Emissions trading and multiple sources of uncertainty

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PRELIMINARY AND INCOMPLETE

Abstract

Firms participating in emissions trading face uncertainty for example in future price of emission permit. In the case of a single uncertainty of emission permit price, risk-averse permit buyers invest more and permit sellers less in the abatement of emissions than in the certain case. In this paper we analyse the effects of multiple sources of uncertainty on optimal investment of more energy efficient production technology in the context of a CO_2-emissions trading. We use a mean-variance approach under the double uncertainty of production price and emission permit price. A risk-averse firm is examined in a static framework. Emissions trading is modelled after the European Union's directive for greenhouse gas emission allowance trading. We show that different factors of uncertainty have divergent effects on optimal investment level depending on cost function properties, relative magnitudes of variance factors or whether the representative firm is a permit buyer or a permit seller.

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1 INTRODUCTION

European Union directive 2003/87/EC establishing a scheme for greenhouse gas emission allowance trading within the Community was adopted in July 2003 and emissions trading in accordance with the directive has started in the beginning of 2005. The first trading period will last to the end of 2007. The EU Emissions Trading Scheme (EU ETS) covers over 11.000 installations in 25 Member States: combustion plants, oil refineries, coke ovens, iron and steel plants, and factories making cement, glass, lime, brick, ceramics, pulp and paper. The aim of the trading is to reduce CO_2-emissions in a cost-effective manner inside the trading sectors.

The regulator sets limits for emissions and allocates permits respectively among the emission trading firms in the beginning of the trading period. At the end of the trading period firms must return equal amount of permits they've had emissions during the period. If firms are going to emit more emissions than they hold permits for, firms may reduce emissions or buy more permits from the permit markets. If firms want to reduce CO_2-emissions they usually have to implement a total new and more energy efficient production technology. This can be very expensive. Thus there may be cases when decreasing the output of production is only possible way to decrease emissions.

Of course firms may sell permits to the markets if their emissions will be lower than their permit holding. Firms tend to minimise the emission reduction costs under the total emission target level. In perfect market conditions every firm will reduce its emissions until the level is reached, where marginal reduction cost equals the permit price. Thus at that point marginal cost is the same for every firm and the solution is cost-effective. It can be shown that under perfect permit market conditions it is irrelevant in respect of cost-effectiveness how the regulator makes the initial allocation of the permits. (Montgomery, 1972.) The cost-effectiveness in emissions trading may be jeopardised because of many reasons. For example monopolistic behaviour changes the competitive character of the emission permit market. This may be the case when there are some players, which hold relatively large share of allocated permits and some of these firms influence the price of the emission permit by their own behaviour. In such conditions the initial allocation of permits may also affect the cost-effectiveness of the final solution. (Hahn, 1984; Missiolek and Elder 1989; von der Fehr, 1993; Tietenberg, 1985.) The production markets may also be imperfect and because of that the cost-effective solution may be unreachable (Malueg, 1990). These market imperfections have influence on market prices as well.

Even if both production and permit markets are completely competitive, from a single firm point of view there are also other sources of uncertainties related to the market prices. For example in the Nordic electricity markets different changes in the demand of electricity, the increases in production capacities as well as stochastic variations of electricity consumption and hydro power production affect the price of electricity as well as the price of emission allowances in the EU emissions trading (Koljonen et al., 2004). Under uncertainty the equality rule of marginal reduction costs and permit price may not be optimal for a firm anymore. The optimal decision may vary depending on firm's attitude to risk and character of the uncertainty.

In this paper we study a risk-averse agent with exponential utility function which has property of constant absolute risk aversion. For example Sandmo (1971) and Baron (1970) have studied production decision of a risk-averse firm under uncertainty. Sandmo showed that under exogenous price uncertainty a competitive, risk-averse firm produces less output than under certainty. At least Ben-David et al. (2000) and Baldursson and von der Fehr (2004) have expanded Sandmo's result in the context of emissions trading.

