Innovation policies and welfare in a small, open economy: a CGE analysis of induced technological change

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Abstract:
Technological change is modelled as results of R&D activities performed by optimising agents. The output of general R&D activity is new technological solutions that are purchased by capital producers in order to supply new varieties of capital equipments. There is monopolistic competition in the capital varieties market and love of capital variety in demand, so that productivity in the final goods industries increases with R&D activity, number of patents and of real capital firms producing different capital varieties. There are external effects of R&D, as knowledge accumulates and enhances the productivity of current R&D activities. We simulate a computable general equilibrium model including these endogenous technological change mechanisms for a small, open economy, Norway. We explore how innovation incentives should be designed in order to achieve the highest welfare growth. We study policy alternatives targeted towards R&D and towards capital formation. Subsidising R&D production directly generates more welfare and economic growth than subsidising the production of capital varieties independent of whether the subsidy is directed to an existing firm or a new firm. The positive welfare effect of subsidising R&D production directly is modified by lower production of each variety compared to a production subsidy to the capital varieties industry, though. This increases the distortion generated by market power and mark-up pricing in the domestic market for capital varieties.

First draft - please do not quote

1. Introduction

The last two decades have seen the emergence of theoretical growth models in which technological change is endogenously specified. The product-variety model by Romer (1990) and the quality-ladder model by Aghion and Howitt (1992) are both well-known examples. The major policy issues were; how can an economy sustain a positive growth rate and how can innovation policy enhance additional growth? In order to analyse total welfare effects of innovation policy, combined with other policy instruments, empirical modelling of endogenous growth mechanisms has emerged. Diao et al (1999) analyses strategic trade policy and innovation policy within a computable general equilibrium (CGE) model for Japan based on the Romer (1990) model. Direct R&D subsidies produces relatively large increases in welfare, while capital subsidies produce a smaller welfare increase and trade policy has the smallest effect and can also reduce welfare. Russo (2004) is another application of the Romer (1990) and Jones (1995) product-variety model for Canada that rank innovation policies by their effects on research and development (R&D) and on welfare. Russo (2004) finds that R&D tax credits are welfare improving, while lower capital tax rate and investment tax credits for capital producers have a positive but smaller effect on welfare. In addition there is a substantial literature analysing the importance of an endogenous specification of technological change for climate-change analysis, Goulder and Schneider (1999), Goulder and Mathai (2000), Popp (2003), Otto et al (2005). Otto et al (2005) formulate a CGE model based on the Romer (1990) and Jones (1995) product-variety model.

Our model approach is inspired by the theoretical model of Romer (1990) and the empirical application by Diao et al. (1999). However, contrary to these contributions, we introduce decreasing returns to scale in the knowledge capital, based on evidence provided in Jones (1995). We model technological change as results of R&D activities performed by optimising agents. The output of general R&D activity is new technological solutions that are purchased by capital producers in order to supply new varieties of capital equipments. There is monopolistic competition in the capital markets and love of capital variety in demand, so that productivity in the final goods industries increases with R&D activity, number of patents and of real capital firms producing different capital varieties. Besides increasing productivity, R&D increases the capacity of domestic productions to absorb knowledge spillovers from international trade. There are external effects of R&D, as knowledge accumulates and enhances the productivity of current R&D activities.

Contrary to Diao et al (1999) and Russo (2004) we model a small open economy where export prices are given at the world market. Both the existence of internal spillovers from investments in R&D and imperfect competition in the market for capital are arguments for subsidising innovations. However, for small, open economies these arguments may empirically turn out to be of less importance. In small open economies technological progress are inevitably dominated by international processes. Hence, cogent arguments for R&D support in small and open economies could rather be absorptive capacity expansion or possible comparative advantages or interests on specific future R&D fields.

We explore how innovation incentives should be designed in order to achieve the highest welfare growth. We simulate two distinct policy alternatives; alternative 1 is a general subsidy towards production of ideas/patents (the R&D industry) and alternative 2 is a general subsidy towards all production of capital varieties. Both policy alternatives are financed by higher lump sum taxes such that public revenues are constant between the reference path and the policy alternatives. In Norway firms performing R&D can withdraw a limited amount of expenses each year. Alternative 1 is an approximation for this kind of R&D support. We simulate a computable general equilibrium model for the small open economy Norway, which includes endogenous technological change mechanisms. We also take into account that several tax wedges riddle the Norwegian economy, allowing for tax interaction effects when introducing new policies.

Our analyses show that subsidising R&D production directly generates more welfare and economic growth than subsidising the production of capital varieties independent of whether the subsidy is directed to an existing firm or a new firm. The positive welfare effect of subsidising R&D production directly is modified by lower production of each variety compared to a production subsidy to the capital varieties industry, though. This increases the distortion generated by market power and mark-up pricing in the domestic market for capital varieties. The share of R&D production in total gross production increases with both policy alternatives, but the effect is strongest with a direct production subsidy towards the R&D industry. But the policy means are not strong enough for the economy to reach the political goal of 3 per cent share of R&D in total production. The positive effect on the competitiveness for the capital varieties industry on the export market is approximately equal in both policy alternatives.

The paper is organized as follows. Section 2 describes the CGE model in more details. The policy alternatives are discussed and numerical simulations and welfare effects are presented in section 3, while section 4 concludes.

2. The model

The model is a numerical Computable General Equilibrium model (CGE-model) with R&D (Research and Development) driven technological change. Our CGE-model is calibrated to the Norwegian economy that is a small, open petroleum producing economy. The R&D driven technological change is based on the Romer (1990) approach that R&D creates new ideas and the ideas is used as input in the production of new capital varieties. Each new idea produced in the R&D industry creates only one new capital variety. Both new capital varieties and the increased stock of ideas/patents contribute to economic growth. Diao (1999) uses the Romer approach in a CGE model for Japan to analyse the effects of trade policy. Following Diao (1999) the model is a direct application of the R&D driven endogenous growth theory. We present the model structure for the private industries, the representative consumer and at the end the equilibrium conditions.

2.1 Industries

There are 10 private industries and 1 governmental industry in the model, see Appendix A for the list of industries. Following Romer (1990) the private goods industries can be separated into final goods industries, intermediate goods industries and R&D industries. The model incorporates both the small open economy assumption of given world market prices, and avoids complete specialization through decreasing returns to scale.

We use the dual approach to the firm's maximization problem. We assume that all firms within the final goods industry *j* are identical and take the prices as given in the input factor markets and in the final goods markets, both in the home market and at the world market. The model captures the fact that especially the home market and the export market may be segmented from each other due to transport costs, market access etc. and we assume separability between deliveries to the domestic and the export market. Each firm has perfect foresight and maximizes the firm value that is equal to the present value of the after tax cash flow. For simplicity we disregard the tax system and the input of other intermediaries, only labour L (measured in efficient man hours) and capital K are considered as inputs. There are no fixed costs in production.[1] The description of the main structure of the behaviour of industries is partly based on the description in Heide et al (2004).

2.1.1 Final goods industries

The value of the representative firm in final goods industry *j* (*j* is suppressed here) in period 0 is given by

$$(1)\ V_0 = \int_0^\infty e^{-rt}\left(\pi_t - P_{Jt} J_t\right)dt$$

$J = \dot{K} + \delta K$ is gross investment and P_J is the price of the investment good. δ is the depreciation rate. Operating profit is defined as

$$(2)\ \pi = P^H X^H + P^W X^W - wL .$$

[1] Except for the intermediate goods industry producing capital varieties, see section 2.3.

X^H is output delivered to the domestic market, X^W is output delivered to the export market, P^H is the domestic market price, P^W is the exogenous world market price and w is the wage rate.

The transformation function between input (the technology of production) and output has the separable structure

$$(3)\quad \left[(X^H)^\rho + (X^W)^\rho\right]^{1/\rho} = \left[f\left(\frac{L}{\tau_L}, \frac{K}{\tau_K}\right)\right]^s$$

s is the scale elasticity, $0 < s \leq 1$ and ρ is the substitution parameter between deliveries to the domestic and the foreign market. τ_i is a factor specific exogenous productivity parameter, i=L,K. In the long run we assume balanced Hicks-neutral technological change. The production technology is given by a nested input factor tree of CES aggregates, see figure 1.

