CVI: the PSS_v, all MSS_c including the CSS_c, and all LS^1_{cv} and LS^2_{cv}.
These networks are in all cases strict Nash.

Proposition 2 *For the incomplete information treatments the following are all Bayesian Nash networks, given any feasible allocation of types:*

VN: all PSS and the EN.
CN,CVN: all PSS, all MSS_c including the CSS_c, and all LS^1_{co} and LS^1_{cv}.
These networks are in all cases strict Bayesian Nash.

[14] Instructions can be found in Appendix B with the parts that differ between treatments emphasized. Instructions for all treatments can be downloaded from http://www1.fee.uva.nl/creed/people/ule/index.htm.

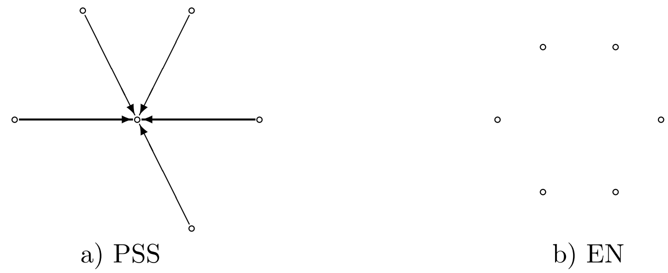


Figure 2: The (Bayesian) Nash networks for treatments BI and VN.

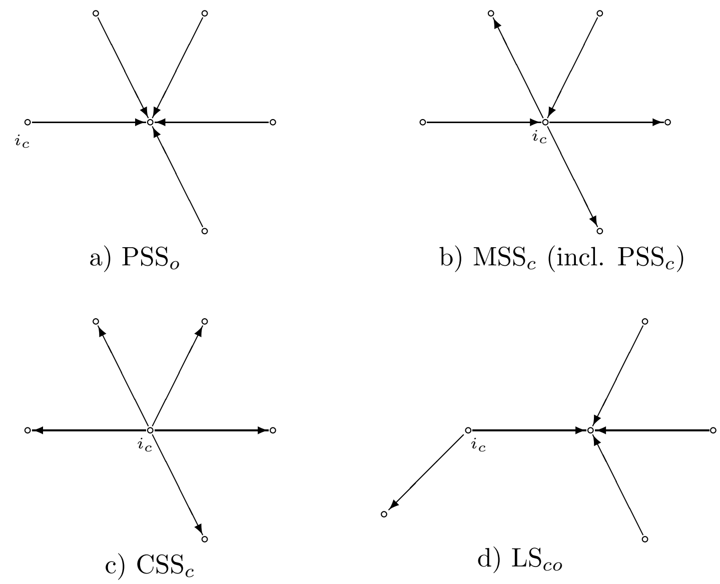


Figure 3: The (Bayesian) Nash networks for treatments CI, CN and CVN.

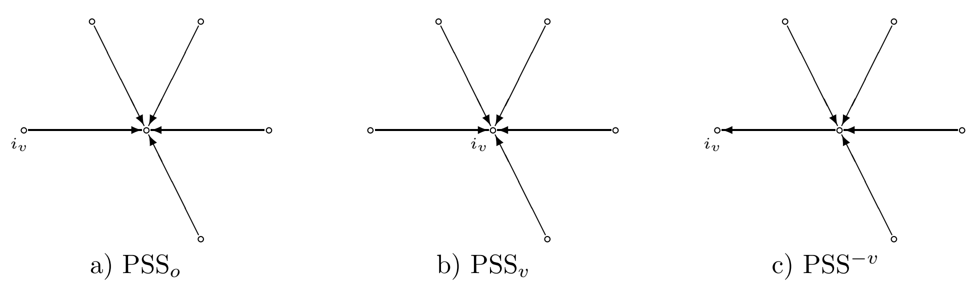
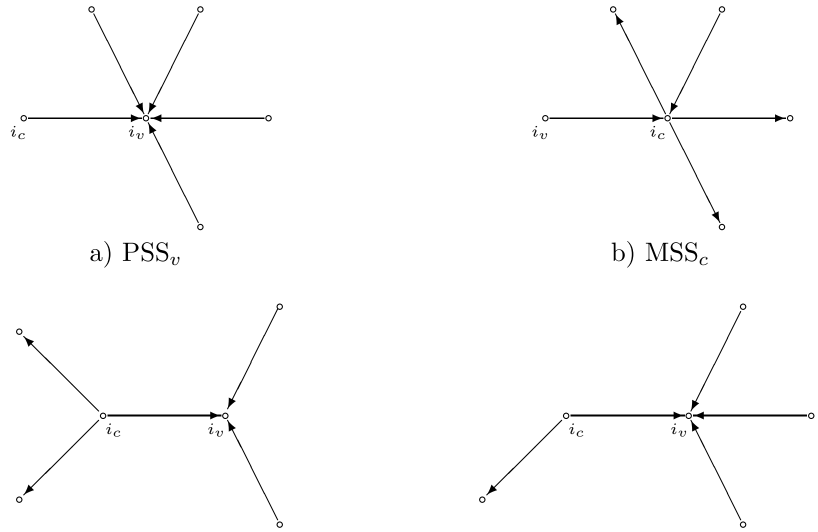


Figure 4: Nash networks for treatment VI.



a) PSS_v b) MSS_c c) LS_{cv} d) LS_{cv}

Figure 5: Nash networks for treatment CVI.

Predictions regarding the impact of information on network formation are obtained by comparing the set of equilibrium networks across the treatments with identical type distributions. Limiting the information changes the set of equilibrium networks in the treatments with a high-value agent, but not in the treatment with only one low-cost agent. This suggests, in particular, that behavior in CI should be similar to that in CN. With repetition, however, agents may overcome the uncertainty about types in any incomplete information treatment. We are therefore interested whether in later rounds behavior in an incomplete information treatment is similar to behavior in the corresponding complete information treatment.

Equilibrium networks differ with respect to *payoff inequality*, which we define as the ratio between the maximal and the minimal payoff in the network g, $\max_{i\in N}\pi_i(g)/\min_{i\in N}\pi_i(g)$.[15] In equilibrium, the payoff inequality in treatments CI, CN, CVI and CVN is lowest in CSS_c, while in treatments VI and VN it is lowest in PSS^{-v}. The payoff inequality in CSS_c in treatments CI and CN is almost equal to that in PSS_v in treatments VI, VN, CVI and CVN.

Efficiency can be measured with the sum of agents' payoffs. Let $w : \mathcal{G} \to \mathbb{R}$ be defined as $w(g) = \sum_{i=1}^{n}\pi_i(g)$. A network g is *efficient* if $w(g) \geq w(g')$ for all $g' \in \mathcal{G}$. For the linear payoffs in equation (1) the efficient network is the one which maximizes the total benefits of all agents, less the aggregate cost of links of the network. Proposition 3 shows that star networks have the best ratio between the number of links and the aggregate distance between agents.

[15]Payoff inequalities in some prominent star networks are: (CI,CN) 1.42 in CSS_c, 2.00 in any PSS, (VI,VN) 1.38 in PSS^{-v}, 1.43 in PSS_v, 2.40 in any PSS_o, (CVI,CVN) 1.25 in CSS_c, 1.43 in PSS_v, 1.71 in PSS^{-v}, 2.40 in any PSS_o and PSS_c.

Proposition 3 *The following are all efficient networks for the different treatments.*
BI: all MSS.
VI, VN: all MSS_v.
CI, CN, CVI, CVN: the CSS_c.

The proof of Proposition 3 is provided in the Appendix. In treatment BI each periphery-sponsored star network is Nash and efficient. In all other treatments there is a unique efficient (Bayesian) Nash network. To summarize, in every treatment the efficient equilibrium network is a star network. In order to assess how close to the efficient networks the actually formed networks come we define the *relative efficiency* of a network $g \in \mathcal{G}$ to be the ratio $w(g)/w(g^*)$, where $g^* \in \mathcal{G}$ is the efficient network. The two benchmark relative efficiencies are 0 for the empty network and 1 for the efficient network.

3 Results

We first present results concerning the occurrence of equilibrium networks in the different treatments. Then we investigate whether star networks are formed and how the frequency of stars (if any) vary across treatments and over time. We briefly discuss the efficiency and stability of the observed networks. Finally, we study how individual behavior accounts for the observed networks. For convenience we use the classification of star architectures introduced in section 2.1. If not otherwise stated all statistical tests are based on the independent observations, i.e., groups.

