The Paradox of New Members in the EU Council of Ministers: A Non-cooperative Bargaining Analysis

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This version: March 2010

Abstract

Power indices suggest that adding new members to a voting body may increase the power of an existing member, even if the number of votes of all existing members and the decision rule remain constant. This phenomenon is known as the paradox of new members. This paper uses the leading model of majoritarian bargaining and shows that the paradox is predicted in equilibrium for past EU enlargements. Furthermore, a majority of members would have been in favor of the 1981 enlargement even if members were bargaining over a fixed budget.

Keywords: majoritarian bargaining, weighted voting, power measures, EU enlargement, paradox of new members.

J.E.L. Classification Numbers: C71, C72, C78.

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1 Introduction

This paper takes a noncooperative approach to modelling voting in the EU Council of Ministers using the Baron-Ferejohn (1989) model of majoritarian bargaining. In this model, the voters bargain over the division of a fixed budget by making and voting on proposals, and a voter's power can be measured by its expected equilibrium payoff.[1] The equilibrium of the bargaining game is analyzed for the Council of Ministers in 1958, 1973 and 1981. Comparing the countries' expected payoffs before and after each enlargement, it is observed that at least one existing member is better-off in each of the two enlargements even under the extreme assumption that the total pie remains constant after enlargement.

The possibility that adding new members to a voting body may increase the power of an existing member even if the number of votes of all existing members and the decision rule remain constant was first raised by Brams and Affuso (1976). In later papers (Brams and Affuso, 1985a, 1985b) they showed that the paradox has theoretically occurred in the EEC (now EU) Council of Ministers. Brams and Affuso based their analysis on the application of Shapley and Banzhaf power indices to weighted voting games. Because power indices do not have clear strategic foundations, one may be tempted to dismiss the paradox as a pathological feature of power indices. The contribution of this paper is to show that the paradox is predicted for past EEC enlargements using the leading model of strategic bargaining.

Not only can the paradox occur as an equilibrium feature of a bargaining game, but it can be even more extreme than suggested by power indices. The countries that gain with the 1981 enlargement had a majority in the 1973 Council. Thus, if qualified majority voting had been used to decide on enlargement, the new member would have been admitted even if the countries were bargaining over a fixed pie.

[1] This concept of power is sometimes labelled P-power (Felsenthal and Machover, 1998).

2 The noncooperative bargaining procedure

There is a budget of size 1 to be divided by majority rule between n players. Player i has w_i votes and q votes are needed to achieve a majority. We will denote a weighted majority game by $[q; w_1, ..., w_n]$. A group of players S with $\sum_{i \in S} w_i \geq q$ is called a *winning coalition*; a winning coalition such that $\sum_{j \in S \setminus \{i\}} w_j < q$ for all i is called a *minimal winning coalition.* A player that does not belong to any minimal winning coalition is a *dummy player.* A player that belongs to all minimal winning coalitions is a *veto player.*

Bargaining proceeds as follows: At every round $t = 1, 2, ...$, Nature randomly selects a proposer (each player is selected with probability $\frac{1}{n}$). This player proposes a distribution of the budget $(x_1, ..., x_n)$ with $x_i \geq 0$ for all i and $\sum_{i \in N} x_i = 1$. The proposal is voted upon immediately (closed rule). If the sum of votes in favor of the proposal is at least q, the proposal is implemented and the game ends; otherwise the game proceeds to the next period in which Nature selects a new proposer (again each player is selected with probability $\frac{1}{n}$). Players are risk neutral and do not discount future payoffs.

A (pure) strategy for player i is a sequence $\sigma_i = (\sigma_i^t)_{t=1}^{\infty}$, where σ_i^t, the tth round strategy of player i, prescribes

1. A *proposal* x^i.
2. A *response function* assigning "yes" or "no" to all possible proposals by the other players.

The solution concept is *stationary subgame perfect equilibrium* (SSPE). Stationarity requires that players follow the same (possibly mixed) strategy at every round t regardless of past offers and responses to past offers. Banks and Duggan (2000) show that an SSPE always exists in this type of bargaining model. Eraslan and McLennan (2006) show that all SSPE lead to the same expected equilibrium payoffs.

2.1 Three voting bodies

Table 1, adapted from Felsenthal and Machover (2001), shows the weighted majority voting games associated to the original European Community in

Country	**1958**	**1973**	**1981**
Germany	4	10	10
Italy	4	10	10
France	4	10	10
Netherlands	2	5	5
Belgium	2	5	5
Luxemburg	1	2	2
UK	-	10	10
Denmark	-	3	3
Ireland	-	3	3
Greece	-	-	5
Quota	12	41	45
Total votes	17	58	63
Quota (%)	70.59	70.69	71.43

Table 1: Weights and quota in the Council of Ministers

1958, 1973 and 1981.