For tractability purpose we assume that $\frac{1}{\rho} = s$, see Holmøy and Hægeland (1997). The variable cost function of the representative firm then takes the form

$$(4)\quad C = c\left[(X^W)^{1/s} + (X^H)^{1/s}\right].$$

c is the dual price index (unit cost function) of the CES-composite of labour and capital input given by

$$(5)\quad c = \left[\delta_L\left(\frac{w}{\tau_L}\right)^{(1-\sigma)} + (1-\delta_L)\left(\frac{P^K}{\tau_K}\right)^{(1-\sigma)}\right]^{\frac{1}{1-\sigma}}.$$

σ is the elasticity of substitution between labour and capital and δ_L is the cost share of labour. P^K is the unit price of the machinery capital composite, consisting of the composite of machinery capital varieties K_V and other machinery K_O. Following Heide et al (2004) after integrating (by parts) equation (1), the value of the firm can be written as

$$(6)\quad V_0 = \int_0^\infty e^{-rt}\left(\pi_t - P_t^K K_t\right)dt + K(0)P(0).$$

Equation (6) implies that maximising total discounted value of the firm is equivalent with maximising $\pi_t - P_t^K K_t$ in each period. By using appropriate substitutions the dynamic maximization problem of the firm can be transformed to a sequence of static problems where the firm maximizes

$$(7)\quad \pi' = P^H X^H - c\left(X^H\right)^{1/s} + P^W X^W - c\left(X^W\right)^{1/s}$$

w.r.t. X^H and X^W. From the first order conditions of the firm's profit maximization we have the following marginal conditions

$$(8)\quad P^H = \frac{c}{s}\left(X^H\right)^{\frac{1-s}{s}}$$

$$(9)\quad P^W = \frac{c}{s}\left(X^W\right)^{\frac{1-s}{s}}$$

Equations (7) and (8) state that price must equal marginal costs in both the domestic and the export market. The price on the world market, P^W, is exogenous, while the price in the domestic market, P^H, is determined by equilibrium in the domestic market, given the cost structure.

The production technology is given by a nested CES structure, see figure 1. In the CGE model the production technology is represented by the dual cost functions for the different CES aggregates. This will determine the different factor shares, which together with total production, determines the use of each input factor. The final goods producers' demand for the differentiated capital varieties is determined by this nested CES structure where total machinery capital (K) is a composite of machinery varieties (K_V) and other machinery (K_O).

$$(10)\quad K = \left[\delta_{ko}\left(\frac{K_O}{\tau_{ko}\delta_{ko}}\right)^{\left((\sigma_k-1)/\sigma_k\right)} + (1-\delta_{ko})\left(\frac{K_V}{\tau_{kv}(1-\delta_{ko})}\right)^{\left((\sigma_k-1)/\sigma_k\right)}\right]^{\left(1/(\sigma_k-1)\right)}$$

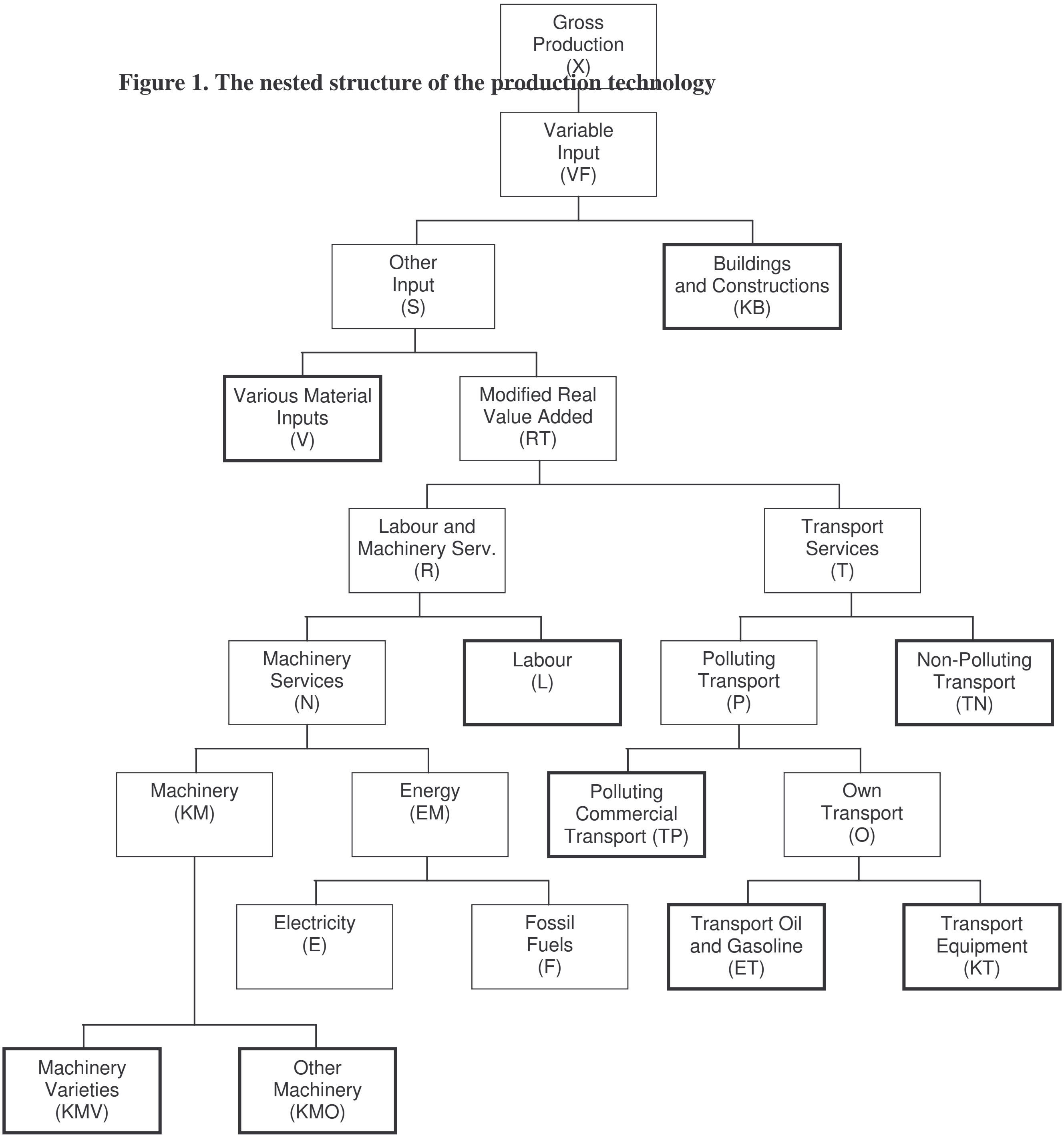
σ_k is the elasticity of substitution between machinery varieties and other machinery. The composite of machinery varieties is only used as capital input in the final goods industries, not in the R&D industries or the intermediate goods industries (industries producing the different capital varieties). Depreciation of capital varieties implies replacement investments of the existing varieties.

We assume so-called Spence-Dixit-Stiglitz preferences for the composite of machinery capital varieties.

$$(11)\quad K_V = \left[\int_0^R \left(K_{V\,i}\right)^{\theta_k} di \right]^{1/\theta_k}$$

$\theta_k = \dfrac{\sigma_{kv} - 1}{\sigma_{kv}}$ and σ_{kv} is the elasticity of substitution between the different capital varieties.

Figure 1. The nested structure of the production technology



2.1.2 Production of R&D services/patents

We assume that the R&D industry only deliver the production of new ideas/patents to the domestic market. The production of new ideas/patents in one time period is given by X_R^H. The transformation function between input (the technology of production) and output has the same structure as for the final goods industries given in equation (3), but since there are only deliveries to the domestic market, the production structure is modified as follows

$$(3')\quad X_R^H = \left[A^I R\right]^{s_1}\left[f\left(\frac{L}{\tau_L},\frac{K}{\tau_K}\right)\right]^s$$

In addition there are *spillovers from abroad* represented by the term A^I that is an exogenous productivity parameter that captures absorption of new technologies from abroad. We will come back to the modelling of such an absorption process. The factor specific exogenous productivity parameter τ_i can capture imported exogenous technological change as well. There is free entry to this sector that implies no rents. The accumulated stock of knowledge R, $R = R_{-1} + X_R^H$, generate *internal spillovers* to the R&D industry that is assumed to be free for the participants in the R&D industry. s_1 denotes the elasticity of spillover, independent of whether it is internal or external.

The production technology is given by a nested CES structure, see figure 1. We assume that the R&D service industries only use other machinery K_O as machinery input in the production process, not the differentiated capital variety aggregate (K_V).

