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The Role of Warnings in Regulation: Keeping Control with Less Punishment

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Abstract

Regulatory agencies frequently present violators with warnings, not pursuing prosecution if the violation ceases upon receipt of the warning. We show how such warnings may help regulators to keep control: Prosecution is costly for the regulator, and insufficient prosecution efforts yield low penalties. Thus, with a limited regulatory budget, threats of harsh sanctions are credible only if the number of violators is low. This produces multiple Nash equilibria. If firms may make mistakes, the economy can accidentally switch from one equilibrium to another. Warnings reduce substantially the probability of such accidental switches from the high to the low compliance equilibrium.

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1 Introduction

According to Gary Becker's (1968) approach to crime and punishment, economic agents will only comply to laws and regulations as long as the marginal cost of doing so is less than the expected marginal penalty. With this in mind, one might expect that regulatory agencies would put much emphasis on strict prosecution of violators; and, if they did not, widespread disobedience of regulations would be the result.

Nevertheless, in many cases, enforcement of regulatory requirements seems to be surprisingly lax: Formally, strict penalties may be available to the regulator; but rather than prosecuting violators, regulators frequently just issue a warning of some kind, and if violators move into compliance upon receipt of the warning, no further penalties are imposed (or penalties are insubstantial). In fact, such warnings are familiar from other parts of everyday life as well, for example in school classrooms, or in sports. If potential violators know that they will get a second chance, why should they bother to comply at all before receipt of the warning? And why wouldn't regulators - or school teachers - rather sanction every violator immediately, thus providing stronger incentives to comply? In this paper, we will suggest one possible explanation for the popularity of warning policies among regulators.

Our attention was initially drawn to this topic by observing the enforcement policy of the Norwegian Pollution Control Authority (henceforth NPCA). When an NPCA inspection reveals a violation of a firm's pollution permit requirements, NPCA's standard procedure is to send a warning letter to the firm, containing information about the nature of the violation and about possible sanctions against the firm if it stays out of compliance (NPCA 2001). Unless the case is of an extraordinarily severe nature, violators face prosecution only if they fail to comply after this warning. Hence, violating firms almost invariably get a second chance. In most cases, the warning letter appears to be sufficient to make firms comply. Severe sanctions, such as plant closure, confiscation of profits, fines and imprisonment, are formally available, but are hardly ever imposed. Most cases are never taken to court at all.[1]

NPCA's practice does not seem to be unique. In a much cited paper, Harrington (1988) claimed that in the United States, Environmental Protection Agency audits were relatively rare; fines or other

[1] For data on firm compliance in Norway and a more detailed description of NPCA's enforcement practice, see Telle and Nyborg (2003).

penalties were seldom imposed on violators; but still, most firms were believed to be in compliance a large part of the time. Heyes and Rickman (1999) later named this the "Harrington paradox". According to Harrington, the by far most common measure taken against detected violators was to send a Notice of Violation, ordering the firm to return to compliance, but taking no further action.[2] Russell (1990) notes that many US states pursue a policy where penalties are ordinarily not levied unless firms refuse to correct violations, or otherwise prove notably uncooperative. The environmental monitoring and enforcement policy in the UK has also been one of conciliation and repeated warnings, where prosecution has been saved for cases of non-cooperativeness and persistent failures to comply (Hawkins 1984, p. 130ff).

In this paper we will show that prosecuting all violators may imply a higher probability of losing control altogether than prosecuting only those who fail to comply after a warning. Our model will focus on firms' compliance to an environmental regulation, but we believe that the model is applicable to many other cases as well. The crux of our argument is that the regulator's ability to impose sanctions on individual violators is decreasing in the number of violators. In our model this is due to the regulator's limited budget for prosecution costs; however, one could also envisage similar mechanisms in the case of social sanctions, where the social disapproval for breaking a norm may be higher the lower the share of violators in society (as in Lindbeck et al. 1999). Hence, when there are few violators, the threat of harsh punishment is a credible one; but when the number of violators exceeds a critical level, the sanction becomes insufficient to effectively deter violation. This yields multiple Nash equilibria, including one of high compliance and one of low compliance. In the low compliance equilibrium, the regulator has lost control; it spends all resources available for sanctioning on this purpose, but still, most firms violate.

There is a considerable literature on enforcement of regulations, but we know of no analysis explicitly analyzing the role of warnings. Heyes and Rickman (1999) show that in a difficult enforcement environment, it may be rational for the regulator to tolerate non-compliance in some contexts to induce compliance in others. Garvie and Keeler (1994) investigate the regulator's trade-off between monitoring and sanctioning, and show how the optimal choice depends crucially on the regulator's enforcement technology. Livernois and McKenna (1999) discuss how self-reports by firms can enable regulators to keep compliance high even with low fines. The role of warnings, however, seems largely ignored in the

[2] See also EPA (2003).

economics literature; for example, warnings are not mentioned in Polinsky and Shavells (2000) excellent survey on the economic theory of the public enforcement of law.

In section 2, we present the basic version of our model, in which violators are prosecuted immediately and there are no warnings. In section 3, we formalize the notion of firms' mistakes, and discuss the probabilities of accidental shifts from the high to the low compliance Nash equilibrium. We then use the model with mistakes to analyze the effects of a warnings policy, and show that under certain conditions, warnings reduce substantially the probability that the regulator loses control altogether. Section 5 provides a brief discussion of how similar mechanisms may occur if compliance is motivated by social rather than legal sanctions. Section 6 concludes.

2 The basic model

We will first present the simplest version of our model, in which there are no warnings and all violators are prosecuted routinely, and firms make no mistakes. We will not treat the regulator as an explicitly strategic player, but rather compare the effects of alternative exogenous regulatory policies. The policy regime is thus determined by factors outside the model, such as political or legal institutions.

To fix ideas, think of the regulation as consisting of a *procedural requirement*; for example, that the firm should apply one specific production technology rather than less costly, but more polluting ones; that internal environmental audits, or maintenance procedures for cleaning equipment, should be performed and documented regularly; or that the firm should incorporate environmental risk adequately in its daily routines and/or employee training. This is an uncommon assumption in analyses of environmental regulation, where the focus is usually on emission caps. However, in their audits, the NPCA puts substantial emphasis on inspecting compliance to the institutional and/or procedural requirements specified in the firm's pollution permit (NPCA 2002); measurement of actual emission levels is just one part of the audits, and does not even appear to be considered the most important part. One reason is that compliance to emission caps as such is very difficult to verify, partly because permitted levels may be specified as for example maximum emissions per year, allowing substantial variation over the year, in which case emission levels during the period of an audit may be of little interest anyway. Institutional or procedural requirements are relatively easy to monitor, and although their violation may have small direct effects on environmental quality, violation of such requirements is seen as a strong indication that actual environmental damage has taken place or will likely take place.

