Social interaction in an exchange economy

Christian Ghiglino [*] Sanjeev Goyal[†]

VERY PRELIMINARY DRAFT
January 9, 2008

Abstract

We consider a pure exchange economy in which individual utility depends on own consumption as well as on the consumption of friends, colleagues and relatives, i.e., the neighbours. These effects may be due to social rank considerations. We model these local consumption externalities in terms of a network of social ties. Our aim is to study how prices, allocations and welfare are shaped by social interaction in the general equilibrium of such an economy. The role of wealth inequality is also explored.

We first study an example in which there are two goods, one of which is subject to consumption externalities while the other is not. In this setting we show that equilibrium prices can be expressed as a function of the sum of social network centralities of the agents. We then show that individual allocations are proportional to individual network centralities. New social connections lead to a larger sum of centrality and this generates higher prices for goods which exhibit externalities. Finally, additional social connections give rise to changes in consumption of different agents which are a function of their network centrality.

We then discuss alternative specifications of consumption externalities, e.g., where the externalities depend solely on the average of neighbours, or the maximum of neighbours, or where externalities arise in the consumption of both goods. We find that in such economies there exists equilibria which are insensitive to patterns of social interaction.

---

[*] Department of Economics, University of Essex, Colchester CO4 3SQ, UK. E-mail:ghiglino@essex.ac.uk

[†] Faculty of Economics, University of Cambridge, Cambridge CB3 9DD, UK. E-mail: sg472@econ.cam.ac.uk

1 Introduction

One of the recurrent themes in the study of individual well being is that, in addition to own consumption, it appears also to depend on the consumption of others with whom we interact and compare ourselves. But the happiness and therefore the consumption of these "close by" others in turn depends on the consumption of their friends and so on. Individual decisions on consumption are therefore shaped by the overall pattern of connections which obtain in the society. In a market economy these decisions also reflect the prices of different goods. In order to understand consumption and welfare we therefore need a framework which takes into account social structure along with the familiar fundamentals of the economy – endowments, technology and preferences.

In this paper, we propose a simple example, in which we extend the classical pure exchange model of general equilibrium to allow for social interaction. There is a finite group of individuals who all have identical initial endowments and are price takers. Each of the individuals is also directly affected by the consumption of a subset of the other individuals, whom we shall refer to as her *neighbours*. The structure of individual neighbourhoods is modeled as a network. In this network, two individuals i and j have a link if and only if they are mutually affected by each other's consumption of a good which displays externality. For a fixed set of prices, individual demand depends on consumption of neighbours, whose demand in turn depends on the demand of their neighbours and so on. This suggests that an agent with more neighbours may face more externality as compared to someone with few connections. However, the choices of neighbours will reflect the connections of the neighbours and so on. In particular, notice that a change in the demand of agent i will have an effect on the demand of her neighbours which will in turn affect the demand of their neighbours, and these changes will in turn feedback on agent i. Thus a study of individual consumption requires us to take into account paths of different lengths via which individual agents are linked in the network. It turns out that a well known concept of centrality in sociology, due to Bonacich (1987), defines the centrality of a node precisely as the weighted sum of paths of different lengths. We are thus led to investigate the relationship between this form of centrality and individual consumption and equilibrium prices.

We start with the result that individual consumption of the two goods can be expressed as a function of (an affine transformation of) her Bonacich network centrality. Second, we show that equilibrium price for the good is proportional to the sum of network centralities and is indeed increasing in this sum. Thus more connected networks will exhibit higher prices of the good subject to consumption externality. Thirdly, we show that an increase in the aggregate supply of a good leads to an increase in consumption which is proportional to the centrality of different agents. Fourth, we study how adding new social connections affects consumption and welfare. To see the issues at work here, it is useful to focus on the case where starting at a network, a link is added between two agents i and j. Our main result is that changes in the consumption of these agents can be expressed as a function of their network centrality: in particular, the consumption of the “positional” good 2 goes up for both agents, but the increase in consumption is larger for the agent who started with the lower centrality in the original network. The consumption of good 2 goes down for all other agents. The decrease being larger for agents with larger centrality. The converse holds for good 1.

We then discuss alternative specifications of consumption externalities, e.g., where the externalities depend solely on the average of neighbours, or the maximum of neighbours, or where externalities arise in the consumption of both goods. We find that in such economies there exists equilibria which are insensitive to patterns of social interaction.

There is a vast literature in economics, as well as in other disciplines, on the importance of relative consumption for individual well being. There are a range of models which have been proposed over the years, see e.g., Abel (1990), Arrow and Dasgupta (2007), Easterlin (1974), Cres, Ghiglino and Tvede (1997), Frank (1985), Frey and Stutzer (2002), Hopkins and Kornienko (2004), Layard (2005), Veblen (1899) and Dussenbery (1949). While these models differ in many ways, they share one common feature: they suppose that individual utility or well being depends on own consumption and *average* social consumption. However, the motivation for relative consumption effects in this literature typically arises at a local level, i.e., when we compare our consumption with the consumption of friends, colleagues and relatives.[1] This formulation of average consumption is thus restrictive on two counts: one, it precludes the study of size effects which is likely to be important when we are faced with ”physical externalities” where what matters is the sum of the individual effects and not the average. Two, the average formulation precludes the study of changing patterns of social interaction and comparison which has been the topic of public debate. These considerations motivate our attempt at developing a framework in which local consumption effects can be studied systematically.[2]

Our paper builds on two earlier papers, Ballester, Calvo-Armengol and Zenou (2006) and Tan (2006). Tan (2006) studies the effect of social networks in a general equilibrium model. There are two main differences between the papers. One, he looks at specific networks – such as star and regular networks – while we allow for arbitrary networks. Our study of general equilibrium in arbitrary networks is made possible due to our key result that the social network effects are summarized in a simple measure of network centrality. In obtaining this result on network centrality, we exploit

---

[1] For an interesting empirical paper on neighbourhood income effects, see Luttmer (2005).

[2] Our interest is in consumption externalities; there is an interesting line of research which examines the effects of trading restrictions, modeled in terms of networks, on equilibrium outcomes. See e.g., Gale and Kariv (2007) and Kakade, Kearns, and Ortiz (2005).

a simplifying feature of our example: for fixed prices, individual demand is a linear function of neighbours' consumptions.

Our paper is similar to Ballester et al. (2006) in one respect: both the papers use the linearity of individual actions as a function of their neighbours actions to obtain the connection between network centrality and equilibrium. However, the nature of the results is quiet different. In particular, our interest is in studying the effects of social interaction on market prices and allocations and we show that equilibrium price is an increasing function of the sum of network centralities. Moreover, our study of the effects of adding connections yields novel insights about changes in prices as well as the relative rates of increase and decrease in the consumption of different goods by agents as a function of their respective network centralities. By contrast, in Ballester et al. (2006), the games have a single dimensional effort/strategy and the addition of links raises the sum total of efforts.

The rest of the paper is organized as follows: section 2 sets out the basic model of network based consumption externality. Section 3 solves the model and obtains our main results. Section 4 discusses some alternative formulations and concludes.

2 Model

We consider a pure exchange economy populated with N consumers, $i = 1, ..., N$. Let $N(i)$ be the set of neighbours of consumer i and let $n_i = |N(i)|$. There are two consumption goods. We denote by p the price of good 2, good 1 being the numeraire. Consumers care about their own consumption of good 1 and good 2. We note x_i the consumption of good 1 by agent i and y_i the consumption of good 2 by agent i. Consumers also care about the consumption of good 2 by their direct neighbours, i.e. consumer i cares also about $\{y_j\}_{j \in N(i)}$. Consumer i is endowed with a bundle of the two goods (ω_i, ν_i). We will represent the pattern of neighbourhoods by G, which is a $n \times n$ matrix of 1's and 0's. An $\{i, j\}$ square in this matrix takes value 1 if and only if i and j are direct neighbours. We will assume that in this matrix the diagonal terms are all set equal 0. The complete network will be denoted by G^c.

In order to model interpersonal comparisons in consumption we let the utility depends on own consumption in the two goods as well as the consumption of the neighbours in the second good. More specifically, we let

$$U_i(x_i, y_i, y_{-i}) = u_i(x_i, \Phi(y_i, y_{-i}))$$

where $\Phi : R \times R^{n_i} \rightarrow R$ and $-i$ is the set of neighbours to agent i. The function Φ is expected to be increasing in y_i and decreasing in each element of the vector y_{-i}. It is also natural to assume that when all neighbours consume y_i, that is each component of y_{-i} is set to y_i, then the effect of the neighbours vanishes, i.e. $\Phi(y_i, y_i) = y_i$.

As all agents have identical preferences and endowments we put ourselves in a situation in which the variations in consumption are small. Subject to this condition we may linearize Φ in respect to y_{-i} near y_i and keep only the first order terms. We then have

$$\Phi(y_i, y_{-i}) \simeq \Phi(y_i, y_i) + \sum_{j \in N(i)} \partial_j \Phi(y_i, y_i)(y_j - y_i)$$

where $\partial_j \Phi(y_i, y_i)$ stands for the partial derivative of Φ in respect to jth neighbour's consumption, evaluated at the point where all consumptions are y_i, i.e. $y_{-i} = y_i$. As all agents are identical we may assume that $\partial_j \Phi(y_i, y_{-i}) = -\alpha$ where $-\alpha < 0$ is the common value of all the partial derivatives of Φ with respect to $-i$. Assuming that α is small we get

$$\begin{aligned} \Phi(y_i, y_{-i}) &\simeq y_i - \alpha \sum_{j \in N(i)} (y_j - y_i) \\ &\simeq y_i(1 + n_i \alpha) - \alpha \sum_{j \in N(i)} y_j \end{aligned}$$

Finally, we assume that u_i has the familiar form

$$u_i(x, y) = x^\sigma y^{1-\sigma}$$

with $0 < \sigma < 1$. The utility function can then be written as

$$u_i(x_i, y_i, y_{-i}) = u_i(x_i, y_i, \{y_j\}_{j \in N(i)}) = x_i^\sigma \left(y_i[1 + \alpha n_i] - \alpha \left[\sum_{j \in N(i)} y_j \right] \right)^{1-\sigma} \tag{1}$$

The specific form of the utility function is a consequence of the introduction of interpersonal comparisons. The function can be written in a way that allows for a more intuitive interpretation in which the effect of departures from the average is weighted by the number of neighbours, i.e. the social pressure. Indeed, we can write

$$u_i(x_i, y_i, y_{-i}) = u_i(x_i, y_i, \{y_j\}_{j \in N(i)}) = x_i^\sigma \left(y_i + \alpha n_i \left[y_i - \frac{1}{n_i} \sum_{j \in N(i)} y_j \right] \right)^{1-\sigma} \tag{2}$$

The utility function in (2) is derived from the general form $u_i(x_i, \Phi(y_i, y_{-i}))$ assuming that the effect of interpersonal comparisons are small. However, we could just as well assume that the utility takes the form (2) without any restriction. The utility function u_i does not explicitly vary across individuals so we might be tempted to drop the subscript i. However, we should be aware that the set of neighbours $N(i)$ do depend on i.

