Welfare effects of dynamic non-Pigouvian taxes in a context of interactions with an ecosystem

Rui Pedro Mota[1], Tiago Domingos, Environment and Energy Section, DEM, Instituto Superior Técnico

Abstract

This papers addresses the problem of welfare effects of using non-Pigouvian taxes for two economies – one harvesting resources from an ecosystem, and the other interacting with the ecosystem through pollution emissions. We present a specific ecosystem dynamics that allows for interesting interactions with the economy. Particularly important is the ecosystem feature of allowing for regime shifts or thresholds, which have important consequences for the welfare measures for the economy. We show in what conditions the approximations of the Pigouvian taxes improve welfare to the controlled economy in comparison to an uncontrolled economy, where the economy misestimates both the marginal utility from ecosystem services and the marginal ecosystem productivity.

Keywords: Environmental taxes; Ecosystem dynamics; non-Pigouvian taxes; Ecosystem thresholds.

1. Introduction

The literature on green accounting has grown enormously over the last few decades. Recent times have witnessed a greatly heightened awareness of the interactions between economic, social and environmental issues (Weitzman, 2003). There has appeared a widespread interest in the idea of extending the concepts and measurement of national income to include important

[1] Address: Avenida Rovisco Pais, 1, 1049-001 Lisboa, Portugal. Phone: +351 - 218 419 441, Fax: +351-218 417 365, E-mail: rmota@ist.utl.pt

nonmarket activities in related areas that affect welfare and productivity - in particular, environmental goods and services.

Many questions have been raised about augmented (or comprehensive) national income accounting ranging from the broad concerns about its welfare foundations, to the basic issues of the design of green national income accounts. As Weitzman (2003) puts it, at the core of this branch of economic analysis runs a common strand attempting to connect a currently observable index of comprehensive net national income or product (NNP) with some appropriate but not observable welfare measure of future power to consume, which typically has a "sustainability-like" flavor. These ideas are close to the ideas of classic economists such as Fisher, Lindahl, Hicks, Pigou, Hayek and many others interested in the concept of income, and in linking this concept to other important concepts in capital theory (Weitzman, 2003).

Our analysis is based on an interpretation of sustainability that is based on the maintenance of social welfare. Arrow et al. (2003) show that the requirement that economic development be sustainable implies, and is implied by, the requirement that the economy's productive base (institutions and capital assets, including human capital and natural capital) be maintained. This result is the basis for the World's Bank definition of genuine savings (now, adjusted net savings).

However to be able to go from theory to practical application, it is important to recognize that data are typically generated by imperfect economies. The presence of imperfections implies that market data alone are not sufficient for measuring welfare. For example, if externalities remain uninternalized in the general equilibrium, welfare measurement requires knowledge of the welfare contribution of these externalities, i.e. the present value of the marginal external effect (Aronsson et al. 2004).

We extend the work of Aronsson et al. (2004) to include a more realistic (and still somewhat simple) ecosystem dynamics and a misestimation of ecosystem dynamics that are essential for the definition of dynamic Pigouvian taxes. As is know this taxes have twofold uses. They put the economy at its first best path, but they can also be useful in social accounting (Aronsson and Löfgren ,1998). Pigouvian taxes can be estimated by using willingness-to-pay techniques imposing a bias on the estimations, which we analyze.

In the next section we introduce the ecosystem dynamics and present the main qualitative dynamics. Section 3 deals with the welfare measure of tax changes for a general equilibrium economy which uses ecosystems resources for production. Section 4 deals with these welfare changes in an economy that alters the structure of the ecosystem due to pollution emissions. Section 5 concludes.

2. Ecosystem dynamics

We interpret the dynamics of biomass as following the dynamics of ecological succession. It is generally assumed that ecological succession drives ecosystems' biomass along a logistic curve and net production along a U-inverted curve. Following this, we assume that the regeneration function for biomass is given by the logistic curve,

$$\frac{dN}{dt} = rN(CC - N)\,, \tag{1}$$

where N is ecosystem biomass, rCC is the growth rate per unit biomass at low biomass values and CC is the ecosystem carrying capacity. The important feature of ecological succession depicted by the logistic function is the existence of a "reasonably directional development, which culminates in a stabilized ecosystem climax". Ecosystem carrying capacity is the value of biomass attained at this state, established by the surrounding environment. It reflects the ecosystem's organization that best allocates the available resources.

If a new unit of ecosystem biomass can alter the ecosystem carrying capacity, this may be understood as a structural change, since an alteration of the efficiency of the allocation of resources has occurred.

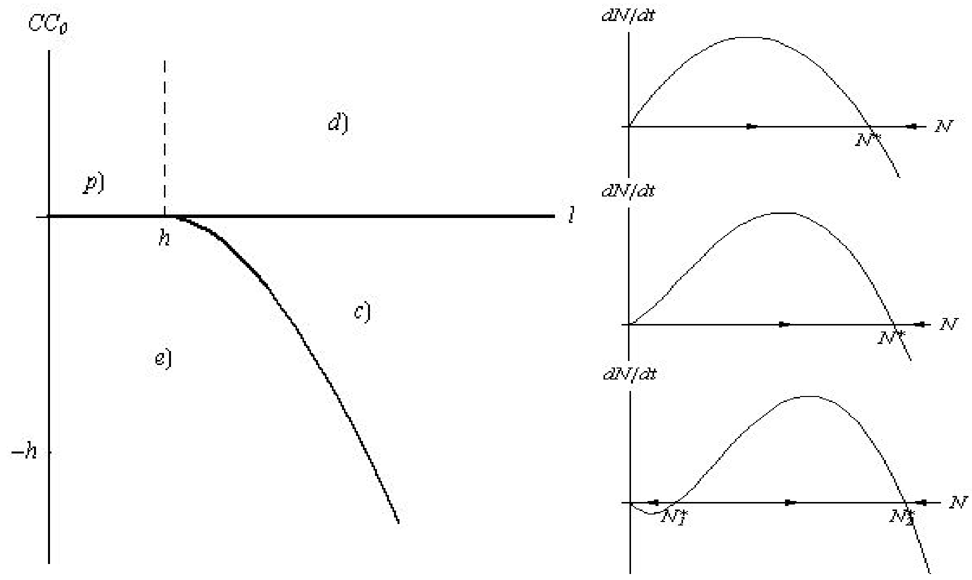
The basic feedback we wish to introduce, follows from the observation that environmental change modifies ecosystem biomass dynamics, while organisms are a know source of environmental change in ecology (bioengineering or niche construction). Since ecosystem carrying capacity accounts for properties of the surrounding environment, we argue that succession should be seen as a variable approaching a variable rather than a constant. So, the simplest obvious way to formally model this succession is to use a density dependent ecosystem carrying capacity in the logistic equation. Accordingly, we assume the following flow niche construction function:

$$\frac{dCC}{dt} = \frac{l}{N+h}\frac{dN}{dt}, \tag{2}$$

This way the environment’s capacity to supply organisms with resources depends on the capacity of those organisms to modify their environments. The parameter l is the bioengineering capability of the ecosystem and h is the feedback saturation. Now, a rise in ecosystem biomass causes a rise in ecosystem carrying capacity, which creates a greater potential for biomass to grow (provided that $N < CC$). The ecosystem production function is then:

$$\frac{dN}{dt} = rN\left(CC_0 + l\ln\left(\frac{N+h}{h}\right) - N\right). \tag{3}$$

Additionally, ecosystem thresholds are internalized, and can thus be estimated. The ecosystem may have three growth modes: pure compensation (concave ecosystem regeneration function), depensation (convex-concave regeneration function) and critical depensation (additionally having negative growth rates for low biomass). We conclude that, feedbacks can make the degraded (arid) system resilient to restorative change. The dynamics of the degraded state are very different from those in the pristine or target state and that the trajectory to recovery will probably be different from that of degradation (hysteresis). This is a very important observation for the design of efficient restoration policies.



a) b)

Figure 3.1 – a) Bifurcation diagram for the ecosystem dynamics for the case where. *p)* represents pure compensation; *d)* pure depensation and *c)* critical depensation. **b)** Ecosystem dynamics: Pure compensation; Pure depensation; Critical depensation, respectively from top to bottom.

3. Welfare and Green NNP in First and Second Best Frameworks - Harvesting

In this section we introduce the basic model which shall be used as a baseline for comparisons with the imperfect economy dealt in the rest of the paper. We present the social planner's problem of optimizing the representative agent's welfare considering an economy harvesting an ecosystem with niche construction in order to produce a homogenous consumption good.

Neglect population growth and normalize population equal to one. We assume that it is possible to define a utility function for the economy – citizens' preferences are identical and depicted by the representative consumer's utility function, $u(t) = u(c(t), a(N(t)))$, where $c(t)$ is consumption, $a(N(t))$ is a measure of the ecosystem's or environmental quality, where N is the ecosystem's biomass, and $u(\cdot)$ is a strictly concave and twice continuous differentiable function, increasing in both arguments. Harvesting will influence the utility function through interactions with the ecosystem dynamics. A continuous and infinite overlapping succession of individuals will behave rationally so as to maximise their utility function throughout their lives. Hence, we are considering that future decisions feedback into the present to give an overall maximum discounted sum of the consumers' utility flow,

$$V(K_0, N_0, 0) = \int_0^{\infty} e^{-\delta t} u(c, a(N)) dt \,, \tag{4}$$

where δ is the discount or haste rate.

3.1. Harvesting resources in a perfect economy

This economy produces a homogenous consumption good by using capital, $k(t)$, and harvested resources, $q(t)$ (think of forest resources, though as stated in the previous section the dynamics considered can also be applied to aquatic and other ecosystems) in such way described by the production function $y(t) = f(k(t), q(t))$. Production is assumed to be strictly concave, twice continuous differentiable, increasing and with decreasing marginal productivity in both arguments. This production function is net of capital depreciation.

We consider the unperturbed dynamics of an ecosystem along ecological succession using the formulation discussed in the previous section. Hence in the context of interactions with the economy the ecosystem dynamics is,

$$\frac{dN}{dt} = rN\left(CC_l + l \ln\left(\frac{N+h}{h} \right) - N \right) - q \equiv R(N) - q \,. \tag{5}$$

Though the main results of this paper could easily be extended to a generic ecosystem regeneration function we focus on this particular one since, among other things, it offers the possibility of incorporating pollution in a new and somewhat simple manner with the, we believe, more realistic interpretation of not affecting the ecosystem biomass directly but through its structure or carrying capacity.

Net investments in physical capital are determined according to

$$\frac{dk}{dt} = f(k,q) - c\,. \tag{6}$$

So, the optimization problem the social planner faces consists in choosing the temporal trajectories for the consumption $c(t)$ and the extraction rates $q(t)$ in a context of interaction with the environment, in order to achieve the maximum inter-temporal utility benefit. Formally, we want to:

$$\max_{c,q} \int_0^{\infty} u(c, a(N)) e^{-\delta t} dt\,,$$

subject to the state equations for capital and ecosystem biomass – equations (5) and (6). We also assume that the economy has fully known and fixed initial stocks $N(0) = N_0, k(0) = k_0$ and $N(t), k(t) \geq 0$, $c(t), q(t) \geq 0$.

We use optimal control theory to characterize optimal solutions. Here the maximum is taken over the set of piecewise continuously differentiable functions $q(t)$. In agreement with Pontryagin's maximum principle, the optimal candidate solutions maximise the present value Hamiltonian,

$$H(N,k,q,c,\lambda,\mu) = u(c,a(N))e^{-\delta t} + \lambda \dot{k} + \mu \dot{N}\,, \tag{7}$$

where the λ, μ are the co-state variables or shadow prices of physical capital and ecosystem biomass respectively. The shadow prices are equal to the marginal value of the general capital stock at time *t*, i.e., if the stock level or ecosystem biomass is reduced by one unit, its value at time *t* will be reduced by $\lambda(t)$ or $\mu(t)$ respectively. The Hamiltonian is the total rate of increase of total assets in units of the objective functional (utility units in this case), and the sum of the terms that have shadow prices is the value flow from investment in capital assets.

Shadow prices are, therefore, multiplier factors that alter the units from capital units to units of the objective functional.