Hence, assume that there are N identical firms, where N is a large, but finite number. To comply to the environmental regulation in period t, the firm has to pay a fixed compliance cost, C, in every period t; for example by performing an extensive internal environmental audit. P^t is the penalty for a violation in period t, i.e. for not paying C in period t.[3]

In general, of course, the *expected* penalty for a violation is determined by the probability of getting caught, the probability of being punished if caught, and the size of the penalty. To make our argument as simple as possible, however, we assume, firstly, that firms are risk neutral. Secondly, we will assume that every firm is monitored with certainty in every period; and, moreover, that monitoring is perfect in the sense that once an audit has taken place, the regulator knows with certainty whether a violation has occurred or not. With these assumptions, a violating firm has to pay P^t with certainty. These assumptions are certainly strict, but allow us to focus on *sanctioning* rather than detection.[4]

Firm i's costs in period t associated with the environmental regulation are thus given by

$$\omega_i(x_i^t) = C(1 - x_i^t) + P^t x_i^t \tag{1}$$

where $x_i^t = 1$ if the firm is a violator in period t and $x_i^t = 0$ if it is compliant.

The regulator's monitoring costs are fixed, and will be disregarded for the purpose of simplification. However, we will assume that prosecuting a firm is costly to the regulator. For example, the regulator must prepare verifiable evidence, take bureaucratic steps legally required to impose sanctions, and possibly hire legal expertise to pursue the case in court.

To begin with, we will assume that the regulator's policy is to prosecute every violator. Nevertheless, if insufficient effort is spent on prosecuting the case of a detected violation, the sanction imposed will be less than the maximum penalty $\overline{P}$ ascribed by law.

[3] We suppress superscripts on some variables for which changes over time will not be crucial in the analysis.

[4] This simplification plays no role in the first part of our analysis, since the certain penalty could be reinterpreted as the certainty equivalent of an uncertain, but higher penalty. However, it does play a role for the analysis of warnings. We will return to this below. In cases where detection rather than prosecution is the main problem, warnings may be less useful than implied by our model.

The regulator has a budget B available for sanctioning expenditures, and its budget constraint is hence

$$B \geq n^t e^t \tag{2}$$

where e^t is the resources spent by the regulator on sanctioning per violating firm in period t, and $n^t = \sum_{i=1}^{N} x_i^t$ is the number of violators.[5]

The maximum follow-up expenditure per violating firm is determined by the budget available per violator. However, the regulator does not waste sanctioning expenditures. Expenses on prosecuting any single case will thus never exceed $\overline{e}$, the expenditure required to impose the maximum penalty $\overline{P}$. Accordingly, the sanctioning expenditure per violator in period t is given by

$$e^t = \begin{cases} \frac{B}{n^t} \text{ if } \frac{B}{n^t} \leq \overline{e} \\ \overline{e} \text{ if } \frac{B}{n^t} > \overline{e} \end{cases} \tag{3}$$

and the penalty imposed on any given violator in period t is

$$P^t = \begin{cases} F(e^t) \text{ if } e^t < \overline{e} \\ \overline{P} \text{ if } e^t \geq \overline{e} \end{cases} \tag{4}$$

where $F(.)$ is continuous and $F(\overline{e}) = \overline{P}$. We assume that $F_e > 0$ and $F_{ee} \leq 0$ (where the subscripts denote first and second derivatives, respectively). (3) and (4) yield

$$P^t = p(n^t) = \begin{cases} F(\frac{B}{n^t}) \text{ if } n^t \geq \frac{B}{\overline{e}} \\ \overline{P} \text{ if } n^t < \frac{B}{\overline{e}} \end{cases} \tag{5}$$

i.e. once the number of violators is sufficiently large for the regulator's budget to be binding, the penalty is decreasing in the share of violators. This is illustrated in Figure 1.

[FIGURE 1 ABOUT HERE]

In the following, we will assume that $\overline{P} > C$, and that $\frac{B}{N} < \overline{e} < B$. These assumptions assure, first, that the maximum expected penalty implied by the legal rules is high enough to make compliance pay; i.e. there is at least a *formal* possibility of providing strong enough economic incentives to make firms

[5] We assume that an equal amount of resources is spent on every violating firm, and that the regulator does not spend resources on sanctioning or other extraordinary follow-up of compliant firms.

comply. The second assumption ensures that the regulator has enough resources to impose the maximum penalty on at least one violator, but that it will not be able to pay the sanctioning expenditures required to impose the maximum penalty on *all* firms. With these assumptions, we have that $p_n < 0$ if $n^t > \frac{B}{\overline{e}}$,[6] and $p_n = p_{nn} = 0$ if $n^t < \frac{B}{\overline{e}}$, where subscripts denote derivatives. Throughout the article, firms are assumed to disregard the impact on P of their own behavior, since there is a large number of firms.[7][8]

The cost-minimizing choice for a firm in any given period depends on what other firms do in the same period: If no firms violate, the regulator's budget constraint is not binding, and the penalty for a potential violator is $\overline{P}$, which exceeds the compliance cost. On the other hand, if all firms violate, the penalty will equal $p(N) = F(\frac{B}{N})$. Let us now assume that $F(\frac{B}{N}) < C$. Firms act simultaneously; thus, they must make their decisions without knowing P^t. Since $C < \overline{P}$, it can easily be seen that $x_i = 0$ for all i (everybody complies) is a Nash equilibrium: In this situation, no firm can decrease its costs by becoming a violator. However, since $C > F(\frac{B}{N})$, it is also the the case that $x_i = 1$ for all i is a Nash equilibrium: When every firm violates, the regulator's resources are spread so thinly that the penalty becomes lower than the compliance cost.

Then we have the following (proofs of all Propositions are found in the Appendices):

Proposition 1 *Assume that $F(\frac{B}{N}) < C < \overline{P}$. Then, in each period t, there are two pure strategy Nash equilibria in the above model: (i) The* full compliance equilibrium*, in which $x_i^t = 0$ for all i; and (ii) the* no compliance equilibrium*, in which $x_i^t = 1$ for all i.*

Let ν be the number of firms such that if $n = \nu$, firms would be just indifferent between compliance and violation, i.e. $p(\nu) = C$[9]. When we now turn to the dynamics of the model, it is worth noting that

[6] $p_n = (-B/(n^t)^2)F_e < 0$, while $p_{nn} = \frac{B}{n^3}[2F_e + \frac{B}{n}F_{ee}] \gtrless 0$.

[7] This assumption is not required for all propositions, but is kept thoughout for simplicity.

[8] To simplify, we will disregard discounting. Unless otherwise stated, this does not matter for the analysis; when it does matter, we will explain how a reinterpretation of variables is required to take discounting into account.

[9] If ν is an integer, there is a third class of pure strategy Nash equilibriua; namely the situation where ν firms violate and $N-\nu$ firms comply. This corresponds to a mixed strategy Nash equilibrium where each firm i plays $x_i^t = 1$ with probability α_1 and $x_i^t = 0$ with probability $1-\alpha_1$, where $\alpha_1 = \frac{\nu}{N} \in (0,1)$; or the class of mixed strategy Nash equilibria where different firms choose different mixed strategies leading to the above outcome. However, while playing $x_i = 0$ and $x_i = 1$ are evolutionary stable strategies (see Weibull 1995, Ch.2), any of the other Nash equilibrium strategies are evolutionary unstable. Similarly, with the best response dynamic used below, the full compliance and no compliance equilibria are

the point where $n = \nu$ is important as a *tipping point*: As long as $n < \nu$, it pays to comply, but as soon as $n > \nu$, violation is the least costly alternative for firms.

