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Abstract

This paper shows that the use of hyperbolic discounting in environmental regulation can have unfortunate consequences. In a three-period model we demonstrate that a planner who 'naively' employs hyperbolic discounting and fails to anticipate problems of dynamic inconsistency, can oversee a collapse of a renewable resource. If the regeneration rate of the resource is within a given range, and stock levels are close to the 'minimum viable population', then an unforeseen collapse will result.

This basic result is shown to hold in an infinite-horizon, continuous-time model with hyperbolic discounting of the sort examined in Barro (1999) and Li and Löfgren (2001). Here, the naive planner does not anticipate extinction of its resource stock because it always plans to lower consumption (but it never does). Two conclusions follow from these results. First, the model provides an explanation for resource collapses such as that of the Peruvian anchovy and Atlantic cod. Second, governments should think carefully before they employ hyperbolic discounting in policymaking.

JEL Classification Numbers: Q21, Q28, E61.

Keywords: hyperbolic discounting, time-inconsistency, renewable resources

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1 Introduction

Seven species of Atlantic cod collapsed in the early 1990s, resulting in a fishing moratoria and 'one of the worst social and economic nightmares in the nation's history', according to Walters and Maguire (1996). The collapse was caused in part by over-optimistic estimates of current stock levels and systematically over-optimistic forecasts of future stock levels. Indeed, over-optimistic forecasts continued to be made even when, year after year, it emerged that previous forecasts were incorrect. Given the huge economic costs of the collapse, such naively optimistic planning is surprising.

The collapse of the Peruvian anchovy, once the world's largest fishery, is also puzzling. Anchovy fishing fleet capacity increased over the 1960s to the extent that the authorities feared a collapse of anchovy stocks, according to Aguero (1987). They responded by restricted the length of the fishing season. Experts such as the Instituto del Mar del Peru (1981) warned that the fishery should be closed, or it would not survive the next shock from El Niño. In 1972-73, El Niño arrived, the authorities opened the fishery and the anchovy population was all but destroyed. The result was an 'anchovy crisis' with worldwide effects on food prices.[1] Why did the authorities open the fishery, contrary to expert advice, knowing that a collapse was likely? They clearly did not want the fishery to collapse; they had tried to protect stocks prior to 1973. Nor is the problem merely a 'tragedy of the commons', as access to the fishery was regulated.[2] Was running the risk of a collapse simply irrational?

This paper suggests a novel explanation for these two situations. It shows that the use of hyperbolic discounting can lead an otherwise rational planner to manage a resource to the brink of collapse. It therefore provides food for thought for governments around the world, such as the UK Government, who are considering employing declining discount rates in long-term policymaking.

The paper proceeds as follows. In section 2, we review recent work on hyperbolic discounting and declining discount rates. In section 3, we propose a simple, three-period model of consumption of a renewable resource. With the assumption of quasi-hyperbolic discounting, we show that an optimising but 'naïve' planner can experience an unforeseen resource collapse. In section 4, we develop a continuous-time hyperbolic discounting model. This model follows the form of Barro (1999) and Li and Löfgren (2001). However, unlike those papers, we find the consumption and resource paths which would result from naive hyperbolic discounting. We apply predictions from our model to the case studies outlined above. Conclusions and policy recommendations are presented in section 5.

2 The rise of hyperbolic discounting

Since Samuelson (1937) introduced the discounted utility model, most standard economic models have employed a constant and positive discount rate to value a flow of future consumption. As economists are well-aware, a major problem with this assumption is that costs or benefits in the

[1] See Idyll (1973).

[2] While Hardin (1968) coined the term 'tragedy of the commons', the problem has been understood for centuries and considered by philosophers from Aristotle (1992, 2. 3.1261b33-37) in *The Politics*, to Hume (1739, Book 3). For instance, in Hume's *A Treatise on Human Nature*, he contrasts the ease with which two neighbours may drain a jointly owned meadow, with the difficulty of a thousand persons organising to drain a commons.

distant future are discounted to almost nothing. For instance, with constant discounting, the benefits of climate change mitigation and the costs of dealing with nuclear waste are so small that they are often categorised as being more ‘political’ than ‘economic’ issues.

Three relatively recent strands of literature, however, suggest that the discount rate should be a declining function of time.[3] First, Sozou (1998) and Weitzman (1998) showed that where the discount rate is constant but uncertain, the ‘certainty-equivalent discount rate’ will decline over time to the lowest possible rate. For instance, when discount rate uncertainty is characterised by a gamma distribution the discount rate will decline asymptotically to zero, as Sozou (1998) demonstrated.[4] The intuition behind this idea is that scenarios with a higher discount rate are given less weight as time passes, precisely because their discount factor is falling more rapidly. Second, ideas in Chichilnisky (1996) have been developed by Li and Löfgren (2000), who show that if one insists that the current generation must not have a ‘tyranny’ over future generations, the resulting program will have a utility discount rate which is a declining function of time. Third, experimental evidence suggests that people discount the future hyperbolically, employing a higher discount rate to trade-offs now than to trade-offs in the future. While evidence supporting hyperbolic discounting appears to be relatively strong,[5] there good reasons to recommend a cautious interpretation of this evidence.[6]

The three approaches suggest that planners should, and individuals do, employ declining discount rates in making intertemporal decisions. So doing increases the relative weight placed on costs and benefits accruing in the distant future, and solves the problem noted above. The arguments for declining discount rates therefore seem extremely persuasive. Indeed, the force of these arguments has now been acknowledged by the UK Government, and HM Treasury (2003) official guidance on the appraisal of investments and policies requires the use of declining discount rates.

Unfortunately, however, declining discount rates generally give rise to time-inconsistent plans, as Ramsey (1928) and Strotz (1956) observed long ago.[7] Time inconsistency implies that plans made today will not be carried out tomorrow unless a commitment mechanism is available. As Strotz (1956) notes, the desire to commit one’s future self to avoid calamity is not a novel human experience:

> *[Y]ou must bind me hard and fast, so that I cannot stir from the spot where you will stand me . . . and if I beg you to release me, you must tighten and add to my bonds.*

Homer, *The Odyssey*, Book XII, (*c* 800 BC).

Because hyperbolic discounting results in such time-inconsistent planning, economists have used it to model ‘irrational’ behaviour. Indeed, since Akerlof (1991) used hyperbolic discounting in

[3]Critical reviews of this literature are contained in Groom et al. (2003) and Pearce et al. (2003).

[4]Weitzman (2001) also used this feature in a playful paper based on results from a survey of 2000 economists.

[5]Harris and Laibson (2001a) note that a large number of experiments has been conducted, with a variety of rewards such as money, durable goods, sweets, relief from noise and so on. For instance, see Thaler (1981), Kirby (1997) and a review in Ainslie (1992). Dasgupta and Maskin (2002) cite evidence for hyperbolic discounting from animal studies.

[6]See, for instance, the interpretation of Rubinstein (2001) and Read (2001), and the comments of Mulligan (1996).

[7]The sole exception is logarithmic discounting, as shown in Heal (1998).

the context of procrastination, drug addiction and organisational failure, it has been applied to a large range of economic phenomena.[8]

However, precisely because hyperbolic discounting results in such 'irrational' behaviour, one might feel uneasy with its use in government policymaking. Some economists do not feel this unease. Heal (1998), for instance, questions the desirability of time consistency in individual behaviour and argues that social decisions are not, and need not be, time consistent. Nevertheless, the models developed in the next two sections suggest that embedding time inconsistency in our policy decision-making process might be unwise. The first of the models examines a planner using quasi-hyperbolic discounting in managing a renewable resource over three periods. It shows that the naive social planner may unwittingly manage the resource into a collapse. The underlying intuition is fairly straightforward. In period one, the naive planner determines an optimal consumption plan (for all three periods). However, in period two the planner recalculates and discovers that optimal second period consumption is higher than originally planned. Indeed, under certain conditions, it is now optimal to consume the entirety of the resource in period two. An unforeseen resource collapse is the result.

3 The Three-Period Model

Consider a planner who manages the harvesting of a renewable resource (eg fish), where E_i denotes the resource stock at the beginning of period i. The planner permits consumption c_i of the resource in period i. The resource is regenerated between the end of each period and the beginning of the next. The regeneration dynamics of renewable resources typically show two features; a 'carrying capacity', representing a limit on population growth, and a 'minimum viable population', the population below which the species is doomed to extinction. The second of these features is important here. A tractable linear specification capturing this is:

$$E_{i+1} = \begin{cases} (E_i - c_i)(1+g) & \text{if } (E_i - c_i)(1+g) \geq E^* \\ 0 & \text{otherwise} \end{cases} \tag{1}$$

where g is the natural rate of regeneration of the resource. Equation (1) implies that if resource stocks are below the minimum viable population, E^*, at the beginning of a period, then the population of the resource collapses to zero.

Instantaneous utility of consumption is assumed to be isoelastic:

$$u(c) = \frac{c^{1-\gamma} - 1}{1-\gamma}, \quad \gamma < 1 \tag{2}$$

where γ is the Arrow-Pratt coefficient of relative risk-aversion.[9] Note that with $\gamma \geq 1$, consumption can never be zero and hence a resource collapse is ruled out.[10] Otherwise, the model

[8]Laibson (1994), Laibson (1997) and Laibson et al. (1998) have considered the problem of undersaving in depth. Harris and Laibson (2001a) and Harris and Laibson (2001b) extend this work to model buffer-stock saving. Retirement timing is considered by Diamond and Koszegi (1998). Drug addiction is examined by Gruber and Koszegi (2000), while O'Donoghue and Rabin (1999a), O'Donoghue and Rabin (1999b) and Benabou and Tirole (2000) have examined procrastination. Cropper and Laibson (1999) consider the effect of hyperbolic discounting in environmental project evaluation.