Equation (3') can be rewritten as

$$(3''')\quad \left(X_R^H\right)^{1/s} = (A^I R)^{s_1/s} f\left(\frac{L}{\tau_L},\frac{K}{\tau_K}\right)$$

The variable cost function (when f(.) is a CES-function) is given by

$$C = cf\left(\frac{L}{\tau_L},\frac{K}{\tau_K}\right).$$

c is the unit cost function. Combining the expression for the variable cost function with equation (3'''), the variable cost function for the representative firm takes the form

(4') $$C = \frac{c}{(A^I R)^{s_1/s}} \left[X_R^H \right]^{1/s}.$$

Spillovers from abroad, A^I, and domestic spillovers, R, will both contribute to reduce the unit cost of production. c is the dual price index (unit cost function) of the CES-composite of labour and capital input given by

(5') $$c = \left[\delta_L \left(\frac{w}{\tau_L} \right)^{(1-\sigma)} + (1-\delta_L) \left(\frac{P^K}{\tau_K} \right)^{(1-\sigma)} \right]^{\frac{1}{1-\sigma}}.$$

As for the final goods industries the dynamic maximization problem of the firm can be transformed to a sequence of static problems where the firm maximizes

(7') $$\pi' = P_R^H X_R^H - \frac{c}{(A^I R)^{s_1/s}} \left(X_R^H \right)^{1/s}$$

w.r.t. X_R^H. From the first order conditions of the firm's profit maximization we get the following marginal condition

(8') $$P_R^H = \frac{c}{s(A^I R)^{s_1/s}} \left(X_R^H \right)^{\frac{1-s}{s}}$$

The domestic price P_R^H must equal marginal costs and is determined by equilibrium in the domestic market, given the cost structure.

2.1.3 Production of capital varieties

The firms that produce machinery capital varieties (the so-called intermediate goods (IG) industry following the Romer/Diao terminology) buy patents from the R&D services industry. We assume that the firms in this intermediate goods industry sell their products on both the domestic and the export market with the same assumptions about separability in the cost structure between deliveries to the domestic and the export market as for the final goods industry. The firms have market power in the domestic market, but exhibit no market power in the export market. This implies that, as for the final goods industries, the export prices are given in the world market.

We assume that the cost structure is identical for all the firms/producers within the capital variety industry. Each firm buys only one idea/patent from the R&D industry and produces only one capital variety.

The production technology is given by the nested CES structure, see figure 1. We assume that the firms within the capital varieties industry only use other machinery K_O as machinery input in the production process.

The value of firm *i* in period 0 can be written as

$$(1'')\quad V_{i0} = \int_0^{\infty} e^{-rt}\left(\pi_{it} - P_t^K K_{it}\right)dt - P_0^R + P_0^K K_{i0}$$

P_0^R is the price of buying one patent from the R&D industry in period 0. The cost of buying a patent is a fixed cost for the firm. Operating profits are defined as

$$(2'')\quad \pi_i = P_{ki}^H X_{ki}^H + P_k^W X_{ki}^W - wL_i$$

X_{ki}^H is the production of capital variety *i* delivered to the domestic market, X_{ki}^W is the production of capital variety *i* delivered to the export market, P_{ki}^H is the domestic market price and P_k^W is the exogenous world market price.

As for the final goods industries the transformation function between input (the technology of production) and output has the separable structure

$$(3'')\quad \left[\left(X_{ki}^H\right)^{\rho} + \left(X_{ki}^W\right)^{\rho}\right]^{1/\rho} = \left[f\left(\frac{L_i}{\tau_L}, \frac{K_i}{\tau_K}\right)\right]^s$$

We assume that $\frac{1}{\rho} = s$ and the variable cost function of the representative firm takes the form

$$(4'')\quad C_i = c\left[\left(X_{ki}^W\right)^{1/s} + \left(X_{ki}^H\right)^{1/s}\right]$$

c is the dual price index (unit cost function) of the CES-composite of labour and capital input given by

$$(5'')\quad c=\left[\delta_L\left(\frac{w}{\tau_L}\right)^{(1-\sigma)}+(1-\delta_L)(\frac{P_{ko}}{\tau_{ko}})^{(1-\sigma)}\right]^{\frac{1}{1-\sigma}}$$

P_{ko} is the user cost of other machinery capital. Remind that the machinery capital varieties are not used as input in the production of machinery capital varieties.

From the firm's value function given in equation (1'') it is easily seen that the dynamic maximization problem of the firm, given that the firm has entered the industry by buying a patent, can be transformed to a sequence of static problems where the firm maximizes

$$(7'')\quad \pi_i' = P_{ki}^H\left(X_{ki}^H\right)X_{ki}^H - c\left(X_{ki}^H\right)^{1/s} + P_k^W X_{ki}^W - c\left(X_{ki}^W\right)^{1/s}$$

w.r.t. X_{ki}^H, given the demand function $P_{ki}^H\left(X_{ki}^H\right)$ and X_k^W. This gives the following first order conditions

$$(8'')\quad \frac{\partial \pi_i'}{\partial X_{ki}^H} = P_{ki}^{H\,\prime}\left(X_{ki}^H\right)X_{ki}^H + P_{ki}^H\left(X_{ki}^H\right) - \frac{c}{s}\left(X_{ki}^H\right)^{\frac{1-s}{s}} = 0$$

$$(9'')\quad \frac{\partial \pi_i'}{\partial X_{ki}^W} = P_k^W - \frac{c}{s}\left(X_{ki}^W\right)^{\frac{1-s}{s}} = 0$$

The domestic demand elasticity for capital variety X_{ki}^H is defined as $\varepsilon_{ki} = -\dfrac{\partial X_{ki}^H}{\partial P_{ki}^H}\dfrac{P_{ki}^H}{X_{ki}^H}$. Inserting this expression into equation (8'') and reorganising gives the following monopoly pricing rule for the domestic price of machinery capital variety *i*.

$$(10)\quad P_{ki}^H = m_{ki}\frac{c}{s}\left(X_{ki}^H\right)^{\frac{1-s}{s}}$$

The mark-up factor is $m_{ki} = \dfrac{\varepsilon_{ki}}{\varepsilon_{ki}-1}$. For deliveries to the export market the world market price equals marginal costs.

$$(11)\quad P_k^W = \frac{c}{s}\left(X_{ki}^W\right)^{\frac{1-s}{s}}$$

The composite of machinery varieties is used as input in the final goods industries. We assume a CES nested production technology, see figure 1. It can be shown that the demand elasticity is equal to the elasticity of substitution between the different varieties in the machinery varieties aggregate σ_{kv}, i.e. $\varepsilon_{ki} = \sigma_{kv}$, independent of *i*, see Appendix B for more details. The mark-up factor in the monopoly pricing rule in equation (10) can then be written as

$$(12)\ \ m_k = \frac{\varepsilon_k}{\varepsilon_k - 1} = \frac{\sigma_{kv}}{\sigma_{kv} - 1}\ ,\ \sigma_{kv} > 1$$

The mark up factor is independent of *i*. Together with the assumption of equal production and cost's structure in each firm, the monopoly pricing rule implies that the price in the domestic market is equal for all the capital varieties. Hence, each variety is produced in equal quantity and the user cost of capital for a machinery capital variety is equal for all the varieties and given by

$$(13)\ \ \overline{P}_{kv} = (r + \delta) P_k^H - \dot{P}_k^H$$

We assume that the composite of machinery capital varieties is described by so-called Spence-Dixit-Stiglitz preferences, see equation (11). The dual unit cost function defining the aggregate price of the machinery capital variety composite is given by

$$(14)\ \ P_{kv} = \left[\int_0^R (P_{kvi})^{(\frac{\theta_k}{\theta_k - 1})} di \right]^{\theta_k - 1 / \theta_k}$$

$\theta_k = \dfrac{\sigma_{kv} - 1}{\sigma_{kv}}$ and σ_{kv} is the elasticity of substitution between the different capital varieties. When the user cost of capital is equal for each machinery capital variety as stated in equation (13), the unit cost price of the machinery capital variety composite P_{kv} can be written as

$$(15)\ \ P_{kv} = R^{\left(1 / (1 - \sigma_{kv}) \right)} \cdot \overline{P}_{kv}$$

By combining equations (1’’) and (7’’) the entry/exit condition for each firm is given by

$$(16)\ \ P_0^R = \int_0^\infty e^{-rt} (\pi'_{it}) dt$$

Firms are entering the machinery capital varieties industry until the representative firm's total discounted net profit is equal to the entry costs, i.e. the costs of buying one new idea/patent. In each period new patents are produced and new firms will enter the machinery capital industry. Together with the first order condition in equation (10), the entry/exit condition determines the number of new varities and then the production in the R&D industry. The implementation of the entry/exit condition in the CGE model is described in more details in Appendix D.