Ben-David et al. (2000) analyse risk-averse firms' optimal compliance strategies in emissions permit markets when emission permit price is uncertain. In order to comply with the given emission limits, firms can choose between the level of abatement and the degree of reliance on the permit markets. Profit function of every firm is constituted from firm's received exogenous revenue, abatement cost function and emissions trading cost/revenue function. As a result of the analytical model, risk-averse permit buyers abate more and risk-averse permit sellers less under uncertainty than under certainty. Consequence of this is that demand and supply of permits will decrease. There will be reduction in the number of permits traded, but if buyers and sellers are equally affected, equilibrium permit price may stay unchanged.

Baldursson and von der Fehr (2004) study the efficacy of price and quantity controls of risk-averse agents under stochastic setting. They analyse an intertemporal problem in a static framework, where emission reduction decision is dispersed in two phases. In the first phase firms decide the level of investment on capital-intensive sunk-cost cleaning technology with low variable costs, but long lead-times of investment. In the second phase firms decide the level of abatement. Abatement is made by technology, which can be implemented comparatively quickly with low investment costs, but high variable costs. In quantity control analysis, when uncertainty is extraneous and is exposed through the price of quotas/permits only, the permit price is uncertain in the investment phase. But decision of abatement on the other hand is made when stochastic variables are observed and the price of the permit is known. Thus abatement will be determined so that marginal abatement cost equals the opportunity cost of emission (i.e. permit price). Whereas optimal investment level for risk averse firms is such that emission permit buyers will invest more and permit sellers less than in the certain case. But there exists such an initial allocation of permits where marginal investment cost equals expected permit price. With that allocation investment level for both permit seller and permit buyer would be the same as in the case of certainty.

In preceding papers there is only single source of uncertainty. In this paper we examine the case with two uncertain variables. It is difficult task to analyse theoretically multiple sources of uncertainties. We use mean-variance analysis, which reduces the analysis of risk to two quantities, expected profit and risk. Risk is defined as the variance and covariance of uncertain variables. There are some advantages but also limitations of using mean-variance approach in analysing risk. It is suitable for comparative static analysis of two uncertain variables. Mean-variance approach with constant absolute risk aversion sets the income effect to zero and concentrates fully on substitution effects, as risk changes. Changes in wealth do not affect the choice of the risky portfolio at all.

Mean-variance analysis also incorporates a special case of expected utility hypothesis without contradicting it. On the other hand in order to mean-variance analysis to work uncertain variables have to be normally distributed and agent's choice should not change the form of the distribution of returns. Agent's objective function must include constant absolute risk-aversion. Risks must also be relatively small (i.e. variances must be small and dispersion of the whole distribution must not be too large) compared to the wealth of agents and the objective functions of the agent must be quadratic. Finally the distributions of uncertain variables must be described by compact probabilities so that all the probability distributions converge to a certain outcome.[2]

2 THE STATIC INVESTMENT DECISION MODEL

There are two uncertain variables in the model: price of the production and price of the emission permit. Contrary to Ben-David et al. (2000) and Baldursson and von der Fehr (2004) there will be no abatement technology. In order to comply with emission targets firms can invest in more energy efficient production technology, reduce their production and buy or sell permits on emission permit markets. Optimal behaviour of a competitive, representative firm is analysed. In the first section analysis is made in simultaneous steps. Representative firm decides the level of production and investment at the same time and before uncertainty of different factors is revealed.

The regulator allocates the permits initially in a way that targeted emission level inside the trading sector will not be exceeded. In this paper I analyse single representative firm, which can be either permit buyer ($\alpha(I)y - Z > 0$) or permit seller ($\alpha(I)y - Z < 0$) depending on the initial allocation of permits and production factors of the firm, when y is the output of production, α the rate between output and emissions, I the level of investment and Z initial allocation of permits. It is also assumed that firms do not change the role of being buyer or seller during the trading period. In other words they do not exercise speculative trading during the trading period.

