It is widely acknowledged that positive externalities associated with research and development foster economic growth. This paper expands on models that explore whether negative externalities caused by environmental degradation can have a similar effect. If both labor and environmental quality are included in a standard Ramsey growth model with two kinds of consumption goods, negative externalities cause output to increase. At the same time, welfare declines. Economy-wide technological progress and population growth further widen this gap.

1. Introduction

Growth economists commonly model the environment as a nonrenewable natural resource. Our dependence on oil and other subsoil assets dampens economic growth and, hence, societal welfare. Yet numerous environmental problems are not caused by the exhaustion of nonrenewable resources that have a market value and are used in production. Many, if not most environmental problems, are directly related to renewable resources. Some, such as timber and fish, are traded in the marketplace and are used as productive inputs. Others, like air and water, derive their greatest importance as basic biological support systems, without which life as we know it would not be possible. The common thread between them is that they all provide direct benefits to society and ought to enter the utility function directly.
Once environment is included as an input in the utility function, demand for environmental quality provides a competing force with consumption. To the household, consumption and a clean environment are complements in the sense that the marginal utility of consumption increases with environmental quality, and vice versa.\(^2\) Producing consumer goods, however, causes pollution and thus decreases environmental amenities available to the household. Smulders (2000) explicitly models these environment-growth interactions by adding an index of environmental quality as a stock variable into a Ramsey growth model. He concludes that in such a model, “environment cannot contribute positively to growth in the long run.” On the contrary, biophysical limits restrict growth; “economic growth is ultimately driven by knowledge accumulation.”

This conclusion does not sufficiently address the concerns of those who have long argued that defensive consumption aimed at alleviating negative effects of a deteriorating environment contributes to growth. Output as measured by our national accounts with GDP as the most prominent indicator, does in fact increase because of defensive expenditures.\(^3\) An important characteristic is that these expenditures are not complements to a clean environment, as is the case with overall consumption. Rather, defensive consumption and environmental quality are substitutes. Bartolini and Bonatti (2003) include defensive expenditures into their model of economic growth. The representative household cares about consumption \((c)\) of traditional consumer goods that complement environmental quality, and about defensive consumption \((d)\) that substitute for a clean environment.\(^4\) They find that growth is essentially a substitution process in which free public goods are replaced by costly consumer products.

This paper expands Bartolini and Bonatti’s model to allow for economy-wide technological progress and population growth. In doing so, we can investigate the connection between growth driven by the substitution between environmental quality and defensive expenditures, as well as by knowledge accumulation. In the context of this model, both

\(^2\) All else being equal, we would rather have the same amount of consumer goods and high levels of environmental quality than low ones. Conversely, given a certain level of environmental quality, utility is higher with more consumer goods than with less. Formally, the cross-partial derivative of utility with respect to consumption \(c\) and environmental quality \(E\) is nonnegative, \(u_{cE} \geq 0.\)

\(^3\) There are a number of studies in the area of green or comprehensive income green accounting that highlight these issues. See Bartelmus and Seifert (2003) for a compendium of major works, or Heal and Kriström (2003) for a survey of the theoretical and empirical literature.

\(^4\) In theory, consumption \(c\) is a complement to environmental services, see footnote 2. Bartolini and Bonatti’s (2003) model and this paper assume the two to be additively separable. Hence, they are neither complements nor substitutes. This assumption is made for mathematical tractability without loss of generality.
contribute to an increase in output, while simultaneously decreasing welfare. Technological progress explicitly aimed at pollution abatement technology has the opposite effect. Such a gap between output and welfare poses a problem for policy makers, who largely use output, as measured by GDP, as a proxy for welfare and, thus, a guide for economic and development policies. The conclusion of this paper underlines the importance subtracting defensive expenditure from GDP, rather than the current practice of adding them.

We first set up the basic model and solve for the steady state in this economy. Later, we discuss its implications.

## 2. The Model

We carry out the analysis within the standard Ramsey framework in continuous time.\(^5\) Instantaneous utility is a function of a single consumer good \(c(t)\), work hours \(h(t)\), and a composite good \(X(t)\) representing consumption related to environmental services,

\[
(1) \quad u(c(t), h(t), X(t)) = \beta \ln(X(t)) + \gamma \ln(c(t)) + (1 - \beta - \gamma) \omega(h(t)),
\]

with \(\beta > 0, \gamma > 0, \beta + \gamma < 1\). Disutility of work has the constant-elasticity form,

\[
(2) \quad \omega(h(t)) = -\xi h(t)^{1+\sigma},
\]

where \(\xi > 0\) and \(\sigma \geq 0\).\(^6\)

The composite \(X(t)\) is a linearly separable function of environmental quality \(E(t)\) and defensive consumption \(d(t)\),

\[
(3) \quad X(t) = E(t) + \kappa \ d(t),
\]

where \(\kappa(>0)\) measures the technological feasibility and the household’s preferences of substituting between \(E(t)\) and \(d(t)\). The higher \(\kappa\), the easier it is to substitute defensive expenditures for environmental quality. This may be a statement about the technology underlying the substitution process, or it could be an indication of the household’s preferences over the two different kinds of services.