In the following, we will presuppose that the regulator prefers the full compliance equilibrium to the no compliance equilibrium.[10]

Assume, now, that firms' play is in accordance with what Kandori et al. (1993, p. 38) call the *best reply dynamic*: If n^{t-1} is sufficiently low, then every firm will plan to comply in period t; if, on the other hand, n^{t-1} is sufficiently high, every firm plans to violate in period t. This will be the case if managers minimize expected costs, and if they moreover expect the penalty level in period t to equal the realized penalty in the previous period:

$$\widehat{P}_i^t = P^{t-1} \tag{6}$$

for every i and t, where $\widehat{P}_i^t$ is the manager of firm i's expectation of the penalty level in period t. Then, if $n^0 < \nu$, every firm will expect compliance to be the cost minimizing alternative in period 1; and from period 1 on, the economy will settle in the full compliance equilibrium. Note that in the full compliance equilibrium, the regulator's budget is large enough to make the threat of harsh sanctions credible; however, since no violations actually occur in this Nash equilibrium, *the budget will not be spent.* If $n^0 > \nu$, firms will expect no compliance to be the optimal strategy in period 1, and as a result the economy will stay in the no compliance equilibrium from period 1 on. If $n^0 = \nu$, firms will be indifferent between the strategies of full and no compliance in period 1.[11]

If the economy has come to rest in the full compliance equilibrium, it seems fair to say that the regulator *has control*; similarly, in the no compliance equilibrium we may say that the regulator has *lost control*, since it spends its entire sanctioning budget but still every firm is violating. However, it also seems reasonable to say that the regulator has lost control when n is so large that the number of violators

asymptotically stable, while any other equilibrium is aymptotically unstable. Below, we will focus on the stable Nash equilibria, i.e. full compliance and no compliance.

[10] For firms, the situation is opposite; in the no compliance equilibrium they pay a fine which is lower than the compliance cost, hence costs are lower in this case.

[11] Note that ν is not necessarily an integer; if it is not, $n = \nu$ is impossible, and the economy will always move to one of the pure strategy Nash equilibria from period 1. To simplify the statistical calculations below, it will be assumed that when firms are exactly indifferent they will comply.

is increasing towards no compliance.

To define "being in control", the following concept will be useful: The *basin of attraction* of a state y is that set of initial states that the dynamics pulls toward y (see Weibull 1995, p.245). The basin of attraction of the full compliance Nash equilibrium above, for example, is every $n < \nu$. We will define the concept of losing control as follows: The regulator is *in control* in period t if n^t is within the basin of attraction of the full compliance equilibrium. The regulator has *lost control* in period t if n^t is within the basin of attraction of the no compliance equilibrium.

The existence of two pure strategy Nash equilibria hinges on the assumption that B is exogenous. To our knowledge it is uncommon that regulatory agencies can add collected penalties to their own budgets, perhaps because this may damage the regulator's objectivity towards possible violators. Nevertheless, one may object that if the government observes that the regulator is losing control, the budget will in practice be increased. If the administrative and political processes required to secure such budget increases are time-consuming, however, the economy may still move to the no compliance Nash equilibrium while the regulator is waiting for budget increases. Making the economy return to full compliance after such a temporary loss of control can be very costly. The government can ensure such a return by increasing the budget sufficiently to make $F(\frac{B}{N}) > C$. *No compliance* will then cease to be a Nash equilibrium. However, the budget required for this, although required only for a limited period, is N times higher than the budget required in the full compliance equilibrium. For example, assume that $F(e) = 2e$, $N = 1,000$, and $C = 20,000$ dollars. Then, full compliance will be a Nash equilibrium as long as $B > 10,000$ dollars (and this budget will not be spent in the full compliance equilibrium). To move the economy to full compliance in period t from a situation of no compliance in period $t-1$, however, we must have $B^{t-1} > 10,000,000$ dollars.

3 Losing or gaining control

In this section, we will demonstrate that if firms intending to comply sometimes make mistakes, the economy may easily shift from the full compliance equilibrium to the no compliance equilibrium. Then, in the next section, we will show that warnings can reduce substantially the probability that the regulator loses control.

We will formalize *mistakes* as follows: There is an exogenous, positive probability $q \in \langle 0, \frac{1}{2} \rangle$ that if the manager of firm i decides that the firm should comply in period t, the firm will violate in period t anyway. This may be caused by principal-agent problems within the firm; for example, subordinates may not exert sufficient effort to ensure that regulatory requirements are handled correctly. Let the *behavior intended by firm i's management* be denoted $\widehat{x}_i^t$, while x_i^t denotes the *actual behavior* of the firm. The assumption is thus that if $\widehat{x}_i^t = 0$, then the probability that $x_i^t = 0$ is $(1 - q)$, while the probability that $x_i^t = 1$ is q. We will assume that errors are asymmetric in the sense that if management decides to violate, the firm will never comply by mistake.[12] The probability q is fixed and unaffected by the occurrence of errors in other firms or in previous periods. Managers are assumed to minimize firms' expected costs, taking the possibility of errors into account. Hence, we do not require limited rationality on firm managers' part, only that their control of the firm is imperfect.

In every period, the manager's choice is first made; then the firm pays or does not pay the compliance cost (at which point an error may occur). Then an audit by the regulator takes place, and the actual behavior x_i^t is revealed. The regulator observes the number of violators n^t, which determines P^t. Finally, violators are prosecuted and must pay P^t (punishment takes place in the same period as the violation).

When choosing his or her strategy, the manager minimizes the expected costs of the firm, taking into account that mistakes may occur. The expected costs of a managerial plan $\widehat{x}_i^t$, given that the managers' expectation of the penalty is $\widehat{P}_i^t$, are given by

$$E\left(\omega_i(\widehat{x}_i^t)\right) = \begin{cases} (1-q)\,C + q\widehat{P}_i^t & if\ \widehat{x}_i^t = 0 \\ \widehat{P}_i^t & if\ \widehat{x}_i^t = 1 \end{cases} \tag{7}$$

If all firms' managers decide to violate, i.e. $\widehat{x}_i^t = 1$ for all i, the number of actual violators in period t will equal N, since one cannot (by assumption) comply by mistake. On the other hand, if all managers try to comply, i.e. $\widehat{x}_i^t = 0$ for all i, some firms may make a mistake and violate anyway. The expected number of actual violators becomes qN, which is strictly positive. If the expected penalty at this level

[12] Our main results do not depend on this asymmetry. Symmetric error probabilities complicate the reasoning, but without adding much new insight. A note on the symmetrical error case (i.e. complying by mistake is equally probable as violating by mistake) is available from the authors upon request.

of violation is below the compliance cost C, then full compliance is no longer a Nash equilibrium, so the occurrence of errors implies that we must sharpen the assumptions from Proposition 1 somewhat.

Let $p^0 = E\left[p(n) \mid \hat{x}_i^t = 0 \text{ for all } i\right]$, i.e. the expected penalty when all managers try to comply.[13] Proposition 2 establishes that although we must now distinguish between planned and actual behavior, and the required assumptions on parameter values must be sharpened somewhat, the model with mistakes yields equilibria corresponding roughly to those of Proposition 1[14]:

Proposition 2 *Assume that $p(N) < C < p^0$. Then, in each period t, there are two pure strategy Nash equilibria in the model with mistakes: (i) The* full compliance equilibrium*, in which $\hat{x}_i^t = 0$ for all i; and (ii) the* no compliance equilibrium*, in which $\hat{x}_i^t = 1$ for all i.*

A cost minimizing manager whose expectations of the penalty is in accordance with (6) will plan to violate if n^{t-1} is sufficiently high and comply if n^{t-1} is sufficiently low. Again, the manager is indifferent between the strategy of full compliance and no compliance if $n^{t-1} = \nu$, where $C = p(\nu)$ as in the previous section. Then, if the number of violators in period $t-1$ is less than ν, the cost-minimizing choice in period t for any firm manager (given (6)) is to order compliance. Unless more than ν firms make a mistake, this will be the case in the next period too. If there were more than ν violators in period $t-1$, however, the managerial plan minimizing costs in period t is to violate; and then, the same will be true in the next period.