[2] More from mean-variance analysis on Newbery and Stiglitz (1981, 85-91).

The higher is the rate between output and emissions, the more emissions there will be at the same level of output. On the other hand α will decrease if firm invests on better technology,

$$\alpha(I) \in \left]\underline{\alpha}, \alpha_0\right], \quad \alpha(0) = \alpha_0$$
$$\alpha'(I) < 0, \quad \alpha''(I) > 0$$

The production cost function $c(y)$ and cost function of the investment $C(I)$ are standard cost functions with convexity assumptions,

$$c'(y) > 0, \quad c''(y) \geq 0 \qquad \text{and} \qquad C'(I) > 0, \quad C''(I) \geq 0$$

Each firm will maximise the utility of profits by choosing the levels of output and investment. Value function of the firm ($V(\pi)$) is exponential and has risk aversion properties with constant absolute risk aversion factor $A = -V''/V'$ (>0),

(1) $$V = -e^{-A\pi(y,I)}$$

When price of the production is $\tilde{p}$ and $\tilde{q}$ is the price of the emission permit, profit function of representative firm is,

(2) $$\tilde{\pi}(y, I) = \tilde{p}y - c(y) - \tilde{q}(\alpha(I)y - Z) - C(I) \qquad s.t. \quad y, I \geq 0$$

Both uncertain variables are assumed to be normally distributed. Thus means, variances and covariance of uncertain prices are common knowledge. A bar above the uncertain variable denotes the expected values, variance of the production price is σ_p^2 and variance of the permit price is σ_q^2. Covariance term is denoted by σ_{pq}.

$$\tilde{p} = N(\overline{p}, \sigma_p^2), \quad \tilde{q} = N(\overline{q}, \sigma_q^2)$$
$$\text{cov}(p, q) = \sigma_{pq}$$

If production price and permit price are uncorrelated the covariance is zero. When the prices are correlated, high permit price may induce higher production prices ($\sigma_{pq} > 0$) or lower production prices ($\sigma_{pq} < 0$). When prices are uncertain in the investment phase, future profit is uncertain as well. Due to normal distribution of permit and production prices also profit is normally distributed.

$$\tilde{\pi} = N(\bar{\pi}, \mathrm{var}(\pi))$$

where expected value and variance of future profit can be written as,

$$\bar{\pi} = \bar{p}y - c(y) - \bar{q}(\alpha(I)y - Z) - C(I)$$
$$\mathrm{var}(\pi) = y^2\sigma_p^2 + (\alpha(I)y - Z)^2\sigma_q^2 - 2y(\alpha(I)y - Z)\sigma_{pq}$$

The expected utility of future profit can be found from the moment-generating function of the normal distribution (Newbery and Stiglitz 1981, 85-86) and expected value of (1) can now be written as

(3) $$E(V) = -e^{-A\left(\bar{\pi} - \frac{1}{2}A\,\mathrm{var}(\pi)\right)}$$

The utility certainty equivalent profit is denoted then with,

(4) $$\hat{\pi} = \bar{\pi} - \tfrac{1}{2}A\,\mathrm{var}(\pi)$$

When firm is deciding optimal production and investment simultaneously, it maximises its expected utility. First order conditions are,

(5a)
$$\frac{\partial E(V)}{\partial y} = 0 \quad \Leftrightarrow \quad \bar{p} - c'(y) - \bar{q}\alpha(I) - A\left[y\sigma_p^2 + (\alpha(I)y - Z)\alpha(I)\sigma_q^2 - 2\alpha(I)y\sigma_{pq}\right] = 0,$$

(5b)

$$\frac{\partial E(V)}{\partial I}=0 \quad \Leftrightarrow \quad -\bar{q}\alpha'(I)y - C'(I) - A\left[(\alpha(I)y - Z)\alpha'(I)y\sigma_q^2 - \alpha'(I)y^2\sigma_{pq}\right] = 0 .$$