We now calculate the equilibrium payoffs of the bargaining game for each of these voting bodies. Since equilibrium payoffs are unique, it will suffice to find one equilibrium strategy combination (all other equilibrium combinations lead to the same payoffs). From now on we restrict ourselves to *symmetric strategies*: all players of the same type follow the same strategy and are treated symmetrically by other players' strategies.

In a stationary equilibrium, a player's expected payoff given that a proposal is rejected (the continuation value) equals his expected equilibrium payoff at the beginning of the game. It is optimal for each player to accept any offer that gives him at least his continuation value as a responder. As a proposer, player i looks for the cheapest group of players controlling at least $q - w_i$ votes, and makes a proposal allocating to these players their continuation values and keeping the remainder for himself. Following common practice, we will refer to the proposer together with the players that are offered their continuation values as the "proposed coalition", and, if the proposal is passed, as the "coalition that forms".

Two conditions must be satisfied in equilibrium: strategies must be optimal given expected payoffs, and expected payoffs must be consistent with the strategies. To find the equilibrium expected payoffs, we will make hypotheses about them (e.g., the expected payoff of a player with 4 votes is twice the expected payoff of a player with 2 votes) and then construct strategies that are optimal given the hypotheses and that lead to payoffs satisfying the hypotheses.

2.2 Equilibrium of game [12;4,4,4,2,2,1]

There are four minimal winning coalitions in this game: one coalition of type [444] and three coalitions of type [4422]. The player with 1 vote does not belong to any minimal winning coalition.

Denote expected equilibrium payoffs by x (for a player with 4 votes), y (for a player with 2 votes) and w (for the player with 1 vote).

Suppose equilibrium payoffs are such that $x = 2y$. Under this hypothesis, a player with 4 votes is indifferent between paying $2x$ and forming the coalition of type [444] and paying $x + 2y$ and forming one of the two coalitions of type [4422] to which he belongs. Denote by λ the probability that a given player with 4 votes proposes [444] (conditional on being proposer). The probability of proposing each of the two coalitions of type [4422] is then $\frac{1-\lambda}{2}$. A player with 2 votes needs to buy 10 votes, and the best way to do this is to form a coalition of type [4422] (a coalition of type [4442] would be too expensive under the hypothesis $x = 2y$). There are three such coalitions, each proposed with probability $\frac{1}{3}$. The player with 1 vote needs to buy 11 votes, and is indifferent between forming coalition [4441] and forming a coalition of type [44221]. Denote the probability of proposing [4441] by μ; then each of the three [44221] coalitions is proposed with probability $\frac{1-\mu}{3}$. The following table shows the probability that each player type proposes each of the coalition types, with the number of available coalitions for that player type in parentheses. Because the proposer must be included in the coalition, the number of available coalitions of each type may depend on the proposer's type.

		Coalition type			
		[444]	[4422]	[4441]	[44221]
	[4]	λ (1)	$1-\lambda$ (2)	-	-
Player type	[2]	-	1 (3)	-	-
	[1]	-	-	μ (1)	$1-\mu$ (3)

Expected equilibrium payoffs are determined by these strategies. Consider a player with 4 votes. With probability $\frac{1}{6}$ he is selected to be proposer and obtains a payoff of $1-2x$ (this is the proposer's payoff regardless of whether he proposes [444] or [4422] because $x=2y$). With probability $\frac{2}{6}$, one of the two other players with 4 votes is selected, and the player receives a proposal with probability $\lambda+\frac{1-\lambda}{2}$. With probability $\frac{2}{6}$ one of the two players with 2 votes is selected and proposes each coalition of type [4422] with probability $\frac{1}{3}$. A given player with 4 votes belongs to two of these three coalitions, and thus receives a proposal with probability $\frac{2}{3}$. With probability $\frac{1}{6}$ the player with 1 vote is selected and proposes to the player with 4 votes with probability $\mu+\frac{2}{3}(1-\mu)$. The equations for y and w can be derived analogously. Together with the postulated condition $x=2y$, we have the following system of equations

$$\begin{aligned}
x &= \frac{1}{6}(1-2x)+\frac{2}{6}\left(\lambda+\frac{1-\lambda}{2}\right)x+\frac{2}{6}\frac{2}{3}x+\frac{1}{6}\left(\mu+\frac{2}{3}(1-\mu)\right)x \\
y &= \frac{1}{6}(1-2x-y)+\frac{3}{6}(1-\lambda)y+\frac{1}{6}y+\frac{1}{6}(1-\mu)y \\
w &= \frac{1}{6}(1-3x) \\
x &= 2y
\end{aligned}$$