In the comprehensive national accounting literature the inner product of the shadow prices vector with the economy's capital stocks vector is termed the genuine saving or genuine investment and represents the accounting value of the changes in the stocks of all society's capital assets at *t* (Arrow et al., 2003). The general result applied to our model translates as,

$$\frac{dV}{dt} = \frac{\partial V}{\partial K}\frac{dK}{dt} + \frac{\partial V}{\partial N}\frac{dN}{dt} = \lambda^c \dot{K} + \mu^c \dot{N} , \tag{7}$$

where the shadow prices are in current value, i.e., $\lambda^c(t) = \lambda e^{\delta t}$ and $\mu^c(t) = \mu e^{\delta t}$.

Proposition 1 – Weak sustainability

At constant accounting prices, genuine wealth $V(t)$ *is non-decreasing iff genuine investment* $\lambda^c \dot{K} + \mu^c \dot{N}$ *is non-decreasing.*

This sustainability measure does not necessarily require maintaining the ecosystem at any given time. Even if the ecosystem biomass is drawn down along a consumption path the weak sustainability criterion could be verified if physical capital is to be accumulated in order to offset the ecosystem decline (Arrow et al., 2003).

As stated in section 2, the ecosystem dynamics has thresholds below which despite the harvest rate the ecosystem biomass is declining, hence extinction is inevitable. In this case, what we can say is that, since there are no other sources of growth other than capital accumulation and ecosystem growth, even if the harvest of ecosystem resources is not essential for the production of the consumption good, this economy is non-sustainable in case of crossing ecosystem thresholds.

In addition to equations (5) and (6) and the non-negativity of the state and control variables, the first order conditions for this optimal control problem are,

$$\frac{dH^*}{dc} = u_c(c^*, a(N^*))e^{-\delta t} - \lambda^* = 0\,, \tag{8}$$

$$\frac{dH^*}{dq} = \lambda^* f_q(k^*, q^*) - \mu^* = 0\,, \tag{9}$$

$$\dot{\lambda}^* = -\lambda^* f_k(k^*, q^*)\,, \tag{10}$$

$$\dot{\mu}^* = -u_N(k^*, q^*)e^{-\delta t} - \mu^* R^{'}(N^*)\,, \tag{11}$$

along with the transversality conditions,

$$\lim_{t\to\infty} \lambda^*(t)k^*(t) = \lim_{t\to\infty} \mu^*(t)N^*(t) = 0\,. \tag{12}$$

The first order conditions and the assumptions made on the utility and the production function imply that both the shadow prices of physical and natural capital are positive.

Applying the Hamilton-Jacobi equation we have,

$$\delta V^*(k, N, t) \equiv \delta \int_t^{\infty} u(c^*, a(N^*))e^{-\delta(s-t)}ds = u(c^*, a(N^*)) + \lambda^{c*}\dot{k}^* + \mu^{c*}\dot{N}^*\,, \tag{13}$$

which is, as Weitzman (2003) puts it, the wealth and income version of the maximum principle and states that:

Proposition 2 – Income and Wealth Condition

The (interest on the) present discounted value of the economy's future welfare is measured by the present value Hamiltonian along the optimal path.

Using the first order approximation around zero $u(c,a(N)) \approx u_c c + u_N N$ we have the linearized welfare measure,

$$\frac{\delta V^*(K_0,N_0,t)}{\lambda^{c^*}} \approx c^* + \dot{k}^* + \frac{u_N{}^*}{\lambda^{c^*}} N^* + \frac{\mu^{c^*}}{\lambda^{c^*}} \dot{N}^* . \tag{14}$$

The right hand side (RHS) of equation (14) is the green NNP in real terms for this economy. The first two terms are the conventional NNP (Aronsson and Löfgren, 1999), the third term measures the value of ecosystem at time *t* as valued by the consumers' in real terms and the fourth measures the value of the ecosystem growth as valued as the marginal productivity of natural resources. The latter term can also be understood as the value of depletion of the ecosystem as a source of resources for production.

So, generally, the Hamilton-Jacobi equation provides a mean of measuring the welfare of this economy, defined has the present value of the discounted future utility, assuming that the economy is currently following the optimal path. Given this, conventional NNP figures, $c^* + \dot{k}^*$, can be found in Input-Output tables; the value of the ecosystem, $\frac{u_N{}^*}{\lambda^{c^*}} N^*$, priced at the consumers' valuation of a marginal change of the ecosystem biomass can be estimated by using willingness-to-pay studies; and the value of depleting of the ecosystem for production, $\frac{\mu^{c^*}}{\lambda^{c^*}} \dot{N}^*$, requires data of marginal harvesting costs of ecosystem products which are usually hard to obtain and (wrongly) approximated as an average cost of harvesting (Hartwick, 1990; Neumayer, 2000, 2004). Neumayer (2000, 2004) suggests using the El-Serafy method for an approximation of marginal costs.

The fundamental drawback of this approach to welfare measurements is the assumption of optimality of the real economy path. So, generally, using the Jacobi-Belman equation does not yield the true economy's welfare. In this context, the analysis of the imperfect economy is important in order to know what data to look for.

In a market economy, $\tau^*(t) \equiv \mu^{c*} / \lambda^{c*}$ is the dynamic equivalent to a Pigouvian tax that puts the economy in the social (first) best path. Aronsson and Löfgren (1999) note that the dynamic Pigouvian tax is forward looking since it requires information on future marginal utilities of ecosystem services. Solving equation (11) we get[2],

$$\mu^*(t) = \int_t^{\infty} u_N{}^* e^{-\delta s} e^{\int_t^s R'(N^*(\xi))d\xi} ds . \tag{15}$$

This means that Pigouvian taxes are also useful in a welfare accounting framework, since as Aronsson et al. (2004) suggest, Pigouvian taxes provide additional information needed to calculate the economy's green NNP. Aronsson et al. (2004) go on stating that it is possible to estimate Pigouvian taxes by using willingness to pay methods, but as these methods are not able to catch the preferences of future generations, the approximation may be quite inadequate for constructing Pigouvian taxes. The following question is posed (Aronsson and Lögfren, 1999): Will the approximations of the Pigouvian taxes improve welfare in comparison to an uncontrolled economy?