Remarks: The case where $\alpha = 0$ corresponds to the benchmark no externality model and α may be interpreted as a measure of the strength of comparison (or externality). In principle α could also be assumed to be negative, in which case we would have a situation of a positive externality. At the end of the paper we will also briefly consider situations where individuals care about average consumption of their neighbours, i.e. without introducing the "size effect", and the case in which each agent cares about the maximum consumption of her neighbours.

A recent strand of the literature on conspicuous consumption (Frank (1985), Robson (1992), Hopkins and Kornienko (2004)) assume that status (which gives utility) is determined by the position of the agent in the distribution of consumption of the positional good. A rationale for this is that this quantity gives the probability that the agent consumes more of a positional good than a randomly chosen other agent. The formulation adopted in the present paper is different as interpersonal comparisons are determined by the network and their effect is weighted by the size of the difference in consumption in the positional good.

A consumer i's optimization program reads

$$\begin{array}{cl} Max_{(x_i,y_i)} & u_i(x_i, y_i, \{y_j\}_{j\in N(i)}) \\ s.t. & x_i + py_i = \omega_i + p\nu_i \end{array}$$

Let $(\widehat{x}_i, \widehat{y}_i)$ solve this problem.

A general equilibrium is a strictly positive price p and a vector of allocations $(\widehat{x}_i, \widehat{y}_i)_{i\in N}$ such that

1. Markets clear: $\sum_{i\in N} \widehat{x}_i = \sum_{i\in N} \omega_i$, $\sum_{i\in N} \widehat{y}_i = \sum_{i\in N} \nu_i$.
2. For each $i \in N$, $(\widehat{x}_i, \widehat{y}_i)$ solves the optimization problem outlined above.

At equilibrium we need to have $x_i > 0, y_i > 0$ as well as $y_i + \alpha n_i \left[y_i - \frac{1}{n_i} \sum_{j\in N(i)} y_j \right] > 0$ for all i.

3 Analysis

The first order conditions associated to the optimization problem are

$$\begin{aligned}
0 &= \sigma x_i^{\sigma-1}\left(y_i[1+\alpha n_i]-\alpha\left[\sum_{j\in N(i)} y_j\right]\right)^{1-\sigma}+\lambda \\
0 &= x_i^{\sigma}(1-\sigma)[1+\alpha n_i]\left(y_i[1+\alpha n_i]-\alpha\left[\sum_{j\in N(i)} y_j\right]\right)^{-\sigma}+\lambda p \\
0 &= x_i+py_i-[\omega_i+p\nu_i]
\end{aligned}$$

The demand for good 2 is

$$f_i^2(p,\{y_j\}_{j\in N(i)})=\frac{1-\sigma}{p}\left(\omega_i+p\nu_i+\frac{\sigma}{1-\sigma}\frac{\alpha}{1+\alpha n_i}p\sum_{j\in N(i)} y_j\right) \tag{3}$$

An important property is that the demand depends linearly on the externality. Similarly, the demand for good 1 is

$$f_i^1(p,\{y_j\}_{j\in N(i)})=\sigma\left(\omega_i+p\nu_i-\frac{\alpha}{1+\alpha n_i}p\sum_{j\in N(i)} y_j\right) \tag{4}$$

The first expression above implies that for each i we have

$$y_i-\frac{\alpha\sigma}{1+\alpha n_i}\sum_{j\in N(i)} y_j-\frac{1-\sigma}{p}(\omega_i+p\nu_i)=0 \tag{5}$$

This can be written as

$$y_i-\frac{\alpha\sigma}{1+\alpha n_i}G_i\cdot Y-\frac{1-\sigma}{p}(\omega_i+p\nu_i)=0 \tag{6}$$

where Y is the N-dimensional vector of good 2 consumption, G_i is the ith row of the $N\times N$ matrix of connections, i.e. the adjacency matrix.

The set of equations can be written in matrix form as

$$\left[I-\alpha\sigma G^N\right]Y-\frac{1-\sigma}{p}W=0 \tag{7}$$

where G^N is the $N\times N$ matrix of connections in which every row is normalized so that the sum of the elements add to $\frac{n_i}{1+\alpha n_i}$, and W the N-dimensional vector of individual wealth.

Whenever $\left[I-\alpha\sigma G^N\right]$ is invertible we obtain

$$Y = \frac{1-\sigma}{p}\left[I - \alpha\sigma G^N\right]^{-1} W \tag{8}$$

Assume now that all consumers have identical endowments so that

$$W_i = \omega_i + p\nu_i = \omega + p\nu \tag{9}$$

which means that

$$W = (\omega + p\nu)J \tag{10}$$

where J is the N-dimensional vector of ones. Then

$$Y = \frac{1-\sigma}{p}\left[I - \alpha\sigma G^N\right]^{-1} J(\omega + p\nu) \tag{11}$$

It is now useful to introduce our notion of centrality.

Definition 1 *Let G^N be the $N \times N$ adjacency matrix in which a row i is normalized by $\frac{1}{1+\alpha n_i}$, where n_i is the degree of agent i. Then we define the centrality vector B by*

$$B = \left[I - \alpha\sigma G^N\right]^{-1} J \tag{12}$$

where J is the N dimensional vector of ones.

When $\alpha\sigma$ is smaller than the inverse of the modulo of the largest eigenvalue of G^N, the inverse $\left[I - \alpha\sigma G^N\right]^{-1}$ can be expressed as a power series

$$\left[I - \alpha\sigma G^N\right]^{-1} = \sum_{s=0}^{\infty}\left(\alpha\sigma G^N\right)^s \tag{13}$$

The element (i, j) of this matrix can be written as

$$\left\{\left[I - \alpha\sigma G^N\right]^{-1}\right\}_{(i,j)} = \sum_{s=0}^{\infty}(\alpha\sigma)^s\{(G^N)^s\}_{(i,j)} \tag{14}$$

where $\{(G^N)^s\}_{(i,j)}$ counts the number of path starting in i and ending at j of length s, weighted by the factors defined in Definition 1. This expression provides a nice interpretation of centrality in terms of interactions with neighbours of increasing distance. In fact the centrality of an individual B_i, reflects the weighted sum of paths of all the different possible lengths. We note that this is closely related to the well known measure of network centrality defined in Bonacich (1987). We discuss this measure in greater detail later in this section.

The vector of individual allocations is then collinear to B

$$Y = B\frac{1-\sigma}{p}(\omega + p\nu) \tag{15}$$

In order to find the price p that equates supply and demand we solve the market clearing equation:

$$\sum_{i=1}^{N} y_i = \frac{1-\sigma}{p}(\omega + p\nu)\sum_{i=1}^{N} B_i = N\nu \tag{16}$$

so that

$$\frac{1-\sigma}{p}(\omega + p\nu) = \frac{N\nu}{\sum_{k=1}^{N} B_k} \tag{17}$$

Therefore

$$Y = \frac{B}{\sum_{k=1}^{N} B_k} N\nu = \frac{B}{\overline{B}}\nu \tag{18}$$

with $\overline{B} = \frac{1}{N}\sum_{i=1}^{N} B_i$. Equivalently we have

$$y_i = \frac{B_i}{\sum_{k=1}^{N} B_k} N\nu = \frac{B_i}{\overline{B}}\nu \tag{19}$$

This equation contains the key result that at equilibrium, *a consumer's consumption of good 2 is proportional to her centrality.* Furthermore, her consumption is proportional to the supply of good 2 but does not depend on the supply of good 1. We note that this relation obtains when all agents are identical in preferences and endowments.

Finally note that the equilibrium price is given by the equation

$$(1-\sigma)\frac{\omega}{p} = \frac{N\nu}{\sum_{i=1}^{N} B_i} - (1-\sigma)\nu \tag{20}$$

or

$$\frac{1}{p} = \frac{\nu}{\omega}\left[\frac{1}{1-\sigma}\frac{N}{\sum_{i=1}^{N} B_i} - 1\right] \tag{21}$$

or

$$p = \frac{\omega}{\nu}\left[\frac{1}{1-\sigma}\frac{N}{\sum_{i=1}^{N} B_i} - 1\right]^{-1} \tag{22}$$

This expression shows that the price depends on the sum of centralities.