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\(^5\) Our continuous time setup corresponds to the discrete time model of Bartolini and Bonatti (2003), extended by including population growth \(n\) and, crucially, exogenous technological progress \(x\). We also relax assumptions on the parameters governing environmental quality in equation (4).

\(^6\) See Barro and Sala-i-Martin (2004), chapter 9.4.
The environment is measured by an aggregate index of environmental quality corresponding to flow services valued by the household. The measure $E(t)$ represents both the quantity and quality of the environment. Society benefits from a greater number of trees as well as from healthier forests. Utility exhibits positive diminishing marginal returns in $X(t)$. Consuming $c(t)$ and $d(t)$ is costly for the household, while $E(t)$ is a freely available public good. The environment’s services are non-rival and non-exclusive.\textsuperscript{7}

Environmental quality has a natural rate of renewal relative to its stock and it is harmed by economic activity proportional to output $y(t)$,

$$
\dot{E}(t) = \nu E(t) - \eta y(t).
$$

We would naturally assume that the parameters $\nu$ and $\eta$ are positive and close to zero. The larger the stock, the higher the capacity of the environment to renew itself. Conversely, a larger output should degrade the environment because of increased resource use and higher pollution. It is hard to conceive of why this would not hold for $\nu$, but some evidence suggests that $\eta$ may indeed be negative in later stages of development.\textsuperscript{8} While $\eta$ is positive for some resources and pollutants, such as fresh water and carbon dioxide, respectively, it does not hold for others. Certain resources (e.g. timber) may grow with increasing GDP, while predominantly local or regional air and water pollutants decrease. The literature on the environmental Kuznets curve (EKC) has established inverted U-shaped relationships between levels of income and several such pollutants. With regard to these pollutants, environmental quality decreases in early stages of development, and subsequently improves.\textsuperscript{9}

Having $\nu$ and $\eta$ be exogenous precludes our model from retracing the EKC. We can only hope to calibrate it in order to mimic certain stages along the curve. However, when we consider the extreme level of aggregation in this economy, it would be reasonable to assume that

\textsuperscript{7} This setup abstracts away nonrenewable resources, which in any case would enter the production instead of the utility function. See the discussion in Smulders (2000), which uses a similar aggregate index of environmental quality.

\textsuperscript{8} The natural rate of renewal could follow a logistical growth function, which is commonly used for stocks of renewable resource (Clark 1990). In this case, $\nu$ would be small for low and high levels of environmental quality and increase for median levels. In any case, it would still be positive, although our model holds for negative $\nu$ as well.

η is indeed small and positive. While many developed economies have long surpassed maxima for certain types of pollution, we have not yet decoupled economic growth from most forms of resource use. One of the fundamental indicators of environmental quality is the physical extent of ecosystems. In this regard, all natural ecosystems with the exception of forested land have decreased in the continental United States over the past fifty years. Urban areas, however, have continuously increased in size.\textsuperscript{10} We will assume η to be positive and close to zero for our initial analysis. Later, we will relax this assumption.

A defining characteristic of our model is the substitutability between $E(t)$ and $d(t)$. Defensive consumption is used to offset the degradation of environmental quality. Improvements in environmental quality, conversely, enable a decrease in defensive consumption. In the most general formulation, we would also expect labor to be substitutes with $c(t)$, $d(t)$, and $E(t)$, while $c(t)$ is a complement to both $d(t)$ and $E(t)$. Equation (1) exhibits substitution between $d(t)$ and $E(t)$ with $u_{de} < 0$. It trivially fulfills the other requirements by having all other cross-partial derivatives equal zero. Using this separable utility function makes the problem mathematically tractable, while still capturing the main features of the model.

