Decomposing the Integrated Assessment of Climate Change

Lecture Notes for the EAERE/FEEM/VIU Summer School

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Overview of Talk

1. Literature

2. A canonical model

3. The decomposition idea

4. Application to DICE

5. Using GNUPLLOT with GAMS

6. Some illustrative computational exercises
The Message

- Integrated assessment models for climate policy design can be decomposed using a linear approximation of the climate system.

- This permits models of the economic and natural science components to be processed independently on different time scales.

- Turnpike properties of Ramsey growth model permit a precise representation of post-terminal emissions and to reduce the requisite economic horizon.
• Decomposition accommodates economic modelling in a complementarity format thereby providing a means of incorporating second-best effects, e.g. distortionary taxes.
Integrated Assessment

Integrated models of climate and economy:

- First appeared in the 1980s as a paradigm for integrating science and economic policy instruments to study complex environmental issues.

- Combine complementarity knowledge from various disciplines to produce informed policy analysis.

- Early example: RAINS model of acidification in Europe (Alcamo et al., 1985)
• Variety of models have been developed to study greenhouse issues (see Weyant et al. 1996, Parson and Fisher-Vanden 1997 and Kelly and Kolstad 1999).

• Nordhaus, Peck and Tysberg, and Manne and Richels in the 1990s.
Figure 1: Schematic Structure of Integrated Assessment Models for Climate Change
Two Types of Integrated Assessment Models

1. Policy simulation models for assessment of specific measures (e.g. IMAGE by Rotmans), and

2. Policy optimization models, which seek to characterize optimal policies. (e.g., DICE by Nordhaus or MERGE by Manne and Richels).
Two Difficulties with Policy Optimization IAMs

1. Existing integrated assessment models must be solved over a long time horizon to provide a consistent accounting of both the costs and benefits of climate policy measures.

2. These models are typically solved as centralized planner optimization programs which do not readily admit second-best effects such as pre-existing taxation.
Our Contributions

1. We demonstrate that a tangent approximation of the climate system provides an excellent means of decomposing integrated assessment models,

2. When the climate system is thus approximated, the economic model can be formulated:

   (a) over policy-relevant time horizon,

   (b) in a complementarity format which accommodates a wider range of economic complexities (e.g., more goods, more regions, technical change, tax revenue recycling etc.)
3. When an IAM is decomposed, it becomes a simple matter to compare results from alternative climate/economic components.
A Canonical Integrated Assessment Model

A stylized optimizing IAM model:

$$\max \sum_{t=0}^{\infty} \left( \frac{1}{1 + \rho} \right)^t U(C_t, D_t)$$

s.t. \quad C_t = F(K_t, D_t, E_t) - I_t

K_{t+1} = (1 - \delta)K_t + I_t

K_0 = \bar{K}_0

In which:

$\rho$ is the discount rate,

$U$ denotes instantaneous utility reflecting both final consumption and the disutility of climate damages,

$C_t$ represents consumption in period $t$,
$F$ characterizes aggregate production in period $t$ as a function of capital, damages (with potentially adverse effects on productivity), and emissions,

$K_t$ is the capital stock in period $t$ (with $K_0 = \bar{K}_0$ as the initial capital stock),

$E_t$ are emissions in period $t$,

$I_t$ is investment in period $t$,
\[ T_t^E = H(S_t) \]
\[ S_{t+1} = G(S_t, E_t) \]
\[ D_t = D_t(T_t^E) \]
\[ S_0 = \bar{S}_0 \]

In which:

\( T_t^E \) is the global mean temperature in period \( t \),

\( H \) describes the functional relationship between the climate state and temperature,

\( S_t \) is a vector of the climate state (with

\( D_t \) denotes damages of climate change in period \( t \), \( S_0 = \bar{S}_0 \) as the initial climate state), and

\( G \) characterizes the motion of the climate state as a function of the previous climate state and current anthropogenic emissions.
Approximation

Merge the relationships $T_t^E = H(S_t)$ and $S_{t+1} = G(S_t, E_t)$ into a single equivalent equation

$$T_t^E = \Gamma_t(S_0, E_0, E_1, ..., E_{t-1}),$$

where $\Gamma_t$ relates temperature in period $t$ as a function of the initial climate state and emissions in all previous periods.