We begin with a few preliminary observations. Subjects in our experiments actively link with each other: across all treatments and all rounds we never observe the empty network. Neither do we observe the complete network, which indicates that there is no excessive 'over-linking'. Furthermore, we observe that between 11% and 14% of networks in treatments BI, CI and CN are minimally connected whereas in treatments VI, VN, CVI and CVN their frequencies are between 43% and 60%. Since non-empty equilibrium networks are minimally connected (see Appendix) these frequencies suggest that the presence of a high-value agent facilitates formation of equilibrium networks.

3.1 Do subjects form equilibrium networks?

Table 3 depicts the frequency of (Bayesian) Nash networks in all treatments across all rounds. The differences across treatments are quite striking. In the baseline treatment BI with only homogeneous agents not a single Nash network is formed. A similar conclusion holds for treatments with one low-cost agent. In CI and CN, respectively, only 2.2% and 8.9% of all observed networks are (Bayesian) Nash. However, introducing a high-value agent has a dramatic effect on the frequency of equilibrium networks. In VI and VN, 40.8% and 51.1% of all observed networks are (Bayesian) Nash and for CVI and CVN the numbers are 33.3% and 26.7% respectively.

Considering data from only the last five rounds indicates even larger differences across treatments. The relative frequency of (Bayesian) Nash networks in VI, VN, CVI, and

CVN, respectively, is 75.0%, 83.3%, 95.0%, and 66.7%. In stark contrast, in CI and CN only 10.0% and 16.7% of all networks are (Bayesian) Nash. In BI no Nash networks are formed at all. In the treatments where a high-value agent is present subjects tend to form an equilibrium network whereas this is not the case in the other treatments.[16]

Table 3: Frequency of (Bayesian) Nash networks in the different treatments.

	Equilibrium networks								
Treatment	EN	PSS_o	PSS_v	PSS^{-v}	MSS_c	LS_{co}	LS_{cv}	Total	# obs.
BI	0.0%	0.0%						0.0%	210
CI		0.0%			2.2%	0.0%		2.2%	180
CN		0.0%			8.9%	0.0%		8.9%	180
VI		0.0%	40.8%	0%				40.8%	120
VN	0.0%	0.0%	51.1%					51.1%	180
CVI			33.3%		0.0%			33.3%	120
CVN		0.0%	17.8%		5.0%		3.9%	26.7%	180

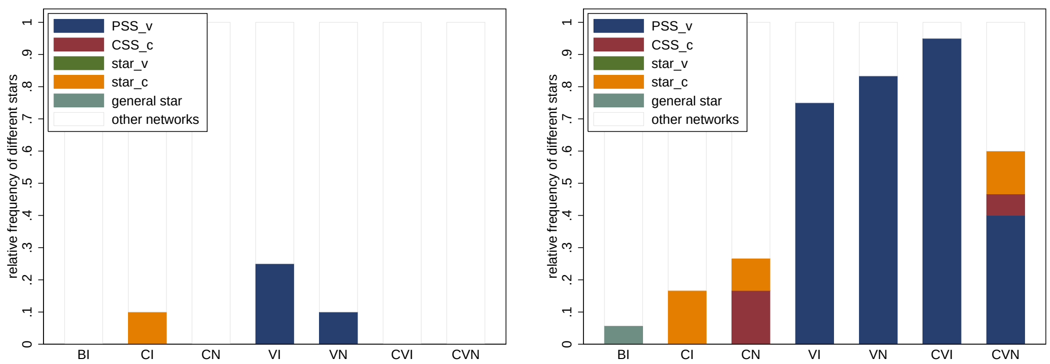
Note: PSS_o (PSS_v): periphery-sponsored star with a normal (high-value) type in the center; PSS^{-v}: star with a normal agent in the center that is periphery-sponsored except for the center sponsoring one link to the high-value agent; MSS_c: minimally-sponsored star with the low-cost agent in the center; LS_{co} (LS_{cv}): linked star with a low-cost agent in one center and a normal (high-value) agent in the other center; EN: empty network. Empty cells indicate the network is not an equilibrium network in the corresponding treatment.

3.2 Do subjects form star networks?

Figure 6 depicts the frequency of the different star architectures for all seven treatments aggregated across the first five rounds (panel(a)) and the last five rounds (panel (b)). Very few star structures are formed in early rounds. In treatments BI, CN, CVI and CVN, no stars are observed at all. In treatment CI only 10% (3/30) of all networks are stars, all of them have the low cost agent in the center with mixed sponsoring. Some stars are formed in the treatments with one high-value agent (25% (5/20) in VI and 10% (3/30) in VN). Interestingly, all observed star networks in VI and VN are efficient equilibrium networks, i.e. periphery-sponsored stars with the high-value agent in the center.

Figure 6(b) highlights the differences across treatments in the last five rounds. Comparing the results to those for the first five rounds indicates that repetition and experience

[16]We calculated the Spearman rank order correlations between the average number of (Bayesian) Nash networks across groups and the round number. For treatments VI, VN, CVI, and CVN this correlation coefficient is larger than 0.86 and significant at $p < 0.0001$ (two-sided tests). This clearly indicates that in these treatments more equilibrium networks are formed as agents gain experience. Interestingly, there is weak evidence of learning in treatment CN, where the percentage of Bayesian Nash networks increases from 0.0% in the first 16 rounds to 16.7% in the last few rounds. This increase is statistically significant ($\rho = 0.84,\ p < 0.0001$; two-sided test). Note, however, that in quantitative terms the number of Bayesian Nash networks in CN is considerably less than in treatments with a high-value agent. No convergence towards Nash networks can be found in treatments BI (because there are no Nash networks at all) and CI where the Spearman correlation coefficient is not significantly different from zero ($p > 0.2935$; two-sided tests).



(a) Rounds 1-5 (b) Rounds 26-30

Figure 6: Frequency of stars in the first five rounds and the last five rounds.

can have a dramatic effect on formation of stars. In the baseline treatment with homogeneous agents only 2 out of 35 networks in the last five rounds are stars (5.7%).[17] A low frequency of stars is observed also in the treatments with one low cost agent, CI and CN. In CI only 5 networks out of 30 are stars (16.7%); in all of them the low-cost agent is in the center. Interestingly, there are slightly more stars in the incomplete information case CN, but their frequency is still low: 26.7% (8/30).[18]

The results for the treatments with a high-value agent are very different. In all four treatments VI, VN, CVI, and CVN, stars are the prevailing network architecture. In VI 75% (15/20) and in VN 83.3% (25/30) of all networks are stars. In CVN the frequency of stars is 60% (18/30) while in CVI their frequency is almost 100% (19/20). Remarkably, in VI, VN, and CVI all formed stars are periphery sponsored stars with the high-value agent in the center. In CVN this network is formed in the majority of cases.[19] From these results it is obvious that the presence of a high-value agent facilitates the formation of stars, but not necessarily efficient ones.[20] Furthermore, as for the case of equilibrium networks, the dynamics of star formation are quite different across treatments. The Spearman rank order correlation of the average number of stars (of any form) across groups with the round is insignificant for treatments BI and CN ($p \geq 0.2255$; two-sided tests). In all other treatments we observe significantly positive correlations ($\rho > 0.80$, $p < 0.001$; two-sided tests).[21]

[17] Interestingly, also other prominent structures do not show up in this treatment. The complete network is never formed and the wheel is formed in only 2 out of 35 cases.

[18] Note that none of the observed stars in CI coincides with the efficient center-sponsored star where the low-cost agent sponsors all links. In CN the majority (62.5%; 5/8) of star networks are efficient CSS_c networks.

[19] The efficient network in these treatments is CSS_c, which do not occur at all in CVI and form only in 6.7% of all cases (2/30) in CVN. In CVN we also observe a few general stars with the low-cost agent in the center (13.3%; 4/30).

[20] It is worth noting that in treatments VI, VN, CVI, and CVN, all stars formed are (Bayesian) Nash networks. In particular, in CVN, the 'general' stars with the low-cost agent in the center ($star_c$) are minimally sponsored. In treatment CI, 3 of the 5 observed $star_c$ are minimally sponsored and, hence, Nash. In CN and BI none of the few observed stars is a (Bayesian) Nash network.