The representative household maximizes

\begin{equation}
W = \int_{0}^{\infty} u(c(t), h(t), X(t)) \ e^{-(\rho-n)t} \ dt
\end{equation}

with respect to $c(t)$, $d(t)$, and $h(t)$, subject to the household’s budget constraint

\begin{equation}
\dot{a}(t) = w(t) \ h(t) + (r(t) - n)a(t) - c(t) - d(t),
\end{equation}

where $a(t)$ represents per capita assets, $w(t)$ the wage per unit of labor, $r(t)$ the competitive interest rate, $\rho$ the time invariant personal discount rate, and $n$ the exogenously given rate of population growth.

Nothing changes for competitive firms from the standard Ramsey setup. They operate with Cobb-Douglas production technology,

\begin{equation}
\dot{y}(t) = A \ \dot{k}(t)^{\alpha},
\end{equation}

\textsuperscript{10} See The state of the nation’s ecosystems (2002).
where \( \hat{y}(t) \) and \( \hat{k}(t) \) are the per capita quantities of output \( y(t) \) and capital \( k(t) \), respectively, per unit of effective labor, \( h(t) \). Capital depreciates at the constant rate \( \delta \geq 0 \). Given \( r(t) \) and \( w(t) \), firms maximize instantaneous profit in each period,

\[
\pi(t) = \left[ A \, \hat{k}(t)^\alpha - (r(t) + \delta)\hat{k}(t) - w(t) \, e^{-x_t}\right]h(t) \, e^{(\alpha+\gamma)x_t},
\]

by setting

\[
r(t) = \alpha \, A \, \hat{k}(t)^{\alpha-1} - \delta
\]

and

\[
w(t) = (1-\alpha)A \, \hat{k}(t)^\alpha \, e^{-x_t}.
\]

Households maximize the present-value Hamiltonian with respect to the controls \( c(t) \), \( d(t) \), and \( h(t) \) and the state variable \( a(t) \),

\[
H = u(c(t), h(t), X(t)) \, e^{-(\rho-n) t} + \lambda(t) \left[ w(t) \, h(t) + (r(t) - n)a(t) - c(t) - d(t) \right],
\]

with \( u \) defined by equations (1) – (3). After replacing \( c(t) \) by consumption per unit of effective labor \( \hat{c}(t) \equiv c(t)/\left( h(t) \ e^{x_t} \right) \) and using equation (9) to substitute for \( r(t) \), we obtain the Euler condition

\[
\frac{\dot{\hat{c}}(t)}{\hat{c}(t)} = \alpha \, A \, \hat{k}(t)^{\alpha-1} - (\delta + \rho + x) \frac{\dot{h}(t)}{h(t)}.
\]

The first-order condition governing the substitution between \( c(t) \) and \( h(t) \), combined with equation (10) yields

\[
\dot{h}(t) = \left( \frac{\gamma \left(1-\alpha \right) \, A \, \hat{k}(t)^\alpha}{(1+\sigma) \left(1-\beta-\gamma \right) \, \xi \, \hat{c}(t)} \right)^{\frac{1}{1+\sigma}},
\]

which we can rewrite in terms of growth rates,

\[
\frac{\dot{h}(t)}{h(t)} = \left( \frac{\alpha}{1+\sigma} \right) \frac{\dot{\hat{k}}(t) - \frac{1}{1+\sigma} \frac{\dot{\hat{c}}(t)}{\hat{c}(t)}}{\frac{\hat{k}(t)}{k(t)} - \frac{1}{1+\sigma} \frac{\hat{c}(t)}{c(t)}}.
\]
When we combine the results of differentiating the Hamiltonian with respect to \( c(t) \) and \( d(t) \) with the identity \( \hat{d}(t) \equiv d(t)/\left(h(t) \ e^{ut}\right) \), we obtain a second first-order condition relating defensive expenditures to consumption and environmental quality,

\[
\hat{d}(t) = \frac{\beta}{\gamma} \hat{c}(t) - \frac{1}{\kappa} \hat{E}(t),
\]

where \( \hat{E}(t) \equiv E(t)/\left(h(t) \ e^{ut}\right) \). This equation divides the aggregate index of environmental quality by effective labor input. It is difficult to interpret \( \hat{E}(t) \) in any meaningful way. We will only use it for ease of calculation and will convert back to \( E(t) \) before drawing any conclusions. Using \( a = k \) and equation (9), the transversality condition is

\[
\lim_{t \to \infty} \left( \hat{k}(t) e^{-\int_{t}^{\infty} A \hat{r} \ t \ \delta + \ n + \ h(t) \ \hat{h}(t) \right) = 0.
\]