The solution to this system of equations is $0\leq\mu\leq 1$, $\lambda=\frac{12-5\mu}{15}$, $x=\frac{10}{42}$, $y=\frac{5}{42}$, $w=\frac{2}{42}$. Notice that even though Luxemburg is a dummy player its expected equilibrium payoff is positive because it is allowed to make proposals.

2.3 Equilibrium of game [41;10,10,10,10,5,5,3,3,2]

The 1973 enlargement changed the voting game from $[12; 4, 4, 4, 2, 2, 1]$ to $[41; 10, 10, 10, 10, 5, 5, 3, 3, 2]$. Three new members were added and the weights of all pre-existing members were multiplied by 2.5, with the exception of the smallest member (Luxemburg), whose votes were multiplied by 2. The percentage of the total votes required to pass a proposal remained essentially constant (keeping it exactly constant would lead to a quota of 40.94, which has the same implications as a quota of 41). If Luxemburg's votes had been multiplied by 2.5, any incumbent being better-off would be an instance of the paradox of new members. The fact that Luxemburg's votes were multiplied by only 2 seems to make it more difficult for Luxemburg to be better-off after the enlargement.[2] However, we will see that Luxemburg's expected equilibrium payoff increases after the enlargement in the Baron-Ferejohn model.

There are 25 minimal winning coalitions of six possible types: [10 10 10 10 5], [10 10 10 10 3], [10 10 10 10 2], [10 10 10 5 5 3], [10 10 10 5 5 2] and [10 10 10 5 3 3].

Expected equilibrium payoffs will be denoted by x (players with 10 votes), y (players with 5 votes), z (players with 3 votes) and w (player with 2 votes). Postulate an equilibrium with $x > y > z > w$, $x = 2y$ and $y + w = 2z$. Then the following types of minimal winning coalitions are the cheapest: [10 10 10 10 2], [10 10 10 5 5 2], [10 10 10 5 3 3]. All other winning coalitions would be too expensive to form. Unlike in the previous example, each player belongs to at least one of the cheapest coalitions.

[2] In fact, $[41; 10, 10, 10, 10, 5, 5, 3, 3, 2]$ and $[41; 10, 10, 10, 10, 5, 5, 3, 3, 1]$ have the same winning coalitions, thus Luxemburg's votes might as well have remained constant.

Equilibrium strategies are summarized by the following table

		Coalition type		
		[10 10 10 10 2]	[10 10 10 5 5 2]	[10 10 10 5 3 3]
	[10]	λ (1)	μ (3)	$1-\lambda-\mu$ (6)
Player type	[5]	-	θ (4)	$1-\theta$ (4)
	[3]	-	-	1 (8)
	[2]	ρ (1)	$1-\rho$ (4)	-

The four equations for expected payoffs together with the two conditions we have postulated form the following system of equations:

$$\begin{aligned}
x &= \frac{1}{9}(1-3x-w)+\frac{3}{9}\left(\lambda+\frac{2}{3}(1-\lambda)\right)x+\frac{4}{9}\frac{3}{4}x+\frac{1}{9}\left(\rho+\frac{3}{4}(1-\rho)\right)x \\
y &= \frac{1}{9}(1-3x-2z)+\frac{4}{9}\left(\mu+\frac{1}{2}(1-\lambda-\mu)\right)y+\frac{1}{9}\theta y+\frac{2}{9}\frac{1}{2}y+\frac{1}{9}(1-\rho)y \\
z &= \frac{1}{9}(1-3x-y-z)+\frac{4}{9}(1-\lambda-\mu)z+\frac{2}{9}(1-\theta)z+\frac{1}{9}z \\
w &= \frac{1}{9}(1-4x)+\frac{4}{9}(\lambda+\mu)w+\frac{2}{9}\theta w \\
x &= 2y \\
2z &= y+w
\end{aligned}$$

Again there are infinitely many solutions for the equilibrium strategies, but a unique solution for x, y, z and w. The (unique) equilibrium expected payoffs are $x=\frac{67-\sqrt{73}}{368}\approx 0.159$, $y=\frac{67-\sqrt{73}}{736}\approx 0.079$, $z=\frac{9\sqrt{73}+133}{2944}\approx 0.071$, $w=\frac{11\sqrt{73}-1}{1472}\approx 0.063$. There are many possible values for the strategies. Setting $\mu=0$ and $\theta=1$ we obtain $\lambda=\frac{11-\sqrt{73}}{8}\approx 0.31$ and $\rho=\frac{\sqrt{73}-8}{3}\approx 0.18$. Because $\lambda+\mu\leq 1$, all probabilities are between 0 and 1.