2.3 Consumer behaviour

We assume an infinitely lived representative consumer that maximizes the distribution of consumption and savings over the time horizon. Labour supply is exogenous. Consumption per capita *c* is measured in efficiency units.

$$(17)\quad c_t = \frac{C_t}{\prod_{s=0}^{t}(1+n_s)}$$

C is total material consumption in the economy and n_s is total population growth. Total material consumption is a CES aggregate of 10 different goods and services, see figure 2. We assume a CRRA utility function for the representative consumer.

$$(18)\quad U_0 = \sum_{t=0}^{\infty}\left[\left(\frac{1}{\prod_{s=0}^{t}(1+\rho_s)}\right)\left(\frac{\sigma_c}{\sigma_c - 1}\right)\left(\frac{C_t}{\prod_{s=0}^{t}(1+n_s)}\right)^{\left(\frac{\sigma_c - 1}{\sigma_c}\right)}\right]$$

σ_c is the intertemporal elasticity of substitution and ρ is the consumer's rate of time preferences. The representative consumer maximizes the utility function (2) given the budget constraint

$$(19)\quad W_0 = \sum_{t=0}^{\infty}\frac{PC_t c_t}{\prod_{s=0}^{t}(1+r_s)}$$

PC is the price index for the material consumption aggregate and r is the nominal interest rate, exogenous given from the world market. Insert c_t from equation (17), summarize over the number of individuals, and derive the corresponding Hamiltonian function for this aggregate maximization problem. The consumption function given in equation (17) is derived from the first order conditions for the maximisation of this Hamiltonian function.

(20) $$C_t = \left[\lambda \cdot PC_t \frac{\prod_{s=0}^{t}(1+\rho_s)}{\prod_{s=0}^{t}(1+r_s)} \right]^{-\sigma_c} \prod_{s=0}^{t}(1+n_s)$$

λ is the marginal utility of wealth. This is independent of t, but endogenous in the determination of the total dynamic model, see Bye and Holmøy (1992,1997) for further details.

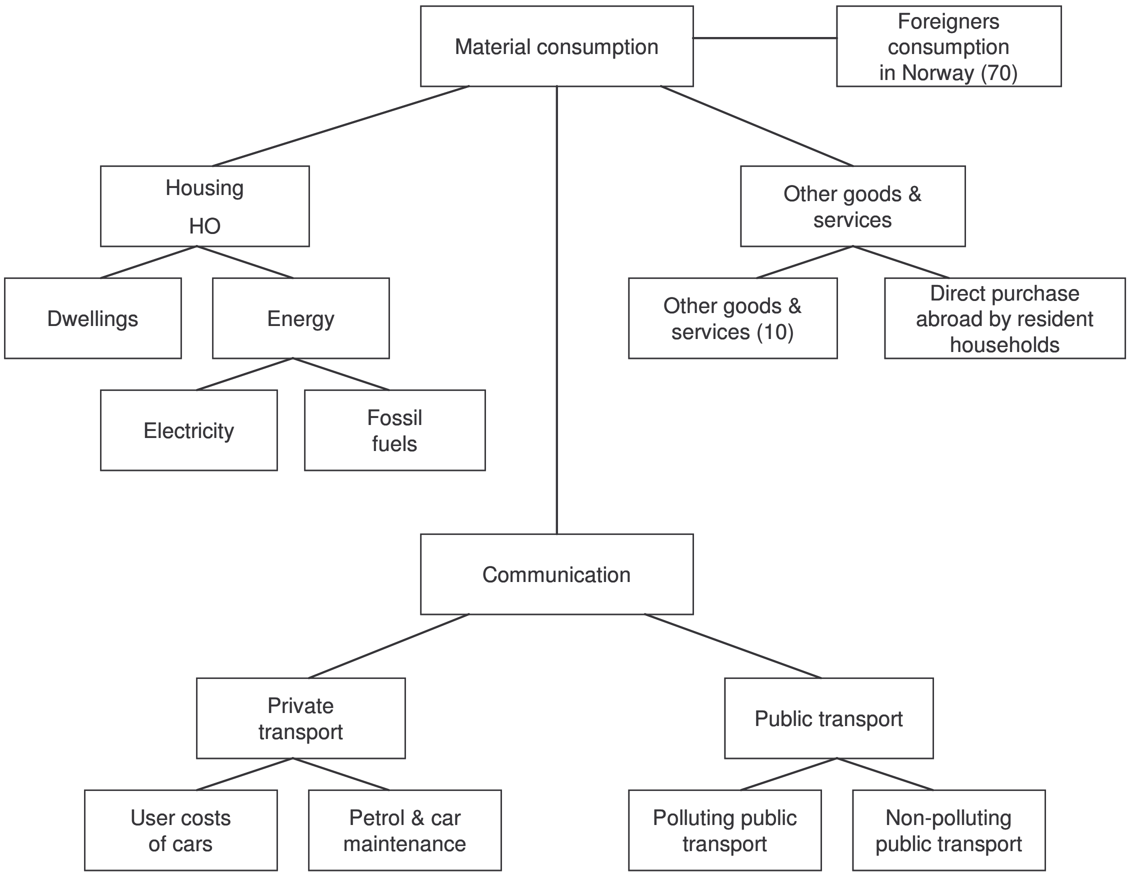
(21) $$\dot{b} = P_t^C C_t - \pi_t - w_t L_t + rb_t + \Omega_t \,.$$

b is net foreign debt, π_t is net profit by the firms and Ω_t is net taxes paid by the consumer. We have the following long run transversality condition

(22) $$\lim_{t\to\infty} be^{-rt} dt = 0$$

This is the so-called Non-Ponzi game condition.

Figure 2. Material consumption



2.4 Equilibrium conditions

2.4.1 Product equilibrium

(23) $$I_j + X_j = \sum_{i=O,T} V_{ij} + \sum_{i=listC} C_{ij} + \sum_{i=listJ} J_{ij} + \sum_{i=listA} A_{ij}$$

j = list of industries

A_{ij} = export of activity i from industry j

I_j = import of product j

Imports are imperfect substitutes in consumption for domestically produced goods, modelled as Armington aggregates.

2.4.2 Governmental budget

The government collects taxes, distributes transfers, and purchases goods and services from the industries and abroad. The model incorporates a detailed account of the government's revenues and expenditures. Direct and indirect taxes, subsidies and transfers follow from the NA.

2.4.3 Balanced long run growth

To ensure a long run balanced growth path, the following conditions must be fulfilled; 1) the rate of technological change for each input factor in each industry must converge to the same, modified exogenous Hicks-neutral rate. Each industry must grow at the same rate. 2) The growth in per capita material consumption equals the Hicks-neutral exogenous growth rate. 3) The population growth rate is constant.

We can derive the conditions for balanced growth in material consumption from the following equation

(24) $$\frac{C_{t+1}}{C_t} = \left[\frac{PC_{t+1}}{PC_t} \frac{(1+\rho)}{(1+r)} \right]^{-\sigma_c} (1+n)(1+g^{ss})$$

To reach a long run balanced growth path the rate of inflation must also be constant

$$1+\pi = \frac{PC_{t+1}}{PC_t}$$

A balanced growth path requires that

(25) $$\frac{C_{t+1}}{C_t} = (1+n)(1+g^{ss})$$

The condition for reaching a balanced growth path is then given by

(26) $$\left[(1+\pi)\frac{(1+\rho)}{(1+r)}\right]^{-\sigma_c} = 1$$

This implies that

(27) $$\rho = \frac{(1+r)}{(1+\pi)} - 1$$

The intertemporal rate of time preferences must equal the real rate of return defined in equation (27), and both r and ρ are constant in the balanced growth path. If there is no inflation the rate of time preferences ρ equals the nominal interest rate r. The nominal interest rate is assumed to be exogenous given from the world market. Dependent on the rate of inflation, the rate of time preferences may fluctuate along the path towards the long run balanced growth path where the rate is constant, following equation (27). Alternatively, the rate of time preferences may be constant along both the transitional and the long run balanced growth path, equal to the exogenous given real interest rate that is determined by the long run equilibrium condition given in equation (27).

The intertemporal budget constraint for households equals the present value of consumption expenditure in the current and all future periods to total wealth (current non-human wealth plus the present value of labour income and net transfers). The value of the marginal utility of wealth that satisfies the intertemporal budget constraint (that follows from the Non-Ponzi game condition) is solved by using the same solution procedure as described in Bye and Holmøy (1997).