The vector of individual demand for good 1, denoted X, can also be computed. From

$$x_i = \omega + p\nu - py_i$$

we get

$$
\begin{aligned}
x_i &= \omega + \frac{\omega}{\nu}\left[\frac{1}{1-\sigma}\frac{N}{\sum_{k=1}^{N} B_k} - 1\right]^{-1}\left[\nu - \frac{B_i}{\sum_{k=1}^{N} B_k}N\nu\right] \\
&= \omega\left(1+\left[\frac{1}{1-\sigma}\frac{N}{\sum_{k=1}^{N} B_k} - 1\right]^{-1}\left[1 - \frac{B_i}{\sum_{k=1}^{N} B_k}N\right]\right) \\
&= \omega\left[1+\frac{(1-\sigma)\sum_{k=1}^{N} B_k - B_i N}{N-(1-\sigma)\sum_{k=1}^{N} B_k}\right] \\
&= \omega\frac{N-(1-\sigma)NB_i}{N-(1-\sigma)\sum_{k=1}^{N} B_k} \\
&= \omega\frac{1-(1-\sigma)B_i}{1-(1-\sigma)\frac{1}{N}\sum_{k=1}^{N} B_k} \\
x_i &= \omega\frac{1-(1-\sigma)B_i}{1-(1-\sigma)\overline{B}}
\end{aligned}
$$

At this stage it is necessary to check that this allocation is indeed an equilibrium. In other words we need to be sure $p>0, x_i>0, y_i>0$ and $y_i(1+\alpha n_i)-\alpha\sum_{j\in N(i)} y_j > 0$ for all i. This requires to evaluate the bounds on B_i for all i. Within the assumption of the present paper it is possible to write B as a converging series. Consider an arbitrary coordinate k

$$
\begin{aligned}
B_k &= 1+\alpha\sigma\{G^N J\}_k + \alpha^2\sigma^2\{(G^N)^2 J\}_k + \alpha^3\sigma^3\{(G^N)^3 J\}_k + ... \\
&= 1+\alpha\sigma\sum_{h=1}^{N} g_{kh}^N + \alpha^2\sigma^2\sum_{q=1}^{N}\sum_{p=1}^{N} g_{kp}^N g_{pq}^N + \\
&\quad \alpha^3\sigma^3\sum_{q=1}^{N}\sum_{p=1}^{N}\sum_{h=1}^{N} g_{kp}^N g_{ph}^N g_{hq}^N + ... \\
&= 1+\alpha\sigma\sum_{j\in N(k)}\frac{1}{1+\alpha n_k} + \alpha^2\sigma^2\sum_{p=1}^{N} g_{kp}^N\sum_{q=1}^{N} g_{pq}^N + \\
&\quad +\alpha^3\sigma^3\sum_{p=1}^{N} g_{kp}^N\sum_{h=1}^{N} g_{ph}^N\sum_{q=1}^{N} g_{hq}^N + ... \\
&= 1+\alpha\sigma\frac{n_k}{1+\alpha n_k} + \alpha^2\sigma^2\sum_{p=1}^{N} g_{kp}^N\frac{n_p}{1+\alpha n_p} + \\
&\quad +\alpha^3\sigma^3\sum_{p=1}^{N} g_{kp}^N\sum_{h=1}^{N} g_{ph}^N\frac{n_h}{1+\alpha n_h} + ... \\
&= 1+\alpha\sigma\frac{n_k}{1+\alpha n_k} + \alpha^2\sigma^2\frac{1}{1+\alpha n_k}\sum_{p\in N(k)}\frac{n_p}{1+\alpha n_p} + \\
&\quad +\alpha^3\sigma^3\frac{1}{1+\alpha n_k}\sum_{p\in N(k)}\frac{1}{1+\alpha n_p}\sum_{h\in N(p)}\frac{n_h}{1+\alpha n_h} + ...
\end{aligned}
$$

The first term is the degree n_k of the agent k scaled down by $1 + \alpha n_k$. The quantity $\frac{n_k}{1+\alpha n_k}$ is increasing in n_k and bounded by 0 and 1. The second term is the sum over the neighbours $N(k)$ of the degrees of the neighbours (also scaled down). This decomposition also allows to define B recursively

$$B_k = 1 + \alpha\sigma \frac{1}{1 + \alpha n_k} \sum_{j \in N(k)} B_j$$

Now, from the series decomposition and since $\frac{n_k}{1+\alpha n_k} < 1$, we have that

$$B_k < 1 + \frac{\alpha n_k}{1 + \alpha n_k} \left[\sum_{s=1}^{\infty} \sigma^s \right] = 1 + \frac{\alpha n_k}{1 + \alpha n_k} \frac{\sigma}{1 - \sigma}$$

We then get the bounds

$$1 < B_k < \frac{1}{1 - \sigma}$$

We are now ready to look at our existence conditions. The condition for the price to be positive is

$$\frac{1}{1 - \sigma} \frac{N}{\sum_{i=1}^{N} B_i} > 1$$

or

$$\frac{1}{1 - \sigma} > \overline{B}$$

which clearly holds as

$$\frac{1}{1 - \sigma} > B_i$$

for all i. The condition $x_i > 0$ is satisfied as both $(1 - \sigma)B_i < 1$ and $(1 - \sigma)\overline{B} < 1$ hold. The condition $y_i > 0$ is automatically satisfied as $B_k > 1$. Finally, we consider the condition $y_i(1 + \alpha n_i) - \alpha \sum_{j \in N(i)} y_j > 0$ which may be rewritten as

$$y_i - \alpha \frac{1}{(1 + \alpha n_i)} \sum_{j \in N(i)} y_j > 0$$

or

$$B_i - \alpha \frac{1}{(1 + \alpha n_i)} \sum_{j \in N(i)} B_j > 0$$

Using the recursive formulation for B, we see that the condition indeed holds

$$\begin{aligned} & 1+\alpha\sigma\frac{1}{1+\alpha n_i}\sum_{j\in N(i)}B_j-\alpha\frac{1}{(1+\alpha n_i)}\sum_{j\in N(i)}B_j \\ = \ & 1+(\sigma-1)\frac{\alpha}{1+\alpha n_i}\sum_{j\in N(i)}B_j \\ > \ & 1+(\sigma-1)\frac{\alpha n_i}{1+\alpha n_i}\frac{1}{1-\sigma} \\ > \ & 1-1 \\ > \ & 0 \end{aligned}$$

The results obtained so far are summarized as follows.

Proposition 2 *Assume that $\alpha\sigma$ is smaller than the inverse of the modulo of the largest eigenvalue of G^N.Let $\overline{B}=\frac{1}{N}\sum_{k=1}^{N}B_k$. Then there exists an equilibrium and this is unique. The equilibrium allocations are*

$$y_i=B_i\frac{1-\sigma}{p}(\omega+p\nu)=\frac{B_i}{\overline{\overline{B}}}\nu \tag{23}$$

and

$$x_i=\omega\frac{1-(1-\sigma)B_i}{1-(1-\sigma)\overline{\overline{B}}}$$

with the equilibrium price being given by

$$p=\frac{\omega}{\nu}\left[\frac{1}{1-\sigma}\frac{1}{\overline{\overline{B}}}-1\right]^{-1} \tag{24}$$

From the proposition we see that conspicuous consumption y_i is proportional to the centrality of agent i and is decreasing in the price p. Consequently, whenever $\overline{B}$ can be considered as given, y_i is proportional to the centrality of the agent i. Symmetrically, the consumption of the standard good x decreases with the centrality.

The implications of the result can be fully appreciated by considering the recursive formulation for B

$$B_k=1+\alpha\sigma\frac{1}{1+\alpha n_k}\sum_{j\in N(k)}B_j$$

Indeed, it shows that the consumption y_i in the positional good by agent i is increasing in the number of direct neighbours and also positively affected by the centrality of these. Recursively, y_i also depends on the number of second level neighbours and their own centrality.

The result also implies that her consumption in good 1 is larger than her initial endowment if and only if her centrality is below the average, i.e. $\sum_{k=1}^{N}B_k>NB_i$ or

$\frac{1}{N}\sum_{k=1}^{N} B_k > B_i$. Of course the converse is true for good 2. The intuition is that the more central she is the more she consumes of good 2 in order to cope with the larger negative externality, i.e. the larger number of comparisons, and the less she consumes of good 1. Furthermore, as the equilibrium price depends on the average centrality it is in general not much affected by the details of the network but is clearly increasing with the overall density of neighbourhood links. The Proposition also implies that the price p is increasing with $\sum_{i=1}^{N} B_i$ while it is decreasing with σ.

The price p (which is the relative price of good 2 over good 1) decreases as the supply of good 2 (relative to good 1) increases. Proposition 2 tells us that equilibrium individual consumption for good 1 increases proportionately with the supply in good 1 according to:

$$\frac{dx_i}{d\omega} = \frac{1-(1-\sigma)B_i}{1-(1-\sigma)\overline{B}} \tag{25}$$

Furthermore, as x_i is independent of ν we get $\frac{dx_i}{d\nu} = 0$. Consequently, an increase in the total supply of good 2 does not affect her consumption of good 1. The effect of a rise in ν on the consumption of good 2 can also be computed. From Proposition 2 we also get

$$\frac{dy_i}{d\nu} = \frac{B_i}{\overline{B}} \tag{26}$$

In this case the effect is straightforward as an increase in the total supply of the externality good 2 increases the demand for good 2 proportionally for all consumers. Note that this demand does not depend on the supply of good 1.

3.1 Wealth heterogeneity

In this section we extend the model to the case with heterogenous agents. The aim is to evaluate the impact of variations in wealth on the patterns of conspicuous consumption. As agents are heterogenous W is not anymore collinear to the vector of ones J, though we still have

$$Y = \frac{1-\sigma}{p}\left[I - \alpha\sigma G^N\right]^{-1} W \tag{27}$$

If we note $M = \left[I - \alpha\sigma G^N\right]^{-1}$ then

$$\begin{aligned} y_i &= \frac{1-\sigma}{p}\sum_{j=1}^{N} M_{ij}W_j = \frac{1-\sigma}{p}\sum_{j=1}^{N} M_{ij}(\omega_j + p\nu_j) \\ &= (1-\sigma)\sum_{j=1}^{N} M_{ij}(\frac{\omega_j}{p} + \nu_j) \end{aligned}$$

Conspicuous consumption depends both on the general structure of the network, via the price effects, and on the local patterns of links, via the M_{ij}'s.

In order to get some intuition consider the series expansion of M

$$\begin{aligned} Y &= \frac{1-\sigma}{p} M W = \frac{1-\sigma}{p} \left[I - \alpha\sigma G^N \right]^{-1} W \qquad (28) \\ &= \frac{1-\sigma}{p} \sum_{s=0}^{\infty} \left(\alpha\sigma G^N \right)^s W \qquad (29) \end{aligned}$$

where $\{(G^N)^s\}_{(i,j)}$ counts the number of paths starting in i and ending at j of length s, weighted by the normalizing factors appearing in G^N. The notion of weighted centrality $\left[I - \alpha\sigma G^N\right]^{-1} W$ used here characterizes then the *sum* of all paths leading to agent k starting at all possible j, weighted by the income of j and of all possible length s.