Substituting equations (9) and (10) into the household’s budget constraint from equation (6) with \( a = k \) results in the capital constraint for the economy,

\[
\frac{\hat{k}(t)}{k(t)} = \frac{A}{\kappa} \hat{k}(t)^{\alpha-1} - (\delta + n + x) \frac{\hat{c}(t)}{k(t)} - \frac{\hat{d}(t)}{k(t)} - \frac{\hat{h}(t)}{h(t)}.
\]

We use equation (4) for environmental quality in conjunction with equation (7) and \( \hat{y}(t) \equiv y(t)/\left(h(t) \ e^{ut}\right) \) to derive the growth rate of \( \hat{E}(t) \),

\[
\frac{\hat{E}(t)}{E(t)} = \nu - x - \frac{\eta}{\kappa} \frac{A \hat{k}^\alpha}{\hat{E}(t)} - \frac{\hat{h}}{h}.
\]

Plugging equations (14) and (15) into (12), (17), and (18), and solving for \( \hat{c}(t)/\hat{c}(t) \), \( \hat{k}(t)/\hat{k}(t) \), and \( \hat{E}(t)/\hat{E}(t) \) finally yields three differential equations governing our economy:

\[
\frac{\hat{c}(t)}{\hat{c}(t)} = \frac{\alpha (\beta + \gamma)}{\gamma \left(\alpha + \sigma\right)} \hat{c}(t) + \alpha A \hat{k}(t)^{\alpha-1} - \frac{\alpha}{\kappa \left(\alpha + \sigma\right)} \hat{E}(t) - \frac{\alpha}{\kappa \left(\alpha + \sigma\right)} \frac{\hat{E}(t)}{\hat{k}(t)} - \frac{\alpha}{\kappa \left(\alpha + \sigma\right)} \frac{(\delta + \rho + x)}{\alpha + \sigma},
\]

\[
\frac{\hat{k}(t)}{\hat{k}(t)} = -\frac{\sigma (\beta + \gamma)}{\gamma \left(\alpha + \sigma\right)} \hat{k}(t) + \sigma A \hat{k}(t)^{\alpha-1} + \frac{\sigma}{\kappa \left(\alpha + \sigma\right)} \hat{E}(t) - \frac{(\delta + \rho + x)}{\kappa \left(\alpha + \sigma\right)} \frac{\hat{E}(t)}{\hat{k}(t)} - \frac{(\delta + n + x)}{\alpha + \sigma},
\]

and
\[
\frac{\dot{E}(t)}{E(t)} = \frac{\alpha}{\gamma} \left( \beta + \gamma \right) \frac{\dot{c}(t)}{E(t)} - \frac{\eta A \dot{k}(t)}{k(t)} - \frac{\alpha}{\kappa (\alpha + \sigma)} \frac{\dot{E}(t)}{k(t)} - \frac{\delta + \rho + x - \alpha (\delta + n + v) + \sigma (x - v)}{\alpha + \sigma}.
\]

We have a closed system of differential equations. The growth rates for \(\dot{c}(t)\), \(\dot{k}(t)\), and \(\dot{E}(t)\) are functions of only these three variables. First, we rewrite the system to treat \(\dot{c}(t)/\dot{k}(t)\) and \(\dot{E}(t)/\dot{k}(t)\) as one variable each. Then, we solve for the unique steady state values of \(\dot{c}^*, \dot{k}^*, \text{ and } \dot{E}^*\).

\[
\dot{c}^* = \frac{\gamma \Psi}{\alpha} \frac{1}{\kappa (\beta + \gamma) (v - x)} \left( \frac{\alpha A}{\delta + \rho + x} \right)^{1 - \alpha},
\]

\[
\dot{k}^* = \left( \frac{\alpha A}{\delta + \rho + x} \right)^{1 - \alpha},
\]

and

\[
\dot{E}^* = \eta A \frac{1}{v - x} \left( \frac{\alpha A}{\delta + \rho + x} \right)^{\alpha}.
\]

where \(\Psi\) in equation (22) is defined as a function of parameters,

\[
\Psi = \eta (\delta + \rho + x) + \kappa (v - x) \left[ (1 - \alpha) (\delta + x) + \rho - n \alpha \right].
\]

We will use this definition for subsequent calculations.