[21] The positive correlation in treatment CI is due solely to the fact that in round 29 two stars are formed

To summarize, stars (whether equilibrium or not) very rarely form in early rounds when subjects have little or no experience (independent of the treatment). This indicates that forming a star with a high-value or a low-cost agent in the center is not an obvious or focal outcome. With repetition, only a few stars are formed when agents are homogeneous or differ only in linking costs. In contrast, in treatments with a high-value agent, experienced agents often form stars, most frequently with the high-value agent in the center.

Given that stars only form in some of the treatments, asking for 'exact' stars may be too restrictive given the huge coordination problem subjects face in the experiment. We therefore also investigate two less restrictive measures: one is based on the notion of 'almost-stars' and the other is based on the 'centrality' of a network.

Almost stars: close-to-star networks, or almost stars, are defined as networks that can be transformed into a star by deleting, adding, or moving a single link. Table 4 depicts the relative and absolute frequencies of general stars and general almost-stars in the seven treatments. The results are akin to those for exact stars. There are more almost stars in later rounds than in earlier rounds and the frequencies of these network structures is clearly largest in treatments where a high-value agent is present. However, there are also some interesting differences compared to the more restrictive exact-star measure. In early rounds of treatment BI there are still only a few almost stars (5.7%), but in the last five rounds their frequency increases to 31.4%. Thus, although subjects almost completely fail to form exact stars it seems that with experience they form almost-stars to some extent. This pattern is more pronounced in treatment CN, where in early rounds only 3.3% of all networks are almost stars and this frequency increases to 56.7% in the last five rounds. No such pattern is observed in treatment CI. This corroborates the earlier finding that in treatments with only a low-cost agent incomplete information facilitates the formation of (almost) stars. It also further confirms that forming stars with the distinctive agent in the center is not an obvious or focal outcome.

Table 4: Frequency of stars and almost stars (of any form) in the different treatments.

Treatment	rounds 1-5		rounds 26-30		all rounds	
BI	5.7%	(2/35)	31.4%	(11/35)	15.2%	(32/210)
CI	23.3%	(7/30)	36.7%	(11/30)	18.9%	(34/180)
CN	3.3%	(1/30)	56.7%	(17/30)	35.0%	(63/180)
VI	40.0%	(8/20)	75.0%	(15/20)	59.2%	(71/120)
VN	13.3%	(4/30)	83.3%	(25/30)	61.1%	(110/180)
CVI	0.0%	(0/20)	100.0%	(20/20)	56.7%	(68/120)
CVN	6.7%	(2/30)	90.0%	(27/30)	51.7%	(93/180)

The tendency towards star like structures in CVN and, especially, CVI is worth noting. In treatment CVN, in early rounds, only 6.7% of all networks are almost stars but in the

and in round 30 three stars are formed while in any of the previous rounds at most 1 star is observed. In the other treatments with a significantly positive correlation coefficient the increase in the number of stars over time exhibits a much stronger pattern.

last five rounds this frequency increases to 90%. For CVI we already saw that in the first five rounds no exact star is formed. Table 4 shows that in the early rounds not even an almost star is observed. Nevertheless, at the end of the game all but one network is a star (see above) and the remaining network turns out to be an almost star.

To test whether the changes in the frequencies of almost-star networks over time are significant we calculate Spearman rank order correlation statistics for all treatments. For treatments VI, VN, CN, CVI, and CVN the correlation coefficient is significantly greater than zero ($\rho > 0.75$, $p < 0.0001$; two-sided tests). For treatment BI the correlation turns out to be only marginally significant ($\rho = 0.31$, $p = 0.0938$; two sided test) and insignificant for treatment CI ($p = 0.4949$; two-sided test).

To summarize, stars are observed mainly in later rounds and only in treatments with a high-value agent. When relaxing the criterion to almost stars, other treatments exhibit some tendency towards star like structures, with the noticeable exception of treatments CI and BI.

Network centrality: the second measure we use to examine whether the observed networks come close to stars is the centrality of a network. Individual centrality is a measure that is often employed in sociological studies to analyze how central (strong, important) agents' positions are in a network regarding, e.g., communication flow, interaction possibilities, or power.[22] One can build upon the individual centrality measures to construct a measure of centrality for the whole network. While various definitions are possible, we use the so called 'degree-centrality' measure, which is defined as the sum of the differences in degrees between the most central agent and all other agents. Formally,

$$cent(g) = \frac{1}{20} \sum_{i \in N} \left[\max_{j \in N} \deg_j(g) - \deg_i(g) \right].$$

Important properties of degree-centrality (or other measures of centrality proposed in the literature) are that it is bounded and, given our normalization, 0 for 'even' networks such as the empty, complete, and wheel networks, and 1 if and only if the network is a star. Increasing centrality can be interpreted as a movement towards star or star-like networks. We are, therefore, particularly interested if the centrality measure increases over time and whether it approaches 1.

Figure 7 depicts the development of average centrality in all seven treatments over rounds (in blocks of 5 rounds). It shows that in early rounds the centrality of networks is very similar across treatments, with the exception of treatment VI. Indeed, a Kruskal-Wallis equality of populations rank test does not reject the hypothesis that the centrality measures across the first five rounds are the same in all treatments ($p = 0.5411$). The development of centrality differs strongly across treatments, however. For treatments CN, VI, VN, CVI, and CVN centrality clearly shows an upward trend. In contrast, in BI and CI no such tendency is observed. Restricting attention to the average centrality in the

[22] See e.g. Freeman (1979) and Wasserman and Faust (1994) for detailed discussions of centrality measures.

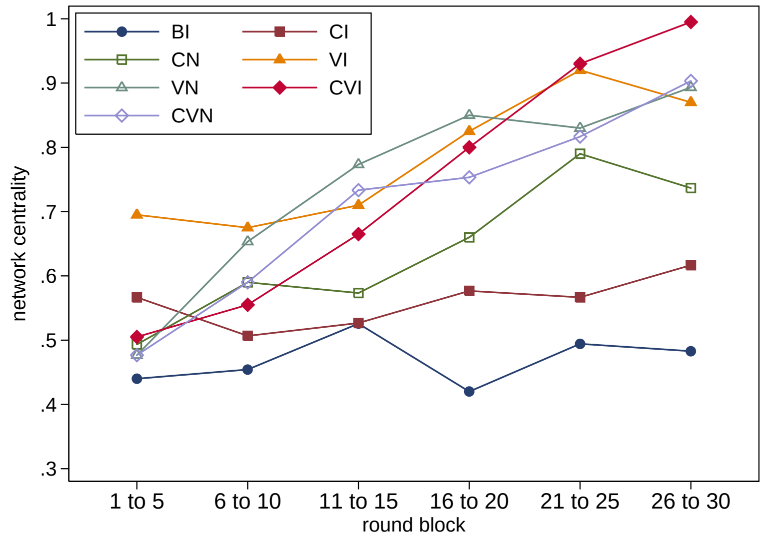


Figure 7: Development of centrality over time.

last five rounds, a Kruskal-Wallis test rejects the hypothesis that centrality is the same in all treatments ($p = 0.0044$).[23]

To test whether the treatments network centrality approaches 1 we conducted Wilcoxon signed-rank tests based on the network centrality averaged across the last five rounds. The null hypothesis is that network centrality is equal to 1. We reject this hypothesis for treatments BI ($p = 0.0178$), CI ($p = 0.0277$), CN ($p = 0.0350$), and marginally also for CVN ($p = 0.0523$). In the other treatments, i.e., VI, VN, and CVI, we cannot reject the hypothesis ($p \geq 0.3173$; all tests two-sided). Hence, our earlier conclusions regarding star formation in the different treatments are corroborated.

3.3 How efficient are the observed networks?

The prevalence of PSS_v in CVI indicates that the formation of stars does not necessarily lead to efficient outcomes. In this treatment the CSS_c network would be efficient but is never observed. Figure 8 shows the development of the relative efficiency of observed networks for the different treatments aggregated over blocks of five rounds. Clearly, efficiency in early rounds (rounds 1-5) is similar across treatments (only in VI more efficient networks are formed).[24] Figure 8 shows that relative efficiencies in treatments VI and VN increase and converge to a common level. A similar pattern holds for treatments CVI

[23] Pairwise comparisons using Mann-Whitney tests show that equality of centrality in BI and CI can not be rejected ($p = 0.2827$; two-sided test) and that centrality in VI, VN, CVI, and CVN is higher than in BI and CI (all comparisons at the 5 percent significance level, except for CI vs. VI where $p = 0.084$; two-sided tests).