In order to obtain a balanced growth path, equation (27) must be satisfied. In addition both the population growth rate n and the rate of technological change must be constant. The rate of technological change is an endogenous variable in these kinds of ITC models. Along the transitional path g may vary, but in the long run balanced growth path case, g must be constant equal to a modified Hicks-neutral exogenous growth rate.

2.5 Data and parameters

The model is calibrated to the 2002 National Accounts. For the production functions the elasticities of substitution between machinery capital varieties and other machinery capital is 1.5, while the elasticity of substitution between the different machinery capital varieties is 5, giving a mark up factor of 1.25 for the price of machinery capital varieties. The other elasticities of substitution in the production technology range from 0.15 at the upper part of the nested tree to 0.5 further down in the nested tree structure, see figure 1.

In the model of producer behavior the elasticities of transformation between deliveries to the domestic and foreign market are set equal to 4. The elasticities of scale in different industries are then calibrated to 0.83, given the elasticities of transformation. The elasticities of substitution between domestic products and imported goods are also equal to 4.

In the consumer model the intertemporal elasticity of substitution, σ_F, equals 0.3, Steigum (1993). Econometric estimates of σ_F vary considerably between different sources, and 0.3 is in the lower end of the range of the estimated parameters. The elasticities of substitution between the different commodities in the demand system for material consumption are all equal to 0.5.

3. Policy analyses and numerical results

3.1 Reference path

The effects of the different policy alternatives are measured as deviations from the reference path (baseline scenario). The model is calibrated to the benchmark year 2002. The steady state solution of the model is path dependent. In the policy simulations both the path and the long run stationary solution differ from the baseline scenario. The tax reforms are implemented immediately, minimizing announcement effects.

3.2 Policy alternatives

We explore how innovation incentives should be designed in order to achieve the highest welfare and growth. We simulate four distinct policy alternatives. To make the four distinct policy alternatives comparable, we assume that the public revenue effect is equal for all the policy alternatives. The public revenue effect is measured as the annuity of the revenue loss generated by the innovation policy measure. The loss in the public revenue, measured as a constant annuity, is approximately 1.3 billion NOK, which corresponds to the total value of the Norwegian tax credit system for R&D support that amount to 1.3 billion NOK in 2004. In each policy alternative we calculate the subsidy rate that is consistent with this revenue effect. All the policy alternatives are implemented in the first year of simulation and kept constant during the rest of the simulation period. We assume that the overall public revenue is unchanged due to implementation of necessary lump sum taxes.

Alternative 1 is a general subsidy α_s towards production of new ideas/patents in the R&D industry. The representative firm's profit function is then given by

(1) $$\pi' = P_R^H (1+\alpha_s) X_R^H - \frac{c}{(A^I R)^{s_1/s}} \left(X_R^H\right)^{1/s}.$$

From the first order conditions of the firm's profit maximization we get the following marginal condition

(2) $$P_R^H = \frac{c}{(1+\alpha_s) s (A^I R)^{s_1/s}} \left(X_R^H\right)^{\frac{1-s}{s}}.$$

The subsidy rate to the R&D industry is 0.052. The numerical results are given in table 1.

Table 1. Alternative 1. Long run effects. Percentage deviation from the reference path.

Model alternatives	Model 1	Model 2
Price of patents (BHA38)	-7.91	-7.59
Production of patents (XH38)	25.23	45.40
Number of capital varieties (GAMMAR)	15.15	21.69
Firm profit, capital varieties (PROFF46)	-6.97	-6.49
Price of capital varieties (BHA46)	-0.82	-0.93
Production of capital varieties, total (X46)	8.95	16.27
Domestic deliveries (XH46)	8.95	12.68
Export deliveries (XA46)	8.95	17.59
Production in each firm (XHF46)	-5.38	-7.41
Price of investment good capital varieties (PJB52)	-0.42	-0.36
Gross product (QREV)	0.85	1.52
Change in gross product (QREV) growth rate*	0.02	0.044
Consumption (CC)	0.09	0.30
Composite price of capital varieties (PKMVC20)	-7.16	-9.61
Domestic price of other machinery (BHA47)	0.39	0.81
Production of other machinery (X47)	0.12	0.32
Domestic deliveries (XH47)	0.26	0.60
Export deliveries (XA47)	-1.51	-3.03
Composite of capital varieties (KMV47)	9.45	13.43
User costs of one unit capital variety (PKMV47)	-0.37	-0.28
Wage rate (PLJUST)	0.62	1.17
Trade surplus (VAVI)	1.24	2.50
Total export (A)	1.45	2.73
Total import (I)	1.13	2.30
Revenue loss R&D subsidy (YTSU38)**	-4852	-5653
Revenue loss R&D subsidy (YTSU38)***	-1174	-1230
Total revenue loss, subsidies (SUMYTSU)**	-5031	-6005

* Absolute change in growth rate

** Absolute change measured in million NOK

*** Measured as total discounted constant annuity, million NOK

We simulate the policy alternatives with two different models. In Model 1 we assume that the export price developments for the capital varieties are equal to the price for domestic deliveries. This implies that the producers execute the same kind of market power both in the domestic and in the export markets. In Model 2 the domestic producers have no market power in the export market. The export price on capital varieties is given from the reference path, while there is mark-up pricing in the domestic market. We concentrate on long run growth effects and disregard short run dynamics.

Alternative 1, Model 2:

The subsidy rate 0.052 is implemented in the first simulation period (2002) and kept constant at this level for the whole simulation period. Subsidising the production of ideas/patents gives a direct reduction in the marginal production costs, and the supply of patents increases immediately, given the patent price. For the capital variety industry to be able to absorb more patents, the production within each firm must decrease, reducing the firms' profit. Total discounted future profit for the representative firm is reduced. From the entry-exit condition that equalizes total discounted future profit with the price of a new patent the patent price is now reduced. New firms are entering the capital varieties industry until the discounted profit for a new firm equals the price of a new patent. In long run equilibrium the production of new patents and the number of capital varieties is higher, but the production and profit within each firm producing a capital variety is lower following the introduction of an R&D production subsidy. For a given marginal cost function, lower production within each firm implies lower domestic price of the capital variety following the mark-up pricing rule. Both the price of the investment good capital varieties and the user costs of a capital variety, fall. Lower price of capital varieties, combined with more capital varieties, stimulates domestic demand for capital varieties, especially through the love-of-variety effect. The overall fall in the composite price of the user costs of capital varieties is 9.61 per cent.

The domestic producers are all price takers in the world market. The considerable increase in the number of firms that reduces the scale of production within each firm implies that more capital varieties can be supplied to the foreign market at a lower marginal cost. This gives a positive effect on competitiveness for the capital variety industry since the export price is unchanged. The total effect on export deliveries is positive and the export deliveries of capital varieties increase considerably more than the domestic deliveries due to given world market price.

Stimulating the production of R&D through an R&D subsidy increases the demand for other inputs as labour and other capital, including other machinery, in both the R&D industry and the capital varieties industry. Since total labour supply is exogenous the wage rate increases, giving higher production costs for all industries. The production costs increases for all industries, even though the price of capital varieties is considerably reduced. Export deliveries for all other products than capital varieties are reduced. There is a small increase in domestic deliveries from the production of other machinery due to higher demand for other machinery in the R&D related industries. In all other industries other machinery is substituted by the composite of capital varieties. The wage rate increase combined with the positive demand effect for more input factors in the R&D related industries and higher consumer demand for goods and services, implies that the domestic prices of other capital equipment and other intermediates increase, contributing to an even higher unit cost of production.

Total gross product increases by 1.52 per cent and the growth rate is 0.04 percent point higher.[2] Total welfare measured as the change in long run material consumption is 0.3 per cent higher. The positive welfare effect is generated by many different sources in this kind of CGE model. Increased R&D production has welfare effects through the following channels; 1) the positive spillover effect in production of patents, 2) the positive love-of-variety effect in the demand for capital varieties and 3) the negative welfare effect of lower production within each monopoly firm producing capital varieties.

The presence of other distortions that are not especially connected to the R&D related industries must also be taken into account when analysing the total welfare effects of a policy alternative. The capital income taxation drives a wedge between private and social return to capital and savings are to low from an efficiency point of view. Intertemporal efficiency is improved since total savings in real and financial capital increase. Housing is lenient taxed compared to other capital, and reallocations of consumption from housing capital towards other goods and services has positive welfare effects (her er det forskjell før/etter 2090). The existence of monopoly power in the domestic market for the producers of capital varieties implies that reallocations of deliveries from foreign to the domestic market have a positive welfare effect. Even though domestic deliveries of capital varieties increases, there is a reallocation of deliveries from the domestic to the foreign market due to the positive effect on the competitiveness for this industry. Reallocations of consumption towards goods and services that have high indirect taxes are also welfare improving.