From equations (13), (22), and (23), we can calculate the steady state value of individual work effort,

\[
h^* = \left( \frac{(1 - \alpha) (\beta + \gamma) \kappa (v - x) (\delta + \rho + x)}{(1 + \sigma) (1 - \beta - \gamma) \xi \Psi} \right)^{1/(1 - \alpha)}.
\]

This in turn allows us to convert \(\dot{c}^*, \dot{k}^*, \text{ and } \dot{E}^*\) into per capita values \(c^*(t)\) and \(k^*(t)\), and the level of environmental quality \(E^*(t)\), which all depend on time because of exogenous technological progress in this economy:

\[
c^*(t) = \frac{\gamma (1 - \alpha) A e^{x t}}{(1 + \sigma) (1 - \beta - \gamma) \xi (h^*)^{\alpha} \left( \frac{\alpha A}{\delta + \rho + x} \right)^{\alpha}}.
\]
\( k^*(t) = e^{\nu t} \ h^* \left( \frac{\alpha \ A}{\delta + \rho + x} \right)^{\frac{1}{1-\alpha}}, \)

and

\( E^*(t) = \frac{\eta \ A \ e^{\nu t} \ h^* \left( \frac{\alpha \ A}{\delta + \rho + x} \right)^{\frac{\alpha}{\beta}}} {v - x} \).

In a similar fashion, we can calculate per capita output,

\( y^*(t) = e^{\nu t} \ h^* \ A \left( \frac{\alpha \ A}{\delta + \rho + x} \right)^{\frac{\alpha}{\beta}}, \)

and, using equation (15), defensive expenditures per capita,

\( d^*(t) = e^{\nu t} \ h^* \left( \frac{\alpha \ A}{\delta + \rho + x} \right)^{\frac{1}{1-\alpha}} \left( \delta + \rho + x \right) \left( v - x \right) (1-\alpha) \beta \kappa + \alpha \beta \kappa (v - x) (\rho - n) - \eta \gamma (\delta + \rho + x) \).

Environmental quality \( E^*(t) \) is an index that can, in theory, take on any value. Given \( v < x \), which we later argue is an appropriate assumption, \( E^*(t) \) will be negative. The highest achievable level of environmental quality is zero, which corresponds to no direct impact of the economy on the environment (\( \eta = 0 \)). This, however, should not be taken to mean that the level of environmental quality is “negative” in any interpretive sense. Negative levels of \( E^*(t) \) may indeed be optimal.

Work effort \( h^* \) is bounded by zero and one. The remaining variables, \( c^*(t), d^*(t), k^*(t) \), and \( y^*(t) \), must be nonnegative for all \( t \geq 0 \). We now investigate the conditions under which this holds true.

In order for \( h^* \), \( c^*(t), d^*(t), k^*(t) \) and \( y^*(t) \) to be positive for all \( \sigma \geq 0 \), we need

\( \frac{(1-\alpha) (\beta + \gamma) \kappa (v - x) (\delta + \rho + x)}{(1+\sigma) (1-\beta - \gamma) } \varepsilon \Psi > 0, \)

where \( \Psi \) is defined in equation (25). Since all parameters are positive (with \( \sigma \) being nonnegative), \( \alpha < 1 \), and \( \beta + \gamma < 1 \), equation (32) holds true if either \( v > x \) and \( \Psi > 0 \), or if \( v < x \) and \( \Psi < 0 \).
If $\nu > x$, we need $\rho > n\alpha$ for $\Psi$ to be unambiguously positive. This last inequality holds under what Barro and Sala-i-Martin (2004) cite as the best parameter estimates: $\rho = 0.02$, $n = 0.01$, and $\alpha = 0.75$. In this scenario, we get the somewhat paradoxical result that $\partial y^*(t)/\partial \eta < 0$ and $\partial u^*(t)/\partial \eta > 0$ for all $t$. An increased impact of the economy on the environment decreases output and increases utility.

Leaving the confines of this model for a moment, it may indeed be possible to argue why pollution would decrease output. We could imagine a scenario where a clean environment is an important factor of production. Polluted water, for example, could decrease the catch of a fishery or increase the cost of doing business for a laundry facility. In both cases the result would be lower output. Tourism-dependent industries may be particularly harmed by a deteriorating environment. Nevertheless, it is difficult to imagine that such a scenario would hold for a large economy with diversified industries. More pollution allowances decrease the cost of doing business for many manufacturing sectors, which would surely increase output. Regardless of the debate about output, higher levels of utility due to more pollution pose a contradictory result that cannot be easily reconciled with reality. Environmental quality is widely regarded as a public good. It is hard to conceive of a scenario in which deteriorating environmental quality would boost utility for a typical household in the economy.

Fortunately, for $\nu < x$ and $\Psi < 0$, the previous conclusions are reversed: $\partial y^*(t)/\partial \eta > 0$ and $\partial u^*(t)/\partial \eta < 0$. Higher levels of pollution intensity raise industry output and decrease utility in all time periods, thus decreasing societal welfare. Let us now investigate the conditions under which this conclusion holds.