Differentiating these again, we get the elements of Jacobian matrix,

(6a) $$\Pi_{yy} = -c''(y) - A\left[\sigma_p^2 + \alpha^2\sigma_q^2 - 2\alpha\sigma_{pq}\right] = -c''(y) - A\left[R_{yy}\right] < 0 ,$$

(6b)

$$\Pi_{yI} = \Pi_{Iy} = -\bar{q}\alpha'(I) - A\left[(2\alpha\alpha' y - Z\alpha')\sigma_q^2 - 2\alpha' y\sigma_{pq}\right] = -\bar{q}\alpha'(I) - A\left[R_{yI}\right] > 0 ,$$

(6c) $$\begin{aligned}\Pi_{II} &= -\bar{q}\alpha''(I)y - C''(I) - A\left[\left[(\alpha' y)^2 + \alpha\alpha'' y^2 - E\alpha'' y\right]\sigma_q^2 - \alpha'' y^2\sigma_{pq}\right] \\ &= -\bar{q}\alpha''(I)y - C''(I) - A\left[R_{II}\right] < 0\end{aligned}$$

I make an assumption that terms *A[Rii]* are not affecting the signs of eq. (6a-c). This is because I assume variance terms to be relatively small. In order (3) to be maximised Jacobian determinant must be positive,

(7) $$|J| = \Pi_{yy}\Pi_{II} - \Pi_{yI}^2 \approx \left(-c''(y)\right)\left(-\bar{q}\alpha''(I)y - C''(I)\right) - \left(\bar{q}\alpha'(I)\right)^2 > 0 .$$

<u>Case 0:</u> *no uncertainty* ($\sigma_p^2 = \sigma_q^2 = \sigma_{pq} = 0$)

When all variance terms are set as zero, we can obtain the benchmark case of certainty,

(8a) $$c'(y) + \bar{q}\alpha(I) = \bar{p} ,$$

(8b) $$C'(I) = -\bar{q}\alpha'(I)y$$

In the optimum the sum of marginal production cost and marginal trading costs is equalised with price of production (eq. (8a)). The optimal certainty investment level will equalise the marginal cost of the investment and the marginal revenue of the investment, which is attained by savings on trading expenses for permit buyers and by extra yield of trading for permit sellers (eq. (8b)).

In the next cases we add uncertainty in the model and do comparative static analysis of the effects of uncertainty on optimal production and investment decisions.

<u>Case 1:</u> *production price uncertainty ($\sigma_p^2 > 0$)*

If only the price of production is uncertain, first order conditions are,

(9a) $$\bar{p} = c'(y) + \bar{q}\alpha(I) + Ay\sigma_p^2,$$

(9b) $$C'(I) = -\bar{q}\alpha'(I)y.$$

Now price of production is in the optimum equal as the sum of marginal production cost, marginal trading costs and the marginal factor of price volatility (eq. (9a)). The condition for optimal investment level do not change from the case of certainty, but the optimal investment level itself will change due to the change of optimal production. The effects on production and investment can be analysing with the comparative statistics[3],

(10a) $$\frac{dy}{d\sigma_p^2} = \frac{1}{|J|}Ay\left[\underbrace{-\bar{q}y\alpha'' - C''}_{-} - \underbrace{A[R_{py}]}_{\approx 0}\right] < 0,$$

(10b) $$\frac{dI}{d\sigma_p^2} = -\frac{1}{|J|}Ay\left[\underbrace{-\bar{q}\alpha'}_{+} - \underbrace{A[R_{pI}]}_{\approx 0}\right] < 0.$$

[3] Using Cramer's rule.

When price of the production is uncertain both risk-averse permit buyers and sellers will produce and invest less than in the certain case.

It can be noticed that the more risk-averse the firm (i.e. the greater the risk-averse factor A is) the stronger is the effect of changing the production and investment level from the case of certainty.