Another drawback of practical Pigouvian taxes is that, as we showed in equation (15), for the correct calculation of Pigouvian taxes to be made it is necessary to have a complete knowledge of the ecosystem dynamics, and at the most we can only have a very rough approximation of it. So it is logical to ask whether the Pigouvian tax is welfare improving, again comparing to an uncontrolled economy, when we do not know the exact ecosystem dynamics. This is the questions we answer in this paper, and we do it in a context of an economy extracting resources in the next subsections, and an economy polluting the ecosystem in the next section.

---

[2] We assume that $\lim_{t \to \infty} k^*(t) > 0$ and apply the corresponding transversality condition.

3.2. The decentralized controlled and uncontrolled market economies

We now consider the case of an uncontrolled market economy. We wish to derive the income and wealth condition in the case of an externality from ecosystem dynamics. In a decentralized economy, the consumer's optimization problem is to,

$$\max_{c} \int_0^\infty u(c, a(N)) e^{-\delta t} dt \text{, s.t.}$$

$$\dot{k} = \pi + rk + w - c \text{,} \tag{16}$$

where we suppress time for simplicity. Equation (16) depicts the fact that in this market economy, the consumer sells one unit of labour at a given real wage *w*, rents capital at the market interest rate *r*, to the representative firm (Aronsson et al. 2004). The term π represents earnings other than those from labour and physical capital renting. The necessary conditions the consumer obeys are

$$u_c(c^0, a(N^0)) e^{-\delta t} = \lambda^0 \text{,} \tag{17}$$

$$\dot{\lambda}^0 = -\lambda^0 r \text{,} \tag{18}$$

where the superscript denotes the market economy optimum.

The firms optimize their profits under perfect competition. So, the representative firm chooses $q(t)$ and $k(t)$ so as to $\max_{q,k} f(k,q) - w - rk$. The first order conditions are $f_k(k^0, q^0) = r$ and $f_q(k^0, q^0) = 0$. The firms use ecosystem resources until its marginal product is zero.

Combining, the conditions for the consumer and the firm we have the general equilibrium for the uncontrolled market economy,

$$\dot{k}^0 = f(k^0, q^0) - c^0 \text{,} \tag{19}$$

$$\dot{\lambda}^0 = -\lambda^0 f_k(k^0, q^0)\,, \tag{20}$$

and equation (17).

To derive the income wealth condition along the market solution we follow Aronsson et al. (2004) and Aronsson and Lögfren, (1998). Writing the Hamiltonian for the consumer's problem evaluated along the general equilibrium we have,

$$H^0 = u(c^0, a(N^0))e^{-\delta t} + \lambda^0 \dot{K}^0\,, \tag{21}$$

Deriving with respect to time using the result $dH^*/dt = \partial H^*/\partial t$ and noting that since the ecosystem biomass is not optimal for the society N depends on *t*, we obtain,

$$\frac{dH^0}{dt} = -\delta u^0(\cdot)e^{-\delta t} + u_N^0(\cdot)e^{-\delta t}\,\dot{N}^0 \tag{22}$$

Using current the value Hamiltonian and integrating until infinity it is straightforward to show,

$$H^{c0}(t) = \delta\int_t^\infty u^0(\cdot)e^{-\delta(s-t)}ds - \int_t^\infty u_N^0(\cdot)e^{-\delta(s-t)}\,\dot{N}^0\,ds \tag{23}$$

This states, as expected, that the correct welfare measure for the uncontrolled economy is the current value Hamiltonian plus the present value of the marginal utility from the externality (Aronsson et al. 2004).

Proposition 3 – Ecosystem externalities in an uncontrolled economy

A welfare measure based solely on the Hamiltonian will result in over (under) estimation of comprehensive welfare if the ecosystem is decreasing (increasing) over time.

Particularly, if some ecological threshold has been crossed, the welfare measured is always an over-estimation of the true welfare. Moreover, the faster the ecosystem is driven to extinction the higher the welfare over estimation is. This is important for policymaking for harvesting or consumption, since an agent may feel that he (she) can increase its consumption or extraction leading the economy faster to extinction. So, knowing the ecosystem dynamics and mostly its thresholds is indispensable for an optimal design of policies for sustainable development.

Now we turn to the controlled market economy. We suppress the time argument for simplicity. The consumer optimization problem is,

$$\max_{c} \int_0^{\infty} u(c, a(N)) e^{-\delta t} dt \text{, s.t.}$$

$$\dot{k} = \pi + rk + w + T - c \,, \tag{24}$$

where T is a time dependent lump-sum transfer from the government which is assumed to have a balanced budget. The first order conditions are equations (17) and (18). The representative firm optimization problem is, again, to choose $q(t)$ and $k(t)$ so as to $\max_{q,k} f(k,q) - w - rk - \tau q$, where τ is a resource tax. The first order conditions are $f_k(k^0, q^0) = r$ and $f_q(k^0, q^0) = \tau^0$. The government balanced budget condition translates to $T = \tau q$. Summing, the general equilibrium conditions for the controlled economy are equations (17), (19), (20) and $f_q(k^0, q^0) = \tau^0$. It is now straightforward to show that:

Proposition 4 – Perfectly controlled market economy

If the controlled market economy tax is defined in a Pigouvian manner $\tau^0 = \tau^* \equiv \mu^* / \lambda^*$, $\mu^*(t) = \int_t^{\infty} u_N{}^* e^{-\delta s} e^{\int_t^s R'(N^*(\xi))d\xi} ds$ *or if it obeys* $\dot{\mu}^* = -u_N(k^*, q^*) e^{-\delta t} - \mu^* R'(N^*)$ *in the above setting, then the imperfect market economy will coincide with the social planer's optimal problem.*

As noted by Aronsson et al. (2004) this proposition allows the welfare to be measured by using the 'Hamiltonian' that supports the perfectly controlled market economy. This is the same to say that the Pigouvian tax internalizes the ecosystem externality and puts the economy in the first best path.