We are now ready to analyze the role of wealth and centrality at a local level. To simplify the analysis assume that the price is fixed. Consider coordinate k of Y. Then

$$\begin{aligned} y_k \left[\frac{1-\sigma}{p} \right]^{-1} &= \sum_{s=0}^{\infty} \alpha^s \sigma^s \left[(G^N)^s W \right]_k \\ &= 1 + \alpha\sigma \{G^N W\}_k + \alpha^2\sigma^2 \{(G^N)^2 W\}_k + \alpha^3\sigma^3 \{(G^N)^3 W\}_k + ... \\ &= 1 + \alpha\sigma \sum\nolimits_{q=1}^{N} g_{kq}^N W_q + \alpha^2\sigma^2 \sum\nolimits_{p=1}^{N} \sum\nolimits_{q=1}^{N} g_{kp}^N g_{pq}^N W_q + \\ &\quad \alpha^3\sigma^3 \sum\nolimits_{p=1}^{N} \sum\nolimits_{h=1}^{N} \sum\nolimits_{q=1}^{N} g_{kp}^N g_{ph}^N g_{hq}^N W_q + ... \\ &= 1 + \alpha\sigma \frac{1}{1+\alpha n_k} \sum\nolimits_{q \in N(k)} W_q + \\ &\quad + \alpha^2\sigma^2 \frac{1}{1+\alpha n_k} \sum\nolimits_{p \in N(k)} \frac{1}{1+\alpha n_p} \sum\nolimits_{q \in N(p)} W_q + \\ &\quad + \alpha^3\sigma^3 \frac{1}{1+\alpha n_k} \sum\nolimits_{p \in N(k)} \frac{1}{1+\alpha n_p} \sum\nolimits_{h \in N(p)} \frac{1}{1+\alpha n_h} \sum\nolimits_{q \in N(h)} W_q + ... \end{aligned}$$

Denote $I_k = \sum_{q=1}^{N} g_{kq}^N W_q = \sum_{q \in N(k)} \frac{W_q}{1+\alpha n_k}$ the weighted degree of agent k, i.e. the sum of incomes of the neighbours to agent k normalized by $1 + \alpha n_k$. Then

$$\begin{aligned} y_k \left[\frac{1-\sigma}{p} \right]^{-1} &= \sum_{s=0}^{\infty} \alpha^s \sigma^s \left[(G^N)^s W \right]_k \\ &= 1 + \alpha\sigma I_k + \alpha^2\sigma^2 \frac{1}{1+\alpha n_k} \sum\nolimits_{p \in N(k)} I_p + \\ &\quad + \alpha^3\sigma^3 \frac{1}{1+\alpha n_k} \sum\nolimits_{p \in N(k)} \frac{1}{1+\alpha n_p} \sum\nolimits_{h \in N(p)} I_h + ... \end{aligned}$$

This expression shows that for a given price, the consumption of the conspicuous good is increasing with the number and wealth of the neighbours as well as the number and wealth of the neighbours of the neighbours, and so on. For small $\alpha\sigma$ the series decreases very fast and the direct effect is expected to dominate. Consequently, an individual immersed in a rich and dense neighbourhood will be induced to consume more of the conspicuous good. The results obtained so far are summarized as follows

Proposition 3 *For a given price p, the consumption y_i in the positional good is increasing in the centrality and the wealth of the agent i, in the centrality and wealth of his neighbours, of his neighbours' neighbours, etc.*

We now consider the effect of local redistributions of endowments on the conspicuous consumption. Clearly, for a given network the equilibrium price p is a continuous function of $(\omega_i, \nu_i)_{i=1}^N$. Therefore we may again assume that p stays constant for any small local change in (ω_i, ν_i). The previous expression shows that a redistribution of incomes within neighbours of the agent k has no effect on his conspicuous consumption. The same is true for a redistribution among the neighbours of a *given* neighbour h of agent k. However, more general redistributions will affect conspicuous consumption of agent k. For example, a redistribution from a neighbour of a neighbour h (of agent k) to a neighbour of a neighbour h' (of agent k) will have an effect. Conspicuous consumption y_k will increase if the degree of h is larger than the degree of h'. Note that this logic only applies to situations in which the neighbourhoods do not intersect.

Proposition 4 *Assume that the considered neighbourhoods are disjoints. For a given equilibrium price p, redistributions among neighbours of an agent k does not affect his consumption of the positional good. The same is true for redistributions among any given neighbourhood of any of the neighbours or neighbours of neighbours (and so on) of agent k. On the other hand, redistributions across neighbourhoods affect the consumption. Furthermore, a redistribution from a neighbour to a "far" agent reduces the conspicuous consumption of agent k.*

We now consider the general equilibrium effects channelled by the equilibrium price. From the expression giving y_i it is clear that an increase in the price reduces the conspicuous consumption. To see how the equilibrium price depends on the wealth distribution we need to find an expression for p. In order to find the price p

that equates supply and demand we solve the market clearing equation:

$$\begin{aligned}
\sum_{i=1}^{N} \nu_i &= R_\nu = \sum_{i=1}^{N} y_i && (30)\\
&= \frac{1-\sigma}{p} \sum\nolimits_{i=1}^{N} \sum\nolimits_{j=1}^{N} M_{ij}(\omega_j + p\nu_j) && (31)\\
&= (1-\sigma) \sum\nolimits_{i=1}^{N} \sum\nolimits_{j=1}^{N} M_{ij}(\frac{1}{p}\omega_j + \nu_j) && (32)\\
&= \frac{1}{p}(1-\sigma) \sum\nolimits_{i=1}^{N} \sum\nolimits_{j=1}^{N} M_{ij}\omega_j + (1-\sigma) \sum\nolimits_{i=1}^{N} \sum\nolimits_{j=1}^{N} M_{ij}\nu_j && (33)
\end{aligned}$$

Therefore

$$\frac{1}{p} = \frac{R_\nu - (1-\sigma)\sum_{i=1}^{N}\sum_{j=1}^{N} M_{ij}\nu_j}{(1-\sigma)\sum_{i=1}^{N}\sum_{j=1}^{N} M_{ij}\omega_j} = \frac{R_\nu}{(1-\sigma)\sum_{i=1}^{N}\sum_{j=1}^{N} M_{ij}\omega_j} - \frac{\sum_{i=1}^{N}\sum_{j=1}^{N} M_{ij}\nu_j}{\sum_{i=1}^{N}\sum_{j=1}^{N} M_{ij}\omega_j}$$

To simplify the analysis suppose that only endowments in the positional good are redistributed. Then the price is an increasing function of $\sum_{i=1}^{N}\sum_{j=1}^{N} M_{ij}\nu_j$. This is evaluated as follows.

$$\begin{aligned}
\sum\nolimits_{i=1}^{N} \sum\nolimits_{j=1}^{N} M_{ij}\nu_j. &= \sum\nolimits_{i=1}^{N} \sum\nolimits_{j=1}^{N} \left[\sum_{s=0}^{\infty} \alpha^s \sigma^s (G^N)^s\right]_{ij} \nu_j\\
&= \sum\nolimits_{i=1}^{N} (1 + \alpha\sigma\Upsilon_i + \alpha^2\sigma^2 \frac{1}{1+\alpha n_i} \sum\nolimits_{p\in N(i)} \Upsilon_p +\\
&\quad + \alpha^3\sigma^3 \frac{1}{1+\alpha n_i} \sum\nolimits_{p\in N(i)} \frac{1}{1+\alpha n_p} \sum\nolimits_{h\in N(p)} \Upsilon_h + ...)
\end{aligned}$$

where

$$\Upsilon_k = \sum\nolimits_{q=1}^{N} g_{kq}^{N}\nu_q = \frac{1}{1+\alpha n_k} \sum\nolimits_{q\in N(k)} \nu_q$$

Of course, at a global level the assumption of disjoints neighbourhoods should be dropped. In fact, as central agents have many neighbours they automatically belongs to several neighbourhoods simultaneously. Consequently, a redistribution of endowments from the less connected to the more connected would certainly rise the price of the positional good.

Proposition 5 *For a given network the equilibrium price p decreases when resources are redistributed from more central agents to less central agents.*

Finally the expression for p can be inserted in the expression for y_i. We get

$$\begin{aligned}
y_i &= (1-\sigma)\sum\nolimits_{j=1}^{N} M_{ij}(\frac{1}{p}\omega_j+\nu_j) \\
&= \frac{1}{p}(1-\sigma)\sum\nolimits_{j=1}^{N} M_{ij}\omega_j + (1-\sigma)\sum\nolimits_{j=1}^{N} M_{ij}\nu_j \\
&= \left[\frac{R_\nu}{(1-\sigma)\sum_{i=1}^{N}\sum_{j=1}^{N} M_{ij}\omega_j} - \frac{\sum_{i=1}^{N}\sum_{j=1}^{N} M_{ij}\nu_j}{\sum_{i=1}^{N}\sum_{j=1}^{N} M_{ij}\omega_j}\right] \\
&\quad (1-\sigma)\sum\nolimits_{j=1}^{N} M_{ij}\omega_j + (1-\sigma)\sum\nolimits_{j=1}^{N} M_{ij}\nu_j \\
&= R_\nu\frac{\sum_{j=1}^{N} M_{ij}\omega_j}{\sum_{i=1}^{N}\sum_{j=1}^{N} M_{ij}\omega_j} - (1-\sigma)\left[\frac{\sum_{i=1}^{N}\sum_{j=1}^{N} M_{ij}\nu_j}{\sum_{i=1}^{N}\sum_{j=1}^{N} M_{ij}\omega_j}\sum\nolimits_{j=1}^{N} M_{ij}\omega_j - \sum\nolimits_{j=1}^{N} M_{ij}\nu_j\right]
\end{aligned}$$

3.2 Changes in the network

We now analyze how modifications in the network affect the equilibrium demand and price. We first consider the effects of the addition of a link between two agents.

