The production and consumption accounting principles as a guideline for designing environmental tax policy

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Abstract

This paper evaluates two alternative tax policies aimed at reducing atmospheric pollutant emissions. One based upon an environmental tax that burdens directly firms' emissions, and the other one that burdens both directly and indirectly household consumption's emissions. Applying input-output approach, we reallocate the emissions generated in the economy according to the responsibility definition, i.e. the production or the consumption accounting principle. Afterwards, we analyse the effects on the products' prices of implementing an ad-quantum environmental tax based on the *Producer Pays Principle* (PPP) and/or on the *User Pays Principle* (UPP). The results obtained show that both a PPP and an UPP environmental tax has the same effect on the final products' prices. However, the price of the intermediate products is only affected by the PPP environmental tax, whereas the UPP environmental tax keeps the prices unchanged.

Keywords: input-output analysis, environmental taxes, atmospheric pollutants.
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1. Introduction

Since the validation of the Kyoto Protocol in 1997, various countries are concerning in reducing their emissions of some atmospheric pollutants. Concretely, for the greenhouse gases the European Union (EU) as a whole is committed to keeping the average emissions in the period 2008-2012 to a level 8% lower than those of the base year considered, i.e. 1990[1]. This fact has come out an increasing interest in analysing the efficiency and feasibility of different policy mechanisms to achieve a given environmental target. It is well known that some economists in the economic literature have advocated by the use of public policy intervention in order to control pollutant emissions (Pigou, 1920) or by creating other market mechanisms (Coase, 1960). At this respect, it has been designed diverse measures of environmental protection, i.e. subsidies for pollution abatement; tradable emission permits markets; and/or environmental taxes. In sum, all of these instruments are designed in such a way that externalities yielded by pollution activities can be internalised into market prices.

Concretely, environmental taxes, among others, are claimed to be market based instruments of environmental policy. This is so, because these sorts of taxes allow policy makers to raise firms' costs which in turn increase the price of polluting intensive goods. With these resulting prices, the market will reallocate the economic resource in such a way that atmospheric pollutant emissions can be reduced. Furthermore, in comparison with other policy instruments, environmental taxes have the property of being less cost-effective. That is, giving a specific environmental objective, the total cost of reducing emissions is minimised, because each polluter is free to choose the most efficient way to comply with environmental requirements. This is the so called price-standard-approach which dates back to Baumol and Oates (1988).

Furthermore, generally both the Kyoto's national targets and these environmental instruments have been established on the basis of the well known Polluter Pays Principle (PPP). According to this principle, the polluter should be the agent who is primarily accountable for measures to maintain desired environmental quality levels[2]. However, when analysing the relationships between economic activity and environmental pressures, the PPP would raise a new question about who is actually the polluter, whether the producer or the consumer. That is, it should be important distinguishing between which economic activity generates the atmospheric pollutant and which is responsible

[1] The commitment refers to the aggregation of six gases measured in CO_2 equivalent units. These six gases are: carbon dioxide (CO_2), methane (CH_4), nitrous oxide (N_2O), sulphur hexafluoride (SF_6), hydrofluorocarbons (HFCs) and perfluorocarbons (PFCs).

[2] This economic principle has been established in the 1970's and afterwards, accepted by the Organization for Economic Cooperation and Development (OCDE) and the European Community (EC); nowadays it is a long standing feature of the EU environmental policy (O'Connor, 1997).

for them (Proops *et al.*, 1993). As a consequence of the production vs consumption responsibility distinction, other "pay-principles" have come up, i.e. the User Pays Principle (UPP) and/or the Polluter and User Pay Principle (PUPP). The former advocates that the user of the higher pollutant intensity commodities should pay, whereas the latter shares the responsibility between the producer and the user of these goods and services (Steenge, 1999).

From an accountant point of view, the conceptual PPP and UPP are translated into two methodological accounting principles, which allocate pollution responsibility to a different economic activity: the production accounting principle and the consumption accounting principle. According to the former, the producer is responsible for the atmospheric pollutant emissions caused by the production of energy, goods and services. In that way, emissions are allocated to those processes actually emitting them to the atmosphere (i.e. industrial production, energy production household fuels consumption). In contrast, according to the consumption accounting principle, the responsible for the pollutant emissions is not the economic agent who produces energy, goods or services, but who demands them (Munksgaard and Pedersen, 2001).

This paper evaluates two alternative tax policies aimed at reducing atmospheric pollutant emissions. One based upon an environmental tax that burdens directly firms' emissions, and the other one that burdens both directly and indirectly household consumption's emissions. In this paper, applying input-output approach, we firstly reallocate the emissions generated in the economy according to the responsibility definition, i.e. the production or the consumption accounting principle. Afterwards, we analyse the effects on the products' prices of implementing an ad-quantum environmental tax based on the PPP and/or based on the UPP. The results obtained show that both a PPP environmental tax and an UPP environmental tax has the same effect on the final products' prices, i.e. they raise its prices in the same percentage. However, the price of the intermediate products is only affected by the PPP environmental tax, whereas the UPP environmental tax keeps the prices unchanged.