Alternative 1, Model 1:

Many of the same effects as with Model 2 are present when using Model 1. The most striking difference is the negative terms of trade effect for the capital variety industry since the export price in this alternative is determined by the mark-up pricing rule in the domestic market. The price for domestic deliveries is reduced as with Model 2, and the export price is now also reduced. The negative

[2] Gross product is measured exclusive of the petroleum production that is exogenous in the model.

terms of trade effect has a negative effect on export deliveries, and total growth and welfare in the economy is lower, compared to the alternative Model 2 where the domestic producers of capital varieties are price takers in the world market.

Alternative 3 is a direct subsidy towards the production of new capital varieties in the capital variety industry. The representative firm's profit function is then given by

$$(4)\ \ \pi_i' = P_{ki}^H\left(X_{ki}^H\right)(1+\alpha_s)X_{ki}^H - c\left(X_{ki}^H\right)^{1/s} + P_k^W(1+\alpha_s)X_{ki}^W - c\left(X_{ki}^W\right)^{1/s}$$

The first order condition for deliveries to the domestic market is given by

$$(5)\ \ P_{ki}^H = m_{ki}\frac{c}{(1+\alpha_s)s}\left(X_{ki}^H\right)^{\frac{1-s}{s}}$$

The mark-up factor is $m_{ki} = \frac{\varepsilon_{ki}}{\varepsilon_{ki}-1}$. For deliveries to the export market the world market price equals marginal costs.

$$(6)\ \ P_k^W = \frac{c}{(1+\alpha_s)s}\left(X_{ki}^W\right)^{\frac{1-s}{s}}$$

The entry/exit condition is given by

$$(7)\ \ P_0^R = \int_0^\infty e^{-rt}\left(\pi'_{it}\right)dt$$

Table 2. Alternative 3. Long run effects. Percentage deviation from the reference path.

Model alternatives	Model 1	Model 2
Price of patents (BHA38)	-0.21	-0.57
Production of patents (XH38)	2.09	16.83
Number of capital varieties (GAMMAR)	1.19	7.57
Firm profit, capital varieties (PROFF46)	-0.06	-0.06
Price of capital varieties (BHA46)	-0.71	-0.91
Production of capital varieties, total (X46)	1.22	8.27
Domestic deliveries (XH46)	1.22	4.99
Export deliveries (XA46)	1.22	9.48
Production in each firm (XHF46)	0.03	-2.40
Price of investment good capital varieties (PJB52)	-0.48	-0.49
Gross product (QREV)	0.85	0.70
Change in gross product (QREV) growth rate*	0.0017	0.019
Consumption (CC)	0.017	0.24
Composite price of capital varieties (PKMVC20)	-1.06	-4.02
Domestic price of other machinery (BHA47)	0.05	0.44
Production of other machinery (X47)	0.07	0.27
Domestic deliveries (XH47)	0.09	0.42
Export deliveries (XA47)	-0.16	-1.59
Composite of capital varieties (KMV47)	1.36	5.51
User costs of one unit capital variety (PKMV47)	-1.06	-0.28
Wage rate (PLJUST)	0.07	0.58
Trade surplus (VAVI)	0.06	1.24
Total export (A)	0.22	1.46
Total import (I)	0.14	1.27
Revenue loss R&D subsidy (YTSU46)**	-4311	-4728
Revenue loss R&D subsidy (YTSU46)***	-998	-1047
Total revenue loss, subsidies (SUMYTSU)**	-4302	-4862

* Absolute change in growth rate

** Absolute change measured in million NOK

***Measured as total discounted constant annuity, million NOK

Alternative 3, Model 2:
The subsidy rate 0.01 is implemented in the first simulation period (2002) and kept constant at this level for the whole simulation period. Subsidising the production of machinery capital varieties shifts the cost curve downwards and the production within each firm increases. This creates an excess supply in the domestic market and the price must fall to establish new market equilibrium. The price of capital varieties falls. The subsidy is offered to all the producers of capital varieties independent of whether the firms already exist or are newly entered firms. The subsidy reduces the production costs for both existing producers and possible new entrants. The demand for new patents increases and new firms are entering the market, reducing production and profits for the existing firms in the capital variety industry. The production of patents increases. The spillover effect in the production of patents dominates and the production of patents increases even though the equilibrium price of patents is lowered. There is entry of new firms in the capital variety industry until the expected discounted profit for a new firm equals the entry costs equal to the price of a patent. Since production and profit within each firm producing a capital variety is reduced, the expected discounted profit falls, and in equilibrium the price of a new patent is lower.

In long run equilibrium the production of new patents and the number of capital varieties is higher, but the production and profit within each firm producing a capital variety is lower when the production of capital varieties is subsidized. Combined with the negative shift in the marginal cost function, lower production within each firm implies lower domestic price of the capital variety following the mark-up pricing rule. Both the price of the investment good capital varieties and the user costs of a capital variety, fall. Lower price of capital varieties, combined with more capital varieties, stimulates domestic demand for capital varieties, especially through the love-of-variety effect. The overall fall in the composite price of the user costs of capital varieties is 4.02 per cent.

The negative shift in the marginal cost curve combined with an increase in the number of firms that reduces the scale of production within each firm implies that more capital varieties can be supplied to the foreign market at a lower marginal cost. As with policy alternative 1 this generates a positive effect on competitiveness for the capital variety industry since the costs of domestic deliveries is reduced, while the export price is unchanged. The total effect on export deliveries is positive and the export deliveries of capital varieties increase more than the domestic deliveries due to given world market price.

Stimulating the production of capital varieties through a production subsidy increases the demand for other inputs as labour and other capital equipment in both the capital variety industry and R&D industry. Since total labour supply is exogenous the wage rate increases, giving higher production costs for all industries. The production costs increases for all industries, even though the price of capital varieties is considerably reduced. Export deliveries for all other products than capital varieties are reduced. There is a small increase in domestic deliveries from the production of other machinery due to higher demand for other machinery in the R&D related industries. In all other industries other machinery is substituted by the composite of capital varieties. The wage rate increase combined with the positive demand effect for more input factors in the R&D related industries and higher consumer demand for goods and services, implies that the domestic prices of other capital equipment and other intermediates increase, contributing to an even higher unit cost of production.

Total gross product increases by 0.70 per cent and the growth rate is 0.02 percent point higher.[3] Total welfare measured as the change in long run material consumption is 0.3 per cent higher. The positive welfare effect is generated by many different sources in this kind of CGE model. Increased R&D production has welfare effects through the following channels; 1) the positive spillover effect in production of patents, 2) the positive love-of-variety effect in the demand for capital varieties and 3) the negative welfare effect of lower production within each monopoly firm producing capital varieties.

The presence of other distortions that are not especially connected to the R&D related industries must also be taken into account when analysing the total welfare effects of a policy alternative. The capital income taxation drives a wedge between private and social return to capital and savings are to low from an efficiency point of view. Intertemporal efficiency is improved since total savings in real and financial capital increase. On the other hand the consumption of housing increases, contributing negatively to total welfare. There is also a reallocation of deliveries from the domestic to the foreign market due to the positive effect on the competitiveness for the capital varieties industry, contributing negatively to welfare.

Alternative 3, Model 1:
(skift3abhabby.db)

Many of the same effects as with Model 2 are present when using Model 1. The most striking difference is the lack of the positive terms of trade effect for the machinery variety capital industry since the export price in this alternative is determined by a mark-up pricing rule in the domestic market. The price for domestic deliveries is reduced as with Model 2, and the export price is now also reduced. The negative terms of trade effect has a negative effect on export deliveries, and total growth and welfare in the economy, compared to the alternative Model 1 where the domestic producers are price takers in the world market.

[3] Gross product is measured exclusive of the petroleum production that is exogenous in the model.

3.3 Comparisons of the policy alternatives

Table 3. Long run effects, Model 2.