[24] This impression is corroborated by a Kruskal-Wallis test for equality of populations between all treatments except VI ($\chi^2_{(5)} = 4.575$, $p = 0.4699$, two-sided test). Mann-Whitney tests comparing (pairwise) early-round efficiency in VI with other treatments shows significant differences in all cases at least at the 5 percent level, except for VI vs. VN where the difference is only marginally significant ($p = 0.0550$; all tests two-sided).

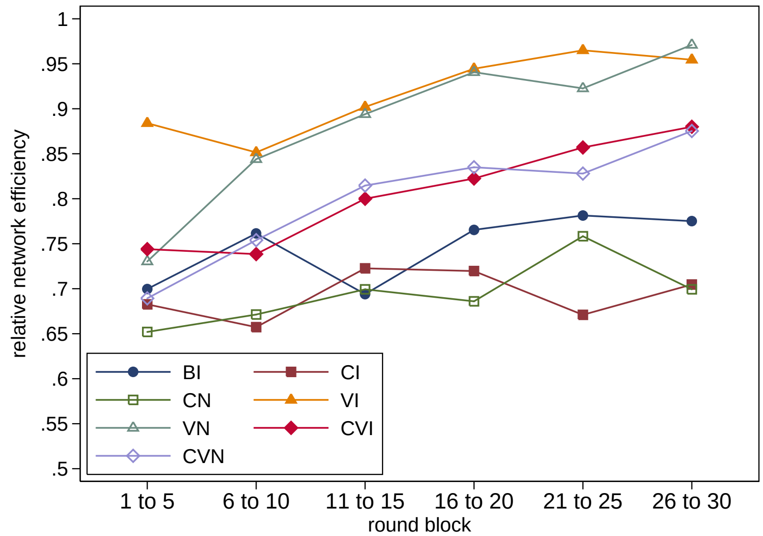


Figure 8: Development of efficiency over time.

and CVN. Over time the lowest efficiency levels occur in treatments CI and CN, with BI ending up between the latter and CVI and CVN. Using data of only the last five rounds, a Kruskal-Wallis test clearly rejects the hypothesis that the efficiency levels for the seven treatments are the same ($\chi^2_{(6)} = 21.034,\ p = 0.0018$, two-sided test).[25]

In summary, observed network efficiency in the different treatments parallels the results regarding star formation. Efficiency levels are lowest in treatments where no high-value agent is present. In treatments with a high-value and a low-cost agent, there is some tendency to reach lower efficiency levels than in treatments with only a high-value agent.

3.4 How stable are the observed networks?

We define the relative stability of a network in round t as the number of agents who do not change any links from round $t-1$ to round t divided by the total number of agents. Hence, if no agent changes a link, relative stability is 1 while it is 0 if all agents change at least one link.

Figure 9 depicts the development of network stability over time for all treatments (in blocks of five rounds). There is a clear separation between treatments with and without a high-value agent. In the last few rounds, networks in treatments CVI, CVN, VI, and VN approach full stability. On average, the stability measure for these treatments is between 0.922 (in CVN) and 0.983 (in CVI). So, roughly speaking, towards the end of the experiment at most one person changes her behavior every other round. Relative stability is not significantly different from one for treatments VI, VN, and CVI ($p \geq 0.1590$;

[25] Furthermore, in the last five rounds, Mann-Whitney pairwise comparisons show that efficiency levels are not significantly different for treatments BI, CI, CN ($p \geq 0.1985$; two-sided). Efficiency levels in each of these treatments are significantly lower than in CVI and CVN ($p \leq 0.0430$; one-sided), which are not different from each other ($p = 0.3367$; two-sided). Efficiency in VN is significantly larger than in CVI and CVN ($p < 0.0297$; one-sided) whereas the levels between VI and VN, and between VI, CVI and CVN do not significantly differ ($p \leq 0.1939$; two-sided).

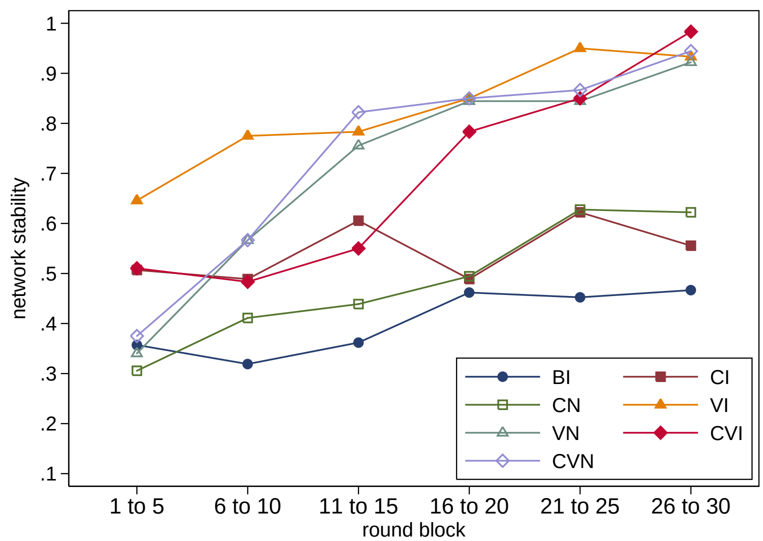


Figure 9: Development of network stability over time.

two-sided Wilcoxon signed-rank test). In treatment CVN relative stability is also large but marginally different from one ($p \geq 0.0516$; two-sided Wilcoxon signed-rank test), indicating that network formation did not completely settle down.

In the treatments without a high-value agent, networks remain unstable over time and there is no tendency towards stability. In the last five rounds, average relative stability is between 0.467 (in BI) and 0.622 (in CN), meaning that on average between 2.3 and 3.2 agents change behavior in each round. In the last five rounds, stability does not significantly differ across treatments BI, CI, and CN ($\chi^2_{(2)} = 3.018$, $p = 0.2211$; Kruskal-Wallis test, two-sided). Mann-Whitney tests show that compared to the treatments with a high-value agent, stability is significantly smaller (pair-wise comparisons: $p < 0.05$ in all cases except CN vs. VI where $p = 0.0521$; two-sided tests).

In summary, network stability strongly improves in the presence of a high-value agent and networks reach (almost) full stability at the end of the experiment. In stark contrast, networks remain unstable in treatments without a high-value agent.

3.5 Determinants of individual linking behavior

The observed network structures are ultimately the outcome of individual linking decisions. To understand why the observed networks are so different across treatments - unlike the theoretical predictions - we study possible determinants of individual behavior.

Recent models of generalized preferences assume that subjects' behavior not only depends on their material income but also on their relative standing towards others (e.g., Fehr and Schmidt (1999) and Bolton and Ockenfels (2000)).[26] In addition, behavior may be influenced by the "well-being" of the relevant reference group as a whole. Charness

[26] Fehr and Schmidt (1999) model agent i's utility, given a profile of monetary payoffs $(x_1, \ldots, x_i, \ldots, x_n)$, as $u(x_1, \ldots, x_i, \ldots, x_n) = x_i - \frac{\alpha_i}{n-1} \sum_{j \neq i} \max\{x_j - x_i, 0\} - \frac{\beta_i}{n-1} \sum_{j \neq i} \max\{x_i - x_j, 0\}$, where α_i is an envy coefficient and β_i a guilt coefficient.

and Rabin (2002) incorporate the latter idea in their model as a "concern for efficiency." Both motives, inequality aversion and efficiency concerns, may have consequences for the network formation process. Indeed, Falk and Kosfeld (2005) find evidence for inequality aversion being an important reason for the absence of stars in their experiments. Extending their ideas we investigate if linking behavior is influenced by envy, guilt and/or efficiency concerns. Like Falk and Kosfeld (2005), we control for standard best-reply behavior, which may also be interpreted as a proxy for the influence of a subject's own earnings on behavior. We conduct probit regressions to examine how a subjects' likelihood to stick to the same choice in round t as in round $t-1$ depends on their relative standing towards others in the group as well as total group earnings, where we control for previous best-reply play and time (assuming a linear trend). We conduct these regressions for all treatments but pool the data of the complete and incomplete information conditions, and measure its effect with a dummy variable. Since we observe very different networks in the different treatments, we are particularly interested whether individual behavior differs across treatments and agent types.