Then compute a linear approximation of the climate response:

$$T_t^E \approx \bar{T}_t^E + \sum_{\tau=0}^{t} \gamma_{t\tau}(E_{\tau} - \bar{E}_{\tau})$$

For a simple application, we can do this by “brute force”:

$$\gamma_{t\tau} \approx \frac{T_t^E - \Gamma_t(S_0, E_0, ..., \bar{E}_\tau + \epsilon, ..., \bar{E}_{t-1})}{\epsilon}.$$
Figure 2: Basic Decomposition Approach
Comments

• Numerical differencing is only computationally tractable for small-scale climate models with solution times measured in seconds, however for larger scale models *adjoint codes* may be used for the same purpose.

• A benefit we perceive is that we can use this approach to provide a decomposition of the relative importance of climate and economic models in a given policy assessment.

• A more subtle advantage of the decomposition relates to differences in the nature of time scales for economic and climate models.
The DICE Climate Model

parameters

\[ E(t) \quad \text{Anthropogenic carbon emissions (economic input)} \]
\[ m(t) \quad \text{CO2-equiv concentration billion t} \]
\[ \text{forc}(t) \quad \text{Radiative forcing - W per m}^2 \]
\[ \text{forcoth}(tc) \quad \text{Exogenous forcings from other greenhouse gases,} \]
\[ \text{te}(t) \quad \text{Temperature - atmosphere C} \]
\[ \text{tl}(t) \quad \text{Temperature - lower ocean C} \]
\[ \text{termv} \quad \text{Terminal value of atmosphere} \]
\[ \text{deltaE} \quad \text{Difference interval} /0.001/; \]

\[ m(t) = m0; \quad \text{te}(t) = t0; \quad \text{tl}(t) = tl0; \quad \text{forcoth}(tc) = 1.42; \]

loop(t,
\[ m(t) = 590 + \text{atret} \times E(t) + (1-\text{deltam}) \times (m(t-1)-590) + m0 \times \text{tfirst}(t); \]
\[ \text{forc}(t) = 4.1 \times (\log(m(t)/590)/\log(2)) + \text{forcoth}(t); \]
\[ \text{te}(t) = \text{te}(t-1) + c1 \times (\text{forc}(t-1)-\text{lam} \times \text{te}(t-1)-c3 \times (\text{te}(t-1)-\text{tl}(t-1))) + t0 \times \text{tfirst}(t); \]
\[ \text{tl}(t) = \text{tl}(t-1) + c4 \times (\text{te}(t-1)-\text{tl}(t-1)) + tl0 \times \text{tfirst}(t); \]
\[ \text{teref}(t) = \text{te}(t); \] )
Post-Terminal Projection

The Ramsey model, which provides the basis for nearly all policy-oriented IAMs, is an “exogenous growth model” (see Barro and Sala-i-Martin, Chapter 2).

Policy measures affect levels but not growth rates.

We can therefore easily extrapolate carbon emissions from the terminal period off the end of the model:

\[-p_t \frac{\partial F}{\partial E_t} = \sum_{\tau=t}^{\infty} \frac{\partial \Gamma_{\tau}}{\partial E_t} p_{\tau}D_t \approx \sum_{\tau=t}^{T} \frac{\partial \Gamma_{\tau}}{\partial E_t} p_{\tau}D_t + \frac{\tilde{p}_{\tau}D_t}{(1 + r)^{\tau - T}} \sum_{\tau=T+1}^{\infty} \frac{\partial \Gamma_{\tau}}{\partial E_t} \]

and

\[E_t \approx E_T \frac{L_t}{L_T} \quad \forall t > T\]
Figure 3: Time Structure of Returns to Economic and Climate Investments
Figure 4: Sensitivity of Emission Control Rate
Figure 5: Welfare Impact of 1% Increase in Abatement
Using **GNUPLOT** from **GAMS**