[9]See Pratt (1964) and Arrow (1965).

[10]This implies that a collapse is impossible with log utility, as the isoelastic family $u(c) = \frac{c^{1-\gamma}-1}{1-\gamma}$ simplifies to log utility as $\lim_{\gamma \to 1}$ by L'Hôpital's rule. However, in section 3.3 a (reasonable) additional assumption is proposed which overcomes this limitation.

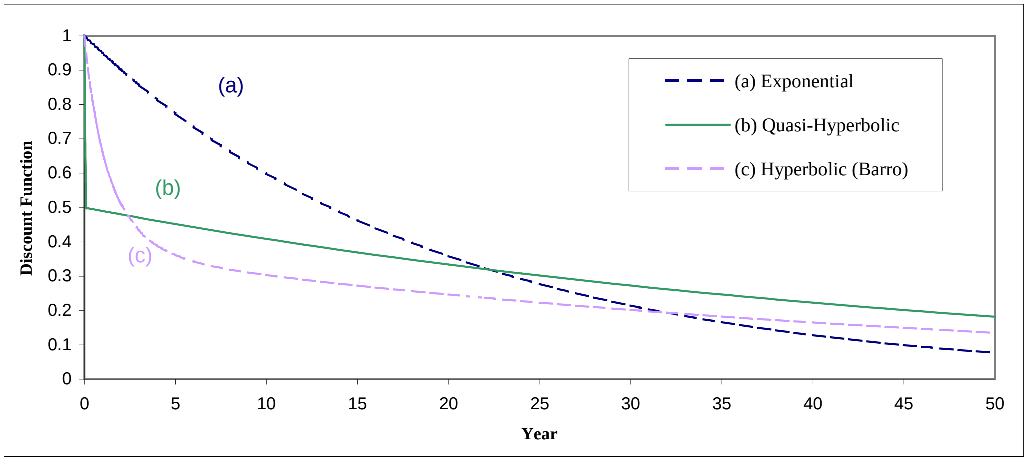

Figure 1: Exponential, hyperbolic and quasi-hyperbolic functions[a]

[a]For (a) the exponential function ($e^{-\rho t}$), $\rho = 5\%$, (b) the quasi-hyperbolic function, $\beta = 0.5$ and $\delta = 0.98$, and(c) the Barro (1999) hyperbolic function, $e^{-[\rho t + \phi(t)]}$, we use the parameters from section 4.

is robust to changes in specification of the utility function,[11] and equivalent results are found with other classes of utility function such as the negative exponential: $u(c) = 1 - \exp(-\omega c)$.

The social planner employs 'quasi-hyperbolic' discounting, using the discount factors $\{1, \beta\delta, \beta\delta^2\}$, where $\beta, \delta \in (0, 1)$. This form of discounting is shown in Figure 1(b).[12] Quasi-hyperbolic discounting is a tractable way of modelling the qualitative properties of the more general hyperbolic function. The utility of consumption over all three periods for consumers in the first period is therefore:

$$U(c) = u(c_1) + \beta\delta u(c_2) + \beta\delta^2 u(c_3) \tag{3}$$

In section 3.1, we derive the conditions under which the naive social planner will oversee an unanticipated resource collapse. The impact of enabling the planner to commit to a consumption plan is considered in section 3.2. Limitations of the model are presented in section 3.3.

3.1 The Naive Planner

The optimal consumption plan is found by standard backwards induction, with a little twist. First, the *actual* second period consumption is determined. The conditions giving rise to a 'collapse' — when the resource is consumed to extinction in period two — are found in Theorem 1. Next, the second period consumption as *anticipated* in period one is determined, and the conditions under which no collapse is anticipated are presented in Theorem 2. Finally, in Theorem 3 we prove that there is an overlap in the conditions required in Theorems 1 and 2: that is, an unanticipated resource collapse may occur.

[11]This robustness arises from the discontinuity in the budget set. Without this discontinuity, the utility function would have to satisfy $\lim_{c \to 0} u'(c) \neq \infty$ in order to give rise to a resource collapse. These issues are discussed further in section 3.3.

[12]This appears to have been originally employed by Phelps and Pollak (1968), then Akerlof (1991) and has been popularised by Laibson (1997).

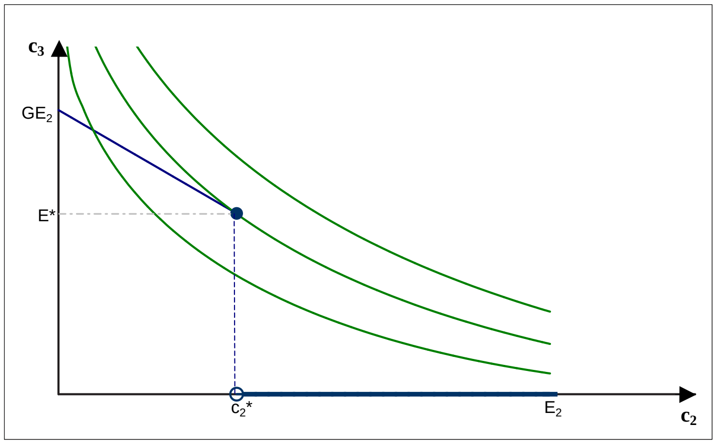

Figure 2: Intertemporal budget constraint.

In the third and final period, the entire stock of the resource will be consumed unless the stock has already been depleted below the minimum viable population. Hence:

$$c_3 \leq \begin{cases} E_3 = (E_2 - c_2)G & \text{for } E_3 \geq E^*, \\ 0 & \text{for } E_3 < E^* \end{cases} \tag{4}$$

where $G = 1 + g$ for ease of notation.

The budget constraint between the second and third periods given by equation (4) is shown in Figure 2. The discontinuity at the minimum viable population, E^*, implies that if consumption in period two exceeds a certain maximum level, the resource will become extinct and $c_3 = 0$. This maximum level of second period consumption is given by:

$$c_2^* \equiv E_2 - \frac{E^*}{G} \tag{5}$$

Utility derived by the second period planner is given by:

$$U_2 = \frac{c_2^{1-\gamma} - 1}{1-\gamma} + \beta\delta\frac{c_3^{1-\gamma} - 1}{1-\gamma} \tag{6}$$

Assuming an optimum on the upper part of the budget constraint (ie. $c_2 \leq c_2^*$, which we shall call an 'interior solution'), the budget constraint in equation (4) may be expressed as:

$$E_2 \geq c_2 + \frac{c_3}{G} \tag{7}$$

The Lagrangian for the optimisation program is therefore given by:

$$\mathcal{L} = \frac{c_2^{1-\gamma} - 1}{1-\gamma} + \beta\delta\frac{c_3^{1-\gamma} - 1}{1-\gamma} + \lambda_2\left(E_2 - c_2 - \frac{c_3}{G}\right) \tag{8}$$

where λ_2 is a Lagrange multiplier. After applying the Kuhn-Tucker conditions we have the following Euler equation:

$$c_3 = (\beta\delta G)^{\frac{1}{\gamma}} c_2 \tag{9}$$

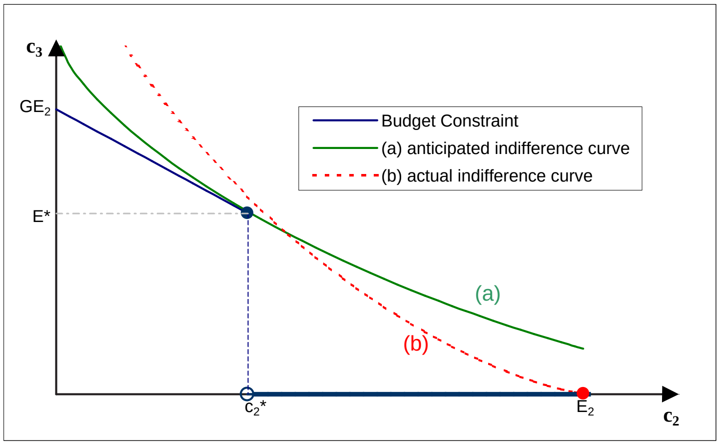

Figure 3: Boundary solutions: (a) extinction is avoided; and (b) extinction is optimal.