3.3. Cost-Benefit Analysis for a change in the ecosystem harvest tax

In this section we examine the welfare implications of using a non-Pigouvian tax to control the economy. As discussed above we have informational problems calculating Pigouvian taxes, whether from not knowing the real future marginal utility of ecosystem, or the ecosystem dynamics. Hence, in this section we derive the cost-benefit rule for a small change in the ecosystem harvest tax and analyze its welfare improving compared with the uncontrolled market by using a non-Pigouvian tax where the used marginal utility from ecosystem stock and the know ecosystem dynamics have a time dependent bias to the real values.

Assume there is a small increase in the harvest tax so that the tax is now $\tau^0(t)+\alpha$. The imperfect economy works in the same setting as in section above. Hence, the value function is,

$$V(K_0,0;\varepsilon)=\int_0^\infty u(c^0,a(N^0))e^{-\delta t}dt\ , \tag{25}$$

where ε is a parameter vector with α as one of its elements. We assume, as Aronsson et al. (2004), that this value function is differentiable with respect to α. The cost benefit rule for a change in the harvest tax is (derived in Appendix A.1),

$$\frac{\partial V(K_0,0)}{\partial\alpha}=\int_0^\infty u_N(c^0(t),a(N^0(t)))\frac{\partial N^0(t)}{\partial\alpha}e^{-\delta t}+\lambda^0(t)\tau^0(t)\frac{\partial q^0(t)}{\partial\alpha}dt\ , \tag{26}$$

which is equivalent to the cost benefit rule derived in Aronsson and Löfgren (1999) and Aronsson et al. (2004) but now applied to an ecosystem harvesting tax. The second term of equation (26) is negative since $\tau^0>0$ and $\partial q^0/\partial\alpha<0$, and it represents the cost of the loss of consumption due to a decrease in the ecosystem resources harvested (due to an increase in the harvest tax). The first term is positive since $u_N > 0$ and $\partial N^0 / \partial \alpha > 0$ [3]. Hence, if $\tau^0 > 0$ then $\partial V(K_0, 0) / \partial \alpha > 0$, which means that introducing a small positive tax in the uncontrolled economy is always welfare improving.

Now, extending the analysis of Aronsson and Löfgren (1999) and Aronsson et al. (2004) to incorporate a bias in the estimation of the ecosystem dynamics we define the following non-Pigouvian tax,

$$\tau^0(t) = \left[\int_t^\infty (u_N{}^0 + \beta(s)) e^{-\delta s} e^{\int_t^s R^{'}(N^0(\xi)) + \gamma(\xi) d\xi} ds \right] / \lambda^{c0}, \tag{27}$$

where $R^{'}(N^0) + \gamma(t)$ is the incorrect measure of ecosystem dynamics and $u_N^0(\cdot) + \beta(t)$ is the incorrect measure of the consumer's marginal utility of ecosystem stock. So if $\beta(t)$ and $\gamma(t)$ are both zero we have the Pigouvian of proposition 4 and the controlled economy is at its first best path. To know the welfare effect of these biases in the non-Pigouvian tax for the controlled economy we derive in Appendix A.3 the following result,

Proposition 5 – Welfare effects of non-Pigouvian harvest tax

If the harvest tax takes the form of equation (27) for all t, the cost benefit rule for α is,

$$\frac{\partial V(K_0, 0)}{\partial \alpha} = -\int_0^\infty \frac{\partial N^0(t)}{\partial \alpha} \left[\lambda^0(t) \tau^0(t) \gamma(t) + \beta(t) e^{-\delta t} \right] dt. \tag{28}$$

If the controlled economy uses the Pigouvian harvest tax then a small increase in the harvest tax does not affect the welfare level. If both the measured marginal utility from the ecosystem and

---

[3] See Appendix A.2 for the proof of the second inequality.

of marginal ecosystem productivity overestimate the true respective values, then a permanent small increase in the harvest tax reduces the welfare level. By the same token, reducing the harvest tax is welfare improving in this case. On the other hand, if we underestimate the marginal ecosystem productivity ($\gamma(t) < 0$) and $-u_N(\cdot) < \beta(t) < -\lambda^0(t)\tau^0(t)\gamma(t)$[4], then increasing harvest tax is always welfare improving.

The term $\lambda^0(t)\tau^0(t)\gamma(t)$ is equal to $u_c{}^0(t)\, f_q{}^0(t)\gamma(t)$ and can be interpreted as the change in consumers' utility due to a change in the harvest rate. So in order to asses the welfare effects of imperfect taxes, the productivity of the economy as well as the marginal utility of consumption play an important role.

An interesting effect is that there is the possibility of tradeoffs between the estimates to generate welfare improving increases in the harvest tax. If, for instance, we are overestimating the ecosystem marginal productivity ($\gamma(t) > 0$), for the controlled economy to be welfare superior we need to underestimate the marginal utility from the ecosystem. This possibility is plausible if the ecosystem is near or has crossed a threshold.

Corollary 1

If the harvest tax takes the form of equation (27) for all t, with $\gamma(t) < 0$ *and* $-u_N(\cdot) < \beta(t) < -\lambda^0(t)\tau^0(t)\gamma(t) > 0$*, then the controlled market economy is always welfare superior to the uncontrolled . If* $\gamma(t) > 0$ *and* $-u_N(\cdot) < \beta(t) < -\lambda^0(t)\tau^0(t)\gamma(t) < 0$ *then the controlled market economy is always welfare superior.*

Though the marginal utility from ecosystem services can be estimated using static willingness to pay methods, the marginal productivity of the ecosystem is much more difficult to estimate.

[4] This implies that we have a tax and not a subsidy.

In theory, with historical data of ecosystems' biomass we could estimate equation (3) and calculate among other things threshold biomasses.