`$title Generate an illustrative GNUPLOT figure`

set  t         Time periods in the model /0*40/,
   tlab(t) Labels for the graph / 0 1990, 10 2000, 20 2010, 30 2020, 40 2030/;

set j /a,b,c,d/;
parameter a(t,j) My model output;

\[ a(t,j) = 2.5 \times \frac{\text{ord}(t)}{\text{card}(t)} + 0.4 \times \text{uniform}(-1,1); \]

* Here are the GNUPLOT commands:

\`
$setglobal gp_opt1 "set key outside"
$setglobal gp_opt2 "set title 'Graph of Random Time Series’"
$setglobal gp_opt3 "set yrange [0:3]"
$setglobal gp_opt4 "set xlabel 'Year -- time step annual’"
$setglobal gp_opt5 "set ylabel 'Value’"

$setglobal domain t
$setglobal labels tlab
$batinclude plot A
\`
Climate 1-2-3

GAMS programs climate1.gms, climate2.gms, and climate3.gms provides a simplified illustration of ideas in our paper. Your programming task involves replicating the graphs which illustrate these calculations.

Here are a few extra credit questions for students who are able to produce the plots with relative ease:

1. The abatement timing model presented here embodies the assumption that in the initial year of climate action abatement cannot exceed 10% of baseline emissions, and thereafter it may grow at only 20% per decade. From a qualitative perspective, how are the optimal policies affected by these assumptions?

2. How are economic costs affected when the stabilization target (specified here as 2 degrees) varies from 1.5 to 5 degrees?

3. How sensitive is the optimal abatement policy affected by the intertemporal discount rate?

4. How do changes in the baseline emissions assumptions affect the estimated cost of climate stabilization?
In `climate1.gms` you will need to introduce four `$setglobal` statements and one `$batinclude plot` statement to produce the following graph:
In climatem2.gms you will need to declare and define a parameter in order to produce the following graph:
In climate3.gms you will need to declare and define a reporting parameter and include two additional solve statements to produce the following figure. $a_1$, $a_2$ and $a_3$ report abatement as a fraction of baseline emissions in the first, second and third optimization problems:
climate1.gms

$title climate1.gms Data and an emissions growth path

SET t Time periods /1*40/;

parameter

  m0  CO2-equiv concent. 1965 billion tons carbon /677/,
  tl0 Lower stratum temperature (C) 1965 /.10/,
  t0  Atmospheric temperature (C) 1965 / .2/,
  atret Marginal atmospheric retention rate / .64/,
  c1  Coefficient for upper level / .226/,
  lam Climate feedback factor / 1.41/,
  c3  Coefficient trans upper to lower stratum / .440/,
  c4  Coeff of transfer for lower level / .02/,

  r  Rate of social time preference per year / .03/,
  gl0 Growth rate of population per decade / .223/,
  dlab Decline rate of population growth per dec / .195/,
  deltam Removal rate carbon per decade / .0833/,
  ga0 Initial growth rate for technology per decade / .15/,
  dela Decline rate of technology per decade / .11/,
gsigma Growth of sigma per decade \(-.1168/\),
sig0 CO2-equiv-GWP ratio \(.519/\),
sigma(t) Emissions-output ratio,
L0 1965 world population millions \(3369/\),
k0 1965 value capital billions 1989 US dollars \(16.03/\),
gamma Capital elasticity in output \(.25/\),
a0 Initial level of total factor productivity \(.00963/\),
L(t) Level of population and labor,
al(t) Level of total factor productivity (TFP),
ga(t) Growth rate of TFP from 0 to T,
gl(t) Growth rate of labor 0 to T,
gsig(t) Cumulative improvement of energy efficiency
ebau(t) Baseline emissions;

gsig(t) = (gsigma/dela)*(1-EXP(-dela*(ORD(t)-1)));
sigma(t)=sig0*EXP(gsig(t));
gl(t) = (gl0/dlab)*(1-EXP(-dlab*(ORD(t)-1)));
L(t)=L0*EXP(gl(t))* .9;
ga(t)= (ga0/dela)*(1-EXP(-dela*(ORD(t)-1)));
al(t) =a0*EXP(ga(t));
ebau(t) = 10 * sigma(t) * al(t) * (k0*L(t)/L0)**gamma * L(t)**(1-gamma);

set t200(t) /1*20/,
    tbll(t)    /5 2050, 10 2100, 15 2150, 20 2200/;
climate2.gms