Proposition 3 *Assume that firms' expectations of the penalty level are in accordance with (6), and that all firm managers play the strategy which, given these expectations, minimizes expected costs. Then the following strategy will be played by all firms in period t: $\widehat{x}_i^t = 0$ (cf. the full compliance equilibrium) if $n^{t-1} \leq \nu$, and $\widehat{x}_i^t = 1$ (cf. the no compliance equilibrium) if $n^{t-1} > \nu$.*

In the case with mistakes, then, if the economy is in the full compliance equilibrium, there is a strictly positive probability that the regulator loses control. Some firms will violate due to errors; and if this happens to more than ν firms in any period t, the penalty in period t will be lower than the compliance

[13] The penalty function $p(n)$ is not generally linear, so we may have $E(p(n)) \neq p(E(n))$. In the special case of a linear penalty function we would have that $E(p(n)) = p(E(n)) = p(qN)$ when $\widehat{x}_i^t = 0$ for all i.

[14] Like before, there may also be Nash equilibria where exactly enough firm managers violate and the rest tries to comply to make every firm perfectly indifferent, but such Nash equilibria will be unstable (see footnote **??**).

cost. Consequently, in the next period, the a priori cost minimizing choice for every firm is to violate; and the economy moves to the no compliance equilibrium from period $t + 1$ on. However, the instance the economy comes to rest in the no compliance equilibrium, it will never move back to full compliance: The regulator loses control.

The probability that the regulator loses control can be calculated using the binomial distribution.[15] In other words, what is the probability that at least ν firms violate in period t, given that the economy is in the full compliance equilibrium in the previous period? For example, assume (as above) that $F(e) = 2e$, $N = 1,000$ and $C = 20,000$ dollars; and assume further that $q = 0.008$. Let the regulator's budget for sanctioning purposes be $B = 100,000$ dollars. Then, we get $\nu = 10$. Hence, in this case (where the expected number of violators is 8), the regulator loses control in period t if more than 10 firms make an error simultaneously in period t (assuming that the economy is in the full compliance equilibrium in period $t - 1$). The probability that this happens, calculated by the binomial distribution, is 0.18. The probability that $n > 10$ has *not* yet occurred after 15 periods is less than 0.05. With these numbers, the probability of losing control within a quite limited time period is thus substantial.[16]

4 Warnings: A formal interpretation

In light of the analysis above, one would expect that a regulator being in control will look for devices which can reduce the probability that the economy accidentally switches to the no compliance equilibrium. As we will demonstrate below, when firms make mistakes, the practice of issuing warnings prior to prosecution can indeed be interpreted as such a device.

We will now stick with the assumption that firms may make mistakes, and consider how warnings

[15] The generation of errors satisfies the conditions of a Bernoulli process, hence n^t will be distributed according to the binomial distribution. See Appendix D.

[16] Note that if *complying* by mistake also occurred with a probability q, i.e. symmetrical error probabilities, the economy might also switch from the no to the full compliance equilibrium. Switches in both directions would not generally be equally likely, however: In the numerical example above, an accidental switch from the no to full compliance requires that $N - \nu = 990$ (of 1000) firms make errors simultaneously! The probability of such an event would be close to zero (the exact probability in this example is 2.8×10^{-2053}). With symmetrical error probabilities, a shift from high compliance to low compliance is more likely than the reverse as long as $\nu < N/2$, which implies that L is the *risk dominant* equilibrium in our model (Harsanyi and Selten 1988). This result is a special case of Theorem 3 in Kandori, Mailath and Rob (1993, p. 44).

may influence the probability that the regulator loses control. The regulatory policy we consider below implies auditing every firm every period, as before; but after the audit, violating firms are presented with a warning. If they move into compliance upon receipt of this warning, but before the end of the period, they are not prosecuted.

In the beginning of every period t, every firm manager either decides to pay or not to pay the compliance cost. With a probability q a firm intending to comply makes a mistake, while firms intending to violate always violate. Then there is an audit, revealing the firm's action. If the audit reveals that the firm is in compliance, nothing more happens until the next period. If the audit reveals that the firm is in violation, the regulator issues a warning, which is assumed to be costless to the regulator. If the firm then presents evidence that it has moved into compliance after receipt of the warning, at a cost $C + V$, it is not prosecuted. $V > 0$ is a verification cost borne by the firm. If evidence of compliance is not presented before the period ends, the firm is prosecuted and must pay the penalty P^t.[17] Again, there is a probability q that a firm intending to comply errs. However, note that if a firm complies in the beginning of a period, it will necessarily also comply after the audit, since C has in fact been paid.

The strategies of firm managers now become somewhat more complicated, since they may exploit the fact that they will get a second chance, and may for example plan to violate before, but not after the warning. We will keep a definition of a violator as someone who must be punished. The concepts of "compliance" and "violation" will thus now be defined as follows. Let $x_i^t \in \{0,1\}$ denote firm i's actual behavior *at the end of* period t. Hence, the number of violators who are to be prosecuted in period t is $n^t = \sum_{i=1}^{N} x_i^t$, implying that the penalty function (5) still holds. Further, let $\widehat{x}_{1i}^t \in \{0,1\}$ be the management's *planned* behavior before the audit, while x_{1i}^t denotes firm i's *actual* behavior before the audit, which is revealed at the audit in period t. If $x_{1i}^t = 0$, there is no further choice to make for the manager in this period, since C is a "per period sunk cost". Hence we have $x_{1i}^t = 0 \Rightarrow x_i^t = 0$. However, if $x_{1i}^t = 1$, the firm receives a warning, and the manager is faced with another decision, namely whether

[17] In the case with warnings, discounting may matter, since discounting makes a delay in the payment of C more attractive. This may be taken into account by interpretating P^t and V^t as present values viewed from the beginning of period t. V^t then consists of two elements; the verification cost as such, minus the gains caused by delaying payment of C. Our assumption that $V^t > 0$ can thus be taken to mean that the verification cost as such is larger than any gains from delaying the payment of C.

to try to move into compliance or not. Hence, let $\widehat{x}_{2i}^t$ denote the manager's plan of what to do after the audit *if* the audit reveals that the firm is in violation (i.e. if $x_{1i}^t = 1$). The payoff structure for the firm in every period is depicted in Figure 2.

[FIGURE 2 ABOUT HERE]

There are now four possible pure strategies $S = (\widehat{x}_{1i}^t, \widehat{x}_{2i}^t)$ for a firm manager:

$S^I = (0,0)$ Full compliance: The manager's plan is to pay the compliance cost.

$S^{II} = (0,1)$ Conditional compliance: The manager plans to comply, but if there is an error he does not try to correct it.

$S^{III} = (1,0)$ Waiting: The manager plans to comply, but he plans to wait until after receipt of the warning.