Luxemburg has stopped being a dummy player, and this increases its equilibrium payoffs. This result is natural but not obvious because Luxemburg was already earning a positive payoff as a proposer in the previous game and it is now less likely to propose.

Perhaps surprisingly, expected payoffs for countries with 2, 3 and 5 votes do not differ much. Intuition dictates that a country with 5 votes and a combination of two countries with 3 and 2 votes respectively are interchangeable and ought to have the same expected payoff. However, the set of minimal winning coalitions is not rich enough for this to be feasible: there are no minimal winning coalitions in which a player with 3 votes and a player with 2 votes appear together.[3] Minimal winning coalitions including a player with 5 votes already include the player with 2 votes (coalitions of type [10 10 10 5 5 2]) or both of the players with 3 votes (coalitions of type [10 10 10 5 3 3]) or are too expensive to be relevant (in [10 10 10 10 5] and [10 10 10 5 5 3] a player with 5 votes could be replaced by a combination of two players, but then the player with 2 votes would be superfluous).

2.4 Equilibrium of the game [45;10 10 10 10 5 5 5 3 3 2]

In 1981 Greece entered the European Community with 5 votes and the quota was raised to 45. The voting weights of all other countries were left unchanged, and the percentage of votes required to achieve a majority was essentially unchanged since $\frac{41}{58} \times 63 \approx 44.53$.

This new game is radically different to the previous one and easier to analyze. First, a player with 3 votes and a player with 2 votes have become interchangeable. The new voting game is equivalent to the game [18;4,4,4,4,2,2,2,1,1,1]. Second, the possibility of replacing a player with 5 votes by a combination of players with 3 and 2 votes (or 3 and 3 votes) has become relevant.

As before, we denote expected payoffs by x, y and z. Since players with 3 votes and players with 2 votes have become interchangeable, they both have the same expected payoff z. To simplify the search for equilibrium, we limit ourselves to strategies in which the players with 3 votes and the player with 2 votes follow the same strategy and are treated symmetrically by other players.

If we postulate $x = 2y$ and $y = 2z$, these two equations together with $4x + 3y + 3z = 1$ uniquely determine expected payoffs. The unique solution is $x = 0.16$, $y = 0.08$, $z = 0.04$. All we need is to verify that there are

[3] This can only happen if the weighted majority game is not strong (i.e., we can divide the players in two groups in such a way that both groups have less than q votes). In strong games, any two players share membership of at least one minimal winning coalition.

equilibrium strategies supporting those payoffs.

Under the hypotheses $x = 2y$ and $y = 2z$, all minimal winning coalitions are equally cheap. There are 46 minimal winning coalitions[4] of 6 possible types (4 types if we take into account that players with 2 and 3 votes are interchangeable): [10 10 10 10 5], [10 10 10 10 3 3], [10 10 10 10 3 2], [10 10 10 5 5 5], [10 10 10 5 5 3 3], [10 10 10 5 5 3 3]. The table below pools players with 2 and 3 votes.

	[10 10 10 10 5]	[10 10 10 10 3 3] [10 10 10 10 3 2]	[10 10 10 5 5 5]	[10 10 10 5 5 3 3] [10 10 10 5 5 3 2]
[10]	λ (3)	μ (3)	θ (3)	$1-\lambda-\mu-\theta$ (27)
[5]	ρ (1)	-	σ (4)	$1-\rho-\sigma$ (24)
[3/2]	-	τ (2)	$-$	$1-\tau$ (24)

We can simplify the search further by looking for equilibria with $\mu = \theta = \rho = \tau = 0$. The strategy table becomes

	[10 10 10 10 5]	[10 10 10 10 3 3] [10 10 10 10 3 2]	[10 10 10 5 5 5]	[10 10 10 5 5 3 3] [10 10 10 5 5 3 2]
[10]	λ (3)	$-$	$-$	$1-\lambda$ (27)
[5]	$-$	-	σ (4)	$1-\sigma$ (24)
[3/2]	-	$-$	$-$	1 (24)