<u>Case 2:</u> *permit price uncertainty ($\sigma_q^2 > 0$)*

If only the price of the emission permit is uncertain the firs order conditions are

(11a) $$\bar{p} = c'(y) + \bar{q}\alpha(I) + A(\alpha(I)y - Z)\alpha(I)\sigma_q^2,$$

(11b) $$C'(I) = -\bar{q}\alpha'(I)y - A(\alpha(I)y - Z)\alpha'(I)y\sigma_q^2.$$

Now both optimal conditions are affected by uncertainty and the effects depends on whether the firm is a permit buyer or seller. Comparative static analysis looks like,

(12a)

$$\frac{dy}{d\sigma_q^2} = \frac{1}{|J|} A\underbrace{[\alpha y - Z]}_{+/-}\left\{\left[\bar{q}y\underbrace{\left((\alpha')^2 - \alpha\alpha''\right)}_{\approx 0} - \underbrace{\alpha C''}_{-}\right] - A\underbrace{[\alpha R_{II} - \alpha' y R_{yI}]}_{\approx 0}\right\} > (<)0\,; \quad [\alpha y - Z] < (>)0$$

(12b) $$\frac{dI}{d\sigma_q^2} = \frac{1}{|J|} A\underbrace{[\alpha y - Z]}_{+/-}\left\{\left[\underbrace{\alpha'(-yc'' + \bar{q}\alpha)}_{+/-}\right] - A\underbrace{[\alpha' y R_{yy} - \alpha R_{yI}]}_{\approx 0}\right\} >< 0 \quad ?$$

The effect of permit price uncertainty depends now on the market position of the firm. Assumption ($(\alpha')^2 - \alpha\alpha'' = 0$) in eq. (12a) comes from the definition of energy efficiency function $\alpha(I)$. Assumption holds if function for energy efficiency $\alpha(I)$ is for example,

$$\alpha(I) = \bar{b}\,\alpha^{-I}; \quad \alpha(0) = \bar{b}; \quad \lim_{I\to\infty} \alpha(I) = 0 .$$

Permit buyer: $\alpha(I)y - Z > 0$

When emission permit price is uncertain risk-averse permit buyers will produce less than in the certain case.

Permit seller: $\alpha(I)y - Z < 0$

The results for risk-averse emission permit seller is opposite. When only emission permit price is uncertain it is optimal for risk-averse permit sellers to produce more than in the certain case.

Effect on optimal investment (eq. (12b)) is a bit more complicated. The first sum term in brackets of eq. (12b) is critical. Denote $f(y) = q\alpha(I) - yc''(y)$. Integrating and differentiating $f(y)$ in respect of y (when I is fixed) we get,

(13)

$$\begin{aligned} f(y) &= \frac{\partial}{\partial y}\left[\int_0^y f(\gamma)d\gamma\right] = \frac{\partial}{\partial y}\left[\int_0^y q\alpha(I)d\gamma - \int_0^y \gamma c''(\gamma)d\gamma\right] \\ &= \frac{\partial}{\partial y}[q\alpha(I)y] - \frac{\partial}{\partial y}\left[yc'(y) - 0c'(0) - \int_0^y c'(\gamma)d\gamma\right] \\ &= \frac{\partial}{\partial y}[q\alpha(I)y] - \frac{\partial}{\partial y}[yc'(y) - c(y)] \end{aligned}$$

Examining the last form of eq. (13) it can be noticed that when firm is a permit buyer (seller) the first term describes the marginal effect of production on trading costs (revenue), whereas the second term describes the marginal effect of production on producer's surplus without trading. We call the first effect later a marginal trading effect and the second a marginal production cost effect. If the marginal production cost effect is greater in the neighbourhood of the optimum,

(14) $$yc''(y) > q\alpha(I)$$

then energy efficiency investments have relatively small effect on total production. This can be the case for example, if trading costs are relatively small compared to firm's total costs.