$S^{IV} = (1,1)$ No compliance: The manager plans to violate both before and after the warning.

Let $E\Omega_i(S^j)$ denote the expected costs of strategy S^j for firm i $(j = I, II, III, IV)$. Then we have

$$E\Omega(S^I) = (1-q)C + q[(1-q)(C+V) + q\widehat{P}_i^t] \tag{8}$$

$$E\Omega_i(S^{II}) = (1-q)C + q[\widehat{P}_i^t] \tag{9}$$

$$E\Omega_i(S^{III}) = [(1-q)(C+V) + q\widehat{P}_i^t] \tag{10}$$

$$E\Omega_i(S^{IV}) = \widehat{P}_i^t \tag{11}$$

S^{III} is strictly dominated by S^{II}; these strategies yield the same expected costs except that S^{III} implies a higher probability of having to pay the verification cost V. Hence, exploiting the "second chance" provided by the warnings by just postponing payment of the compliance cost until after the warning is never optimal with these assumptions.

With reasonable restrictions on the verification costs, it turns out that the Nash equilibria of this game have quite similar properties as in the case without warnings: There is one pure strategy Nash equilibrium of full compliance, and one of no compliance.

Note first that if all managers play S^{II}, the number of actual violators is distributed according to the binomial distribution with N trials and a "success" probability q. This implies that if every firm plays conditional compliance in the case *with* warnings, the number of violators and hence also the penalty level, is distributed in exactly the same way as in the full compliance equilibrium in the case *without* warnings. Consequently, we have that $E\left[p(n) \mid S = S^{II} \text{ for all } i\right] = E\left[p(n) \mid \hat{x}_i^t = 0 \text{ for all } i\right] = p^0$.

Proposition 4 *Assume that the regulator follows the policy with warnings described above, that* $p^0 > C + V$*, and that* $p(N) < C$*. Then both* full compliance, *i.e. the situation where every firm manager plays* S^I*; and* no compliance, *i.e. the situation where every firm manager plays* S^{IV}*, are Nash equilibria.*

The important point to note here is that for the full compliance equilibrium to exist, the expected penalty when everybody tries to comply must not be too small. This implies that the error probability q should not be too large: Even when every firm intends to comply, there will be some violators left after the warning due to errors; and if the error probability is large, the number of mistaken violators may easily be large enough to make expected penalties fall below compliance cost. If so, full compliance cannot be a Nash equilibrium.[18]

What is now the cost minimizing strategy for a manager whose expectations of the penalty is in accordance with (6)? Let the assumptions of Proposition 4 hold. The answer is very similar to the results summarized in Proposition 3 above: If the number of violators is low, every manager will play full compliance; if the number of violators is high, managers will play violate.

However, there is now also a small region close to ν where it is optimal to play S^{II}, conditional compliance; i.e. to try to comply, but abstain from correcting an error should it occur. The reason for this is that when the number of violators is very close to (but less than) ν, the manager does not care much whether he must pay the penalty or the compliance cost, since in this area these two are of roughly the same size; so when n is approaching ν, the important thing is to avoid having to pay the verification cost V.

Consider Figure 3, where ν' is defined by $p(\nu') = C + V$, while as before, $p(\nu) = C$. Note that $\nu' \leq \nu$ because the penalty is decreasing in n. (In the figure we have assumed that $p(qN) = p\left(q^2N\right) = \overline{P}$; this need not be the case.)

[18] In the special case where $p(N) < C$, but $C < p^0 < C + V < p^{SI}$, where $p^{SI} = E\left[p(n) \mid S = S^I \textit{ for all } i\right]$, then there is an additional pure strategy Nash equilibrium, namely the situation where all firms play S^{II}. Moreover, if V is very high so that $C + V < p^{SI}$, then S^{II} will be an equilibrium strategy, but not S^I. To make sure that full compliance is a Nash equilibrium, but not conditional compliance, V must not be too high. (As above, there will also exist unstable mixed strategy Nash equilibria, in which firms are exactly indifferent.)

[FIGURE 3 ABOUT HERE]

First, for $n < \nu'$ the penalty is higher than the sum of compliance and verification costs. Thus S^I is the best reply. When $n > \nu$, the penalty is less than the compliance costs. The best reply to $n > \nu$ is thus to violate (S^{IV}). Finally, if $\nu' < n < \nu$, the best reply is S^{II}. Still, the situation where every firm plays S^{II} cannot be expected to be stable: If everybody plays S^{II}, the expected number of violators is $qN < \nu'$ with our assumptions. Thus, if everybody plays S^{II} in period t, the most likely outcome is that every firm plays the Nash equilibrium strategy S^I in period $t+1$. With the best reply dynamic, firms will thus play as follows[19]:

Proposition 5 *Assume that the regulator follows the policy with warnings described above, that the firms' expectations of the penalty level are in accordance with (6), that all firm managers play the strategy which, given the expectations formation, minimizes expected costs, and that the assumptions of Proposition 4 hold. Then the following strategy will be played by all firms in period t:* S^I *(cf. full compliance equilibrium) if* $n^{t-1} \leq \nu'$, S^{II} *(conditional compliance) if* $\nu' < n^{t-1} \leq \nu$, *and* S^{IV} *(cf. no compliance equilibrium) if* $n^{t-1} > \nu$.

Let us now compare the probability that the regulator loses control in the models with and without warnings. We will focus on those situations in which there are indeed both a full compliance Nash equilibrium and a no compliance Nash equilibrium both in the case with and without warnings. Let us first establish the conditions for this. As above, let $p^0 = E\left[p(n) \mid \hat{x}_i^t = 0 \text{ for all } i\right]$.

Proposition 6 *Assume that* $p^0 > C + V$ *and* $p(N) < C$. *Then there exists a full compliance Nash equilibrium and a no compliance Nash equilibrium both in the case where the regulator does not use warnings and in the case where warnings are used.*

In the discussion below we will assume that the conditions given in Proposition 6 hold. It turns out that when firms play according to the best response dynamics, the probability that the regulator loses control is unambiguously lower in the case with warnings.

First, note that both in the case with and without warnings, the regulator is in control when $n^{t-1} < \nu'$: In this case, firms playing according to the best response dynamics will comply in the model without

[19] In line with the assumption in footnote 11, a manager being completely indifferent is assumed not to play no compliance.

warnings, and play strategy S^I in the model with warnings. The probability of $x_i^t = 1$ for any given firm i then equals q in the model without warnings, and q^2 $(< q)$ in the model with warnings.

Second, when $\nu' < n^{t-1} < \nu$, firms will still comply in the model without warnings (see Proposition 3) with an error probability of q; while in the model with warnings, firms will comply conditionally (Proposition 5), yielding a probability of violation for any single firm which is also q.

Finally, if $n^{t-1} > \nu$, every firm will violate with certainty, both in the case with and without warnings. Consequently, the probability of individual violations in period t is weakly lower when warnings are used for any value of n^{t-1}. Since the cumulative binomial distribution is strictly increasing in the success probability (i.e. in our case, the probability of individual violation), we know that for any given $n^{t-1} < \nu$, the warning device reduces the probability that n^t exceeds the common tipping point ν.[20] Consequently, we can conclude that under our assumptions, *giving violators a warning weakly reduces the probability that the regulator loses control.*

Proposition 7 *Assume that the firms' expectations of the penalty level are in accordance with (6), that all firms play the strategy which, given the expectations formation, minimizes costs, and that the assumptions of Proposition 6 hold. Then the probability that the regulator loses control is weakly smaller if the regulator uses warnings.*

Consider the example above where $N = 1000$, $\nu = 10$, and $q = 0.008$. The warning device practically eliminates the probability that the regulator loses control in this example: The probability of losing control is more than 1.1×10^{20} times higher without than with the warning device (0.18 and 1.7×10^{-21}).