If, for instance, the bias of the ecosystem dynamics is constant and only in one parameter, say l the bioengineering capability of the ecosystem, so that the parameter introduced in the ecosystem dynamics equation (3) is $l+\gamma$, it is straightforward to show that the cost benefit rule can be written as,

$$\frac{\partial V(K_0,0)}{\partial \alpha} = -\int_0^{\infty}\left\{\frac{\partial N^0(t)}{\partial \alpha}\left[\lambda^0(t)\tau^0(t)\frac{\gamma}{N(t)+h}+\beta(t)e^{-\delta t}\right]\right\}dt \, .$$

The ecosystem biomass now also plays a role in the welfare effects of non-Pigouvian taxes. If the ecosystem biomass is decreasing (increasing) what decides the welfare usefulness of the tax is the first (second) term inside the brackets. Now, imagine that the economy behaves as if the ecosystem did not have any bioengineering capability along ecological succession. This translates to $\gamma = -l$. If the ecosystem path as crossed a threshold this implies that whatever the responses of willingness-to-pay studies it becomes welfare improving to raise the harvest tax. Also, the welfare improvement is becoming higher with the decreasing of the ecosystem biomass. The reverse happens if the ecosystem is growing.

4. Welfare and Green NNP in First and Second Best Frameworks - Polluting

We now turn to the analysis of an economy that, instead of harvesting the ecosystem for the production of the consumption good, emits a homogenous pollutant to the ecosystem. The novelties (so far as we know) in this analysis are the assumptions that the ecosystem follows equation (3) and that emissions from production affect directly the ecosystem structure, or its carrying capacity. This is particularly easy to introduce in our ecosystem dynamics. As in section 3 we first present the social planner's optimization problem and evolve to the case of the decentralized economy, where we want to know the effects of non-Pigouvian emission taxes for the welfare level of the controlled economy.

4.1. The perfect economy

We follow the assumptions of Aronsson et al. (2004) for the production function. The economy produces a homogenous good using physical capital and energy, where we assume that emissions equal the input of energy. Hence, $y(t) = f(k(t), g(t))$, where $g(t)$ is the emissions from the production of the consumption good. The accumulation of physical capital for this economy follows $\dot{k} = f(k, g) - c$.

We consider the following perturbed ecosystem dynamics,

$$\frac{dN}{dt} = rN\left(CC_l + l\ln\left(\frac{N+h}{h}\right) - e(g) - N\right) \equiv R(N, e(g)), \tag{29}$$

and $e(\cdot)$ is the effect of emissions on the ecosystem carrying capacity[5].

Note that emissions do not affect ecosystem biomass directly, but through its carrying capacity. For future work, this ecosystem production function offers the possibility of direct structural perturbation of ecosystem dynamics due to emission or even harvesting practices, and direct effects of both emissions and harvesting on the ecosystem biomass. For now, we are interested in the welfare effects of structural damages of emissions on the ecosystem dynamics.

The social planner's optimization problem consists of choosing consumption and emissions rates in order to maximize the discounted sum of the consumers' utility flow. We assume that the consumer values the ecosystem services rather than the pollution emissions. This way the consumer does not need to know specific information about the different kinds of pollutants and its effects on the ecosystem. We believe this is a more 'clean' way of addressing the problem of measuring marginal utilities from emission effects.

---

[5] This effect is increasing with emissions.

Formally,

$$\max_{c,g} \int_0^{\infty} u(c, a(N)) e^{-\delta t} dt \,, \tag{30}$$

subject to the dynamics of the ecosystem and the accumulation of physical capital. We also assume that the economy has fully known and fixed initial stocks $N(0) = N_0, k(0) = k_0$ and $N(t), k(t) \geq 0$, $c(t), g(t) \geq 0$.

The present value Hamiltonian for this problem is

$$H(N,k,g,c,\lambda,\mu) = u(c, a(N)) e^{-\delta t} + \lambda \dot{k} + \mu \dot{N} \,, \tag{31}$$

and the necessary conditions for the optimal path are,

$$u_c(c^*, a(N^*)) e^{-\delta t} = \lambda^* \,, \tag{32}$$

$$f_g(k^*, g^*) = -\frac{\mu^*}{\lambda^*} R_g(N, e(g)) \,, \tag{33}$$

$$\dot{\lambda}^* = -\lambda^* f_k(k^*, g^*) \,, \tag{34}$$

$$\dot{\mu}^* = -u_N(k^*, g^*) e^{-\delta t} - \mu^* R_N(N^*, e(g^*)) \,, \tag{35}$$

along with the transversality conditions,

$$\lim_{t \to \infty} \lambda^*(t) k^*(t) = \lim_{t \to \infty} \mu^*(t) N^*(t) = 0 \,. \tag{36}$$

Applying the Hamilton-Jacobi equation and using the first order approximation for the utility function we obtain the linearized welfare measure in real terms,

$$\frac{\delta V^*(K_0, N_0, t)}{\lambda^{c^*}} \approx c^* + \dot{k}^* + \frac{u_N{}^*}{\lambda^{c^*}} N^* - \frac{f_g{}^*}{R_g{}^*} \dot{N}^* \,. \tag{37}$$

The RHS of equation (37) is the green NNP in real terms for this economy. The difference for the harvesting economy is the last term is the value of the ecosystem growth as valued as the marginal depletion of ecosystem structure due to emission from production. The dynamic Pigouvian tax for this economy is defined as $\tau^* = -\frac{\mu^*}{\lambda^*} R_g(N, e(g))$.

4.2. The decentralized controlled market economy

In this section we present the imperfect market economy where, again, the externality comes from the ecosystem dynamics. This setting is somewhat more close to the standard setting of analysing emissions but now, with the ecosystem interacting with the emissions flow in a much more realistic way than considering a constant ecosystem dynamics.

The consumers solve the problem,

$$\max_c \int_0^\infty u(c, a(N))e^{-\delta t} dt \text{, s.t.}$$

$$\dot{k} = \pi + rk + w + T - c \text{,} \qquad (38)$$

where we suppress time for simplicity. The first order conditions are,

$$u_c(c^0, a(N^0))e^{-\delta t} = \lambda^0 \text{,} \qquad (39)$$

$$\dot{\lambda}^0 = -\lambda^0 r \text{,} \qquad (40)$$

where the superscript denotes the market economy optimum.

The representative firms chooses $g(t)$ and $k(t)$ so as to $\max_{g,k} f(k, g) - w - rk - \tau g$, where τ is, now, the emission tax. The first order conditions are $f_k(k^0, g^0) = r$ and $f_q(k^0, g^0) = \tau^0$.