	Alternative 1	Alternative 3
Total welfare*, per cent deviation from the reference path	0.37	0.26
Growth rate, per cent points deviation from the reference path	0.044	0.019
Share of R&D in total production	1.3	1.06
Share of machinery varieties in total production		

* Measured as total discounted consumption

Alternative 1, a direct subsidy towards the production of ideas/patents, i.e. to the R&D industry that direct stimulates entry of new firms is welfare superior compared to a production subsidy to the capital varieties industry. The production of R&D increases by more than 45 per cent in policy alternative 1, while only by 16.8 per cent in policy alternative 2, implying that the number of firms in the capital varieties industry increases by considerably more in policy alternative 1 than policy alternative 2. The production of capital varieties increases by 16 per cent in alternative 1 and 8 per cent in alternative 2. On the other hand, the production within each firm falls by 7.41 per cent in alternative 1 and only 2.40 per cent in alternative 2. Lower production within each monopoly firm in alternative 1 contributes negatively to welfare compared to alternative 2. So - even though the effect on the growth rate in total gross product is more than doubled in policy alternative 1 compared to policy alternative 2, the effect on total welfare is only approximately 30 per cent larger, which is also the case for the share of R&D in total production.

Compared to the reference path, the share of R&D production in total gross production increases from 0.91 to 1.30 per cent in policy alternative 1 and to 1.06 per cent in policy alternative 3. These are both lower than the political goal of 3 per cent (Ministry of Finance (2004)). There are at least two issues that are important to bear in mind when we discuss such R&D policy goals. First, we only consider R&D delivered by private industries. If we also include R&D performed by the institute- and public sector the share of R&D in the total economy is higher. Second, the supply of the scarce resource labour is crucial for the possibilities of expanding the R&D industry in special and the economy in general. If the policy does not generate more labour supply, and especially more supply of high skilled labour, the expansion possibilities are very limited. This particular study does not consider labour supply responses or the supply of skilled labour, but these issues will definitely influence the effects of R&D policy.

So - what are the policy advices of these analyses? Subsidising R&D production directly generates more welfare and economic growth than subsidising the production of capital varieties in general independent of whether the subsidy is directed to an existing firm or a new firm. But - the positive effect of policy alternative 1 is modified by lower production of each variety compared to policy alternative 3, increasing the distortion generated by market power and mark-up pricing in the domestic market for capital varieties market. The positive effect on the competitiveness for the capital varieties industry on the export market is approximately equal in both policy alternatives.

Alternative 4 is a direct subsidy towards domestic purchase of machinery capital varieties.
The unit cost price of the machinery capital variety composite P_{kv} can then be written as

$$(8)\ P_{kv} = R^{\left(1/1-\sigma_{kv} \right)} \cdot \overline{P}_{kv}$$

$\overline{P}_{kv}$ is the user cost of capital of each machinery capital variety. With a purchaser price subsidy the user cost of capital is defined as

$$(9)\ \overline{P}_{kv} = (r+\delta)\frac{P_k^H}{(1+\alpha_s)} - \frac{\dot{P}_k^H}{(1+\alpha_s)} = (r+\delta)\left(P_k^H - \dot{P}_k^H\right)\left(\frac{1}{1+\alpha_s}\right)$$

4. Concluding remarks

We model technological change as results of R&D activities performed by optimising agents. The output of general R&D activity is new technological solutions that are purchased by capital producers in order to supply new varieties of capital equipments. There is monopolistic competition in the capital markets and love of capital variety in demand, so that productivity in the final goods industries increases with R&D activity, number of patents and of real capital firms producing different capital varieties. Besides increasing productivity, R&D increases the capacity of domestic productions to absorb knowledge spillovers from international trade. There are external effects of R&D, as knowledge accumulates and enhances the productivity of current R&D activities. Our model approach is inspired by the product-variety model of Romer (1990) and the empirical application by Diao et al. (1999). However, contrary to these contributions, we introduce decreasing returns to scale in the knowledge capital, based on evidence provided in Jones (1995).

Contrary to Diao et al (1999) and Russo (2004) we model a small open economy where export prices are given at the world market. Both the existence of internal spillovers from investments in R&D and imperfect competition in the market for capital are arguments for subsidising innovations. However, for small, open economies these arguments may empirically turn out to be of less importance. In small open economies technological progress are inevitably dominated by international processes. Hence, cogent arguments for R&D support in small and open economies could rather be absorptive capacity expansion or possible comparative advantages or interests on specific future R&D fields.

We explore how innovation incentives should be designed in order to achieve the highest welfare and economic growth. Our analyses show that subsidising R&D production directly generates more welfare and economic growth than subsidising the production of capital varieties independent of whether the subsidy is directed to an existing firm or a new firm. The positive welfare effect of subsidising R&D production directly is modified by lower production of each variety compared to a production subsidy to the capital varieties industry, though. This increases the distortion generated by market power and mark-up pricing in the domestic market for capital varieties. The share of R&D production in total gross production increases with both policy alternatives, but the effect is strongest with a direct production subsidy towards the R&D industry. But the policy means are not strong enough for the economy to reach the political goal of 3 per cent share of R&D in total production. The positive effect on the competitiveness for the capital varieties industry on the export market is approximately equal in both policy alternatives.

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Appendix A

Table A.1: Production Activities in ITC-model

ITC Code	Production Activities
24	Other Products and Services
30	Manufacture of Metals
32	Polluting Transport Services
33	Non Polluting Transport Services
40	Transport Oils
42	Heating Fuels
46	Machinery Varieties
47	Other Machinery
50	Building of Ships, Oil Drilling Rigs, Oil Production Platforms etc.
55	Construction, excl. of Oil Well Drilling
62	Ocean Transport - Foreign, Services in Oil and Gas Exploration
66	Crude Oil
67	Natural Gas
69	Pipeline Transport of Oil and Gas
71	Production of Electricity
81	Wholesale and Retail Trade
90	Government Input Activities

Appendix B - The CES production technology and production of machinery capital varieties

The CES production technology structure is described in figure 1. Disregarding all other inputs than labour and machinery capital, the CES production function is given by

$$(B.1)\ f\left(\frac{L}{\tau_L},\frac{K}{\tau_K}\right)=\left[\delta_L\left(\frac{L}{\tau_L\delta_L}\right)^{\left((\sigma-1)/\sigma\right)}+(1-\delta_L)\left(\frac{K}{\tau_K(1-\delta_L)}\right)^{\left((\sigma-1)/\sigma\right)}\right]^{\left(1/(\sigma-1)\right)}$$

Real capital K is a composite of machinery capital varieties defined as K_V further down, and other machinery capital K_O.

$$(B.2)\ K=\left[\delta_{ko}\left(\frac{K_O}{\tau_{ko}\delta_{ko}}\right)^{\left((\sigma_k-1)/\sigma_k\right)}+(1-\delta_{ko})\left(\frac{K_V}{\tau_{kv}(1-\delta_{ko})}\right)^{\left((\sigma_k-1)/\sigma_k\right)}\right]^{\left(1/(\sigma_k-1)\right)}$$

The variable cost function of the representative firm takes the form

$$(B.3)\quad C_i=c\cdot f\left(\frac{L}{\tau_L},\frac{K}{\tau_K}\right)$$

c is the dual price index (unit cost function) of the CES-composite of labour and the machinery capital aggregate given by

$$(B.4)\quad c=\left[\delta_L\left(\frac{w}{\tau_L}\right)^{(1-\sigma)}+(1-\delta_L)\left(\frac{P^K}{\tau_K}\right)^{(1-\sigma)}\right]^{\frac{1}{1-\sigma}}$$

P^K is the unit cost for the machinery capital aggregate given by (following the nested CES input structure in the model presented in Figure 1)

$$(B.5)\quad P^K=\left[\delta_{ko}\left(\frac{P_{ko}}{\tau_{ko}}\right)^{(1-\sigma_k)}+(1-\delta_{ko})\left(\frac{P_{kv}}{\tau_{kv}}\right)^{(1-\sigma_k)}\right]^{\frac{1}{1-\sigma_k}}$$

P_{ko} is the user cost of other machinery capital while P_{kv} is the unit price of the composite of machinery capital varieties. We assume so-called Spence-Dixit-Stiglitz preferences for the composite of machinery capital varieties.

$$(B.6)\ K_V=\left[\int_0^R (K_{V\,i})^{\theta_k}\,di\right]^{1/\theta_k}$$

The dual unit cost function defining the aggregate price of the machinery capital variety composite is then given by

$$(B.7)\quad P_{kv}=\left[\int_0^R (P_{kvi})^{\left(\frac{\theta_k}{\theta_k-1}\right)}\,di\right]^{\theta_k-1/\theta_k}$$

Where $\theta_k = \frac{\sigma_{kv} - 1}{\sigma_{kv}}$ and σ_{kv} is the elasticity of substitution between the different capital varieties.

By using Shepard's lemma on the total cost function (B.3) we derive the demand for machinery capital variety *i*.