More precisely, we estimate the following probit regression separately for the different heterogeneity conditions (pooled over information conditions):[27]

$$\begin{aligned} \mathrm{P}(\text{no change}_{i,t}) \quad = \quad & \Phi\big(\alpha_0 + \alpha_{1,i}\, BR(t-1)_i + \alpha_{2,i}\, \mathrm{eff}(t-1) + \\ & \qquad \alpha_{3,i}\, \mathrm{envy}(t-1)_i + \alpha_{4,i}\, \mathrm{guilt}(t-1)_i + \alpha_5\, t + \alpha_6\, \mathrm{info}\big), \end{aligned}$$

Table 5 displays the results of the regression analysis. Note that in all treatments, normal agents (o) have a strong tendency to stick to their previous round behavior if this was a best reply. This indicates that individual earnings maximization is an important motive for normal agents in all treatments.[28] Individual earnings also seem important for high-value agents (see the results for treatments VI&VN and CVI&CVN). For the low-cost agent, results are less clear-cut. In CVI&CVN these agents show a significant tendency to stick to their behavior if it was best-reply in the last round but this is not the case when no high-value agent is present.

Concern for efficiency is an important motive for all agent types. For normal (high-value) agents the regression coefficient for our efficiency measure is significantly positive in all treatments (where they are present). For low-cost types the corresponding coefficient is (marginally) significant when a high-value type is present but seems to be unimportant if this is not the case.

[27]Here $i = o, c, v$ denotes the agent type (normal, low-cost, and high-value respectively), t denotes the round number, and no change$_{i,t}$ is 1 if i made the same links in round t as in round $t-1$ and 0 otherwise. On the right hand side, $\Phi(\cdot)$ denotes the standard normal cumulative, $BR(t-1)_i$ equals 1 if agent i played a best response in round $t-1$ and 0 otherwise, $\mathrm{eff}(t-1)$ equals the sum of earnings in agent i's group in round $t-1$, $\mathrm{envy}(t-1)_i$ is defined as $\frac{1}{5}\sum_j \max\{x_j - x_i, 0\}$ and $\mathrm{guilt}(t-1)_i$ as $\frac{1}{5}\sum_j \max\{x_i - x_j, 0\}$ where x_j is subject j's monetary payoff, and, finally, info equals 1 for the complete information treatment and 0 otherwise. Best responses in the latter treatments are computed as if agents' types are known, since subjects in the experiment were able to resolve the incomplete information about types fast.

[28]This motive appears strongest in the treatments with a high-value agent (VI&VN and CVI&CVN). The coefficients for these treatments are about twice as large as for BI and CI&CN.

Table 5: Determinants of the likelihood of no change in linking (probit regressions).

	dependent variable: likelihood to keep same links in $t-1$ as in t			
treatments	BI	CI&CN	VI&VN	CVI&CVN
constant	-.6572***	-.2786	-3.2323***	-1.4675***
	(.1948)	(.2278)	(.6150)	(.3316)
$BR(t-1)_o$	.4378***	.4850***	1.1429***	.8924***
	(.1561)	(.0881)	(.1495)	(.0927)
$BR(t-1)_c$		.1785		.6874**
		(.2101)		(.2867)
$BR(t-1)_v$			2.0391***	.8316**
			(.4401)	(.3418)
eff$(t-1)_o$	.0032***	.0021*	.0088***	.0039***
	(.0005)	(.0011)	(.0019)	(.0009)
eff$(t-1)_c$		-.0001		.0023*
		(.0018)		(.0013)
eff$(t-1)_v$			.0050**	.0031***
			(.0022)	(.0011)
envy$(t-1)_o$	-.0590***	-.0524***	-.0112	-.0328***
	(.0109)	(.0098)	(.0090)	(.0072)
envy$(t-1)_c$		.0445		-.0294
		(.0360)		(.0256)
envy$(t-1)_v$			.0086	.0022
			(.0161)	(.0088)
guilt$(t-1)_o$	-.0254***	-.0345***	.0253***	.0161
	(.0048)	(.0079)	(.0096)	(.0112)
guilt$(t-1)_c$		-.0104		.0078
		(.0138)		(.0176)
guilt$(t-1)_v$			.0280	.0537***
			(.0184)	(.0171)
round t	.0067**	.0097**	.0337***	.0495***
	(.0028)	(.0042)	(.0071)	(.0122)
info		.1879	.3095	-.2910***
		(.1561)	(.1880)	(.0935)
log likelihood	-747.19	-1291.92	-521.15	-658.53
Pseudo R^2	0.0910	0.1065	0.4487	0.3512
# of obs.	1218	2088	1740	1740

Note: *** (**) [*] indicates significance at the 1% (5%) [10%] level; standard errors are corrected for dependence of observations within groups.

Envy is an important behavioral motive only for normal agents. In three of the four regression models the experience of negative inequality significantly increases the likelihood that normal types change their behavior. Interestingly, this effect is statistically insignificant in VI&VN. The effect of guilt is also interesting. For normal types we observe the intuitive result that positive inequality increases the likelihood to change behavior only in treatments without a high-value agent. With a high-value agent present, the likelihood to stick to the current behavior is positively related to guilt and this effect is highly significant in VI&VN but insignificant in CVI&CVN. A similar result is observed for high-value types: in CVI&CVN we find a significantly positive effect of guilt while in VI&VN the guilt coefficient is insignificant.

In all treatments, a subject's likelihood to stick to her previous round behavior significantly increases over time, indicating that behavior stabilizes with repetition. Finally, the information dummy is significant only in CVI&CVN, consistent with the differences in observed equilibrium networks and star-like architectures (Table 3 and Figure 6).

The main lesson to be taken from these results is that linking behavior depends on an agent's own type and the types of other agents in the network. In particular, standard income maximization is a prominent concern for all types of agents only when the high-value agent is present. Likewise, efficiency considerations are important for all types when a high-value agent is present, while this is not the case when the low-cost agent is the only 'different' type in the group. Finally, envy is generally important for normal types but not when faced with opportunity to link to a high-value agent.[29]

4 Conclusion

This paper reports the first experimental study on endogenous network formation with heterogeneous agents. We consider several implementations of individual heterogeneity and allow agents' types to be private information or publicly known. Our simple parametrization of agent heterogeneity simplifies the analysis of equilibrium networks and allows us to measure the effects of a small change in agent types. We provide a complete characterization of the set of (Bayesian) Nash equilibria for all treatments and consider approximations in terms of "almost star networks and by measuring network centrality. Our interest in doing the experiments is not simply in testing theory. Indeed, the main motivation for this work is to uncover factors that are relevant in actual network formation.

Casual empiricism suggests that individual differences may facilitate the creation of uneven, hierarchal structures such as star networks. Some people have lower opportunity costs or a taste for networking, while others may possess skills or information that is relatively more scarce. We conjectured that the presence of these "special" individuals facilitates the creation of star-type networks because it reduces the coordination problem and makes the resulting payoff inequalities more acceptable.

The experimental results corroborate this intuition. We find that almost no stars form among symmetric agents. In contrast, the introduction of different types of agents has a dramatic impact on linking behavior and observed networks: stars are prevalent in most heterogeneous treatments. Stars are not born, however, but grow over time. Early on in

[29] We also estimated a model where agents make logit best-responses against the previous round network. We find similar levels of noise in the baseline and CVI&CVN treatments ($\mu = 9.0$ and $\mu = 8.7$ respectively) but lower levels in the VI&VN and CVI&CVN treatments ($\mu = 4.5$ and $\mu = 2.2$). When the basic logit model is "dressed up" with other-regarding preferences, the estimated envy and guilt parameters vary across treatments, like in Table 5.

the experiment almost no stars are formed, which suggests that a star with a high-value or a low-cost agent in the center is not an obvious or focal outcome. But towards the end of the experiment, stars are the most prevalent architecture in treatments with a high-value agent. This finding is underlined by the evolution of network centrality, network efficiency, and network stability, which all show a strong trend towards stable star networks.

While the introduction of a high-value agent strongly improves star formation, this is not the case for the introduction of a "networker" with lower linking costs. Again this suggests it is not simply "focalness" that determines the empirical likelihood of star formation. An important difference is that with a low-cost agent the focal network is a center-sponsored star where the center may develop a tendency to remove individual links anticipating these will be replaced by the periphery (who lose more than the center from the center's deviation). In contrast, the focal network with a high-value agent is the periphery-sponsored star where the center has no opportunity to deviate by removing links. This difference in "central power" can explain why stability does not evolve over time in the treatments with a low-cost agent and, hence, no stars develop.