Substituting this into the budget constraint in equation (7) yields the optimum second period consumption as a function of the environmental stock at the beginning of the second period:

$$c_2 = \frac{E_2}{1 + G^{-1}(\beta\delta G)^{\frac{1}{\gamma}}} \tag{10}$$

Now, if the result of this optimisation yields (c_2, c_3) such that $c_2 > c_2^*$, then the assumption of an interior solution was incorrect. Employing equations (5) and (10) it can be shown that c_2 will exceed critical consumption c_2^* if the minimum viable population, E^* is such that:

$$E^* > \left(\frac{(\beta\delta G)^{\frac{1}{\gamma}}}{1 + G^{-1}(\beta\delta G)^{\frac{1}{\gamma}}} \right) E_2 \tag{11}$$

If equation (11) holds, an interior solution is inapplicable and the optimum is one of the two boundary solutions, illustrated in Figure 3. Curve (a) shows the case where optimal consumption is the highest possible without extinction. In contrast, curve (b) shows boundary solution where extinction of the resource in the second period is optimal. This case is of interest to us. Extinction of the resource in the second period will occur if and only if:

$$u(c_2^*) + \beta\delta u(E^*) < u(E_2) + \beta\delta u(0) \tag{12}$$

The left-hand side of equation (12) is the utility from consuming as much of the resource as possible without extinction (ie $c_2 = c_2^*$). The right-hand side is the utility from consuming the resource to extinction in the second period. For reasonable values of the parameters, a non-empty set of values (E_2, E^*) can be found where extinction occurs.[13] In Theorem 1, the first of three theorems in this section, we find the necessary and sufficient condition for premature extinction of the resource.

[13] For purposes of illustration, assume $\beta = 0.6$ and $\delta = 0.99$, which is roughly consistent with the empirical evidence on intertemporal choice in Ainslie (1992). A 5% regeneration rate for a renewable resource is also plausible. So letting $\beta = 0.6, \delta = 0.99, \gamma = 0.4, g = 0.05$, then a simple calculation shows that unless the planner starts the second period with stocks 44% above the minimum viable population, then it will be optimal to consume everything in the second period, leaving nothing for the third. Hence if the minimum viable population is 100 fish, than an initial stock of above 144 fish is necessary to avoid collapse. In other words, if the resource stock is not much above the minimum viable population, it is optimal to consume it to extinction.

Theorem 1 **[Collapse actually occurs]** *Iff* $G < f(\beta\delta, x, \gamma)$ *then equation (12) holds and a collapse will occur, where* $f(\beta\delta, x, \gamma) = x[1 - (1 - \beta\delta x^{1-\gamma})^{\frac{1}{1-\gamma}}]^{-1}$ *and* $x = E^*/E_2$.

Proof 1 *See Appendix 1.* ∎

The intuition of Theorem 1 is straightforward. If the regeneration rate G is small relative to the discount factor between periods two and three ($\beta\delta$), and if society starts the second period with low levels of the renewable resource — near to the minimum viable population — then society is best off consuming the resource to extinction in period two.

We now turn to examine the *planned* consumption in period two which, with naive hyperbolic discounting, may be different from actual consumption. In period one, the naive planner thinks (erroneously) that it will discount exponentially from the second period onwards. In other words, it anticipates using a discount factor of δ, rather than the actual factor, $\beta\delta$. As such, actual second period consumption turns out to be higher than anticipated consumption. Consider the *anticipated* Euler equation for an interior solution:

$$c_3 = (\delta G)^{\frac{1}{\gamma}} c_2 \tag{13}$$

Consequently, anticipated second period consumption is:

$$c_2 = \frac{E_2}{1 + G^{-1}(\delta G)^{\frac{1}{\gamma}}} \tag{14}$$

In comparison with the *actual* second period consumption given in equation (10), *anticipated* consumption is lower, due to the absence of β in equation (14). Hence the naive social planner might anticipate second period consumption to be low enough to avoid extinction ($c_2 \leq c^*$), and yet actual second period consumption results in the extinction of the resource. Extinction is not anticipated if:

$$u(E_2) + \delta u(0) < u(c_2^*) + \delta u(E^*) \tag{15}$$

The left-hand side of equation (15) is the anticipated utility from consuming the stock to extinction in the second period, while the right-hand side is the (erroneous) anticipated utility from consuming as much as possible without extinction. If this inequality holds, the naive planner does not anticipate extinction at the end of the second period.

Again, for reasonable values of the parameters we can find a non-empty set of values (E_2, E^*) in which an unforeseen extinction occurs.[14] In Theorem 2 we find the necessary and sufficient condition under which the planner does not anticipate extinction of the resource.

Theorem 2 **[No collapse is anticipated]** *Iff* $G > f(\delta, x, \gamma)$ *then equation (15) holds and no collapse is anticipated, where* $f(\delta, x, \gamma) = x[1 - (1 - \delta x^{1-\gamma})^{\frac{1}{1-\gamma}}]^{-1}$ *as in Theorem 1.*

[14] For the parameter values noted in footnote 13: $\beta = 0.6, \delta = 0.99, \gamma = 0.4, g = 0.05$, extinction will occur and will not be foreseen for $E^* = 100$ and $100 < E_2 < 144$.

Proof 2 *See Appendix 1.* ■

The intuition of Theorem 2 is as follows. Even if stock levels are close to the minimum viable population at the start of the period, a collapse will not be anticipated if the regeneration rate G is large enough relative to the anticipated discount factor (δ). In other words, if the resource grows quickly enough, anticipated returns from conserving the resource in period two are high enough that the planner anticipates saving some of the resource for period three. Hence no collapse is foreseen.

Now, to complete the mathematical triptych, notice that if equations (12) and (15) both hold, then extinction will take place without the naive planner foreseeing it. For ease of reference the two conditions are:

$$\begin{aligned} u(c_2^*) + \beta\delta u(E^*) &< u(E_2) + \beta\delta u(0) \\ u(E_2) + \delta u(0) &< u(c_2^*) + \delta u(E^*) \end{aligned} \tag{16}$$

Combining Theorems 1 and 2, we now show that if the regeneration growth rate of the resource is within a certain range and stock levels are close to the critical threshold, an unforeseen resource collapse will occur.

Theorem 3 [Unforeseen collapse occurs] *Iff* $f(\delta, x, \gamma) < G < f(\beta\delta, x, \gamma)$*, then equation (16) holds, implying an unforeseen collapse o the stock in period two.*

Proof 3 *See Appendix 1.* ■

The underlying intuition of these three theorems can be seen by considering the isoelastic utility function when $\gamma = 0$. This restriction gives us a linear utility function: $u(c) = c - 1$. Theorem 3 tells us that provided $x[1 - (1 - \delta x^{1-\gamma})^{\frac{1}{1-\gamma}}]^{-1} < G < x[1 - (1 - \beta\delta x^{1-\gamma})^{\frac{1}{1-\gamma}}]^{-1}$, an unforeseen extinction of the resource will occur in the second period. For $\gamma = 0$ this requirement is:[15]

$$\frac{1}{\delta} < G < \frac{1}{\delta\beta} \tag{17}$$

With linear utility, the anticipated marginal rate of substitution (MRS) between periods two and three consumption is $1/\delta$.[16] The actual MRS, $1/\beta\delta$, is higher than the anticipated MRS. In other words, the actual indifference curve is steeper than anticipated. Finally, G is slope of the budget constraint, which can also be thought of as the marginal rate of transformation (MRT). Equation (17) tells us that an unforeseen collapse occurs if the actual indifference curve is steeper than the budget constraint, which is steeper than the anticipated indifference curve:

$$\text{anticipated MRS} < \text{MRT} < \text{actual MRS} \tag{18}$$

Figure 4 nicely illustrates this situation. The planner anticipates saving the renewable resource entirely for consumption in period three, but it turns out to be optimal to consume the resource

[15] As the size of the resource endowment approaches zero under negative exponential utility, the same linear condition is required to guarantee the possibility of an unforeseen collapse applies, as shown in Appendix 2.

[16] Here we are adopting the conventional economic definition that the slope = MRS = $||\frac{dc_2}{dc_3}||$.

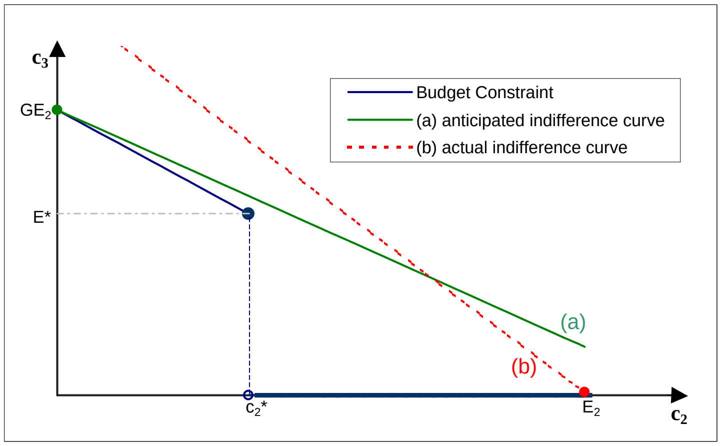

Figure 4: Unforeseen resource collapse with linear utility.

to extinction in period two. The general case for isoelastic utility is shown in Figure 3. We see in Figure 3 that the additional discount factor β means that the anticipated MRS $<$ actual MRS. In other words, the extra β rotates the anticipated indifference curve clockwise once period two arrives. If this rotation is such that the optimum second period consumption becomes E_2, then an unforeseen collapse will occur.

Thus a naive social planner using hyperbolic discounting will condone a consumption plan that could give rise to an unforeseen resource collapse. This result does not rest on any assumption of uncertainty in the minimum viable population. Nor does it require lags between human action and impact on the environment. The mere use of hyperbolic discounting (whether deliberate or accidental) in a standard model of intertemporal optimisation with a renewable resource means that an unforeseen collapse is possible.