[20] As mentioned in footnote 18, there may exist situations (not satisfying the assumptions in Proposition 4) where S^{II} is an equilibrium strategy in the case with warnings. Even in these situations, the warnings policy will not fare worse (from the regulator's point of view) than the policy of no warnings: Since n is distributed according to the binomial distribution with N trials and a success probability of q *both* in the full compliance equilibrium in the model without warnings *and* when all firms play conditional compliance in the model with warnings, the performance of the economy will be equivalent in the two cases.

5 Social sanctions

Above, we have focused on formal prosecution and sanctioning as the main driving force behind firms' compliance; and on the regulator's budget as the limiting factor when there are too many violators. However, in many cases it seems that social disapproval is a more important motivating factor than legal sanctions. Classroom discipline and street crime are obvious examples, but it is quite possible that social sanctions constitute an important motivating factor even for firm managers. Managers may have a direct preference for social status and/or social approval which influences their choices, or social sanctions may affect firm earnings indirectly, via consumer or employee reactions. Nevertheless, the reasoning of our model may be relevant in such cases as well, so let us discuss this very briefly.

Assume, for example, that firm managers fear negative media attention if the firm is prosecuted for violation of environmental regulations. If no other firms are prosecuted, media coverage of the case may be substantial; while if almost every other firm is also prosecuted for similar violations, the expected media attention on each case will be very small. The expected social sanction resulting from prosecution will then be decreasing in the number of violators, much like the penalty function (5). This mimics the mechanisms of our model: Total penalties for violation (legal plus social sanctions) are stronger the fewer violators there are, which may yield multiple equilibria. Indeed, this can make full compliance an equilibrium even if $\bar{P} < C$ (if this holds, no compliance is the only Nash equilibrium in the model(s) above).

Assume further that prosecution produces a stronger social sanction than receipt of a warning. Then, the regulator may be able to keep control even if its budget is very low, because even a small prosecution expenditure e may produce a substantial total sanction for the firm. Nevertheless, if too many firms violate by mistake and are prosecuted simultaneously, the social sanctions erode; and in the next period, the regulator may have lost control. Hence, the regulator can issue warnings to avoid erosion of the social sanctions, and thus increase the probability of staying in control.

Let $s(n)$ be the expected social sanction imposed on each convicted firm as a function of the number of violators n, and assume that s is decreasing in n. Further, define $P(n) = s(n) + p(n)$. Then, if the assumptions in Proposition 7 holds for $P(n)$ (and not only $p(n)$), it follows that if the regulator uses warnings, the probability of a shift from full compliance to no compliance is reduced.

More generally, the N decision makers may not necessarily be firms; the model may also be applicable to individuals trying to minimize their costs associated with some regulation (making, now and then, a mistake). Assume, like in for example Lindbeck et al. (1999) or Rege (2003), that the larger the population share adhering to a social norm, the more intensely this norm is felt by the individual. With a given population size, this yields a social sanction function $s(n)$ which is decreasing in n, the number of violators of the norm. Assume further that there are both social and legal sanctions for breaking the norm, and that the social sanctions for an individual who is prosecuted is larger than for those who only receive a warning. Then there may be multiple equilibria, and, as above, the use of warnings can increase substantially the probability that the regulator stays in control.

Note, however, that although this reasoning implies that informal and formal sanctioning can complement each other, we do assume that there is a formal regulator present. When both formal and informal sanctions exists, and formal sanctions trigger informal sanctions, then the use of formal warnings will increase the probability that the economy stays in the full compliance Nash equilibrium where the informal sanctions are high.

6 Concluding remarks

In this paper we have discussed the effects of two enforcement policies: To prosecute every violator, or alternatively, to issue warnings when a violation is detected, and only prosecute those who fail to comply after receipt of the warning. Using Becker's (1968) theory of rational crime, one may be inclined to think that the latter policy would unambiguously increase violation as compared to the former. However, we have shown that under certain conditions, warnings do not increase violation; on the contrary, they may substantially reduce the probability that the regulator loses control of the situation altogether. This may provide one explanation for the apparent popularity of warnings policies among regulators.

The use of warnings seems to be widespread not only among regulatory agencies, but also in more informal settings where a leader needs to keep control, such as in school classrooms. Regulators or leaders may understand that if they are consistently harsh in their reactions, giving violators no second chance, their sanctioning capacity may easily become exhausted; and the instant this happens, the threat of future harsh sanctions becomes incredible, with lost control as the result.

As always, the conclusions depend on the model assumptions employed. A couple of these deserve mention here. First, we have modelled a situation where compliance is related to a per period fixed cost, and where there is a strictly positive verification cost imposed on those who comply after the warning. This means that firms cannot reduce their net present costs associated with compliance simply by delaying compliance. These assumptions are plausible in many cases, for example if the regulatory requirement is concerned with having performed some task during the period (removing hazardous waste from the site; undertaking internal audit or maintenance procedures; having filed taxes correctly; having done one's homework). Indeed, the control routines of the Norwegian Pollution Control Authority seem to be much more oriented towards firms' institutional arrangements concerning pollution control, such as internal environmental audit schemes, than towards actual pollution measurement. However, there are obviously cases where these assumptions do not hold. If compliance can easily be "switched on and off" with no or low costs, or if net verification costs are negative (i.e. it pays to delay compliance), the above results may not hold, and warnings do not necessarily help the regulator keep control.

Further, we have disregarded the issue of monitoring and detection altogether, since our focus has been on sanctioning. Again, this seems more reasonable when the regulatory requirement is of an institutional character as compared to a requirement like an emission cap, since violation of an institutional requirement appears to be relatively easy to detect even with infrequent audits. It should be noted that in cases where detection probabilities are small, our conclusions may not hold.

A Proof of Proposition 1

Proof. Let $\omega_i(\sigma,\tau)$ denote the costs for firm i if it follows strategy σ (cf. Eq. 1), given that the strategy of all other firms is τ. The strategy profile where all firms play $S \in \{(x=0),(x=1)\}$ is a Nash equilibrium if and only if $\omega_i(S,S) \leq \omega_i(T,S)\, \forall i = \{1,...,N\}$ and $T \in \{(x=0),(x=1)\}, T \neq S$.

(i) $x = 0$ yields the condition $C \leq \bar{P}$, which holds by assumption.

(ii) $x = 1$ yields the condition $F\left(\frac{B}{N}\right) \leq C$, which holds by assumption. ■

B Proof of Proposition 2

Proof. Let $\omega_i(\sigma,\tau)$ denote the costs for firm i if it follows strategy σ (cf. Eq. 7), given that the strategy of all other firms is τ. Let $p^0 = E[p(n) \mid \hat{x}_i^t = 0 \text{ for all } i]$. The strategy profile where all firms play $S \in \{(\hat{x}=0),(\hat{x}=1)\}$ is a Nash equilibrium if and only if $E\omega_i(S,S) \leq E\omega_i(T,S)\, \forall i = \{1,...,N\}$ and $T \in \{(\hat{x}=0),(\hat{x}=1)\}, T \neq S$.