The effects of incomplete information also depend on the types of agents in the network. With an unknown high-value agent, subjects are quick to find this agent and form a periphery-sponsored star. In this case, network formation is similar to the complete-information case. In contrast, with a low-cost agent, incomplete information *raises* the occurrence of stars with the low-cost agent at the center. When both types of agents are present, incomplete information aggravates the coordination problem and fewer stars are formed. Moreover, a significant fraction of the observed stars do not have the high-value agent at the center unlike the complete information case where all stars are of this type.

We find that "other regarding" preferences, such as a taste for efficiency, envy, or guilt, also play a role in actual network formation. In most of our treatments, envy (more than guilt) and a taste for efficiency are significant determinants of individual linking behavior. However, in treatments with a high-value agent, envy appears less important and standard payoff maximization and a taste for efficiency dominate. More generally, our experimental results highlight several empirically relevant factors for network formation.

References

Bala, V. and Goyal, S. (2000). A non-cooperative model of network formation. *Econometrica*, 68:1131–1230.

Barabási, A.-L. (2002). *Linked: The New Science of Networks*. Promeus.

Barabási, A.-L. and Albert, R. (1999). Emergence of scaling in random networks. *Science*, 286(286):509–512.

Bolton, G. and Ockenfels, A. (2000). ERC - a theory of equity, reciprocity and altruism. *American Economic Review*, 90:166–193.

Callander, S. and Plott, C. (2005). Principles of network development and evolution: An experimental study. *Journal of Public Economics*, 89:1469–1495.

Charness, G. and Rabin, M. (2002). Understanding social preferences with simple tests. *Quarterly Journal of Economics*, 117:817–869.

Corbae, D. and Duffy, J. (2003). Experiments with network economies. mimeo.

Falk, A. and Kosfeld, M. (2005). It's all about connections: Evidence on network formation. Working paper, University of Zürich.

Fehr, E. and Schmidt, K. M. (1999). A theory of fairness, competition, and cooperation. *The Quarterly Journal of Economics*, 114:817–868.

Freeman, L. (1979). Centrality in social networks: Conceptual clarification. *Social Networks*, 1:215–239.

Fudenberg, D. and Tirole, J. (1991). *Game Theory*. MIT Press, Cambridge, Massachusetts.

Galeotti, A., Goyal, S., and Kamphorst, J. (2005). Network formation with heterogeneous players. *Games and Economic Behavior*. Forthcoming.

Goyal, S., van der Leij, M., and Moraga-Gonzáles, J. L. (2004). Economics: An emerging small world? mimeo.

Haller, H., Kamphorst, J., and Sarangi, S. (2006). (non-)existence and scope of nash networks. Forthcoming.

Haller, H. and Sarangi, S. (2005). Nash networks with heterogeneous links. *Mathematical Social Sciences*, 50:181–201.

Jackson, M. O. (2004). A survey of models of network formation: Stability and efficiency. In Demange, G. and Wooders, M., editors, *Group Formation in Economics: Networks, Clubs and Coalitions*. Cambridge University Press, Cambridge.

Jackson, M. O. (2005). The economics of social networks. In Blundell, R., Newey, W., and Persson, T., editors, *Proceedings of the 9th World Congress of the Econometric Society*. Cambridge University Press, Cambridge.

Jackson, M. O. and Rogers, B. (2005). Search in the formation of large networks: How random are socially generated networks? mimeo.

Jackson, M. O. and Wolinsky, A. (1996). A strategic model of social and economic networks. *Journal of Economic Theory*, 71:44–74.

Johnson, C. and Gilles, R. P. (2000). Spatial social networks. *Review of Economic Design*, 5:273–299.

Kosfeld, M. (2004). Economic networks in the laboratory: A survey. *Review of Network Economics*, 3:20–42.

McBride, M. (2005). Position-specific information in social networks: Are you connected? mimeo.

Newman, M. E. J. (2004). Coautorship networks and patterns of scientific collaboration. *Proceedings of the National Academy of Sciences*, 101:5200–5205.

Wasserman, S. and Faust, K. (1994). *Social Network Analysis: Methods and Applications*. Cambridge University Press.

A Appendix: Proofs.

In Lemma 4 below we characterize for each incomplete information treatment the expected payoff of each agent in each possible network. We next use this characterization to show in Proposition 5 below that all (Bayesian) Nash networks in all treatments are either minimally connected or empty. With the help of Lemma 4 and Proposition 5 we then prove Propositions 1, 2 and 3.

The decay function used in our experiments can be expressed as $\Phi(v_i, d) = v_i \frac{\nu^\circ(d)}{16}$, with ν given by $(\nu^\circ(1), \nu^\circ(2), \nu^\circ(3), \nu^\circ(4), \nu^\circ(5), \nu^\circ(\infty)) = (16, 12, 9, 7, 5, 0)$, see Table 1b. Hence, the benefit from accessing a normal value agent at distance d is $\Phi(16, d) = \nu^\circ(d)$, while the benefit from accessing a high-value agent at the same distance is $\Phi(32, d) = 2\nu^\circ(d)$. The following Lemma characterizes an agent's expected payoff in the incomplete information treatments.

Lemma 4 *Given a cost/value profile θ, the expected payoffs of different types of agents in the incomplete information treatments are:*
1. for $i \neq i_v$ in VN and CVN: $E_i u_i(g; \theta_i) = \frac{6}{5} \sum_{j \neq i} \nu^\circ(d(i, j; g)) - \mu_i(g)c$,
2. for i_v in VN and CVN and for any i in CN: $E_i u_i(g; \theta_i) = \sum_{j \neq i} \nu^\circ(d(i, j; g)) - \mu_i(g)c$.

Proof. Note from (2) that agent i's payoff is independent of the linking costs incurred by other agents $j \neq i$. Agent i's expected payoff therefore depends only on her beliefs about others' values.

1. Given the realized profile θ, any agent $i \neq i_v$ in treatments VN and CVN knows that any of the other agents is equally likely to be the unique high-value agent. For each $k \neq i$, let $w^k = (w_1^k, ..., w_n^k)$ be one of the five possible value profiles from i's perspective, with $w_k^k = 32$ and $w_j^k = 16$ for $j \neq k$, and $p_i^{\theta_i}[w^k] = \frac{1}{5}$. Agent i's expected payoff, given the network g, is

$$
\begin{aligned}
E_i u_i(g; \theta_i) &= \sum_{k \neq i} p_i^{\theta_i}[w^k] \sum_{j \neq i} \Phi(w_j^k, d(i, j; g)) - \mu_i(g) c_i(\theta) \\
&= \sum_{k \neq i} \frac{1}{5} \sum_{j \neq i} \frac{w_j^k}{16} \nu^\circ(d(i, j; g)) - \mu_i(g) c_i = \frac{1}{80} \sum_{j \neq i} \nu^\circ(d(i, j; g)) \sum_{k \neq i} w_j^k - \mu_i(g) c_i \\
&= \frac{1}{80} \sum_{j \neq i} \nu^\circ(d(i, j; g)) 96 - \mu_i(g) c_i = \frac{6}{5} \sum_{j \neq i} \nu^\circ(d(i, j; g)) - \mu_i(g) c_i.
\end{aligned}
$$

2. Given the realized profile θ, all agents in treatment CN and agent i_v in treatments VN and CVN know the value of any other agent is 16. Hence, for these agents the expected payoff, given the network g, is $E_i u_i(g; \theta_i) = \sum_{j \neq i} \Phi(16, d(i, j; g)) - \mu_i(g) c_i = \sum_{j \neq i} \nu^\circ(d(i, j; g)) - \mu_i(g) c_i$. ■

Proposition 5 *With complete or incomplete information, (Bayesian) Nash networks are either minimally connected or empty.*

Proof. It is easy to see that a (Bayesian) Nash network is minimally sponsored: an agent incurs costs but no additional benefits from making a link to another agent who already maintains this link. It is also straightforward to see that whenever a link is made between $i \neq j$, any other agent k that does not access i or j is willing to sponsor a link with either of them. This implies that a (Bayesian) Nash network is either empty or connected. Next we show that such a network has no cycles. We do this by arguing that in any minimal cycle there is at least one agent who benefits from removing a link from the cycle. Below we use the notation ij to denote a link made by i to j.