3.2 The Committed Planner

Suppose the planner in period one can commit later generations to its desired consumption plan. It is straightforward to show that period one consumption is given by:

$$c_1^c = E_1 \left[1 + \beta\delta X(1 + \delta Y)\right]^{-1} \tag{19}$$

where the c subscript denotes consumption permitted by the committed planner, and where $X \equiv (\beta\delta G)^{\frac{1-\gamma}{\gamma}}$ and $Y \equiv (\delta G)^{\frac{1-\gamma}{\gamma}}$. This can be shown to be the same as the naive first period consumption, for the reason that naive consumption is determined under the misapprehension that later generations can be committed. The second period consumption when a commitment mechanism is available is:

$$c_2^c = E_1 G \frac{\beta\delta X}{1 + \beta\delta X(1 + \delta Y)} \tag{20}$$

In contrast, it can be shown that second period naive consumption is given by:

$$c_2 = E_1 G \frac{\beta\delta X(1 + \delta Y)}{(1 + \beta\delta X)(1 + \beta\delta X(1 + \delta Y))} \tag{21}$$

Comparing the two, it is clear that period two consumption by the naive planner is greater than that of the committed planner:

$$c_2 = \left(\frac{1+\delta Y}{1+\beta\delta X}\right) c_2^c \;=\; \left(\frac{1+\delta(\delta G)^{\frac{1-\gamma}{\gamma}}}{1+\beta\delta(\beta\delta G)^{\frac{1-\gamma}{\gamma}}}\right) c_2^c \;=\; \left(\frac{1+\delta^{\frac{1}{\gamma}} G^{\frac{1-\gamma}{\gamma}}}{1+\beta^{\frac{1}{\gamma}}\delta^{\frac{1}{\gamma}} G^{\frac{1-\gamma}{\gamma}}}\right) c_2^c \;>\; c_2^c \quad \text{as } \beta < 1 \tag{22}$$

Hence in period two, the naive planner without a commitment mechanism permits higher consumption than it would otherwise. Higher consumption in period two implies to lower consumption in period three, corresponding to an increase in the likelihood of a resource collapse. Hence if the naive planner can find a mechanism to commit itself to its planned consumption trajectory in periods two and three, the probability of a resource collapse would be decreased.

After a review of the psychological evidence on hyperbolic discounting, Loewenstein (1996) concludes that people do behave at least partially naively. If such naiveté is also present in environmental policymakers, then mechanisms to commit ourselves to sound future environmental practices — such as investment in renewable technologies perhaps — will be worth investigating.[17] Commitment mechanisms are discussed in more detail in section 5.

3.3 Limitations of the Model

A limitation of the model is that results do not hold for isoelastic utility when $\gamma \geq 1$, because for $\gamma \geq 1$, $\lim_{c\to 0} u(c) = -\infty$, and extinction of the resource can never be on an optimum consumption path. This difficulty can be circumvented if we note that society has other sources of consumption than the renewable resource. For simplicity, suppose the other activities increase consumption in each period by the constant A. Then the budget constraint for the second and third periods is:

$$c_3 = \begin{cases} E_3 = (E_2 - (c_2 - A))G + A & \text{for } E_3 \geq E^*, \\ A & \text{for } E_3 < E^* \end{cases} \tag{23}$$

With this modified budget constraint it is straightforward to prove that, even with $\gamma \geq 1$, an unanticipated resource collapse is possible.

With the exception of this modification required for $\gamma \geq 1$, results are robust to changes in the utility function. Equivalent results are found under negative exponential and quadratic utility. The model is also robust to utility functions with the property that $\lim_{c\to 0} u'(c) = \infty$. In normal consumer optimisation, utility functions with this property (such as the entire isoelastic family) do not permit corner solutions, so a resource collapse would be ruled out. However, because of the discontinuity in the budget set, our model operates with such utility functions without difficulty.

The use of a discrete-time model with three periods is illustrative and allows simple, tractable results. However, it also limits our ability to compare results with those from the standard discounted utilitarian model. As such in the next section, a continuous-time model of renewable resource consumption under hyperbolic discounting is developed, showing that the basic result remains intact.

[17] Whether commitment improves welfare depends upon the weighting of the three temporal selves' welfare, which itself involves an interesting range of issues. We do not consider these here. For a broad treatment of some of these intergenerational equity issues, see Portney and Weyant (1999).

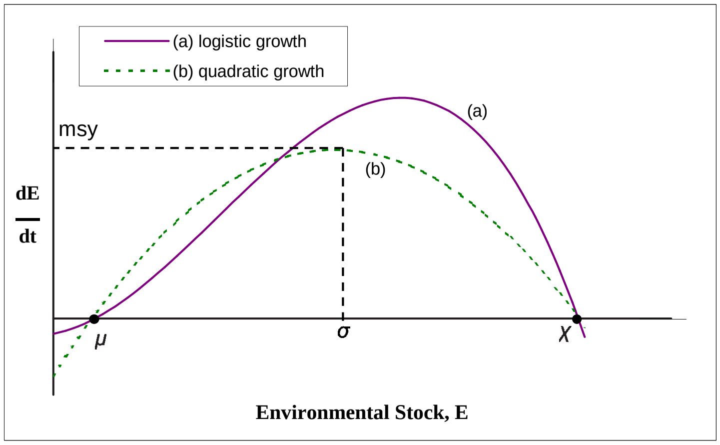

Figure 5: Resource dynamics: (a) logistic and (b) a quadratic approximation.

4 The Infinite-Horizon, Continuous-Time Model

As before, consider the consumption of a renewable resource, E, but this time with marginally more sophisticated dynamics given by a quadratic approximation to the logistic model, illustrated in Figure 5. The carrying capacity, χ, the minimum viable population, μ, and the population at the 'maximum sustainable yield', σ, are the main features of the regeneration dynamics.[18]

Our specification for the resource regeneration rate is:

$$g(E(t)) = -pE(t)^2 + qE(t) - r \tag{24}$$

where $p, r, q > 0$. The resource regeneration dynamics are represented by the parabola in Figure 6 (which is also the $\dot{E} = 0$ isocline), for parameters $(p, q, r) = (0.0001, 0.1, 10)$. Figure 6 illustrates three important properties of the system:

$$\chi = \frac{q + \sqrt{q^2 - 4pr}}{2p} \tag{25}$$

$$\mu = \frac{q - \sqrt{q^2 - 4pr}}{2p} \tag{26}$$

$$\sigma = \frac{q}{2p} \tag{27}$$

Resource stock dynamics are simply regeneration less consumption at time t:

$$\dot{E} = g(E(t)) - c(t) \tag{28}$$

Consumption provides instantaneous utility $u(c(t))$, and the planner finds the consumption path which maximises the discounted present value of the stream of utilities. The utility functional is:

$$U[c(\tau)] = \int_{\tau}^{\infty} u(c(t))D(t - \tau)dt \tag{29}$$

[18] See Clark (1990) for more detail on the generalised logistic model and other biological growth models.

where τ is the current date, and $D(t-\tau)$ is the discount function. In the standard model with a constant discount rate ρ, the discount function is $D(t-\tau) = e^{-[\rho\cdot(t-\tau)]}$. When a planner employs hyperbolic discounting, the discount function must be modified. Barro (1999) employs a tractable specification:

$$D(t-\tau) = e^{-[\rho\cdot(t-\tau)+\phi(t-\tau)]} \tag{30}$$

where the inclusion of $\phi(t-\tau) \geq 0$ allows for hyperbolic discounting. With this discount function, shown in Figure 1(d), the discount rate $\rho + \phi'(v)$ will be declining if $\phi''(v) \leq 0$, where $v = t - \tau$ is the distance from the current time, τ, to a future point in time, t. An exponential function satisfies these requirements:

$$\phi'(v) = be^{-\theta v} \tag{31}$$

where $b = \phi'(0) > 0$ and $\theta > 0$. Hence the discount rate, initially $\rho + b$, declines at the constant rate θ towards ρ as the time horizon increases. Integrating this equation, and normalising so that $\phi(0) = 0$ yields our specification for $\phi(v)$:

$$\phi(v) = \frac{b}{\theta}\left(1 - e^{-\theta v}\right) \tag{32}$$

Using this framework, in section 4.1 we determine the consumption path for an environmental policymaker able to commit itself (and its later selves). We briefly discuss the optimisation problem for the sophisticated planner in section 4.2. In section 4.3, we find actual and anticipated trajectories for the naive planner, and note that an unforeseen collapse is a possible outcome under isoelastic utility.