$title climate2.gms Computation of Climate Response

* Include the preceding file:

$include climate1

parameter
  m(t) CO2-equiv concentration billion t
  forc(t) Radiative forcing - W per m2
  forcoth(t) Exogenous forcings from other greenhouse gases,
  te(t) Temperature - atmosphere C
  teref(t) Reference temperature path
  tl(t) Temperature - lower ocean C
  termv Terminal value of atmosphere
  deltaE Difference interval /0.001/;

set tfirst(t) The first time period; tfirst(t) = yes$(ord(t)=1);

* Initial conditions for climate model:
m(t\text{first}) = m0; \  te(t\text{first}) = t0; \ tl(t\text{first}) = tl0; \ forc\text{coth}(t) = 1.42;

\begin{verbatim}
parameter climate Climate evolution
  er\text{ef}(t) Reference emissions
  E(t) Currently estimated emissions path,
  grad Temperature gradient,
  te\text{init} Initial temperature path;
\end{verbatim}

* Write out two "subroutines" for computing the climate model:

\>$\text{onecho >climatemodel.gms}
\text{loop(t,}

'Atmospheric carbon accumulation:

\begin{verbatim}
m(t) \ = \ 590 + at\text{ret}\cdot\text{er\text{ef}(t)} + (1-\text{deltam})\cdot(m(t-1)-590) + m0\cdot t\text{first}(t);
\end{verbatim}

* This equation relates the stock of atmospheric carbon to
  * forcing, with a climate sensitivity of 4.1 and a
  * pre-industrial carbon concentration of 590 parts per million:
forc(t) = 4.1*(LOG(m(t)/590)/LOG(2)) + forcoth(t);

* These equations relate forcing to climate change. Higher
  * radiative forcings warm the atmospheric layer:

  \[ te(t) = te(t-1) + c1*(forc(t-1) - lam*te(t-1) - c3*(te(t-1) - tl(t-1))) + t0\text{tfirst}(t); \]

* The atmosphere then warms the upper ocean, gradually
  * warming the deep oceans:

  \[ tl(t) = tl(t-1) + c4*(te(t-1) - tl(t-1)) + tl0\text{tfirst}(t); \]

  teref(t) = te(t);

); $offecho

alias (t, tp);

$onecho >jacobian.gms
  eref(t) = E(t);
$include climatemodel
teinit(t) = teref(t);
grad(t, tp) = 0;
loop(tp, eref(tp) = eref(tp) + deltaE;
$include climatemodel
    grad(t, tp)= (teref(t)-teinit(t)) / deltaE;
    eref(tp) = eref(tp) - deltaE;
    teref(t) = teinit(t);
$offecho

E(t) = ebau(t);
$include jacobian
$title climate3.gms  Optimal Abatement

* Include both preceding files:

$include climate2

parameter pv(t) Present value cost;

pv(t) = 1/(1+r)**(ord(t)-1);

variables obj Objective function
    abate(t) Abatement measures;

positive variable abate;

equations objdef, avetemp, ratelimit;

objdef.. obj =e= sum(t, pv(t) * ABATE(t));

ratelimit(t+1).. ABATE(t+1) =l= 1.2 * ABATE(t) + 0.10*Ebau(t);
avetemp(t) = teref(t) + sum(tp, grad(t,tp)*(Ebau(tp)-ABATE(tp)-eref(tp))) = l

model optabate /objdef, avetemp, ratelimit/;

* Solve iteration 1:

solve optabate using nlp minimizing obj;
E(t) = ebau(t) - ABATE.L(t);
$include jacobian