Treatments BI,VI,CI: Let g be a minimally sponsored network with a shortest circle of length $k \geq 3$. It is easy to see that in any such circle at least one link must be sponsored by an agent of normal linking cost whose distance to the high-value agent (if present) does not change if she removes that link. Let $(1, ..., k, 1)$ denote such a circle and $1k$ be the corresponding link. Recall that the linking cost of (normal) agent 1 is $c = 24$.

If 1 removes the link $1k$, yielding the network g_{-1k}, the distances between her and other agents may increase, which lowers agent 1's benefits. It is convenient to define $\Delta\nu^{\circ}(d, \Delta d) = \nu^{\circ}(d) - \nu^{\circ}(d + \Delta d)$ as the loss in benefits that result when the distance (to a normal agent) increases from d to Δd. The loss function $\Delta\nu^{\circ}(d, \Delta d)$ is decreasing in d and increasing in Δd. For each j, let $\Delta_{ik}d(i, j; g) = d(i, j; g_{-ik}) - d(i, j; g)$ denote the increase in the distance between i and j when link ik is removed from g, and let

$$\begin{aligned}
\Delta_k \pi_i(g) &= \pi_i(g) - \pi_i(g_{-ik}) \\
&= c_i + \sum_{j \neq i} \frac{v_j}{16} \left[\nu^{\circ}(d(i, j; g)) - \nu^{\circ}(d(i, j; g_{-ik}))\right] \\
&= c_i - \sum_{j \neq i} \frac{v_j}{16} \Delta\nu^{\circ}(d(i, j; g), \Delta_{ik} d(i, j; g))
\end{aligned}$$

denote the corresponding change in agent i's payoff. When agent 1 removes the link $1k$, the distances between herself and agents $\{2, ..., \lceil \frac{k}{2} \rceil\}$ remains the same, but the distances to all (normal-valued) other agents may increase. Her distance to any agent $j > k$ increases by at most $k - 2$, hence $\Delta\nu^{\circ}(2, k - 2)$ is the maximum possible decrease in benefits for i obtained from a normal agent j with whom i did not have a direct link in g. The change of agent 1's payoff, when she removes the link $1k$, is therefore bounded by

$$\Delta_k \pi_1(g) \geq 24 - \sum_{j=\lceil \frac{k}{2} \rceil + 1}^{k} \left[\nu^{\circ}(j - 1) - \nu^{\circ}(k - j + 1)\right] - (n - k)\Delta\nu^{\circ}(2, k - 2). \qquad (4)$$

For $k = 3, 4, 5, 6$ this bound is strictly positive ($\Delta_3\pi_1(g) \geq 11$, $\Delta_4\pi_1(g) \geq 7$, $\Delta_5\pi_1(g) \geq 5$, $\Delta_6\pi_1(g) \geq 8$), which implies that agent 1 always benefits from removing the link $1k$.

Treatment CVI: In a Nash network the low-cost agent i_c will always be linked with the high-value agent i_v. Indeed, i_c is willing to sponsor the link with i_v in any minimally sponsored g as the loss from removing such a link is at least $2\Delta\nu^{\circ}(d(i_c, i_v; g), \Delta d_{i_c,i_v}(i_c, i_v; g)) \geq$

$2\Delta\nu^{\circ}(1,1) = 8$, which exceeds i_c's linking cost of 7. Since i_c and i_v are linked, in any circle of length $k \geq 4$ at least one link is sponsored by a normal agent whose distance to the high-value agent (if present) does not change if she removes that link. The same conclusion holds for a circle of length $k = 3$ where either i_c or i_v is not part of the circle. In both situations, the proof that applied to the BI, VI, and CI cases can be replicated.

Finally, suppose that both i_c and i_v are part a cycle of length $k = 3$, including a normal agent i who sponsors the link with i_v.[30] By removing this link agent i gains at least $\Delta_{i_v}\pi_i(g) \geq 24 - (32/16)\left[\nu^{\circ}(2) - \nu^{\circ}(1)\right] - 3\Delta\nu^{\circ}(2,1) = 7$.

<u>Treatments VN,CN,CVN</u>: Let θ be a feasible allocation of types. Lemma 4 implies that, compared to the complete information case, normal agents' benefits are multiplied by factor 6/5 in treatments VN and CVN while they are unchanged in treatment CN. Define $\Delta_k E_i u_i(g;\theta_i) = E_i u_i(g;\theta_i) - E_i u_i(g_{-ik};\theta_i)$. Again, let $(1, ..., k, 1)$ be the shortest cycle in g and let agent 1 satisfy the asssumptions above. Following the steps of the proof for treatment BI, inequality (4) becomes

$$\Delta_k E_1 u_1(g;\theta_1) \geq 24 - \frac{6}{5} \sum_{j=\lceil \frac{k}{2} \rceil + 1}^{k} \left[\nu^{\circ}(j-1) - \nu^{\circ}(k-j+1)\right] - \frac{6}{5}(n-k)\Delta\nu^{\circ}(2, k-2).$$

for treatments VN and CVN. The corresponding bounds ($\Delta_3 E_1 u_1(g;\theta_1) \geq 8.4$, $\Delta_4 E_1 u_1(g;\theta_1) \geq 3.6$, $\Delta_5 E_1 u_1(g;\theta_1) \geq 1.2$, $\Delta_6 E_1 u_1(g;\theta_1) \geq 4.8$) are all strictly positive. For treatment CN the bounds found in the proof for treatment BI apply. ■

Proof of Proposition 1. We leave it to the reader to verify that the listed networks are strict Nash. Note that the benefit to a normal cost agent for making a link with an isolated normal value agent is 16 and the cost is 24. In a Nash network (hereafter NN) the normal cost agent therefore never links to a normal periphery agent.

Next consider two distinct periphery agents i and j, each being the sponsor of their (only) link. Let i be linked to i_1 and j be linked to $j_1 \neq i_1$. If the network is Nash, agent i must find linking to i_1 at least as beneficial as linking to j_1. If j removes her link with j_1 then agent i should find linking to i_1 strictly more beneficial than linking to j_1. But then agent j should find linking to i_1 also strictly more beneficial than linking to j_1, which contradicts the assumption that the network is Nash. Thus in a Nash network all periphery agents who sponsor their only link are linked to the same central agent.

This provides some intuition for the predominance of periphery-sponsored stars among the NN. The rest of the proof proceeds by considering each treatment separately. Proposition 5 implies that, in a non-empty NN, each pair of agents is connected by a single path.

<u>Treatment BI</u>: Let g be a non-empty, minimally connected NN. The two agents i and j who are furthest apart in g must be periphery and therefore sponsor their own link. They must therefore be linked to the same agent, which implies that 2 is the maximal distance

[30] If link $i_v i$ is sponsored by i_v, then the proof above applies.

between agents in g. The star network is the only minimally connected network with maximal distance of 2. The only non-empty NN is therefore the periphery-sponsored star.

Treatment VI: Let g be a NN. Normal agents are always willing to sponsor a link with an isolated i_v so g cannot be empty. Let i and j be the two periphery agents who are furthest apart in g. Either both sponsor their own link and g is a star network, or one of them is i_v. Let the path $(i_v, i_1, ..., j_1, j)$ be the longest path in g. If $i_1 \neq j_1$ then j decreases the average distance to other agents by removing the link with j_1 and making a link with i_1. Therefore $i_1 = j_1$, the maximal distance is 2, and g is a star network in which all normal periphery agents sponsor their link.

Treatment CI: Let g be a NN. Agent i_c is willing to sponsor a link with any agent i who, in absence of such link, would be more than 2 links away or isolated. Hence g cannot be empty. Let i and j be the two periphery agents who are furthest apart in g. If both sponsor their own link then g is a star network. If the center of g is a normal agent then g is periphery-sponsored.

If $d(i,j) \geq 3$ then agent i_c must be sponsoring the link with either i or j, say with i. We argued above that $d(i_c, j) \leq 2$ for each j which, together with $d(i_c, i) = 1$ implies $d(i,j) = 3$. Let j be linked with j_1. All other periphery agents sponsoring their own link must be linked with j_1. Agent i_c sponsors the links with all agents not linked to j_1. This defines the linked star $\mathrm{LS}^p_{i_c j_1}$. It is easy to verify that if $p \geq 2$ all other periphery agents would prefer linking to i_c. The star networks PSS and MSS_c and the linked star $\mathrm{LS}^1_{i_c j_1}$ are the only NN.