The following equations must hold

$$\begin{aligned}
x &= \frac{1}{10}(1-3x-y)+\frac{3}{10}\left(\lambda+\frac{2}{3}(1-\lambda)\right)x+\frac{6}{10}\frac{3}{4}x \\
y &= \frac{1}{10}(1-4x)+\frac{4}{10}\left(\frac{\lambda}{3}+\frac{2}{3}(1-\lambda)\right)y+\frac{2}{10}\left(\sigma+\frac{1}{2}(1-\sigma)\right)y+\frac{3}{10}\frac{2}{3}y \\
z &= \frac{1}{10}(1-4x-z)+\frac{4}{10}\frac{2}{3}(1-\lambda)z+\frac{3}{10}\frac{2}{3}(1-\sigma)z+\frac{2}{10}\frac{1}{2}z \\
x &= 2y = 4z
\end{aligned}$$

The solution to this system is $\lambda = \frac{3}{4}$, $\sigma = \frac{5}{6}$, $x = \frac{4}{25}$, $y = \frac{2}{25}$, $z = \frac{1}{25}$.

[4] The number of minimal winning coalitions can be checked using the Powerslave software (Pajala, 2002).

The values of x and y all increase slightly compared with the 1973 values. This means that if enlargement were put to the vote under weighted majority it would be approved!

Expected equilibrium payoffs are summarized in the following table

	1958	**1973**	**1981**
Germany	0.238	0.159	**0.160**
Italy	0.238	0.159	**0.160**
France	0.238	0.159	**0.160**
Netherlands	0.119	0.079	**0.080**
Belgium	0.119	0.079	**0.080**
Luxemburg	0.048	**0.063**	0.040
UK	-	0.159	**0.160**
Denmark	-	0.071	0.040
Ireland	-	0.071	0.040
Greece	-	-	0.080

Table 2. Expected equilibrium payoffs

For comparison, the Shapley value and the Banzhaf index[5] are

	Shapley value			Banzhaf index		
Country	**1958**	**1973**	**1981**	**1958**	**1973**	**1981**
Germany	0.233	0.179	0.174	0.238	0.167	0.158
Italy	0.233	0.179	0.174	0.238	0.167	0.158
France	0.233	0.179	0.174	0.238	0.167	0.158
Netherlands	0.150	0.081	0.071	0.143	0.091	0.082
Belgium	0.150	0.081	0.071	0.143	0.091	0.082
Luxemburg	0	**0.001**	**0.030**	0	**0.016**	**0.041**
UK	-	0.179	0.174	-	0.167	0.158
Denmark	-	0.057	0.030	-	0.066	0.041
Ireland	-	0.057	0.030	-	0.066	0.041
Greece	-	-	0.071	-	-	0.082

[5] The table reports the normalized Banzhaf index; the effects of enlargement according to the absolute Banzhaf index are qualitatively similar.

Power indices like the Shapley value and the Banzhaf index agree with the noncooperative model in that the 1973 enlargement favored Luxemburg, and the 1981 enlargement hurt Denmark and Ireland the most. An important difference is that Luxemburg gains in both enlargements according to the power indices, and loses in the second enlargement according to the noncooperative bargaining model. Also, if countries wanted to maximize their Shapley or Banzhaf power indices and enlargement was subject to weighted majority voting, it would have been rejected.

3 Concluding remarks

The paradox of new members in the EU is not exclusive to power indices, but can occur in a noncooperative model of bargaining over a fixed pie. In fact, it is stronger in the noncooperative model since enlargement can benefit a majority of existing members.

It is difficult to know the real effects of enlargement. Power indices and the legislative bargaining model agree in that the paradox is possible, but differ on which country benefits. The Banzhaf index assumes yes/no voting over exogenous proposals with each country being equally likely to vote yes or no and countries voting independently. The Shapley value may be interpreted as a measure of expected payoffs in bargaining over a fixed pie, though it is difficult to find a compelling bargaining model that yields the Shapley value for weighted majority games.[6] It is clear that neither yes/no voting with a random agenda and random preferences nor pure bargaining over a private good are accurate models of voting in the Council of Ministers. However, the fact that very different assumptions all lead to the paradox of new members seem to indicate that this is a potentially important phenomenon. The paradox has also been observed experimentally under two different bargaining procedures by Montero, Sefton and Zhang (2008) and Drouvelis, Montero and Sefton (2010).

[6] Existing models either assume that all proposals must be passed by unanimity or their results are restricted to a domain that does not include weighted majority games; see the discussion in Drouvelis, Montero and Sefton (2010).

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