4.1 The Committed Planner

With log utility and a planner able to commit itself to follow a consumption plan developed at time τ, the consumption trajectory is the result of the following optimisation:

$$\max_{c} \quad U[c(\tau)] = \int_{\tau}^{\infty} \log c(t) e^{-[\rho\cdot(t-\tau)+\phi(t-\tau)]} dt \tag{33}$$

subject to:

$$\dot{E} = -pE(t)^2 + qE(t) - r - c(t) \tag{34}$$

Applying the Maximum Principle we obtain the optimal growth rate in consumption:

$$\frac{\dot{c}}{c} = q - 2pE(t) - \rho - \phi'(t-\tau) \tag{35}$$

Equations (34) and (35) constitute a non-linear, non-autonomous system of differential equations. This system resembles the standard Ramsey-Cass-Samuelson optimal economic growth problem, but has several important differences. The system can be understood by examining the $\dot{E} = 0$ and $\dot{c} = 0$ isoclines. When $\dot{E} = 0$, consumption must equal the resource regeneration rate:

$$c = g(E) = -pE^2 + qE - r \tag{36}$$

Hence the $\dot{E} = 0$ isocline is given by the parabolic curve in Figure 6. When $\dot{c} = 0$, from equation (35) we see that either:

$$c = 0, \text{ or } E(t) = \frac{q - \rho - \phi'(t-\tau)}{2p} \tag{37}$$

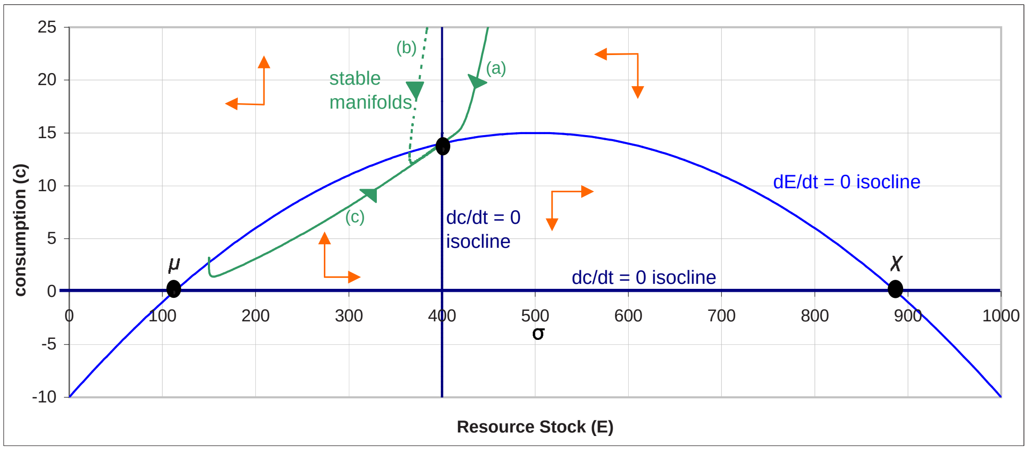

Figure 6: Phase-plane diagram of the nonlinear optimal consumption problem.

Hence the $\dot{c} = 0$ isocline is the horizontal axis and the vertical line given by $E(t)$ in equation (37). Because $\phi'(t-\tau) \to 0$ as $t \to \infty$, from equation (37) it follows that as $t \to \infty$ the vertical $\dot{c} = 0$ isocline is given by:

$$\hat{E} = \frac{q}{2p} - \frac{\rho}{2p} = \sigma - \frac{\rho}{2p} \tag{38}$$

This specifies the value of the resource stock at the equilibrium with non-zero consumption. We find the standard result that the equilibrium resource stock is $\frac{\rho}{2p}$ below the population at maximum sustainable yield, σ. Given the long run equilibrium resource population in equation (38), equilibrium consumption can be found from (36):

$$\hat{c} = g(\hat{E}) = -p\hat{E}^2 + q\hat{E} - r \tag{39}$$

$$= \frac{q^2 - \rho^2}{4p} - r \tag{40}$$

Figure 6 shows the phase-plane for the system, divided into isosectors with three illustrative stable manifolds. The manifolds were calculated for initial resource stocks of $E(\tau) = 500, 400$ and 150 with the following parameter values: $(p, q, r) = (0.0001, 0.1, 10)$, ρ (long run discount rate) $= 0.02$, $b = \phi'(0) = 0.5$ and θ (rate of decay of $\phi'(v)$) $= 0.5$. With these parameters values, equation (38) gives an equilibrium resource population to be $\hat{E} = 400$, and from equation (39) we see that equilibrium consumption is $\hat{c} = 14$. The equilibria at the carrying capacity χ and the minimum viable population μ are stable and unstable respectively, but neither are optimal as they yield zero consumption.

This system has one striking difference to the standard Ramsey model. The system of differential equations is non-autonomous as consumption growth in equation (35) is an explicit function of t. A result of this is that the equation for the vertical $\dot{c} = 0$ isocline gives $E(t)$ as a function of t, meaning that this isocline moves as time passes. This implies that the equilibrium with non-zero consumption is shifting over time.[19] The stable manifolds are shooting for a moving target. For instance, the dotted manifold in Figure 6(b) shows the optimum trajectory given

[19]Note, for instance, that when $t = \tau$, $\phi'(0) = b$, so the vertical isocline starts below its final value (mathematically, it can even be negative).

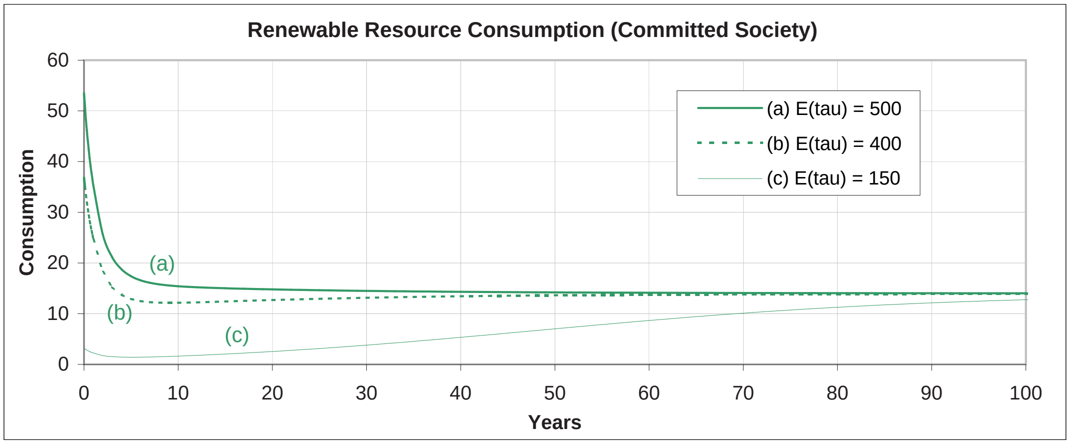

Figure 7: Resource consumption with a committed planner.

an initial stock $E(\tau) = E^* = 400$. Given this initial condition, optimal consumption under exponential discounting is $\hat{c} = 14$, starting and remaining at the equilibrium. With hyperbolic discounting, however, at $t = \tau$ the vertical $\dot{c} = 0$ isocline is mathematically negative. Hence consumption begins as though we are in the top-right isosector, on a pathway heading down and to the left. As time passes, the vertical $\dot{c} = 0$ isocline shifts rightwards until it crosses the dotted manifold, at which point the manifold turns and heads upwards following the isocline until both (asymptotically) reach equilibrium at $\hat{E} = 400$ and $\hat{c} = 14$. Li and Löfgren (2001) prove that the committed hyperbolic discounting pathway asymptotically approaches the constant discounting analogue as the discount rate declines. While this might appear self-evident, their detailed proof involves finding the asymptotic error rate of approximating the non-autonomous path to an autonomous flow, employing theory developed in Benaïm and Hirsch (1996).

This example is illustrative of a more general feature of the system. Because preferences are time-inconsistent, there is no, single, stable manifold as in standard problems. For each given initial condition $E(\tau)$, there is a unique $c(\tau)$ which will lead to the Pareto-dominant equilibrium. For each $(E(\tau), c(\tau))$ pair there is also a unique stable manifold. In other words, there are an infinite number of stable manifolds on the phase plane.

Figures 7 and 8 show time paths for consumption and population levels, for the three initial values of $E(\tau) = 500, 400$ and 150. The assumption of hyperbolic discounting is clearly evident in the shape of the consumption trajectory: the rate of consumption decline is faster in earlier periods, reflecting higher impatience. Notice that for initial resource stock of $E(\tau) = 400$, described by Figure 8(b), the resource starts at its long run equilibrium, but due to hyperbolic discounting and the salience of the present, falls before it returns to its equilibrium level. Essentially, the planner is choosing to commit later periods to lower consumption levels, so that more can be consumed today.

It remains to note one limitation of the committed solution, highlighted by Barro (1999). Our solutions were found for a given starting time, τ, which implies that the ability to commit arose precisely at that starting time. If the possibility of commitment had already existed, all current and future values of consumption would have been determined at the point the commitment mechanism appeared. Indeed, if the commitment mechanism had always existed, then τ is effectively minus infinity and $\phi'(t - \tau)$ is zero for all $t \geq 0$.

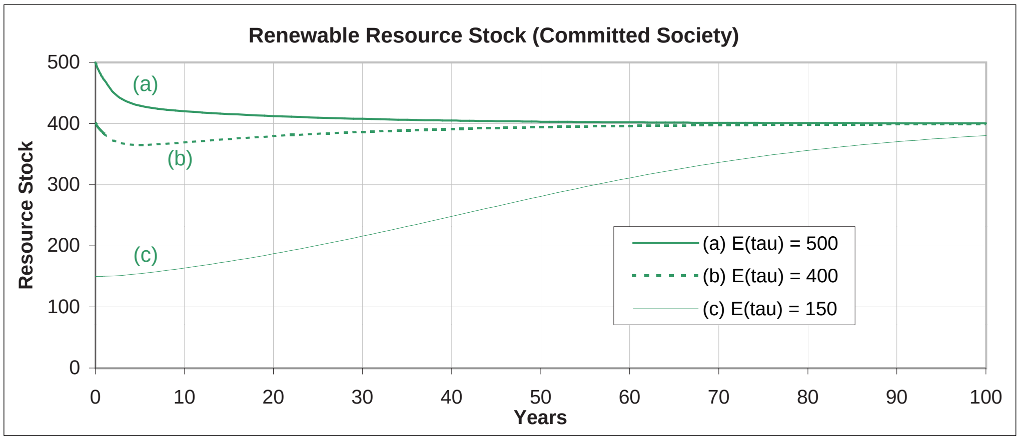

Figure 8: Resource stock evolution with a committed planner.