If $\nu < x$, $\Psi$ is negative under the most plausible parameter estimates as long as $\nu$ and $\eta$ are relatively small and $\kappa$ is large. Assuming $\rho = 0.02$, $n = 0.01$, $\alpha = 0.75$, $\delta = 0.05$, and $x = 0.02$, we need

$$\kappa > \frac{150\eta}{1-50\nu}. \tag{33}$$

Translated into words, it is more likely that environmental pollution increases output while decreasing welfare, if there is less impact of the economy on the environment (small $\eta$),

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11 See Barro and Sala-i-Martin (2004) for justifications of these parameter values.
environmental quality is stable (small $\nu$), and it is easy to substitute consumption goods for free environmental services (large $\kappa$).

This result seems to counter intuition in at least two respects. For one, assuming small $\nu$ and $\eta$ implies that the environment is relatively large and stable compared to the economy. The total amount of environmental quality cannot easily be influenced by the environment’s own rate of renewal nor by economic activity. This reasoning corresponds to Kenneth Boulding’s notion of a “cowboy” economy, where we live off a seemingly infinite stream of resources without having to worry about natural limits or scale. A cowboy is relatively small compared to his surroundings, the infinite plains. On the other end of the scale spectrum is the “spaceman” economy, in which the economy has reached the physical limits of the natural environment. Life in a closed-off capsule in space requires resources to be carefully conserved and measured. In order to survive, the spaceman needs to be in full control of the surroundings. There is no environment not subsumed by the economy.\footnote{Daly (1996), p. 58, explores the distinction between a “cowboy” and “spaceman” economy and attributes the original terms to Kenneth Boulding.} This extreme scenario corresponds to large $\nu$ and $\eta$. Many “limits to growth” arguments are based on the assumption that we are moving ever closer to the “spaceman” economy. In doing so, however, the negative feedback mechanism of pollution increasing the gap between output and welfare may be reversed.

What appears to be a second paradox involves the possibility of substitution between environmental services and consumption goods or defensive expenditures. The coefficient $\kappa$ measures the technological feasibility of substituting human-made consumption goods for freely available environmental services and the preferences of consumers over the two. Environmentalists generally argue for a low $\kappa$. In doing so, they cite the difficulty of substituting consumer goods for natural services both from a technical standpoint and because of consumer preferences. A $\kappa$ too close to zero, however, not only reverses the prediction of an increasing gap between output and welfare due to pollution but is a theoretical impossibility. It would imply $\nu < x$ and $\Psi > 0$, which would result in negative work effort, consumption, capital and output in the model. Hence, we need a certain minimum level of $\kappa$ as approximated by equation (33).

3. The Results

We are primarily interested in the behavior of the economy in relation to its direct impact on the environment $\eta$. Bartolini and Bonatti (2003) find that output increases with $\eta$, whereas
instantaneous utility decreases (Proposition 1). When extending the model to include exogenous technological progress \( x \), the results depend on the relative magnitudes of \( x \) and the capacity of the environment to renew itself \( \nu \).

Assuming \( \nu \) and \( \eta \) are small and \( \kappa \) is large enough to fulfill \( \nu < x \) and \( \Psi < 0 \), we can now analyze the behavior of the economy in greater depth. The following proposition is the main result of analyzing the behavior of the economy in relation to its impact on the environment.

**Proposition 1:** As the negative impact of the economy on the environment increases, output rises while welfare \( W^* \) falls:

\[
\frac{\partial y^*(t)}{\partial \eta} > 0,
\]

and

\[
\frac{\partial W^*}{\partial \eta} < 0.
\]

**Proof of Proposition 1:** We can show the first result by taking the partial derivative,

\[
\frac{\partial y^*(t)}{\partial \eta} = -\left(\frac{\delta + \rho + x}{1 + \alpha} A e^{\alpha} \eta A^{1/\alpha} \left(\frac{\alpha A}{\delta + \rho + x}\right)^{\frac{\alpha}{1 - \alpha}}\right) > 0,
\]

where \( h^* > 0 \) and \( \Psi < 0 \).