The government has balanced budget. Summing, the general equilibrium conditions for the controlled economy are equations (39), (40), $\dot{k}^0 = f(k^0, g^0) - c^0$ and $f_q(k^0, q^0) = \tau^0$.

Proposition 6 – Perfectly controlled market economy

If the controlled market economy tax is defined in Pigouvian manner as $\tau^0 = \tau^* \equiv -R_g \mu^* / \lambda^*$,

$\mu^*(t) = \int_t^\infty u_N{}^* e^{-\delta s} e^{\int_t^s R_N(N^*(\xi), e(g^*(\xi)))d\xi} ds$ *or if it obeys* $\dot{\mu}^* = -u_N(k^*, g^*)e^{-\delta t} - \mu^* R_N(N^*, e(g^*))$

in the above setting, then the imperfect market economy will coincide with the social planer's optimal problem.

4.3. Cost-Benefit Analysis for the emission tax

The purpose of this section is to analyze the welfare implications of using a non-Pigouvian tax. As stated above, we misestimate both the marginal utility of ecosystem services and marginal ecosystem productivity. Similarly to section 3.3 we derive the cost-benefit rule for a small change in the emission tax and analyze its welfare improving compared with the uncontrolled market by using a non-Pigouvian tax where the measured marginal utility from ecosystem stock and the know marginal ecosystem productivity have a time dependent bias to the real values.

Consider a small change in the emission tax so that the tax is now $\tau^0(t) + \alpha$, and that the value function is differentiable with respect to α. Following the steps in Apendix A.1. but now for the emission economy, the cost benefit rule for a change in the emission tax is,

$$\frac{\partial V(K_0, 0)}{\partial \alpha} = \int_0^\infty u_N(c^0(t), a(N^0(t))) \frac{\partial N^0(t)}{\partial \alpha} e^{-\delta t} + \lambda^0(t)\tau^0(t) \frac{\partial g^0(t)}{\partial \alpha} dt \,. \tag{41}$$

The second term of the RHS of equation (26) is negative since $\tau^0 > 0$ and $\partial g^0 / \partial \alpha < 0$, and it is the cost of increasing α in terms of forgone consumption. The first term is positive since $u_N > 0$ and $\partial N^0 / \partial \alpha > 0$[6]. Again we have that if $\tau^0 > 0$ then $\partial V(K_0, 0)/\partial \alpha > 0$, which translates as, introducing a small positive tax in the uncontrolled economy is always welfare improving.

Define the following non-Pigouvian tax,

$$\tau^0(t) = -R_g \left[\int_t^{\infty} (u_N{}^0 + \beta(s)) e^{-\delta s} e^{\int_t^s R_N^0(\cdot) + \gamma(\xi) d\xi} ds \right] / \lambda^{c0}, \tag{42}$$

where $\gamma(t)$ is the bias of estimated marginal productivity and $\beta(t)$ is the bias of the measure of the consumer's marginal utility of ecosystem stock. How these biases affect the welfare of a change in emission tax? To answer this we just applied the demonstration of proposition 5 to our emission economy cost benefit rule and use equation (27) to show the next proposition.

Proposition 7 – Welfare effects of non-Pigouvian emission tax

If the emission tax takes the form of equation (42) for all t, the cost benefit rule for α is,

$$\frac{\partial V(K_0, 0)}{\partial \alpha} = \int_0^{\infty} \frac{\partial N^0(t)}{\partial \alpha} \left[\lambda^0(t) \tau^0(t) \frac{\gamma(t)}{R_g(N^0(t), e(g^0(t)))} - \beta(t) e^{-\delta t} \right] dt. \tag{43}$$

Comparing to the cost benefit rule in equation (28), the only difference is the marginal damage of ecosystem structure due to pollution. Note that $R_g(\cdot)$ is negative. The main conclusions are depicted in the following corollary.

[6] See Appendix A.2 for the proof of the second inequality.

Corollary 2

If the harvest tax takes the form of equation (43) for all t, with $\gamma(t) < 0$ *and* $-u_N(\cdot) < \beta(t) < -\lambda^0(t)\tau^0(t)\gamma(t)/R_g(\cdot) > 0$ *,, then the controlled market economy is always welfare superior to the uncontrolled .*

If $\gamma(t) > 0$ *and* $-u_N(\cdot) < \beta(t) < -\lambda^0(t)\tau^0(t)\gamma(t)/R_g(\cdot) < 0$ *then the controlled market economy is always welfare superior.*

What we conclude in comparison with corollary 1 is that effect of emissions in the ecosystem structure is important in defining welfare improving taxes.

5. Conclusions and future work

In this work we have addressed the problem of finding the conditions for which the definition of non-Pigouvian taxes is welfare improving for the economy. We focused particularly on the effects of misestimating marginal utility from ecosystem services and the marginal ecosystem productivity. We presented an ecosystem dynamics that has interesting and recent ecological fundamentals and still allows for clear analytic conclusions. This ecosystem dynamics is based on the assumption that ecosystem while evolving alter their physical environment and enhance future growth. The ecosystem dynamics considered allows for the existence of thresholds, which have important consequences in welfare analysis, which are not fully understood. The two economies considered here hint at the usefulness of using this ecosystem dynamics for more realistic analysis of welfare accounting in imperfect economies.

Our analysis of non-Pigouvian taxes generalized Aronsson et al. (2004) work in two ways. First we considered a more general ecosystem dynamics, and second we considered wrong estimates for the marginal ecosystem productivity. The main conclusion is that when we consider misestimates of the marginal ecosystem productivity it is possible to have always welfare improving tax changes in either over or under estimations of the marginal utility of ecosystem services, provide certain requirements are meet.

6. References

Aronsson, T., Löfgren, K., G., 1998 . Green accounting in imperfect market economies. Environmental and Resource Economics. 11(3-4):273-287.

Aronsson, T., Löfgren, K., G., 1999 . Pollution tax design and 'green' national accounting. Environmental and Resource Economics. 43:1475-1474.

Aronsson, T., Löfgren, K., G., Backlund, K., 2004. *Welfare measurement in imperfect markets*. Edward Elgar, Cheltenham, UK.

Arrow, K., J., Dasgupta., P., Mäler, K.,G., 2003. Evaluating projects and assessing sustainable development in imperfect economies. Environmental and Resource Economics. 26:647-685.