(i) The above implies that $\hat{x}_i = 0$ for all i is a Nash equilibrium if $E\omega_i(0,0) \leq E\omega_i(1,0) \Leftrightarrow C \leq p^0$, which holds by assumption. Note that when $\hat{x}_i = 0$ for all i, the expected number of violators is qN, but since $p(n)$ is not linear and may be either concave or convex, the expected penalty p^0 may be either smaller or larger than $p(qN)$.

(ii) $\hat{x} = 1$ for all i is a Nash equilibrium if $E\omega_i(1,1) \leq E\omega_i(0,1) \Leftrightarrow p(N) \leq C$, which holds by assumption. ■

C Proof of Proposition 3

Proof. $n^{t-1} < \nu$ implies, by Eq. (6), $\widehat{P}_i^t = P^{t-1} > p(\nu) = C$. Then Eq. (7) gives $E\omega_i(0) = (1-q)C + qP^{t-1} < E\omega_i(1) = P^{t-1}$. Hence, the best response when $n^{t-1} < \nu$ is to play $\widehat{x}_i^t = 0$ (full compliance).

$n^{t-1} > \nu$ implies, by Eq. (6), $\widehat{P}_i^t = P^{t-1} < p(\nu) = C$. In this case Eq. (7) yields $E\omega_i(1) = P^{t-1} < (1-q)C + qP^{t-1} = E\omega_i(0)$. Hence, for all firms it is cost minimizing to play $\widehat{x}_i^t = 1$ (no compliance). ■

D The binomial distribution

The probabilities that the regulator loses control are calculated using the binomial distribution, see e.g. Bhattacharyya and Johnson (1977). Let ν^+ be the lowest positive integer higher than ν. Assume that if a manager is exactly indifferent between compliance and violation, he chooses to comply (assuming that indifferent managers violate would not change the qualitative results, but could change the numbers slightly). Then the probability that the regulator loses control in period t in the case without warnings, given expectations (6) and assuming that $n^{t-1} \leq \nu$, is given by the following expression:

$$\Pr(n^t > \nu \mid n^{t-1} \leq \nu) = \sum_{n=\nu^+}^{N} \binom{N}{n} q^n (1-q)^{N-n} = 1 - \sum_{n=0}^{\nu^+-1} \binom{N}{n} q^n (1-q)^{N-n} \tag{12}$$

E Proof of Proposition 4

Proof. Let $\Omega_i(\sigma, \tau)$ denote the costs for firm i if it follows strategy σ (cf. Eqs. 8 - 11), given that the strategy of all other firms is τ. Let $p^{S\mu} = E\left[p(n) \mid S = S^\mu \text{ for all } i\right]$, where $\mu \in \{I, II, III, IV\}$. The strategy profile where all firms play S^μ is a Nash equilibrium if and only if $E\Omega_i(S^\mu, S^\mu) \leq E\Omega_i(S^j, S^\mu)\ \forall i$ and $\forall j$, where $i \in \{1, ..., N\}$, and $j, \mu \in \{I, II, III, IV\}, j \neq \mu$.

(i) All firms play S^I: This situation is a Nash equilibrium iff

$$E\Omega_i\left(S^I, S^I\right) = (1-q)C + q[(1-q)(C+V) + qp^{SI}] \leq E\Omega_i\left(S^j, S^I\right) \text{ for all } j \in \{II, III, IV\}. \tag{13}$$

For $j = II$ this is equivalent to $(C+V) \leq p^{SI}$. This is true by the assumption that $(C+V) < p^0$ and by the following: When all firms play S^I, n is distributed according to the binomial distribution with N trials and success probability q^2. When all firms play S^{II}, the success probability is $q > q^2$. The cumulative binomial probability distribution is strictly increasing in the success probability, and this implies, since $p(n)$ is weakly decreasing, that $p^{SI} \geq p^{SII} = p^0$.

For $j = III$ (13) holds since S^{III} is strictly dominated by S^{II}.

Moreover, (13) holds for $j = IV$ provided that $C + q(1-q)/(1-q^2)(V) \leq p^{SI}$, which is true because $(C+V) \leq p^0$, $p^{SI} \geq p^0$and $q(1-q)/(1-q^2) < 1$.

(ii) All firms play S^{II}: This is not a Nash equilibrium: $E\Omega_i\left(S^{II}, S^{II}\right) \leq E\Omega_i\left(S^I, S^{II}\right)$ is equivalent to $p^{SII} = p^0 \leq (C+V)$. This is contradicted by the assumption $(C+V) < p^0$.

(iii) All firms play S^{III}: This cannot be a Nash equilibrium since S^{III} is strictly dominated by S^{II}.

(iv) All firms play S^{IV}: This situation is a Nash equilibrium iff

$$E\Omega_i\left(S^{IV}, S^{IV}\right) = p(N) \leq E\Omega_i\left(S^j, S^{IV}\right) \text{ for all } j \in \{I, II, III\}. \tag{14}$$

For $j = I$ this is equivalent to $p(N) \leq C + [q(1-q)/(1-q^2)]V$. This is true since, by assumption, $p(N) < C$ and $C < C + [q(1-q)/(1-q^2)]V$.

For $j = II$ (14) is equivalent to $p(N) \leq (1-q)C + qp(N)$ which is true since by assumption $p(N) < C$.

For $j = III$ (14) holds since S^{III} is strictly dominated by S^{II}. ■

F Proof of Proposition 5

Proof. Recall that ν and ν' are defined implicitly by $C = p(\nu)$ and $C + V = p(\nu')$. It is clear from the assumptions and monotonicity of $p(.)$ that $\nu' \leq \nu$. Firm i will play S^μ in period t if $E\Omega_i(S^\mu) < \Omega_i(S^j)$ for all $j \neq \mu$, where $\mu, j \in \{I, II, III, IV\}$ (see Eqs. 8 - 11), given the expectations of (6). This yields the following results:

1. $n^{t-1} \in (0, \nu') \Rightarrow \widehat{P}_i^t \in \left(C + V, \overline{P}\right) \Rightarrow E\Omega_i(S^I) < E\Omega_i(S^j) \forall j \neq I$, and it is cost minimizing for all firms to play S^I in period t.

2. $n^{t-1} \in (\nu', \nu) \Rightarrow \widehat{P}_i^t \in (C, C + V) \Rightarrow E\Omega_i(S^{II}) < E\Omega_i(S^j) \forall j \neq II$, and it is cost minimizing for all firms to play S^{II} in period t.

3. $n^{t-1} \in (\nu, N) \Rightarrow \widehat{P}_i^t \in (p(N), C) \Rightarrow E\Omega_i(S^{IV}) < E\Omega_i(S^j) \forall j \neq IV$, and it is cost minimizing for all firms to play S^{IV} in period t. ■

G Proof of Proposition 6

Proof. This follows directly from (the proofs of) Propositions 2 and 4. ■

H Proof of Proposition 7

Proof. Proposition 3 states that all firms play the no compliance strategy $(\hat{x}_i^t = 1 \forall i)$ if $n^{t-1} > \nu$; and Proposition 5 states that all firms play the no compliance strategy $\left(S^{IV}\right)$ if $n^{t-1} > \nu$. Hence, $n^{t-1} > \nu$ is a sufficient (and necessary) condition to make every firm play the no compliance strategy both with and without warnings.