For the second part of the proposition, we first look at the household’s instantaneous utility function defined in equations (1)–(3). Using equation (15) for defensive expenditures, we can rewrite utility as a function of \( c^*(t) \) and \( h^* \),

\[
u(c^*(t),h^*) = \beta \ln \left(\frac{\beta \kappa}{\gamma}(\beta + \gamma) \ln(c^*(t)) - (1 - \beta - \gamma) \xi (h^*)^{1/\alpha}\right).
\]

Given that \( \nu < x \) and \( \Psi < 0 \), consumption \( c^*(t) \) declines with \( \eta \),

\[
\frac{\partial c^*(t)}{\partial \eta} = \frac{\gamma \sigma A e^{\alpha} h^* A^{\frac{\alpha}{1 - \alpha}}}{(1 + \sigma)(\beta + \gamma) \kappa (\nu - x) \left(\frac{\alpha A}{\delta + \rho + x}\right)^{1/\alpha}} < 0,
\]
and $h^*$ increases,

$$\frac{\partial h^*}{\partial \eta} = -\frac{(\delta + \rho + x)}{(1 + \sigma)} \Psi > 0. \quad (39)$$

Hence, instantaneous utility decreases in every time period, $\partial u(c^*(t), h^*)/\partial \eta < 0 \ \forall t$. Welfare, defined in equation (5) as the present discounted value of the future stream of utilities, decreases as well.

As the negative impact of output on the environment increases, the economy enters a self-perpetuating growth process. Households substitute defensive expenditures for previously free environmental services, $\partial d^*(t)/\partial \eta > 0$. At the same time, households work more to afford the additional amount of costly consumer goods. Increased labor, in turn, produces more output, which implies a deteriorating environment. Defensive expenditures increase to make up for deteriorating environmental quality. In steady state, we have a larger output than if $\eta$ was zero. At the same time, welfare is smaller compared to a scenario without production externalities. Growth due to the economy’s negative impact on the environment is undesirable.

Economy-wide technological progress further widens the gap between output and welfare.

**PROPOSITION 2:** Over time, technological progress increases output and decreases welfare:

$$\frac{\partial y^*(t)}{\partial x} > 0, \quad (40)$$

and

$$\frac{\partial W^*}{\partial x} < 0, \quad (41)$$

for large $t$.

**Sketch of proof of Proposition 2:** We cannot prove either equation rigorously for general parameter values. Both expressions simplify to inequalities of the form $t > B$, where $B$ is a different finite constant. Without making assumptions about specific parameter values, we are not able to sign $B$ in either case. However, when we plug in our standard parameter values, $\rho = 0.02, \ n = 0.01, \ \alpha = 0.75, \ \delta = 0.05$, and $x = 0.02$, use values for $\nu, \ \eta$, and $\kappa$ that correspond
to equation (33) and let $\sigma$ vary between 0 and 2, $B$ for both expressions is less than 40. Under standard conditions, equations (40) and (41) hold for any $t > 40$. Keeping other parameters constant, this also holds for any values of $x > 0.02$.

This result may be somewhat surprising. Technological progress is commonly seen as positive from a welfare standpoint. It allows the economy to produce more output with the same or less amount of inputs. Consumers are able to enjoy greater amounts of consumption and more leisure. In this economy, however, technological progress increases output, while at the same time decreasing welfare.\(^\text{13}\)

It is important to note that technological progress here is non-discriminatory. It boosts output in all sectors of the economy at equal rates. This is a natural assumption in our model, given that we only have one production technology available to the entire economy. But even in this aggregate setup, we can imagine a different kind of progress. Technological advances could be aimed at decreasing the economy’s negative impact on the environment. Advances in pollution abatement technology may result in a smaller $\eta$. Such targeted technological progress does indeed increase welfare. From equation (35) in Proposition 1, we can see that a decrease in $\eta$ would lessen the negative impact on $W^*$. At the same time, however, decreasing $\eta$ would also lead to a fall in $y^*(t)$ $\forall t$, equation (34). Technological progress aimed at lessening the economy’s negative effect on the environment is welfare-enhancing, but it also depresses output.

Having decreases in $\eta$ go hand in hand with rising welfare and falling output is, in fact, a desirable property of the model. As mentioned above, the Environmental Kuznets Curve literature has established inverted U-shaped relationships between levels of income and several pollutants. Environmental quality with regard to local or regional air and water pollutants decreases in early stages of development, and is subsequently followed by improvements. Many of these improvements are achieved through active environmental policy instituted by governments in developed countries. While output generally does not decrease due to environmental policies, its growth rate often slows. Whether they involve command-and-control measures or market-based policies such as environmental taxes or tradable emissions permits, the majority of policies has in common that they tend to increase societal welfare while decreasing output growth.