Hartwick, J., M., 1990. Natural resources, national accounting and economic depreciation. Journal of Public Economics. 43:291-304.

Neumayer, E., 2000. Resource accounting in measures of unsustainability: Challenging the World Bank's conclusions. Environmental and Resource Economics. 15:257-278.

Neumayer, E., 2004. *Weak versus Strong Sustainability: Exploring the limits of two opposing paradigms*. Edward Elgar, Cheltenham, UK.

Weitzman, M., L., 2003. *Income, Wealth and the Maximum Principle*. Harvard University Press, London, England

A.1. Appendix 1 – Derivation of the cost benefit rule

This proof follows closely the approach by Aronson et al. (2004). Define the Hamiltonian like function,

$$\widetilde{H}^0(t;\varepsilon) = u^0(c^0(t;\varepsilon), a(N(t;\varepsilon)))e^{-\delta t} + \lambda(t)\dot{k}^0(t;\varepsilon)\,, \tag{A1.1}$$

where $\lambda(t)$ does not depend on the vector ε. The value function can now be written as,

$$V(K_0, 0; \varepsilon) = \int_0^\infty \widetilde{H}^0 - \lambda \dot{k}^0 dt\,, \tag{A1.2}$$

where we suppressed the arguments for simplicity. Integrating,

$$V(K_0, 0; \varepsilon) = \int_0^\infty \widetilde{H}^0 - \dot{\lambda} k^0 dt - \lambda k^0 \Big|_0^\infty . \tag{A1.3}$$

The cost benefit rule is derived by differentiating the value function with respect to α and evaluating the resulting derivative along the general equilibrium path (where $\lambda = \lambda^0(t,\varepsilon)$) with $\alpha = 0$. Hence,

$$\frac{\partial V^0(\cdot)}{\partial \alpha} = \int_0^\infty \frac{\partial \widetilde{H}^0}{\partial \alpha} - \dot{\lambda}^0 \frac{\partial k^0}{\partial \alpha} dt - \frac{\partial}{\partial \alpha} \lambda^0 k^0 \Big|_0^\infty . \tag{A1.4}$$

where the last term is zero since $\lim_{t\to\infty} \lambda^0(t) = 0$ and $k(0)$ is fixed.

Now, substituting

$$\frac{\partial \widetilde{H}^0}{\partial \alpha} = u_N^0 \frac{\partial N^0(s)}{\partial \alpha} e^{-\delta t} + \lambda^0 \left(f_k^0 \frac{\partial k^0}{\partial \alpha} + \tau^0 \frac{\partial q^0}{\partial \alpha} \right), \tag{A1.5}$$

into equation (A1.4) and using the first order conditions gives equation (26).

A.2. Appendix 2 – Proof of the sign of $\partial N^0 / \partial \alpha$

Taking the derivative of $\dot{N} = R(N) - q$ with respect to α and using and since $N(t)$ is continuous we get,

$$\frac{\partial}{\partial t}\frac{\partial N}{\partial \alpha} = R^{'}(N)\frac{\partial N}{\partial \alpha} - \frac{\partial q}{\partial \alpha} \tag{A2.1}$$

Noting that this is a linear differential equation it can be solved for $\partial N/\partial \alpha$, yielding,

$$\frac{\partial N(t)}{\partial \alpha} = -\int_0^t \frac{\partial q(s)}{\partial \alpha} e^{\int_t^s R^{'}(N^*(\xi))d\xi} ds\,, \tag{A2.2}$$

which is positive since $\partial q^0/\partial \alpha < 0$.

For the case of the emission economy we have $\dot{N} = R(N, e(g))$. Differentiating with respect to α gives,

$$\frac{\partial}{\partial t}\frac{\partial N}{\partial \alpha} = R_N \frac{\partial N}{\partial \alpha} + R_g \frac{\partial g}{\partial \alpha} \tag{A2.3}$$

Solving for $\partial N/\partial \alpha$, yields

$$\frac{\partial N(t)}{\partial \alpha} = \int_0^t R_g(\cdot) \frac{\partial g(s)}{\partial \alpha} e^{\int_t^s R_N(\cdot)d\xi} ds\,, \tag{A2.4}$$

which is positive since $\partial g^0/\partial \alpha < 0$ and $R_g < 0$.

A.3. Appendix 3 – Proof of proposition 5

Substituting equation (A2.1) in equation (26) the cost benefit rule is written as,

$$\frac{\partial V(K_0, 0)}{\partial \alpha} = \int_0^\infty u_N{}^0 \frac{\partial N^0}{\partial \alpha} e^{-\delta t} + \lambda^0 \tau^0 \left(\frac{\partial R^0}{\partial \alpha} - \frac{\partial^2 N^0}{\partial \alpha \partial t} \right) dt\,, \tag{A3.1}$$

where we have suppressed the arguments for simplicity. Integrating by parts,

$$-\int_0^\infty \lambda^0 \tau^0 \frac{\partial^2 N^0}{\partial\alpha\partial t} dt = \int_0^\infty \frac{\partial^2 N^0}{\partial\alpha\partial t}\left(\dot{\lambda}^0 \tau^0 + \lambda^0 \dot{\tau}^0\right) dt , \tag{A3.2}$$

noting that $\lim_{t\to\infty} \lambda^0(t) = 0$ and $N(0)$ is fixed. Substituting (A3.2) in (A3.1) and making use of the first order conditions we have,

$$\frac{\partial V(K_0,0)}{\partial \alpha} = \int_0^\infty \frac{\partial N^0}{\partial \alpha}\left[u_N{}^0 e^{-\delta t} + \lambda^0 \tau^0 \left(R'^0 - f_k{}^0 \right) + \lambda^0 \dot{\tau}^0 \right] dt . \tag{A3.3}$$

Now, substituting the time derivative of the non-Pigouvian harvest tax,

$$\dot{\tau}^0(t) = \frac{-\left(u_N + \beta\right) e^{-\delta t}}{\lambda^0} + \tau^0 \left(f_k{}^0 - R'^0 - \gamma \right), \tag{A3.4}$$

in equation (A3.3) we obtain equation (28).