We observe that the addition of a link reflects the local externality and this directly affects the demand of the individuals. The change in demand of the two linked individuals then affects the demands of others indirectly and these changes work themselves into the price which in turn is mediated via the network again. We know that demands are related to the centrality of agents, and so it is natural to start by asking how the addition of a link affects B_i. Formally, if a link is added between i and j, the matrix G is modified into $\widetilde{G} = G + \Delta_{ij} + \Delta_{ji}$ where Δ_{ij} is the matrix defined by $\Delta_{ij} = \{t_{pq}\}_{p,q=1}^{N}$ with $t_{ij} = 1$ and $t_{pq} = 0$ otherwise. In order to evaluate how the vector B is modified by the addition of the link it is useful to note that

$$G^N = \left(G^T\Lambda\right)^T$$

where $(.)^T$ means transpose and Λ is defined as

$$\Lambda = \begin{bmatrix} \frac{1}{1+\alpha n_1} & 0 & 0 & 0 \\ 0 & \frac{1}{1+\alpha n_2} & 0 & 0 \\ 0 & 0 & ... & 0 \\ 0 & 0 & 0 & \frac{1}{1+\alpha n_n} \end{bmatrix}$$

Then the addition of a link between i and j implies that G^N becomes $\widetilde{G}^N$ and

$$\begin{aligned}
\widetilde{G}^N &= \left(\widetilde{G}^T\widetilde{\Lambda}\right)^T \\
&= \left((G+\Delta_{ij}+\Delta_{ji})^T\,\widetilde{\Lambda}\right)^T
\end{aligned}$$

with $\widetilde{\Lambda}$ defined as Λ but with the elements at position i and j modified to become $\frac{1}{1+\alpha(n_i+1)}$ and $\frac{1}{1+\alpha(n_j+1)}$.

Therefore, introducing a link between i and j modifies B as follows

$$\begin{aligned} \widetilde{B} &= \left[\sum_{s=0}^{\infty} \alpha^s \sigma^s (\widetilde{G}^N)^s\right] J \qquad (34) \\ &= \left[\sum_{s=0}^{\infty} \alpha^s \sigma^s \left[\left((G+\Delta_{ij}+\Delta_{ji})^T \widetilde{\Lambda}\right)^T\right]^s\right] J \qquad (35) \end{aligned}$$

Clearly, introducing a connection between i and j affects the value of all the elements of the vector B. However, this effect is vanishing with the distance and as $\alpha\sigma \longrightarrow 0$. It is then useful to look at the first terms in the expression for B noting that all terms are positive. We consider an arbitrary coordinate k. As shown before we get for the first three terms

$$\begin{aligned} B_k &= 1+\alpha\sigma\frac{n_k}{1+\alpha n_k}+\alpha^2\sigma^2\sum\nolimits_{p=1}^{N} g_{kp}^{N}\frac{n_p}{1+\alpha n_p}+ \\ &\quad +\alpha^3\sigma^3\sum\nolimits_{p=1}^{N} g_{kp}^{N}\sum\nolimits_{h=1}^{N} g_{ph}^{N}\frac{n_h}{1+\alpha n_h}+... \\ &= 1+\alpha\sigma\frac{n_k}{1+\alpha n_k}+\alpha^2\sigma^2\frac{1}{1+\alpha n_k}\sum\nolimits_{p\in N(k)}\frac{n_p}{1+\alpha n_p}+ \\ &\quad +\alpha^3\sigma^3\frac{1}{1+\alpha n_k}\sum\nolimits_{p\in N(k)}\frac{1}{1+\alpha n_p}\sum\nolimits_{h\in N(p)}\frac{n_h}{1+\alpha n_h}+... \end{aligned}$$

The expansion for B_k allows us to evaluate the effect of the introduction of a link between i and j. Denote this change as ΔB_k. Using the series expansion we then define $\Delta B_k = \sum_{s=1}^{\infty} \Delta B_k^s$ where the suffix indicates the power of $\alpha\sigma$. For the first term in the series we have

$$\begin{aligned} \Delta B_i^1 &= \alpha\sigma\left[\frac{n_i+1}{1+\alpha\{n_i+1\}}-\frac{n_i}{1+\alpha n_i}\right] \\ &= \frac{\alpha\sigma}{[1+\alpha\{n_i+1\}]\,[1+\alpha n_i]} \end{aligned}$$

Let $N(i)$ denote the set of neighbours to agent i before the new link $i \longleftrightarrow j$ is added.

Then the second order term ΔB_i^2 is

$$\begin{aligned}
\Delta B_i^2 &= \alpha^2\sigma^2 \frac{1}{1+\alpha(n_i+1)} \left[\sum\nolimits_{p\in N(i)} \frac{n_p}{1+\alpha n_p} + \frac{n_j}{1+\alpha n_j}\right] \\
&\quad -\alpha^2\sigma^2 \frac{1}{1+\alpha n_i} \sum\nolimits_{p\in N(i)} \frac{n_p}{1+\alpha n_p} \\
&= \alpha^2\sigma^2 \left[\frac{1}{1+\alpha(n_i+1)} - \frac{1}{1+\alpha n_i}\right] \sum\nolimits_{p\in N(i)} \frac{n_p}{1+\alpha n_p} \\
&\quad +\alpha^2\sigma^2 \frac{1}{1+\alpha(n_i+1)} \frac{n_j}{1+\alpha n_j} \\
&= \alpha^2\sigma^2 \frac{-\alpha}{(1+\alpha(n_i+1))\,(1+\alpha n_i)} \sum\nolimits_{p\in N(i)} \frac{n_p}{1+\alpha n_p} \\
&\quad +\alpha^2\sigma^2 \frac{1}{1+\alpha(n_i+1)} \frac{n_j}{1+\alpha n_j} \\
&= \alpha\sigma^2 \frac{1}{1+\alpha(n_i+1)} \left[\frac{\alpha n_j}{1+\alpha n_j} - \frac{\alpha}{1+\alpha n_i} \sum\nolimits_{p\in N(i)} \frac{\alpha n_p}{1+\alpha n_p}\right]
\end{aligned}$$

An upper bound on ΔB_i^2 can be obtained noting that $0 < \frac{\alpha n_p}{1+\alpha n_p} < 1$. We get

$$\begin{aligned}
\Delta B_i^2 &= \alpha^2\sigma^2 \frac{-1}{(1+\alpha(n_i+1))\,(1+\alpha n_i)} \sum\nolimits_{p\in N(i)} \frac{\alpha n_p}{1+\alpha n_p} \\
&\quad +\alpha\sigma^2 \frac{1}{1+\alpha(n_i+1)} \frac{\alpha n_j}{1+\alpha n_j} \\
&< \alpha\sigma^2 \frac{1}{1+\alpha(n_i+1)} \frac{\alpha n_j}{1+\alpha n_j}
\end{aligned}$$

We now evaluate the contribution of the second term relative to the first term. We have

$$\begin{aligned}
\frac{\Delta B_i^2}{\Delta B_i^1} &< \frac{\alpha\sigma^2 \frac{1}{1+\alpha(n_i+1)} \frac{\alpha n_j}{1+\alpha n_j}}{\frac{\alpha\sigma}{[1+\alpha\{n_i+1\}][1+\alpha n_i]}} \\
&< \frac{\sigma \frac{\alpha n_j}{1+\alpha n_j}}{\frac{1}{1+\alpha n_i}} \\
&< \sigma\,(\alpha n_j)\, \frac{1+\alpha n_i}{1+\alpha n_j} \\
&< \sigma\,(1+\alpha n_i)
\end{aligned}$$

Therefore, if $\sigma << 1$ and $\alpha\sigma n_i << 1$ then all the terms beyond the first can be neglected.

Consider now how the centralities of the neighbours of i and j are affected. Denote by h a generical neighbour of i, i.e. $h \in N(i)$. Then, the addition of a link between agent i and j has no effect on n_h so that $\Delta B_h^1 = 0$. The second term in the expression of B_h is $\alpha^2\sigma^2 \frac{1}{1+\alpha n_h} \sum_{p\in N(h)} \frac{n_p}{1+\alpha n_p}$ so that

$$\begin{aligned}
\Delta B_h^2 &= \alpha^2\sigma^2 \frac{1}{1+\alpha n_h}\left[\frac{n_i+1}{1+\alpha(n_i+1)} + \sum\nolimits_{\substack{p\in N(h)\\ p\neq i}} \frac{n_p}{1+\alpha n_p}\right] \\
&\quad -\alpha^2\sigma^2 \frac{1}{1+\alpha n_h}\left[\frac{n_i}{1+\alpha n_i} + \sum\nolimits_{\substack{p\in N(h)\\ p\neq i}} \frac{n_p}{1+\alpha n_p}\right] \\
&= \alpha^2\sigma^2 \frac{1}{1+\alpha n_h}\left[\frac{n_i+1}{1+\alpha(n_i+1)} - \frac{n_i}{1+\alpha n_i}\right] \\
&= \alpha^2\sigma^2 \frac{1}{1+\alpha n_h} \frac{1}{(1+\alpha(n_i+1))(1+\alpha n_i)}
\end{aligned}$$

For further neighbours, only higher order terms are non-zero.