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\(^\text{13}\) Technological progress also increases the output gain due to environmental pollution, while at the same time furthering the loss of welfare; i.e., we have $\frac{\partial^2 y^*(t)}{\partial \eta \partial \lambda} > 0$ and $\frac{\partial^2 W^*}{\partial \eta \partial \lambda} < 0$ for large $t$. 

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We could also consider a third kind of technological progress that combines the previous two. On the one hand, new inventions increase economy-wide technological progress; on the other, some of these inventions also enable decreases in the pollution intensity of the environment. In many ways, this is the best description of reality. The overall effects on output and welfare now depend on the relative magnitude of the two kinds of progress. From the perspective of a national income accountant, who would hope that changes in GDP reflect changes in welfare, the best outcome would be that overall technological progress and advances in abatement technology cancel each other and the economy remains in a constant steady state. If overall technological progress is larger compared to the scenario where the two cancel each other out, we move towards higher output and lower welfare, and vice versa.

The effects of population growth on the gap between output and welfare are essentially the same as for technological progress, except that they hold for any \( t \geq 0 \) regardless of specific parameter values.

**PROPOSITION 3:** Population growth increases the gap between output and welfare:

\[
\frac{\partial y^*(t)}{\partial n} > 0, \tag{42}
\]

and

\[
\frac{\partial W^*}{\partial n} < 0. \tag{43}
\]

**Proof of Proposition 3:** Given that \( v < x \) and \( \Psi < 0 \), the first result follows immediately from differentiating equation (30) with respect to \( n \). For the second result, we substitute equations (26) and (27) for \( h^* \) and \( c^*(t) \), respectively, into equation (37) and again differentiate with respect to \( n \). In contrast to Proposition 2, the result here holds for any \( t \geq 0 \).

An expanding population causes the economy to grow relative to the natural environment. Increases in \( n \) go hand in hand with rising output. Larger output, in turn, results in the familiar feedback mechanism. The expanding economy decreases environmental quality and induces households to increase their work input, causing steady state welfare to decline.\(^{14}\)

\(^{14}\) Population growth also increases the gap between output and welfare caused by negative effects of the economy on the environment; i.e., \( \frac{\partial^2 y^*(t)}{\partial \eta \partial n} > 0 \) and \( \frac{\partial^2 W^*}{\partial \eta \partial n} < 0 \) hold for any \( t \).
5. Conclusion

If we extend a Ramsey growth model by including environmental quality and allow for substitution of environmental services by consumption goods, we obtain a gap between output and welfare. Economic growth causes the environment to deteriorate, which in turn prompts households to replace the lost free amenities by costly consumption activity. Output increases while welfare declines. Adding economy-wide technological progress and population growth amplifies the problem. Instead, we ought to aim technological advances at decreasing the negative effect of economic output on the environment. Nevertheless, the gap between output and welfare persists. Such a gap poses a problem for policy makers, who largely use output, as measured by GDP, as a proxy for welfare and, thus, a guide for economic and development policies.

The results of this paper are largely driven by the assumptions on substitution between environmental quality and consumption. Substitutability in general plays a key role in determining the effect of the natural environment on growth. Environmentalists cite low levels of substitutability as an argument for conservation, while growth advocates tend to emphasize the importance of final goods or services provided and generally regard different inputs as interchangeable.15

Our model splits consumption into two parts. Defensive expenditures $d$ substitute for environmental services, while traditional consumption $c$ is neither a substitute nor complement. It is this high level of substitutability between environment and one form of consumption, $d$, that drives our results of leading to undesirable increases in output. Growth advocates are correct in pointing to high substitutability as an argument for growth. This time, however, it is for decidedly different reasons than the usual argument involving interchangeable production inputs. Substitution now leads to undesirable growth that increases output but decreases societal welfare. As Bartolini (2003) argues, “substitutability guarantees the sustainability of growth, but not of well-being.”

Throughout our discussion, we have thought of the public good affected by growth as environmental quality. This does not necessarily have to be the case. The argument subsumes

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15 Solow (1991) discusses the relationship between substitutability and sustainability. Daly (1996) emphasizes that natural resources and human-made capital should be thought of as complements rather than substitutes.
many sociological interpretations of public goods. Defensive consumption, for example, could be aimed at social institutions that become more scarce as the economy grows. In any case, taking this phenomenon seriously from an economist’s point of view would entail putting monetary values on such public goods. Whether we are comparing the impact of growth on welfare to that of social institutions or environmental quality, any welfare statement relies on having a common denominator as a basis of comparison. In order to validate (or invalidate) this model with something more than anecdotal evidence, we need to look to comprehensive income or green accounting as a way to obtain the data.
REFERENCES