Treatment CVI: Let g be a NN. Agent i_c is willing to sponsor a link to i_v regardless of other links. Therefore, a star network with a normal agent in the center cannot be NN. If g is not a star then it is a linked star with agent i_c in one center, see the proof for treatment CI. If i_v is a periphery agent linked with i_c, all other agents would also link with i_c, see the proof for treatment VI. Agent i_v must therefore be in the other center of the linked star. It is easy to verify that g is NN if all or less than three periphery normal agents are linked with i_c, which excludes all networks but the MSS_c, and any LS^2_{cv}, LS^1_{cv} and $\mathrm{LS}^0_{cv} = \mathrm{PSS}_v$. ■

Proof of Proposition 2. Again, we leave it to the reader to verify that the listed networks are strict Bayesian Nash. The proof closely follows the steps in the proof of Proposition 1, using the expected payoffs characterized by Lemma 4. In particular, characterization of Bayesian Nash networks in treatment VN follows the same steps as those for treatment BI, and characterization of Bayesian Nash networks in treatments CN and CVN follows those for treatment CI. ■

Proof of Proposition 3. The sum of agents' payoffs in a group can be written as

$$w(g) = \sum_{i,j \in N} \Phi(v_j, d(i,j;g)) - \sum_{i \in N} \mu_i(g) c_i.$$

The efficient network therefore maximizes the aggregate benefits while minimizing the linking costs, i.e. it minimizes the distances between agents using a minimal number of links. Since adding a link to connect any two components of the network increases total payoffs, the efficient network is either empty or connected. In the latter case, it has at least $n-1$ links. Of the networks with precisely $n-1$ links the star networks have minimal average distances. In addition, the CSS_c with the low-cost agent i_c in the center minimizes the aggregate linking costs, while a MSS_v with the high-value agent i_v in the center maximizes the aggregate benefits. One can verify that $w(\text{CSS}_c) > w(\text{MSS}_v)$ when both i_c and i_v are present. Hence, among the star networks, CSS_c is efficient whenever i_c is present and MSS_v are efficient when only i_v is present. Any MSS is the efficient star network in BI. In a star network each new link benefits only the two linked agents. Adding links to the CSS_c thus decreases total payoffs: each link costs 24 but adds only 4 to the payoff of one agent and at most 8 to the payoff of the other (if one agent is a high-value agent). Adding links to a MSS_v or MSS_o when i_c is not present also decreases total payoffs. Finally, total payoffs decrease if links are removed from any MSS. The above-mentioned star networks are therefore the efficient networks. ■

B Appendix: Instructions.

[*Most of the text is the same for all treatments. Treatment-dependent text appears in italics and in brackets* "[...]", *where we indicate the treatments in which the text appears.*]

Welcome to this experiment in decision-making. In this experiment you can earn money. The amount of money you earn depends on the decisions you and other participants make. Please read these instructions carefully.

In the experiment you will earn points. At the end of the experiment we will convert the points you have earned into US dollars [*in Europe:* "euros"] according to the rate: **70 points equal $1** [*in Europe:* "**1 euro**"]. You will be paid your earnings privately and confidentially after the experiment.

Throughout the experiment you are not allowed to communicate with other participants in any way. If you have a question please raise your hand. One of us will come to your desk to answer it.

YOUR GROUP

At the beginning of the experiment the computer will randomly assign you - and all other participants - to a **group of 6 participants**. Group compositions do not change during the experiment. Hence, you will be in the same group with the same people throughout the experiment. The composition of your group is anonymous. You will not get to know the identities of the other people in your group: neither during the experiment nor after the experiment. The other people in your group will also not get to know your identity. On your computer screen, you will be marked **Me** and each of the five other people in your group will be marked with a letter: **A**, **B**, **C**, **D**, and **E**. **Each letter will correspond to the same participant in your group during the entire experiment.**

NUMBER OF ROUNDS

The experiment consists of 30 rounds. You will receive 150 points at the beginning of the first round. In each round you can earn additional points. Your total earnings will be the sum of the initial 150 points and your earnings in each of the 30 rounds.

YOUR LINKING DECISIONS (I)

In each round you will make linking decisions. You will have the possibility to link with any other participant in your group. That is, you can make any number of links (0, 1, 2, 3, 4, or 5). **The link(s) you make in a particular round are only valid for that round.**

As we explain below, you can be "**directly linked**", "**indirectly linked**" or "**not linked**" with any other member of your group. You are **directly linked** with another person if:

- you make a link to that person, or
- if that person makes a link to you, or
- if both of you link to each other.

Neighbors: The people in your group that you are directly linked with are called your **neighbors**.

You are **indirectly linked** with another person if that person is not your neighbor but there is a sequence of links between you and that person.

If there is no sequence of links between you and another person then you are **not linked** (neither directly nor indirectly) to that person.

Distance: The distance between you and someone you are linked with is the **minimal number** of links you have to count to go from you to that person. That is, the distance between you and one of your neighbors is always 1. The distance between you and someone directly linked with one of your neighbors but not to you is 2, and so on. Since there are five other people in your group, the distance between you and a person you are linked with is at least 1 and at most 5.

YOUR LINKING DECISIONS (II)

For each (direct) link that you make you have to pay some costs. You also benefit from all your direct and indirect links with other persons. Other persons also benefit from all their direct and indirect links (including their links with you).

Colors: Each person in your group has a color. There are four **green** persons, one **blue** person, and one **purple** person. At the beginning of the experiment, the computer randomly chooses one person in your group to be **blue**, and another person to be **purple**. [*In treatments CI and CN these sentences explain that the group consists of* "five green and one purple person"; *in VI, VN*: "five green and one blue"; *in BI*: "six persons"] The colors of all people in your group remain the same during the entire experiment.

[*Sentence in treatments CI, VI and CVI*:] You receive information about your color and the color of the other people in your group at the beginning of the experiment.

[*Paragraph in treatments CN, VN and CVN*:] At the beginning of the experiment you will receive information about your own color. You will, however, receive no information about the colors of the other persons in your group. **Throughout the experiment you will never see the colors of the other persons in your group.** The other persons in your group will also receive only the information about their own color and will never see your color.

Costs of linking: In every round, each person incurs a cost for each link this person makes.

[*List in treatments CI,CN,CVI and CVN*:] This cost depends on the color of the person:

- Each **green** ["or **blue**" *added in CVI and CVN*] person incurs a cost of **24 points** for **each link** this person makes.
- The **purple** person incurs a cost of **7 points** for **each link** this person makes.

[*Two sentences in treatments BI, VI and VN*:] You incur a cost of **24 points** for each link **you** make. Other persons also incur a cost of 24 points for each link **they** make.

These linking costs remain the same throughout the experiment. You do not incur any cost for the links that other persons make and other persons do not incur any cost for the links that you make.

Benefits of linking: You will earn points from being linked with other persons in your group. The amount you earn from being linked with another person depends on the distance between you and that person, and on the color of that person. Your benefits from your direct and indirect links are determined as follows:

For **each** ["**green**" *added in VI and VN*; "**green** or **purple**" *added in CVI and CVN*] person you are linked with you receive
- 16 points if the distance between you and that person is 1
- 12 points if the distance between you and that person is 2
- 9 points if the distance between you and that person is 3
- 7 points if the distance between you and that person is 4
- 5 points if the distance between you and that person is 5

[*List added in VI, VN, CVI, CVN*:] If you are linked with the **blue person** you receive
- 32 points if the distance between you and that person is 1
- 24 points if the distance between you and that person is 2
- 18 points if the distance between you and that person is 3
- 14 points if the distance between you and that person is 4
- 10 points if the distance between you and that person is 5

Your total benefits in points are therefore computed by summing the benefits you receive from all the persons you are linked with, directly or indirectly. The benefits for other persons in your group are calculated in the same way from their direct and indirect links.

Note: There is a possibility that you are (indirectly) linked with the same person in more than one way. However, you only benefit once from being linked to this person. Your benefits are determined by the shortest connection (the distance) between you and that person.

CONCLUDING REMARKS

You have reached the end of the instructions. It is important that you understand them. If anything is unclear to you or if you have questions, please raise your hand.

To ensure that you understood the instructions we ask you to answer a few control questions. After everyone has answered these control questions correctly the experiment will start.