We are now in a position to evaluate how $\sum_{k=1}^N B_k$ is affected by the new link. Let $\sum_{k=1}^N \widetilde{B}_k$ be the value of the sum after the new link is introduced. If we keep only the first two terms of the expansion we have

$$\begin{aligned}
\Delta \sum_{k=1}^N B_k &= \sum_{k=1}^N \widetilde{B}_k - \sum_{k=1}^N B_k \\
&\simeq \Delta B_i^1 + \Delta B_i^2 + \Delta B_j^1 + \Delta B_j^2 + \sum_{k\in N(i)} \Delta B_k^2 + \sum_{k\in N(j)} \Delta B_k^2
\end{aligned}$$

Now,

$$\begin{aligned}
\Delta B_i^1 + \Delta B_j^1 &\simeq \alpha\sigma\left[\frac{n_i+1}{1+\alpha(n_i+1)} + \frac{n_j+1}{1+\alpha(n_j+1)} - \frac{n_i}{1+\alpha n_i} - \frac{n_j}{1+\alpha n_j}\right] \\
&\simeq \frac{\alpha\sigma}{(1+\alpha(n_i+1))(1+\alpha n_i)} + \frac{\alpha\sigma}{(1+\alpha(n_j+1))(1+\alpha n_j)}
\end{aligned}$$

On the other hand,

$$\begin{aligned}
\Delta B_i^2 + \Delta B_j^2 &\simeq \alpha^2\sigma^2 \frac{-\alpha}{(1+\alpha(n_i+1))(1+\alpha n_i)} \sum\nolimits_{h\in N(i)} \frac{n_h}{1+\alpha n_h} \\
&\quad +\alpha\sigma^2 \frac{1}{1+\alpha(n_i+1)} \frac{\alpha n_j}{1+\alpha n_j} \\
&\quad +\alpha^2\sigma^2 \frac{-\alpha}{(1+\alpha(n_j+1))(1+\alpha n_j)} \sum\nolimits_{h\in N(j)} \frac{n_h}{1+\alpha n_h} \\
&\quad +\alpha\sigma^2 \frac{1}{1+\alpha(n_j+1)} \frac{\alpha n_i}{1+\alpha n_i}
\end{aligned}$$

Finally,

$$\begin{aligned}\sum_{h\in N(i)\cup N(j)}\Delta B_h^2 &= \frac{\alpha^2\sigma^2}{(1+\alpha(n_i+1))(1+\alpha n_i)}\sum_{h\in N(i)}\frac{1}{1+\alpha n_h}\\ &\quad+\frac{\alpha^2\sigma^2}{(1+\alpha(n_j+1))(1+\alpha n_j)}\sum_{h\in N(j)}\frac{1}{1+\alpha n_h}\end{aligned}$$

Therefore,

$$\begin{aligned}\Delta\sum_{k=1}^{N}B_k &= \frac{\alpha\sigma}{(1+\alpha(n_i+1))(1+\alpha n_i)}+\frac{\alpha\sigma}{(1+\alpha(n_j+1))(1+\alpha n_j)}\\ &\quad+\alpha\sigma^2\frac{1}{1+\alpha(n_j+1)}\frac{\alpha n_i}{1+\alpha n_i}\\ &\quad+\alpha\sigma^2\frac{1}{1+\alpha(n_i+1)}\frac{\alpha n_j}{1+\alpha n_j}\\ &\quad+\alpha^2\sigma^2\frac{1}{(1+\alpha(n_i+1))(1+\alpha n_i)}(1-\alpha)\sum\nolimits_{h\in N(i)}\frac{n_h}{1+\alpha n_h}\\ &\quad+\alpha^2\sigma^2\frac{1}{(1+\alpha(n_j+1))(1+\alpha n_j)}(1-\alpha)\sum\nolimits_{h\in N(j)}\frac{n_h}{1+\alpha n_h}\end{aligned}$$

This expression shows an important pattern. Adding a new link between i and j increases the first term in the series expansion of B_i and B_j. The second term of the expansion is reduced but at a rate α^3. On the other hand the second term of the expansion for the neighbours to i and j are increased but at a rate α^2. Consequently, the overall impact on the second term of the series is also positive. By a similar pattern as expected the entire sum increases when a new link is added.

We can now obtain the result concerning the effect of the addition of a new link on the equilibrium price. As the addition of a new link increases $\sum_{i=1}^{N}B_i$ this push up the demand for good 2, everything else equal (including the relative price). Since aggregate endowment is constant, the price for good 2 has to increase to restore market clearing. This intuition is captured in the above result on the relation between equilibrium price for good 2 and the sum of the centralities.

A straightforward implication of this result is that the price of good 2 is minimized in the empty network and maximized in the complete network. We summarize these observations in the following result.

Proposition 6 *Starting from any network $G \neq G^c$, an addition of a link raises p; it therefore follows that this price is minimized in the empty network and maximized in the complete network.*

We now turn to the effects of link addition on different individuals. Denote $\Delta \sum_{k=1}^{N} B_k^1$ the first term of the expansion concerning the effect of the new link on the sum of centralities and $\Delta \sum_{k=1}^{N} B_k^2$ denote the second term. Then, the relative contribution of the different terms of the expansion still satisfies

$$\frac{\Delta \sum_{k=1}^{N} B_k^2}{\Delta \sum_{k=1}^{N} B_k^1} << 1$$

provided $\sigma << 1$ and $\alpha\sigma n_i << 1$. In the sequel we will then keep only the first term of the expansion, which implies that only the effect on the centralities B_i and B_j of the newly connected agents are considered, i.e.

$$\Delta \sum_{k=1}^{N} B_k \simeq \frac{\alpha\sigma}{(1+\alpha(n_i+1))\,(1+\alpha n_i)} + \frac{\alpha\sigma}{(1+\alpha(n_j+1))\,(1+\alpha n_j)}$$

Denote this term by $\Delta\Sigma$.

From this expression we see that the effect of a new link on the equilibrium price p is lower the higher is the degree of the connected agent.

The effect on the allocations can also be worked out. The first order correction to the value of y when a link is added between i and j can then be computed as follows. Let $\widetilde{y}_i$ denote the first-order corrected value of y_i. A similar convention is used for the other variables. For a node h with $h \neq i, j$, we have

$$\widetilde{y}_h = \frac{B_h}{\sum_{k=1}^{N} \widetilde{B}_k} N\nu < y_h \tag{36}$$

because $\Delta\Sigma = \sum_{k=1}^{N} \widetilde{B}_k - \sum_{k=1}^{N} B_k > 0$. This is intuitive as the increase in y_i and y_j need to be balanced by a similar decrease in the conspicuous consumption of the other agents. The extent of this decrease is computed as follows.

$$\begin{aligned}
\widetilde{y}_h - y_h &= \frac{B_h}{\Delta\Sigma + \sum_{k=1}^{N} B_k} N\nu - \frac{B_h}{\sum_{k=1}^{N} B_k} N\nu \\
&= N\nu \left(\frac{(B_h)\left(\sum_{k=1}^{N} B_k\right) - (B_h)\left(\Delta\Sigma + \sum_{k=1}^{N} B_k\right)}{\left(\Delta\Sigma + \sum_{k=1}^{N} B_k\right)\left(\sum_{k=1}^{N} B_k\right)} \right) \\
&= N\nu \left(\frac{-B_h \Delta\Sigma}{\left(\Delta\Sigma + \sum_{k=1}^{N} B_k\right)\left(\sum_{k=1}^{N} B_k\right)} \right) \\
&\simeq -N\nu\Delta\Sigma \frac{B_h}{\left(\sum_{k=1}^{N} B_k\right)^2}
\end{aligned}$$

If $\alpha n_i << 1$ so that $(1+\alpha(n_i+1))(1+\alpha n_i) \simeq 1$ for all i then

$$\widetilde{y}_h - y_h \simeq -2N\nu\alpha\sigma \frac{B_h}{\left(\sum_{k=1}^{N} B_k\right)^2}$$

So the more central is the agent the larger is the decrease in his consumption of the second good when a new connection is added that does not involve him directly. The intuition for this result is as follows. Because central agents consume more of the second good they are more affected by the rise in the price of good 2 induced by the increase in the total number of connections.

We now analyze the effect on the agents who have formed a link. The previous result clearly implies that $\widetilde{y}_i > y_i$ or/and $\widetilde{y}_j > y_j$. However, are both inequalities true? We know that

$$\widetilde{y}_i \simeq \frac{B_i + \frac{\alpha\sigma}{(1+\alpha(n_i+1))(1+\alpha n_i)}}{\Delta\Sigma + \sum_{k=1}^{N} B_k} N\nu \tag{37}$$

and

$$\widetilde{y}_j \simeq \frac{B_j + \frac{\alpha\sigma}{(1+\alpha(n_j+1))(1+\alpha n_j)}}{\Delta\Sigma + \sum_{k=1}^{N} B_k} N\nu \tag{38}$$

Therefore, again

$$\begin{aligned}
\widetilde{y}_i - y_i &\simeq \frac{B_i + \frac{\alpha\sigma}{(1+\alpha(n_i+1))(1+\alpha n_i)}}{\Delta\Sigma + \sum_{k=1}^{N} B_k} N\nu - \frac{B_i}{\sum_{k=1}^{N} B_k} N\nu \\
&\simeq N\nu \left(\frac{\left(B_i + \frac{\alpha\sigma}{(1+\alpha(n_i+1))(1+\alpha n_i)}\right)\left(\sum_{k=1}^{N} B_k\right) - (B_i)\left(\Delta\Sigma + \sum_{k=1}^{N} B_k\right)}{\left(\Delta\Sigma + \sum_{k=1}^{N} B_k\right)\left(\sum_{k=1}^{N} B_k\right)} \right) \\
&\simeq N\nu \left(\frac{\left(\frac{\alpha\sigma}{(1+\alpha(n_i+1))(1+\alpha n_i)}\right)\left(\sum_{k=1}^{N} B_k\right) - (B_i)\,\Delta\Sigma}{\left(\Delta\Sigma + \sum_{k=1}^{N} B_k\right)\left(\sum_{k=1}^{N} B_k\right)} \right)
\end{aligned}$$

If $\alpha n_i << 1$ so that $(1+\alpha(n_i+1))(1+\alpha n_i) \simeq 1$ for all i then

$$\begin{aligned}
\widetilde{y}_i - y_i &\simeq N\nu\alpha\sigma \left(\frac{\sum_{k=1}^{N} B_k - 2B_i}{\left(\Delta\Sigma + \sum_{k=1}^{N} B_k\right)\left(\sum_{k=1}^{N} B_k\right)} \right) \\
&\simeq N\nu\alpha\sigma \frac{\sum_{k=1}^{N} B_k - 2B_i}{\left(\sum_{k=1}^{N} B_k\right)^2}
\end{aligned}$$

The question is then whether $\sum_{k=1}^{N} B_k - 2B_i$ is always positive, i.e. does $B_i \leq \frac{1}{2}\sum_{k=1}^{N} B_k$ always hold? The answer is yes, since every path of any length between two agents l and m generates a path of equal length from m to l.

The most interesting property obtained from the previous expression is that the increase in the consumption of the externality good for the newly connected agents is larger for the less central agent of the two. In general, the effect of a new connection is inversely proportional to the centrality of the agent before the connection is established. The intuition is as follows. The rise in the price of good 2 due to the new connection tends to lower the consumption of good 2. However, the increase in the centrality of agents i and j over-compensate this effect. In fact, the less central is the agent before the new connection is added, the bigger is the relative jump in her centrality and consequently her increase in consumption of good 2.