Moreover, in the context of renewable resources, mechanisms to commit future generations are not immediately obvious. We discuss these in more detail in section 5. For now, we turn to paths in the absence of commitment.

4.2 The Sophisticated Planner

If a commitment mechanism is not available, maximisation using the Pontragin Principle or Ramsey-like perturbation arguments is impossible. Future planners have different preferences and will not adhere to the first-best optimum determined at time τ. As such, we seek a solution analogous to the second-best optimum proposed by Phelps and Pollak (1968). Time τ planner chooses its strategy, $c(\tau)$, to be a best response to future planners' selection of $c(t)$. The planner at time τ therefore needs to understand the impact of changes in $c(\tau)$ on the future level of the renewable resource, $E(t)$, and then work out how this will affect future consumption.

Such an analysis is neatly performed by Barro (1999), who shows that in a decentralised economy with log utility, the consumption program resulting from this intertemporal game is observationally equivalent to a consumption program under exponential discounting.[20] Barro (1999), however, does not analyse the system under the assumption of naivety, and we turn now to the main purpose of the model, which is to show that a naive planner will, up until it is too late, think that it can escape a resource collapse.

4.3 The Naive Planner

The concept of naiveté, developed by Strotz (1956) and Akerlof (1991), does not appear to have been widely used in continuous-time models. Strotz (1956) sketched out several anticipated consumption paths for a naive individual, but no actual consumption path. Neither Barro (1999) nor Li and Löfgren (2001) consider the implications of naivety. Given the mounting evidence that people behave as, at the least, partial naifs, we develop an appropriate model

[20] As ingenious as this approach is, we are inclined to agree with Solow (1999) that '...this does not feel anything like the way policy is talked about or could be talked about in a democracy, especially since any current generation is notoriously bad at guessing what future generations will want or do.'

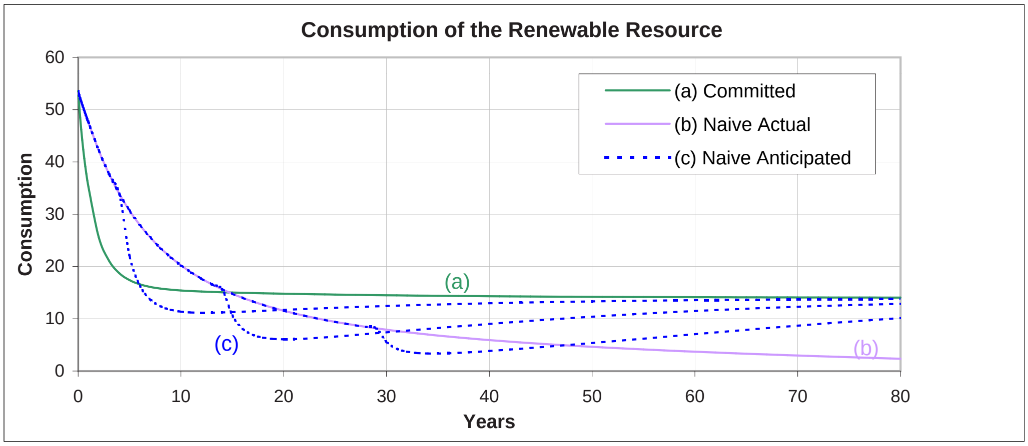

Figure 9: Resource consumption with (a) committed, (b) naive, and (c) anticipated by naive planners.

here.

In continuous-time, a naive social planner believes at time τ that it can commit its future selves to a consumption plan. However, at time $\tau+\epsilon$, the naive planner refuses to respect its preferences at time τ and consumes more than planned. Nevertheless, the planner at time $\tau+\epsilon$ retains the erroneous belief that things will be different from now onwards.

The anticipated trajectories of the naive planner are straightforward. At any time τ, the anticipated trajectory is given by the commitment trajectory for the stock of resources $E(\tau)$. Hence the variation on the Ramsey formula in equation (35) applies:

$$\frac{\dot{c}}{c} = q - 2pE(t) - \rho - \phi'(t-\tau) \tag{41}$$

Figure 9(c) shows the anticipated consumption paths at $\tau = 5, 15$ and 30. These obviously have the same shape as the committed pathway. Indeed trajectory anticipated by the naive planner at $\tau = 0$ is just the committed trajectory. The derivation of the actual time path of the naive planner, shown in Figure 9(b), is a little more difficult. At any given moment in time, the naive planner intends to consume according to the commitment optimum. Hence its level of consumption at time τ is given by the solution to the non-linear system of differential equations in (35) and (34). Rewriting the system here for convenience:

$$\dot{c} = c \cdot g'(E) - c(\rho - \phi'(t-\tau)) \tag{42}$$
$$\dot{E} = g(E) - c \tag{43}$$

Although there is no analytical solution to this non-autonomous and non-linear system, numerical methods provide us with a solution $c(\tau) = f(E(\tau))$. For a given level of the resource stock, the function $f(\cdot)$ tells us the actual consumption by the naive planner. Combining this solution with the budget constraint gives us a differential equation in E which we can also solve numerically:

$$\dot{E} = g(E) - f(E) \tag{44}$$

Solution of equation (44) gives the actual time path for the resource population levels, E. Then, using $c(\tau) = f(E(\tau))$, we calculate the time path for consumption. Illustrative paths, using the

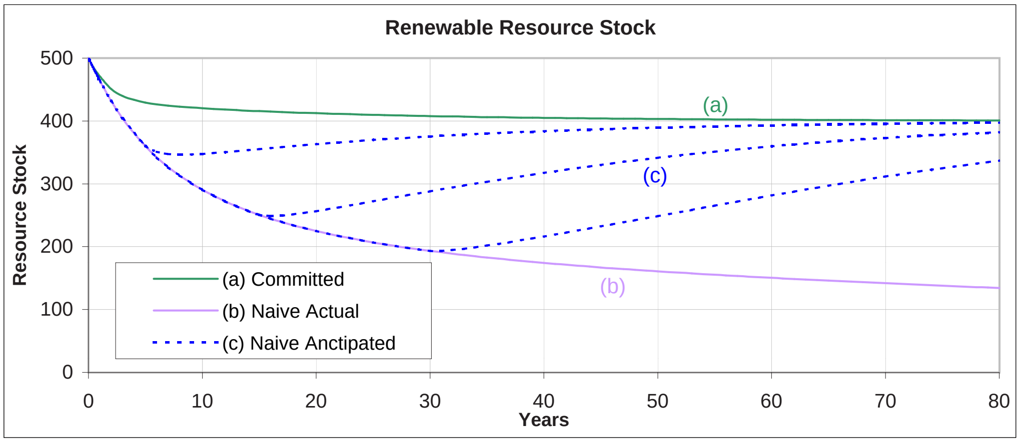

Figure 10: Resource stock evolution with (a) committed, (b) naive, and (c) anticipated by naive planners.

same parameters as before, are shown in Figures 9 and 10. Figure 9 shows that at any time τ, the naive planner anticipates reducing consumption, starting from time $\tau + \epsilon$. Of course, when time $\tau + \epsilon$ arrives, the planned reduction in consumption is no longer optimal!

Obviously, with log utility (as assumed above), the actual resource population levels can never collapse (as $u(0) = -\infty$). Instead, with the naive planner, resource levels asymptotically approach the minimum viable population, μ. Consumption asymptotically approaches zero. With isoelastic utility and $\gamma < 1$, however, a collapse is possible. As the naive planner anticipates (erroneously) that it can commit future selves, a collapse is only anticipated once a committed planner in the same situation would find it optimal to collapse the resource. Note that this does not imply that the resource will be collapsed immediately; with a concave utility function some degree of consumption smoothing will be optimal even with a positive discount rate.

A collapse will inevitably occur once stock levels fall sufficiently close to the minimum viable population, μ, that the utility arising from a 'managed collapse' exceeds the utility from ensuring the conservation of the stock. We define the stock level at which collapse is optimal, $\tilde{E}$, to be ϵ units above the minimum viable population, so that $\tilde{E} = \mu + \epsilon$. Whether or not $\epsilon > 0$ depends upon the specific hyperbolic discounting parameters employed.[21]

Hence no collapse is anticipated by the naive planner while stock level $E(\tau) > \mu + \epsilon$. However, the moment that $E(\tau) = \mu + \epsilon$ the naive planner will be startled to find that it is now optimal to begin the collapse of the resource. Hence, as in the discrete time model, we find that an unforeseen resource collapse is a possible outcome for a naive hyperbolic planner with $\gamma < 1$.

4.4 Applications of the Model

In the introduction two case studies of apparently 'irrational' resource planning — the collapse of the Atlantic Cod and the Peruvian Anchovy — were presented. We are now in a position to consider these collapses with reference to the continuous-time hyperbolic discounting model.

[21]Proof to be completed before commencement of the summer school.