Permit buyer: $\alpha(I)y - Z > 0$

When emission permit price is uncertain and marginal production cost effect is dominating marginal trading effect, as is written in eq. (14) (remembering that $\alpha'(I) < 0$), risk-averse permit buyers will invest more than in the certain case. But if inequality in eq. (14) is opposite and marginal trading effect dominates, it is optimal for risk-averse permit buyers to invest less than in the benchmark case of certainty.

Permit seller: $\alpha(I)y - Z < 0$

The results for risk-averse emission permit seller is opposite. When only emission permit price is uncertain and marginal production cost effect is dominating marginal trading effect, risk-averse permit sellers will invest less than in the certain case. But if $q\alpha(I) - yc''(y) > 0$ and marginal trading effect dominates, it is optimal for risk-averse permit sellers to invest more than in the certain case.

Case 3: *correlation of production and permit prices* ($\sigma_{pq} \neq 0$)

When both production and permit prices are uncertain and there is (negative or positive) correlation between them, the effects determined in the previous cases are still valid. The first order conditions are eq. (5a-b). The extra effect of covariance factor can be analysed separately.

(15a) $$\frac{dy}{d\sigma_{pq}} = \frac{1}{|J|} A\left\{\left[\bar{q}y^2\underbrace{\left(2\alpha\alpha'' - (\alpha')^2\right)}_{+} + \underbrace{2\alpha y C''}_{+}\right] + A\underbrace{\left[2\alpha y R_{II} - \alpha' y^2 R_{yI}\right]}_{\approx 0}\right\} > 0$$

(15b) $$\frac{dI}{d\sigma_{pq}} = \frac{1}{|J|} A\left\{\left[\underbrace{\alpha' y\left(yc'' - 2q\alpha\right)}_{+/-}\right] - A\underbrace{\left[\alpha' y^2 R_{yy} - 2\alpha y R_{yI}\right]}_{\approx 0}\right\} >< 0 \quad ?$$

It can be noticed, that covariance factor affects both permit buyer and seller to move in the same direction. If the prices of production and emission permits are positively (negatively) correlated, the firm will produce more (less) than in the case of no correlation (eq. (15a)).

Whenever *yc''(y) > 2qα(I),* energy efficiency investments have relatively small effect on total production. This is stronger condition than the one determined in the case of permit price uncertainty, because the trading effect is multiplied with term 2. Thus when the marginal production cost effect is at least twice as great as the trading effect in the neighbourhood of the optimum it implies the following. Positive correlation between permit price and production price strengthens the effect of both risk-averse permit buyer and permit seller to under-invest. Negative correlation has an opposite effect.

When *yc''(y) < 2qα(I)* in the neighbourhood of the optimum, positive correlation between permit price and production price weakens the effect of risk-averse permit buyer and permit seller to under-invest compared to the benchmark case of certainty. Negative correlation has an opposite effect.

As a conclusion of preceding cases all the effects are gathered in Table A. and Table B. Plus signs imply the effects to over-produce or over-invest and minus signs the effects to under-produce or under-invest.

Table A. Effects of different kind of uncertainties on the level of optimal production when emission permit buyer or permit seller is risk-averse.

Condition	$\sigma_p^2 > 0$	$\sigma_q^2 > 0$	$\sigma_{pq} \neq 0$
Permit buyer	–	–	+
Permit seller	–	+	+

Table B. Effects of different kind of uncertainties on the level of optimal investment when emission permit buyer or permit seller is risk-averse.

Condition	$\sigma_p^2 > 0$	$\sigma_q^2 > 0$		$\sigma_{pq} \neq 0$	
		$yc''(y) > q\alpha(I)$	$yc''(y) < q\alpha(I)$	$yc''(y) > 2q\alpha(I)$	$yc''(y) < 2q\alpha(I)$
Permit buyer	–	+	–	–	+
Permit seller	–	–	+	–	+

3 SIMULATION MODEL

Draft of GAMS simulation model of emissions trading sector will be added later.

4 CONCLUSIONS

TBA

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