We now examine the effects on demand of good 1, when a single connection is added to the network between i and j. Assume that $(1+\alpha(n_i+1))\,(1+\alpha n_i) \simeq 1$ or equivalently that $\alpha n_i << 1$, for all i then. For agent i, we have

$$\begin{aligned} \frac{1}{\omega}\widetilde{x}_i &= \frac{N-(1-\sigma)N\widetilde{B}_i}{N-(1-\sigma)\sum_{k=1}^{N}\widetilde{B}_k} \\ &\simeq \frac{N-(1-\sigma)N(\alpha\sigma+B_i)}{N-(1-\sigma)[2\alpha\sigma+\sum_{k=1}^{N}B_k]} \end{aligned}$$

Therefore

$$\begin{aligned} \frac{1}{\omega N}\Delta x_i &= \frac{1}{\omega N}(\widetilde{x}_i - x_i) \\ &= \frac{1-(1-\sigma)(\alpha\sigma+B_i)}{N-(1-\sigma)[2\alpha\sigma+\sum_{k=1}^{N}B_k]} \\ &\quad -\frac{1-(1-\sigma)B_i}{N-(1-\sigma)\sum_{k=1}^{N}B_k} \end{aligned}$$

After reducing to a common denominator, the numerator becomes

$$\begin{gathered} Num = [1-(1-\sigma)(\alpha\sigma+B_i)][N-(1-\sigma)\sum\nolimits_{k=1}^{N}B_k] \\ -[1-(1-\sigma)B_i][N-(1-\sigma)[2\alpha\sigma+\sum\nolimits_{k=1}^{N}B_k]] \end{gathered}$$

Let

$$A_1 = 1-(1-\sigma)B_i \ \text{ and } A_2 = N-(1-\sigma)\sum\nolimits_{k=1}^{N}B_k$$

Then

$$\begin{aligned} Num &= [A_1-(1-\sigma)\alpha\sigma)][A_2]-[A_1][A_2-(1-\sigma)2\alpha\sigma] \\ &= [-(1-\sigma)(\alpha\sigma)]A_2 + A_1(1-\sigma)2\alpha\sigma \\ &= [(1-\sigma)\alpha\sigma][2A_1-A_2] \end{aligned}$$

Then

$$\begin{aligned}
Num &= [(1-\sigma)\alpha\sigma] \\
&\quad [2 - 2(1-\sigma)B_i - N + (1-\sigma)\sum\nolimits_{k=1}^{N} B_k] \\
&= [(1-\sigma)\alpha\sigma]\left(2 - N + (1-\sigma)\left[\sum\nolimits_{k=1}^{N} B_k - 2B_i\right]\right)
\end{aligned}$$

Finally we get

$$\Delta x_i = \omega N[(1-\sigma)\alpha\sigma]\frac{2 - N + (1-\sigma)[\sum_{k=1}^{N} B_k - 2B_i]}{\left(N - (1-\sigma)\sum_{k=1}^{N} B_k\right)^2}$$

Note that the condition for a positive price is $(1-\sigma)\sum_{k=1}^{N} B_k < N$ (see page 9) and that $B_i > 1$ by construction. Therefore, $\Delta x_i < 0$. It is worth noticing that the loss in the consumption of good 1 is larger for more central agents. We observe that this effect is the mirror image of the effect of an additional link on the demand of good 2.

For a node h with $h \neq i, j$, the increase in consumption of first good is computed as follows.

$$\begin{aligned}
\frac{1}{\omega}\Delta x_h &= \frac{1}{\omega}(\widetilde{x}_h - x_h) \\
&= \frac{N - (1-\sigma)NB_h}{N - (1-\sigma)[2\alpha\sigma + \sum_{k=1}^{N} B_k]} \\
&\quad - \frac{N - (1-\sigma)NB_h}{N - (1-\sigma)[\sum_{k=1}^{N} B_k]}
\end{aligned}$$

First observe that $\Delta x_h/\omega > 0$, as $N - (1-\sigma)[\sum_{k=1}^{N} B_k] > 0$. After reducing to the same denominator, the numerator becomes:

$$\begin{aligned}
Num &= [N - (1-\sigma)NB_h] \\
&\quad \left[\left(N - (1-\sigma)[\sum_{k=1}^{N} B_k]\right) - \left(N - (1-\sigma)[2\alpha\sigma + \sum_{k=1}^{N} B_k]\right)\right] \\
&= [N - (1-\sigma)NB_h]\,[(1-\sigma)2\alpha\sigma] \\
&= N(1-\sigma)2\alpha\sigma - (1-\sigma)^2 2\alpha\sigma N B_h \\
&= N(1-\sigma)2\alpha\sigma\,[1 - (1-\sigma)B_h]
\end{aligned}$$

Therefore the increase in x_h decreases with the (relative) centrality of h; in other words, for fixed $\sum B_i$, an increase in B_h reduces the increase in demand for good 1. These findings are summarized in the following proposition and the related corollary.

Proposition 7 *Assume that a new link is added between agents i and j. Then the first order correction to the consumption of agent i is*

$$\Delta x_i \simeq \omega N[(1-\sigma)\alpha\sigma]\frac{2-N+(1-\sigma)[\sum_{k=1}^N B_k - 2B_i]}{\left(N-(1-\sigma)\sum_{k=1}^N B_k\right)^2} \leq 0$$

and

$$\Delta y_i \simeq N\nu\alpha\sigma\frac{\sum_{k=1}^N B_k - 2B_i}{\left(\sum_{k=1}^N B_k\right)^2} \geq 0$$

Similar expressions obtain for agent j. The first order correction for the agent $h \neq i, j$ are

$$\Delta x_h \simeq 2\alpha\sigma\omega N(1-\sigma)\frac{1-(1-\sigma)B_h}{\left(N-(1-\sigma)[\sum_{k=1}^N B_k]\right)^2} \geq 0$$

and

$$\Delta y_h \simeq N\nu\alpha\sigma\frac{-2B_h}{\left(\sum_{k=1}^N B_k\right)^2} \leq 0$$

The first order correction is dominant whenever $\sigma << 1$ and $\alpha n_i << 1$.

Corollary 8 *For a sufficiently small values of α and σ the effect of a new link $i \leftrightarrow j$ on the equilibrium consumption is as follows:*

- *i) y_i increases and the effect is smaller for more central agents*
- *ii) For $h \neq i, j$, y_h decreases and the effect is larger for more central agents*
- *iii) x_i decreases and the effect is larger for more central agents*
- *iv) For $h \neq i, j$, x_h increases and the effect is smaller for more central agents*

From the Corollary we see that the densification of some of the regions of the network increases the consumption in the positional good in those regions, the larger increase affecting the less connected (central) agents. On the other hand, the general equilibrium effect channeled by the price decreases the conspicuous consumption of the agents non affected by the densification, the larger effect being on the more central of these agents.

Remark: Of course there is a second order effect on the gents in the neighbourhood of agent i and agent j. For these the reduction in conspicuous consumption due to the rise in the price is less noticeable because of the offsetting effect of the increase in the centrality of their neighbours (this can be seen from the recursive formulation of B).

Welfare effects: We now turn to the study of welfare effects of additional new links. The individual welfare in logarithmic terms is

$$\begin{aligned}
& \log u(x_i, y_i, \{y_k\}_{k\in N(i)}) \\
= & \sigma \log x_i + (1-\sigma)\log(y_i(1+\alpha n_i) - \alpha \sum_{k\in N(i)} y_k) \\
= & \sigma \log \left(\omega \frac{N-(1-\sigma)NB_i}{N-(1-\sigma)\sum_{h=1}^{N} B_h}\right) \\
& +(1-\sigma)\log\left(\nu \frac{N}{\sum_{h=1}^{N} B_h}\left[(1+\alpha n_i)B_i - \alpha \sum_{k\in N(i)} B_k\right]\right)
\end{aligned}$$

Note that $y_i + \alpha n_i \left[y_i - \frac{1}{n_i}\sum_{j\in N(i)} y_j\right]$ needs to be positive for the utility to be defined. Prior to the new connection, agent j was not a neighbour to i, i.e. $j \notin N(i)$. The effect on welfare of increases in total supply is straightforward. The effect of an increase in the supply of good 1 is clearly beneficial. An increase in ν is also beneficial for the utility of consuming good 2. The overall effect is positive.

The role of centrality in defining the change in welfare can now be analyzed. The effect of the new connection on agent's i welfare comes from the change in his consumptions of good 1 and good 2 but also from the change in the comparison with the neighbours, i.e. in $\sum_{k\in N(i)} B_k$. This term changes as a consequence of changes in the B_k and also because now agent i has one more neighbour, $j \in \widetilde{N}(i)$, where $\widetilde{N}(i)$ is the updated set of neighbours. In fact, an increase in the centrality of an agent, everything else equal, seems to have an ambiguous effect as it increases his equilibrium utility from consuming good 2 (provided α is small enough so that $(1+\alpha n_i)B_i - \alpha \sum_{j\in N(i)} B_j$ increases) but reduces her utility from consuming good 1. A formal analysis is as follows. Let as before $\sum_{k=1}^{N} B_k = N\overline{B}$. Consider agent i.

$$
\begin{aligned}
& \Delta \log u_i(x_i, y_i, \{y_k\}_{k\in N(i)}) \\
= \; & \frac{\sigma}{x_i}\Delta x_i + \frac{1-\sigma}{(1+\alpha n_i)y_i - \alpha \sum_{k\in N(i)} y_k}\left[(1+\alpha n_i)\Delta y_i - \alpha \sum\nolimits_{k\in N(i)} \Delta y_k - \alpha y_j\right] \\
= \; & \sigma\left(\omega\frac{1-(1-\sigma)B_i}{1-(1-\sigma)\overline{B}}\right)^{-1} \omega N[(1-\sigma)\alpha\sigma]\frac{2-N+(1-\sigma)[N\overline{B}-2B_i]}{\left(N-(1-\sigma)N\overline{B}\right)^2} + \\
& +(1-\sigma)\left(\frac{\nu}{\overline{B}}\left[(1+\alpha n_i)B_i - \alpha \sum\nolimits_{k\in N(i)} B_k\right]\right)^{-1} \\
& \left[N\nu\alpha\sigma(1+\alpha n_i)\frac{N\overline{B}-2B_i}{\left(N\overline{B}\right)^2} - \alpha \sum\nolimits_{k\in N(i)} N\nu\alpha\sigma\frac{-2B_k}{\left(N\overline{B}\right)^2} - \alpha y_j\right]
\end{aligned}
$$