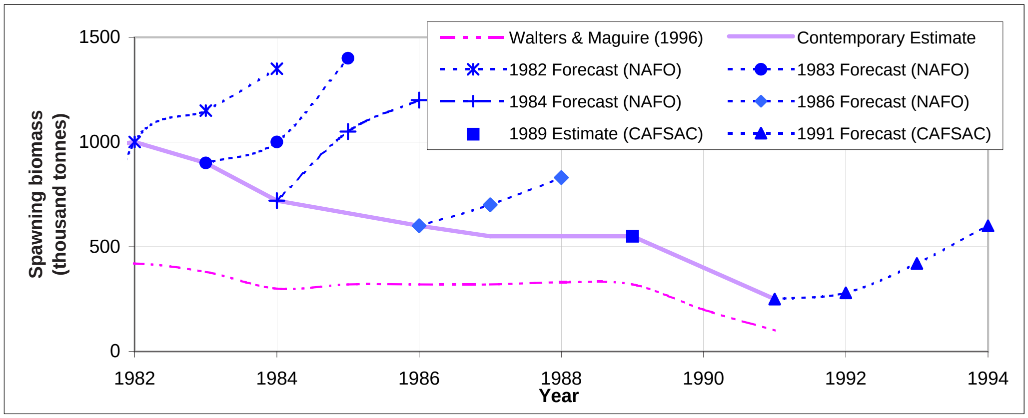

Figure 11: Comparison between forecast and actual populations of northern Atlantic cod.

In 1977, Canada extended its jurisdiction and took a wider role in management of fish stocks of Newfoundland. During the course of the 1980s, the Northern Atlantic Fisheries Organisation (NAFO) and the Canadian Atlantic Fisheries Scientific Advisory Committee (CAFSAC) employed virtual population analysis to estimate and forecast Atlantic cod stock levels, including the northern cod shown in Figure 11. After the collapse, Walters and Maguire (1996) used more sophisticated virtual population analysis to show that NAFO had significantly overestimated the true population levels. These overestimates of current population encouraged harvesting at higher than optimal levels and was one of the factors in the demise of the northern Atlantic cod.

The other significant factor, and one of greater relevance to this paper, was NAFO's consistently over-optimistic predictions of future stock levels, also shown in Figure 11. The NAFO forecast in 1982 shows steady population increases. It became clear a year later that the 1982 prediction for 1983 population levels had been over-optimistic. In 1983, predictions for the next two years show another rise in stock levels. Estimates in 1984, however, again revealed the 1983 predictions to have been too optimistic. The pattern repeats itself for the data available in 1984, 1986 and 1991.

How could these continual overestimates have occurred? Walters and Maguire (1996) note that the 'abundance indices' were inaccurate. Irrespective of the mathematics, one wonders why modelling strategies were not improved in the light of consistently over-optimistic forecasts. It is hinted that the reasons were as much political as scientific. Walters and Maguire (1996) admit that:

> *results are presented in complex, delicate, and often confused environments of debate between fishing interests and regulatory authorities over appropriate policy and resource allocation decisions. In such fora, we are often under more or less explicit pressure from fishery managers to understate the uncertainty in our assessments...*

Fishing interests and policy makers, while wanting to ensure a sustainable catch in the long run, also want the analysis of fishing scientists to justify high catches in the present. In other words, like people in all endeavours, policymakers weight the present in a particularly 'salient' way, to use the terminology of Akerlof (1991). These are precisely the characteristics expected when hyperbolic discounting is employed. Indeed, the trajectories of actual and anticipated northern cod stocks in Figure 11 is not unlike the theoretical predictions for a continuous-time, hyperbolic discounter in Figure 10.

One could similarly argue that the Peruvian authorities managed the Peruvian anchovy in a manner consistent with naive hyperbolic discounting. Upon the advice of experts such as the Instituto del Mar del Peru (1981), they initially made efforts to avoid collapse of the stock, presumably with a view to sustainable harvesting from a valuable natural asset. In 1972-73, with the arrival of El Niño, experts advised the government that opening the stock to fishing would lead to a collapse. Reversing their preference for maintenance of the anchovy revealed in previous years, the authorities opened the fishery and stock levels collapsed.

This decision could be explained by a number of reasons, among them uncertain science and management failure. However, a time-inconsistent preference for the short term is also very plausible. In the short term of 1972-73, the authorities were subject to pressure from an increasingly industrialised and powerful anchovy industry to open the resource. The political benefits of opening the resource would accrue immediately (i.e. in 1972-73), whereas the costs — cessation of anchovy fishing in later years — accrued in the future. The salience of the present dominated the decision-making process, resulting in a resource collapse. Based upon our hyperbolic discounting model, one might speculate that if the Peruvian authorities had been able to commit to a plan of action several years in advance which accounted for stochastic events such as El Niño, the collapse might have been avoided.

Unfortunately, another application for the model may be emerging at a global level. Countries agreed to targets under the Framework Convention on Climate Change and its Kyoto Protocol which many, including the USA, now seem unwilling to implement. Under the Framework Convention in 1990, developed countries agreed to stabilise their emissions at 1990 levels by 2000. (In large part, they failed to do so.) Under the subsequent Kyoto Protocol, developed countries agreed to reduce their greenhouse gas emissions to 95% of 1990 levels by 2008-2012. Despite undoubtedly genuine intentions at the time of signature in December 1997, performance to date has been less than encouraging. Moreover, Pearce (2003) argues that where targets have been met, this was more by accident than design.

While many factors may be partially responsible for the impending failure to meet the climate change challenge, it could also be explained by the hypothesis that our political leaders are behaving as naive hyperbolic discounters. The cost of emission reductions are borne in the present, while the benefits accrue relatively far into the future. The naive discounter would plan to achieve its reductions, starting soon. Given the higher discount rate on the present, however, it would not be optimal to start now. If hyperbolic discounting has played a role, then our model suggests that a real commitment mechanism with teeth stronger than the penalties currently applicable under international law is critical. This, of course, is more easily said than done.

5 Conclusion

This paper demonstrates that using hyperbolic discounting in environmental policy may have unintended and disastrous consequences. This is not to suggest, however, that the current policy shift, from constant to declining discount rates, is unambiguously bad. On the contrary, it represents an important leap forward for both environmental and economic theory and policy. All the same, time inconsistency may prove to be a thorn in the side of this policy change, which will otherwise lead to better provision for future generations.

Given the evidence suggesting that individuals discount the future in a naively hyperbolic way, it is plausible that policymakers will suffer, and possibly have already suffered, from the same temptation. Perhaps the best antidote to the risk of collapse demonstrated in this paper is simply an awareness that such problems may arise. The sophisticated hyperbolic discounter will not experience an unforeseen collapse, and hence will be able to construct regulation accordingly. It is only a naive planner who will fail to anticipate a resource collapse until it is too late.

The other remedy to the naivety problem is to establish appropriate commitment mechanisms. Legal commitment mechanisms are not all that promising at a national level; even less so at an international level.[22] Few nations have constitutionally entrenched environmental laws, and even constitutionally entrenched provisions can always be amended by later referendum.

Economic commitment mechanisms are probably more appealing. The possibility of irreversible investment in environmental technologies would appear to provide a mechanism to commit later generations. Consider the following example. Suppose that once the fixed costs of renewable energy research, development and infrastructure provision have been incurred, the relative marginal costs between renewables and fossil fuel power generation would favour renewable generation. Sinking irreversible investment into research, development and infrastructure, then, would compel future generations to use renewable energy.

In the absence of such solutions, further research into the significance of the time inconsistency problem is necessary. Perhaps this will show that a form of logarithmic discounting — the only form of declining discount rate to be intertemporally consistent — should provide the foundation for long-term policymaking. No doubt time will tell.

[22]Barrett (1990), however, examines the conditions under which treaties are self-enforcing.

6 Appendices

6.1 Appendix 1: Proof of Theorems

Proof of Theorem 1

Necessary and Sufficient Condition. Substituting from equations (2) and (5) into equation (12) yields the condition that a collapse will occur if and only if:

$$\frac{c_2^{*1-\gamma}-1}{1-\gamma}+\beta\delta\frac{E^{*1-\gamma}-1}{1-\gamma} < \frac{E_2^{1-\gamma}-1}{1-\gamma}+\beta\delta\frac{-1}{1-\gamma} \quad (45)$$

$$(E_2-E^*/G)^{1-\gamma}+\beta\delta E^{*1-\gamma} < E_2^{1-\gamma} \quad (46)$$

Simplification yields:

$$1-\beta\delta x^{1-\gamma} > (1-x/G)^{1-\gamma} \quad (47)$$

where $x = E^*/E_2$ denotes the ratio of the minimum viable population to the stock in the second period. Rearrangement gives:

$$G < x\left[1-(\beta\delta x^{1-\gamma})^{\frac{1}{1-\gamma}}\right]^{-1} \quad (48)$$

Therefore, satisfaction of equation (49) is necessary and sufficient for satisfaction of equation (12) and for a collapse to occur. This completes the proof of Theorem 1. ■