Let $\Pr^W$ denote the probability that the economy switches from the full compliance equilibrium to the no compliance equilibrium in the model with warnings, and $\Pr^M$ denote the probability that the economy switches from the full compliance equilibrium to the no compliance equilibrium in the model without warnings. Then,

$\Pr^W \left(n^t > \nu \mid n^{t-1} \leq \nu'\right) = \sum_{n=\nu^+}^{N} \binom{N}{n} q^{2n} \left(1 - q^2\right)^{N-n}$

$\Pr^M \left(n^t > \nu \mid n^{t-1} \leq \nu'\right) = \sum_{n=\nu^+}^{N} \binom{N}{n} q^{n} \left(1 - q\right)^{N-n}$, where ν^+ is the lowest positive integer higher than ν.

Since the cumulative binominal distribution is strictly decreasing in success probability, it follows that $\Pr^W < \Pr^M$ when $n^{t-1} < \nu'$, i.e. the probability that the regulator loses control is strictly smaller in the model with warnings.

However, we must also consider the case when $\nu' < n^{t-1} \leq \nu$. In the model without warnings, firms will then comply $(\hat{x}_i^t = 0 \forall i)$, implying that $\Pr^M \left(n^t > \nu \mid \nu' < n^{t-1} \leq \nu\right) = \Pr^M \left(n^t > \nu \mid n^{t-1} \leq \nu'\right)$. In the model with warnings, firms play S^{II} when $\nu' < n^{t-1} \leq \nu$ (Prop. 5), implying that $\Pr^W \left(n^t > \nu \mid \nu' < n^{t-1} \leq \nu\right) = \Pr^M \left(n^t > \nu \mid n^{t-1} \leq \nu'\right)$.

Consequently, the probability that the regulator loses control is weakly smaller in the model with warnings. ■

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Figure 1. Sanctions *p(n)* as a function of the number of violators *n*.

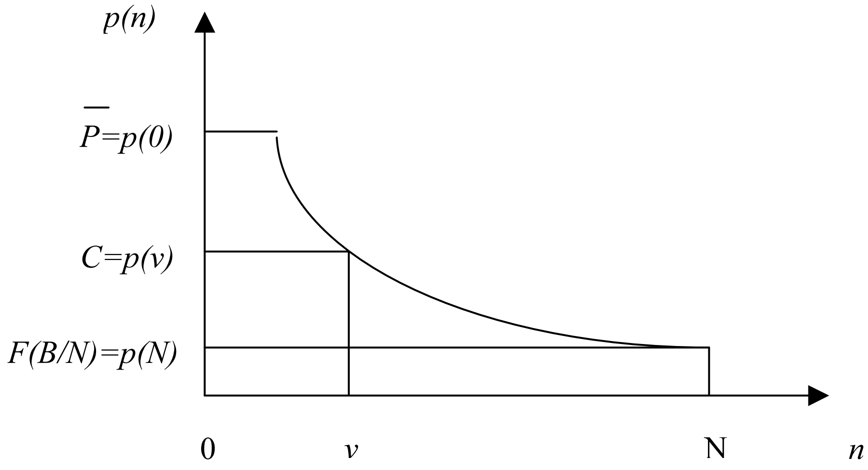

Figure 2. Decision tree in period *t* for a firm, when the regulator uses warnings. Costs (negative payoff) at all endpoints.

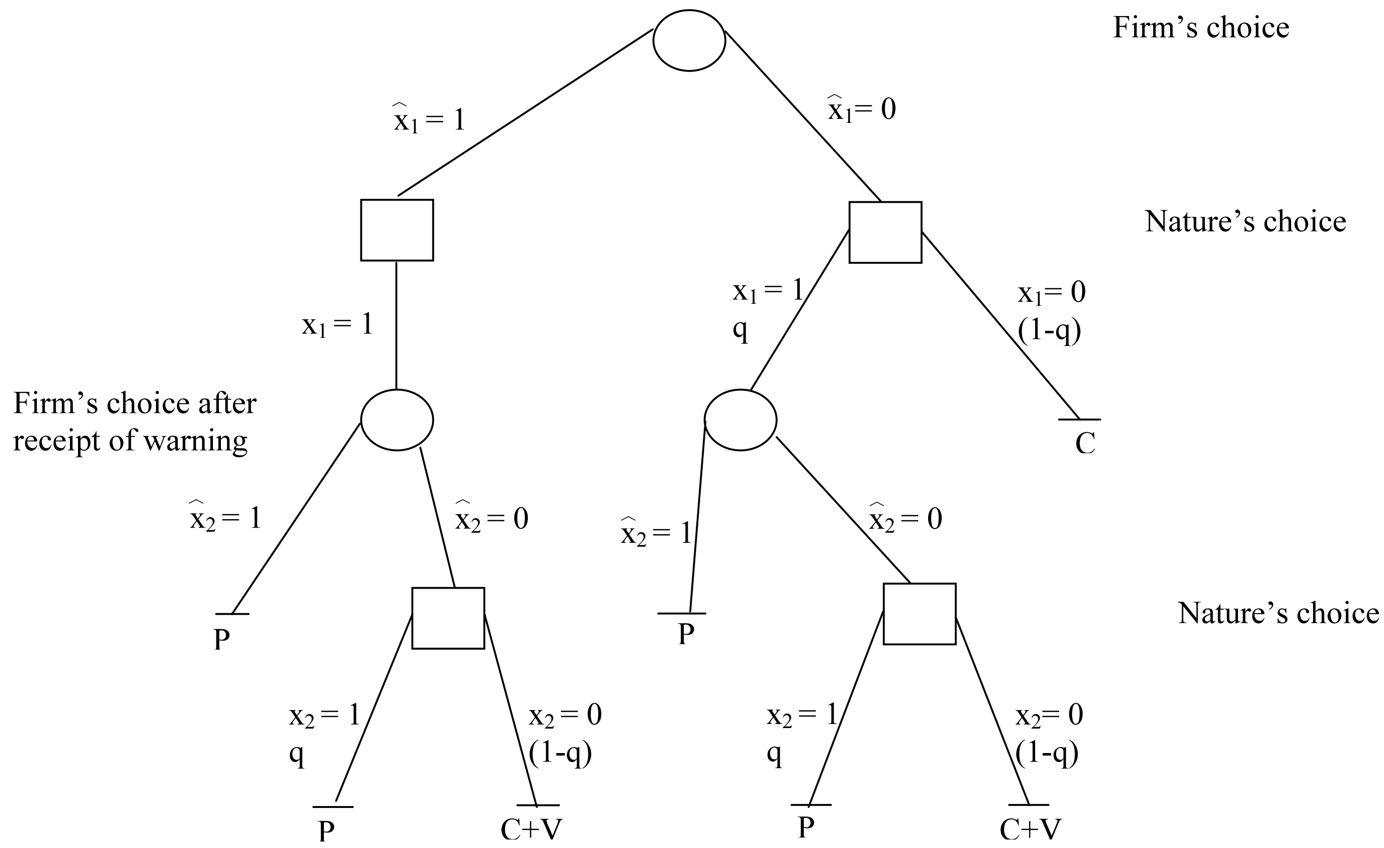

Figure 3. Sanctions, p(n), as a function of the number of violators, *n*.