Let us rewrite this in order to highlight the terms specific to agent i

$$
\begin{aligned}
& \Delta \log u_i(x_i, y_i, \{y_j\}_{j\in N(i)}) \\
= \; & \sigma^2\omega N(1-\sigma)\alpha\left(\frac{\left(N-(1-\sigma)N\overline{B}\right)^2}{1-(1-\sigma)\overline{B}}\right)^{-1} \\
& \left[\frac{2-N+(1-\sigma)[N\overline{B}]}{1-(1-\sigma)B_i} - \frac{2(1-\sigma)B_i}{1-(1-\sigma)B_i}\right] + \\
& (1-\sigma)\left(\frac{\nu}{\overline{B}}\left[(1+\alpha n_i)B_i - \alpha \sum\nolimits_{k\in N(i)} B_k\right]\right)^{-1} \\
& \left[N\nu\alpha\sigma(1+\alpha n_i)\frac{N\overline{B}-2B_i}{\left(N\overline{B}\right)^2} - \alpha \sum\nolimits_{k\in N(i)} N\nu\alpha\sigma\frac{-2B_k}{\left(N\overline{B}\right)^2} - \alpha y_j\right]
\end{aligned}
$$

Now, the expression

$$
\begin{aligned}
& \left[N\nu\alpha\sigma(1+\alpha n_i)\frac{N\overline{B}-2B_i}{\left(N\overline{B}\right)^2} - \alpha \sum\nolimits_{k\in N(i)} N\nu\alpha\sigma\frac{-2B_k}{\left(N\overline{B}\right)^2} - \alpha y_j\right] \\
= \frac{N\nu\alpha\sigma}{\left(N\overline{B}\right)^2} \quad & (1+\alpha n_i)\left(N\overline{B}-2B_i\right) + 2\alpha \sum\nolimits_{k\in N(i)} B_k - \frac{\alpha}{N\nu\alpha\sigma}\left(N\overline{B}\right)^2\left(\frac{N\nu B_j}{N\overline{B}}\right) \\
= \frac{N\nu\alpha\sigma}{\left(N\overline{B}\right)^2} \quad & \left[(1+\alpha n_i)\left(N\overline{B}-2B_i\right) + 2\alpha \sum\nolimits_{k\in N(i)} B_k - \frac{1}{\sigma}B_j N\overline{B}\right]
\end{aligned}
$$

We are not able to find out whether or not the change in welfare is in general positive or negative. However, some interesting results can still be obtained.

First, from the above expression it is clear that the more central is j the worst, or less beneficial, is the addition of the new connection $i \longleftrightarrow j$ as far as the the welfare of i is concerned. The incentive for an agent i is then to connect with non-central agents. The role of the centrality of agent i on her welfare is more difficult because it causes the term in x_i to rise and the term in y_i to fall. However, when $\sigma \to 0$ the above expression tends to

$$= \frac{N\nu\alpha\sigma}{\left(\sum_{k=1}^{N} B_k\right)^2}\left[-\frac{1}{\sigma}B_j \sum\nolimits_{k=1}^{N} B_k\right] = -\frac{N\nu\alpha B_j}{\sum_{k=1}^{N} B_k}$$

which is a fixed negative number when $\sigma \to 0$. We then get that

$$\begin{aligned} &\lim_{\sigma\to 0} \Delta \log u_i(x_i, y_i, \{y_j\}_{j\in N(i)}) \\ = \; & -\left(\frac{\nu}{\overline{B}}\left[(1+\alpha n_i)B_i - \alpha \sum\nolimits_{k\in N(i)} B_k\right]\right)^{-1} \frac{N\nu\alpha B_j}{\sum_{k=1}^{N} B_k} \end{aligned}$$

Note that the denominator is positive (otherwise the utility would not be defined). We summarize the findings as follows.

Proposition 9 *Given a network g. For any given pair i, j and any sufficiently small value of α there exists $1 > \overline{\sigma} > 0$ such that for all $0 < \sigma \leq \overline{\sigma}$, adding a connection between i and j reduces the welfare of agents i and j.*

4 Alternative specifications and open issues

In this section we comment on alternative formulations of the externality.

Suppose that individuals care about their consumption relative to the average consumption of their neighbours. In this case,

$$u(x_i, y_i, \{y_j\}_{j\in N(i)}) = x_i^{\sigma}(y_i - \frac{\alpha}{n_i}\sum_{j\in n_i(g)} y_j)^{1-\sigma} \tag{39}$$

The average formulation suggests a natural outcome: every consumer chooses the same bundle of goods. Clearly then the externality faced by every consumer is the same and so the optimal consumption bundle under a common set of prices will be the same as well. The market clearing equations are then $nx_i = n\omega$ and $ny_y = n\nu$. The equilibrium allocation is then (ω, ν). The equilibrium price can be obtained from eq. (3)

$$\omega = \sigma\left(\omega + p\nu + \alpha\frac{1}{n_i}p\sum_{j\in N(i)} \nu\right) = \sigma\left(\omega + p\nu + \alpha p\nu\right) \tag{40}$$

giving

$$\omega(1-\sigma) = p(\sigma\nu + \sigma\alpha\nu) = p\sigma(1+\alpha) \tag{41}$$

so that

$$p = \frac{1-\sigma}{\sigma(1+\alpha)}$$

Note that it is independent of the network, as is to be expected since demands are independent of the network connections. However, note that the equilibrium price is related to the magnitude of the externality, and is indeed decreasing in the value of α.

We now turn to another possibility: comparison with the maximum of neighbour's consumption. In this case, we can write utility as:

$$u(x_i, y_i, \{y_j\}_{j\in N(i)}) = x_i^\sigma (y_i - \alpha \max_{j\in N(i)} y_j)^{1-\sigma} \tag{42}$$

Now again we can check that there is an equilibrium in which as in the previous case, individuals all choose the same demand and the equilibrium price is given $p = \frac{1-\sigma}{\sigma(1+\alpha)}$.

Finally suppose that consumption externalities are relevant in a symmetric way for both goods. In that case, the utility is given by

$$\hat{u}_i(x_i, x_{-i}, y_i, y_{-i}) = \left(x_i[1+\alpha n_i] - \alpha\left[\sum_{j\in N(i)} x_j\right]\right)^\sigma \left(y_i[1+\alpha n_i] - \alpha\left[\sum_{j\in N(i)} y_j\right]\right)^{1-\sigma} \tag{43}$$

Then there exists an equilibrium which is identical in prices and allocations to an equilibrium in the economy with no consumption externalities. This is analogous to a point made in a recent paper by Arrow and Dasgupta (2007). They consider a dynamic model of work, leisure and savings, and find that if consumption and leisure are equally susceptible to consumption externalities then there is no distortion in equilibrium.

The discussion in this section has shown that if externality generated by individual consumption takes an average form or a maximum form then the network structure does not matter; the equilibrium price and allocation is independent of the network of interaction. However, the price is related to the magnitude of the externality parameter α. In a pure exchange framework with identical agents this plays no role. However, in a model with heterogenous agents or production the change in the relative price is expected to matter for the equilibrium allocation.

5 References

1. Abel, A. (1990), Asset Prices under habit formation and catching up with Jonesses, *American Economic Review*, 80, 2, 38-42.
2. Arrow, K. and P. Dasgupta (2007), Conspicuous consumption; inconspicuous leisure. mimeo Cambridge University.
3. Ballester, Calvo-Armengol and Zenou (2006), Who's Who in Networks. Wanted: The Key Player, *Econometrica*, vol. 74(5), pages 1403-1417.
4. Bonacich, P. (1987), Power and Centrality: A Family of Measures, *The American Journal of Sociology*, 92, 5, 1170-1182
5. Cres, H. (1996), Symmetric Smooth Consumption Externalities", *Journal of Economic Theory*, vol. 69, pp. 334-366.
6. Cres, H., C. Ghiglino and M. Tvede (1997), Externalities, Internalization and Fluctuations, *International Economic Review,* 38(2), 465-477.
7. Dussenbery, J. (1949), *Income, savings and the theory of consumer behavior.* Harvard University Press, Cambridge Mass.
8. Easterlin, R. A. (1974), Does Economic Growth Improve the Human Lot? in Paul A. David and Melvin W. Reder, eds., Nations and Households in Economic Growth: Essays in Honor of Moses Abramovitz, New York: Academic Press, Inc.
9. Frank, R. (1985), *Choosing the right pond.* Oxford University Press. Oxford.
10. Frey, B. and A. Stutzer (2002), What can economists learn from happiness research? *Journal of Economic Literature*, XL, 402-435.
11. Gale, D. and S. Kariv (2007), Trade in networks: a normal form experiment. mimeo New York University.
12. Goyal, S. (2007), *Connections: an introduction to the economics of networks.* Princeton University Press.
13. Kakade, M., M. Kearns, and L. Ortiz (2005), Graphical Economics, Proceedings COLT.
14. Hopkins, E. and Kornienko (2004), Running to keep in the same place: consumer choice as a game of status, *American Economic Review*, 94, 4, 1085-1107.
15. Layard, R (2005), *Happiness: lessons from a new science*, Penguin Books. London.

16. Luttmer, E. (2005), neighbours as Negatives: Relative Earnings and Well-Being, *Quarterly Journal of Economics*, 120, 3, 963-1002.

17. Robson, A. (1992), Status, the distribution of wealth, private and social attitudes to risk, *Econometrica,* 60 (4), 837-857.

18. Tan, Hi-Lin (2006), Prices in Networks, mimeo,

19. Veblen, T. (1899), *The theory of the leisure class.* Macmillan.