(B.8) $K_{Vi} = \frac{\partial C\left(w, P^K\right)}{\partial P_{kvi}} = c'_{P_{kvi}} \cdot f\left(L, K\right)$

For convenience we disregard the exogenous technological change terms τ_i. The demand function for machinery capital varieties is then given by

(B.9) $K_{Vi} = f\left(L, K\right)\left(\frac{c}{P^K}\right)^{\sigma}\left(\frac{P^K}{P_{kv}}\right)^{\sigma_k}\left(\frac{P_{kv}}{P_{kvi}}\right)^{\sigma_{kv}}$

The gross investment in machinery capital variety *i* (equal to the deliveries of machinery capital variety *i* to the domestic market) is given by

(B.10) $J_{kvi} = \dot{K}_{Vi} + \delta K_{Vi} = X_{ki}^{H}$

The domestic demand elasticity for machinery capital variety *I*, ε_i, is equal to the domestic demand elasticity for the production of machinery capital variety *i*, defined as $\varepsilon_i = -\frac{\partial K_{Vi}}{\partial P_{kvi}} \frac{P_{kvi}}{K_{Vi}} = -\frac{\partial X_{ki}^{K}}{\partial P_{ki}^{H}} \frac{P_{ki}^{H}}{X_{ki}^{H}}$.

ε_i is derived from the demand for machinery capital varieties in the final goods industry.

The demand elasticity derived from equation (B.9) is given by

$$\varepsilon_i = \frac{-1}{\theta_k - 1}$$

that is independent of *i*. It can be shown that the elasticity of substitution between two different varieties in the composite given in equation (B.6) is

$$\sigma_{kv} = \frac{-1}{\theta_k - 1}.$$

In this case the substitution elasticity is equal to the domestic demand elasticity, and we have the following expression for the mark-up factor

(B.11) $m = \frac{\varepsilon}{\varepsilon - 1} = \frac{\sigma_{kv}}{\sigma_{kv} - 1}$, $\sigma_{kv} > 1$.

With a nested CES production technology the mark-up factor only depends on the partial demand/substitution elasticity in the capital composite. The relationship between the mark-up factor in the monopoly pricing rule of the machinery capital varieties and the elasticity of substitution between the different capital varieties, implies that low substitution between the different varieties gives a high mark-up factor and vice versa. The mark-up factor is calibrated given an appropriate size of the elasticity of substitution.

The mark up factor is independent of *i*. Together with the assumption of equal production and cost structure in each firm, the monopoly pricing rule implies that the price in the domestic market is equal for all the capital varieties. This implies equal quantity of each variety. The unit cost price of the machinery capital variety composite P_{kv} can then be written as

$$\text{(B.12)}\ P_{kv} = R^{\left(1/1-\sigma_{kv}\right)} \cdot \overline{P}_{kv}$$

$\overline{P}_{kv}$ is the user cost of capital of each machinery capital variety. The machinery capital variety composite is given by

$$\text{(B.13)}\ K_V = R^{\left(\sigma_{kv}/(\sigma_{kv}-1)\right)} \cdot \overline{K}_V$$

$\overline{K}_V$ is the quantity of each machinery capital variety.

Inserting equation (B.12) into equation (B.5) gives the following expression for the unit price of the machinery capital aggregate

$$\text{(B.14)}\ P^K = \left[\delta_{ko} \left(\frac{P_{ko}}{\tau_{ko}} \right)^{(1-\sigma_k)} + (1-\delta_{ko}) R^{\left(\frac{1-\sigma_k}{1-\sigma_{kv}}\right)} \left(\frac{\overline{P}_{kv}}{\tau_{kv}} \right)^{(1-\sigma_k)} \right]^{\frac{1}{1-\sigma_k}} .$$

The factor share of machinery capital varieties in the machinery capital aggregate is given by

$$\text{(B.15)}\ \frac{K_V}{K} = (1-\delta_{ko}) R^{\left(\frac{-\sigma_k}{1-\sigma_{kv}}\right)} \left(\frac{\overline{P}_{kv}/\tau_{kv}}{P^K} \right)^{-\sigma_k} .$$

The corresponding factor share of other machinery capital in the machinery capital aggregate is given by

$$\text{(B.16)}\ \frac{K_O}{K} = \delta_{ko} \left(\frac{P_{ko}/\tau_{ko}}{P^K} \right)^{-\sigma_k} .$$

Appendix C - The entry/exit condition in the machinery capital variety industry

The entry/exit condition for the representative firm in the machinery capital variety industry is given by equation (16). This condition can be rewritten in discrete time as follows

$$\text{(C.1)}\ P_0^R = \sum_{t=0}^{\infty} \frac{1}{(1+r)^t} \pi_{it}' .$$

Along a stationary path, the interest rate r and the profit are both constant. The entry/exit condition can then be written as

(C.2) $P^R = \left(\frac{1+r}{r}\right)\overline{\pi}'$.

The price of a new idea/patent is also constant along a stationary path. Due to the small open economy assumption, the interest rate r is exogenous and also assumed to be constant along the whole simulation path. The exit/entry condition is implemented in the CGE model by the following approximation

(C.3) $P_t^R = \left(\frac{1+r}{r}\right)\pi_{it}^{'}$.

In each time period there will be entry of new firms until the discounted value of the new firm's profit is equal to the price of the new idea/patent.

Appendix D - Calibration

Calibration of the entry/exit condition for the machinery capital variety industry and the accumulated stock of patents

We assume decreasing returns to scale in production both within the representative firm in the industry and for the industry as a whole. Since all firms in the machinery capital variety industry are equal and produce the same amount of each capital variety, we assume that the total industry is represented by one representative firm in the base year, i.e. $X_{ki}^H = X_k^H$ and $X_{ki}^W = X_k^W$ and X_k^H and X_k^W is the total industry's deliveries to the domestic and the export market, respectively. This implies that we can transform the number of firms/varieties into the following index that equals 1 in the base year.

(D.1) $\gamma_R = \frac{R_{t-1} + X_{Rt}^H}{R_{.0}}$.

$R_{.0}$ is the base year value of the number of machinery capital varieties. In the base year (t=0) γ_R=1. The entry/exit condition determines the number of new varieties (new firms) in the machinery capital industry that is equal to the production of new ideas in the R&D industry. The entry/exit condition in each period only yields for the new firms in the machinery capital varieties industry. The only profit that should be included in the entry/exit condition is the share of the total industry's profit that stems from the new firms. Therefore, both the entry/exit condition and the cost functions for the machinery capital variety industry must be formulated at the firm level. We define output delivered to the home market for the representative firm as $\frac{X_k^H}{\gamma_R}$ and correspondingly output delivered to the export market for the representative firm as $\frac{X_k^W}{\gamma_R}$. This gives the following entry/exit condition that is implemented in the CGE model

$$\text{(D.2)}\quad \left(\frac{1+r}{r}\right)\left[P_k^H \frac{X_k^H}{\gamma_R} + P_k^W \frac{X_k^W}{\gamma_R} - c\left(\frac{X_k^H}{\gamma_R}\right)^{1/s} - c\left(\frac{X_k^W}{\gamma_R}\right)^{1/s}\right]\frac{X_R^H}{R_{.0}} = P_R^H X_R^H \ .$$

The first-order conditions for profit maximization is given as

$$\text{(D.3)}\quad P_k^H = m\frac{c}{s}\left(\frac{X_k^H}{\gamma_R}\right)^{\left(\frac{1}{s}-1\right)}$$

$$\text{(D.4)}\quad P_k^W = \frac{c}{s}\left(\frac{X_k^W}{\gamma_R}\right)^{\left(\frac{1}{s}-1\right)}$$

(D.3) is the first order condition for deliveries to the home market and (D.4) is the first order condition for deliveries to the export market. The cost functions that determine the use of inputs in the industry must be aggregated over the number of firms, i.e.

$$\text{(D.5)}\quad C_k^H = c\left(\frac{X_k^H}{\gamma_R}\right)^{1/s}\gamma_R$$

$$\text{(D.6)}\quad C_k^W = c\left(\frac{X_k^W}{\gamma_R}\right)^{1/s}\gamma_R$$

C_k^H and C_k^W is the total production costs for deliveries to the domestic and export market, respectively.

Calibration of input and output for the R&D industry

The National Accounts have no separate data for R&D, either as input into production processes or as output from an industry. We have calibrated input and output data for the R&D industry in the following way. By using input factor shares (labour and capital) and output shares from the R&D Statistics for year 2000, we calculate the shares of R&D related activity from both the input factors and the output for all other industries than the R&D industry in the model. These numbers for input and output is then reallocated to the R&D industry. Total output of R&D is bought by the machinery capital variety industry.