Sufficient Condition. The proof above states the necessary and sufficient condition for a collapse to occur. However, we need only show that a collapse is possible, so finding a sufficient condition is enough. Simpler sufficient conditions than equation (49) are available. For instance, a first order Taylor approximation around the right hand side of equation (48) reveals that:

$$(1-x/G)^{1-\gamma} < 1-(1-\gamma)x/G \quad (49)$$

And therefore, using equations (48) and (50), an alternative sufficient condition for a collapse is:

$$1-\beta\delta x^{1-\gamma} > 1-(1-\gamma)x/G \quad (50)$$

Simplifying this expression gives:

$$x > \left(\frac{G\beta\delta}{1-\gamma}\right)^{\frac{1}{\gamma}} \quad (51)$$

And therefore,

$$G < \left(\frac{1-\gamma}{\beta\delta}\right)x^{\gamma} \quad (52)$$

Hence if the regeneration rate G of the renewable resource is **small** enough to satisfy equation (53), then the inequality in equation (12) will hold and the resource stock will collapse in the second period. Equation (53) therefore provides a sufficient condition for collapse. ■

Proof of Theorem 2

Necessary and Sufficient Condition. Analogously to Theorem 1, substitution from equations (2) and (5) into equation (15) gives:

$$E_2^{1-\gamma} < c_2^{*1-\gamma} + \delta E^{*1-\gamma} \quad (53)$$

$$E_2^{1-\gamma} < (E_2 - E^*/G)^{1-\gamma} + \delta E^{*1-\gamma} \quad (54)$$

Simplification yields:

$$1 - \delta x^{1-\gamma} < (1 - x/G)^{1-\gamma} \quad (55)$$

where $x = E^*/E_2$ denotes the ratio of the minimum viable population to the stock in the second period. Rearrangement gives:

$$G > x\left[1 - (\delta x^{1-\gamma})^{\frac{1}{1-\gamma}}\right]^{-1} \quad (56)$$

Therefore, satisfaction of equation (57) is necessary and sufficient for satisfaction of equation (15) to hold and for no collapse to be foreseen. This completes the proof of Theorem 2. ∎

Sufficient Condition. As before, we are primarily interested in the sufficient condition. To simplify, note that in the relevant range for x the following inequality holds:

$$(1 - x/G)^{1-\gamma} > 1 - x/G \quad (57)$$

Therefore a sufficient condition is given by:

$$1 - \delta x^{1-\gamma} < 1 - x/G \quad (58)$$

Simplifying gives:

$$x < (G\delta)^{\frac{1}{\gamma}} \quad (59)$$

And therefore,

$$G > \left(\frac{1}{\delta}\right) x^{\gamma} \quad (60)$$

Hence provided the regeneration rate of the renewable resource is **large** enough to satisfy equation (61), then the inequality in equation (15) will hold and the planner will not anticipate a collapse in the second period. ∎

Proof of Theorem 3

Necessary and Sufficient Condition. From Theorem 1, we know that provided $G < x[1 - (\beta\delta x^{1-\gamma})^{\frac{1}{1-\gamma}}]^{-1}$, the top inequality of equation (16) holds. From Theorem 2 it follows that provided $G > x[1 - (\delta x^{1-\gamma})^{\frac{1}{1-\gamma}}]^{-1}$, then the bottom inequality of equation (16) is also satisfied. Hence if:

$$x\left[1 - (\delta x^{1-\gamma})^{\frac{1}{1-\gamma}}\right]^{-1} < G < x\left[1 - (\beta\delta x^{1-\gamma})^{\frac{1}{1-\gamma}}\right]^{-1} \quad (61)$$

then both inequalities hold and equation (16) is satisfied, implying an unforeseen extinction of the resource stock in the second period. This completes the proof of Theorem 3. ∎

Sufficient Condition. Analogously, a sufficient condition for a unforeseen collapse of the stock in the second period follows from combining the sufficient conditions found in Theorems 1 and 2. Hence if:

$$\frac{x^{\gamma}}{\delta} < G < \left(\frac{1-\gamma}{\beta}\right)\frac{x^{\gamma}}{\delta} \tag{62}$$

then equation (16) is satisfied, implying an unforeseen extinction of the resource stock in the second period. Note that this can only be satisfied for $\beta + \gamma < 1$, which is relatively restrictive. Nevertheless, equation (63) provides a sufficient condition for an unforeseen collapse. ∎

6.2 Appendix 2: Collapse under Exponential Utility

In this appendix we derive the conditions for a resource collapse under negative exponential utility in order to illustrate that our results are relatively robust to changes in utility function. We also show that, if the environmental stock and the minimum viable population are very small, the condition for collapse under exponential utility approximates that under linear utility. This finding corroborates results in the main paper. Beyond these purposes, however, Appendix 2 does not contain highly significant material and may be skipped without great loss.

Our utility function is given by:

$$u(c) = 1 - \exp(-\omega c) \tag{63}$$

By optimisation equivalent to that under isoelastic utility we have the Euler equation for an interior solution:

$$c_2 = c_3 - \frac{\log \beta\delta G}{\gamma} \tag{64}$$

Substituting this into the budget constraint in equation (7) yields the optimum second period consumption as a function of the environmental stock at the beginning of the second period:

$$c_2 = \frac{G\gamma}{G\gamma + \gamma + \log \beta\delta G}.E_2 \tag{65}$$

If this yields (c_2, c_3) such that $c_2 > c_2^*$, then the assumption of an interior solution is unjustified. As before, we note that extinction of the resource in the second period will occur if:

$$u(c_2^*) + \beta\delta u(E^*) < u(E_2) + \beta\delta u(0) \tag{66}$$

The naive planner will not anticipate extinction if:

$$u(E_2) + \delta u(0) < u(c_2^*) + \delta u(E^*) \tag{67}$$

Noting that $u(0) = 0$ we can combine equations (67) and (68) to get a single condition for an unanticipated extinction:

$$u(c_2^*) + \beta\delta u(E^*) < u(E_2) < u(c_2^*) + \delta u(E^*) \tag{68}$$

As with isoelastic utility, with negative exponential utility and reasonable values of the parameters, we can always find a non-empty set of values (E_2, E^*) resulting in an unforeseen extinction. For instance, for $\beta = 0.6, \delta = 0.99, \omega = 1, g = 0.05$, and also $E_2* = 1$, then an unanticipated extinction in the second period will be the optimal course of action if the minimum viable population is $0.6 < E^* < 1$. In Theorem 4 we show that unforeseen extinction of the resource is always possible if $\frac{1}{\beta} < G < \frac{1}{\beta\delta}$.

Theorem 4 [Unforeseen Collapse under exponential utility] *If* $\frac{1}{\beta} < G < \frac{1}{\beta\delta}$*, then* $\exists$ $\Omega = \{(E_2, E^*) : E_2 > E^* > 0\ \} \neq \emptyset$ *satisfying equation (69).*

Proof 4 *Examining the left-hand side of equation (69) and substituting from equations (5) and (64) yields the condition for a collapse:*

$$\exp\left(-\omega(E_2 - \frac{E^*}{G})\right) + \beta\delta\left(\exp(-\omega E^*) - 1\right) \quad > \quad \exp(-\omega E_2) \tag{69}$$

$$\rightarrow \quad e^{-\omega E_2}\left(e^{\frac{\omega E^*}{G}} - 1\right) \quad > \quad \beta\delta\left(1 - e^{-\omega E^*}\right) \tag{70}$$

As $E^* \rightarrow E_2$ *we have:*

$$\frac{1}{\beta\delta} > \frac{\left(e^{\omega E_2} - 1\right)}{\left(e^{\frac{\omega E_2}{G}} - 1\right)} \tag{71}$$

Applying similar operations to the right-hand side of (69) yields the condition for a collapse to be unforeseen:

$$\frac{1}{\delta} < \frac{\left(e^{\omega E_2} - 1\right)}{\left(e^{\frac{\omega E_2}{G}} - 1\right)} \tag{72}$$

Combining these two conditions together gives:

$$\frac{1}{\delta} < \frac{\left(e^{\omega E_2} - 1\right)}{\left(e^{\frac{\omega E_2}{G}} - 1\right)} < \frac{1}{\beta\delta} \tag{73}$$

This equation represents the requirement for an unforeseen resource collapse under negative exponential utility. Interestingly, if we consider the requirement as $E_2 \rightarrow 0$*, using L'Hôpital's rule:*

$$\frac{1}{\delta} < \lim_{E_2 \rightarrow 0} \frac{\left(e^{\omega E_2} - 1\right)}{\left(e^{\frac{\omega E_2}{G}} - 1\right)} < \frac{1}{\beta\delta} \tag{74}$$

$$\frac{1}{\delta} < \lim_{E_2 \rightarrow 0} \frac{\omega e^{\omega E_2}}{\omega/G e^{\omega E_2 G}} < \frac{1}{\beta\delta} \tag{75}$$

$$\frac{1}{\delta} < G < \frac{1}{\beta\delta} \tag{76}$$

As this holds by assumption, we conclude that there exists a non-empty set of pairs (E_2, E^*) *satisfying equation (69).* ■

Hence, we observe that as $E_2 \rightarrow 0$, the condition for a resource collapse under negative exponential utility is that found under linear utility. For small values of E_2 (and hence E^*) the condition approximates that under linear utility. The intuition for this can be shown by noting that the negative exponential utility function is well approximated by a linear function for values of consumption near zero. The first order Taylor approximation is $u(c) = 1 - exp(-\omega c) \approx \omega c$. This, then, corroborates our results for isoelastic utility in the main paper, where linear utility was presented as a special case.

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