This study is of relevance for three reasons. First, unlike standard input-output works, we combine the quantity and the price input-output models, showing that the analytical benefit from using the quantity and price input-output models in a complementary way are greater than using them separately. Second, in this paper we develop a theoretical methodology in order to evaluate the effects of two alternative environmental taxes based on the emission responsibility, whether the producer or the consumer. And finally, although the most of the studies introduced an ad-valorem environmental tax, in this paper we consider an ad-quantum environmental tax.

Most of the literature about environmental taxes is based on the PPP and it deals essentially with two issues: the debate on environmental tax reform analysing the way in which environmental tax revenue redistribution takes place, and the analysis of the environmental tax effects on household consumption pattern. On the one hand, some economists have argued that an environmental tax reform consisting of taxing polluting emissions and recycling the so-obtained revenue by reducing other distorting taxes, in such a way that public revenue remains unchanged, can give rise to two potential dividends or benefits. Thus, we can identify two effects yielded by this kind of taxes: on one hand, the dividends originated by improving environmental quality, and on the other, the dividends derived by reducing other existing distortion taxes that may have a positive impact on economic growth, employment or technological development. The first of these dividends is not controversial, but it is very difficult to calculate, whereas the relevance and magnitude of the second is not so clear, and it has opened up a debate and given rise to a large literature labelled as the double dividend hypothesis. Theoretical arguments favouring and neglecting the viability of the double dividend are clearly and concisely stated in Goulder (1995), Bohm (1997), Bovenberg (1999) and De Mooji (1999). The existing empirical studies draw evidence corroborating those ideas Bovenberg and Goulder (1996), Bye (2000), Wender (2001), Kumbaroglu (2003) and Manresa and Sancho (2005).

On the other hand, even when environmental taxes are appealing for pure efficiency grounds, their consequences in terms of competitiveness and distributive impacts may be a fundamental issue determining their political acceptability and, therefore, their actual implementation. That means that the effectiveness of an environmental tax should be evaluated not only in terms of its environmental impact, e.g. the reduction of CO_2 emissions achieved, but also in terms of its effects on firms' structure and household consumption pattern. In consequence, if policies makers are interested in reaching a given environmental target, they should take into account that environmental taxes generate important substitution effects. This may depend on the sensitiveness of pollutant intensive goods to price changes. Thus, if a certain good is relatively insensitive to price changes, emissions will not decrease sufficiently to obtain a given abatement objective. Therefore, in order to favouring the consumption of cleaner goods, it should be important not only charging those pollutant intensive goods, but also charging those with relatively high demand elasticity. Relevant literature analyse the environmental tax distributive impacts and the effects of this environmental instrument on the household consumption structure (Symons *et al.*, 1994; Labeaga and Labandeira, 1999; Labandeira *et al.*, 2004; and Tiezzi, 2005).

Even though the PPP has been accepted as a background principle for environmental policy and management in many countries, some doubts about its effectiveness have sprung from. Thereby, several works have shifted the attention to the user's responsibility, reflecting the idea that environmental issues should be concern of the entire society rather than just the polluter. Therefore, the adoption of the UPP or the PUPP points to other new issues such as the analysis between household consumption pattern and pollutants emission, and/or the emissions embodied in international trade. However, there is few literature combining the UPP and taxes. For the time being and up to our knowledge, only Wier et al. (2005) analyses the distributional effects and the regressiveness of a CO_2 tax imposed on energy consumption in both households and industries.

Regarding the environmental effects of household consumption structure, the general idea is to study the relative responsibility of different types of households in some environmental pressures. Heredeen and Tanaka (1976) and Herendeen *et al.* (1981) were seminal works which studied the energy cost of living for different types of households in USA. They took into account not only the direct demand of energy products but also the even more important indirect energy requirements, i.e. the energy used to produce and distribute goods and services demanded by households. Afterwards, numerous empirical studies draw evidence of the importance of this issue analysing the same question for other countries (Herendeen, 1978; Peet *et al.*, 1985; Wier, 1998; Wilting *et al.*, 1999; Munksgaard *et al.*, 2000; Lenzen, 2001; and Lenzen *et al.*, 2006). Others have gone further on including in the analysis household characteristics, e.g. education level or socioeconomic status, (Vringer and Blok, 1995; Duchin, 1998; Lenzen, 1998; Biesiot and Noorman, 1999; Weber and Perrels, 2000; and Wier *et al.*, 2001).

On the matter of trade and emissions it is especially significant apply the distinction mentioned above, i.e. distinguishing between the economic agent who generates the atmospheric pollutant and who is responsible for them. Clearly, through international trade the consumption of one country is linked to the emissions produced in other countries and therefore, the emissions produced in one country do not have to be the same as the emissions actually generated by its consumption. Within the trade and emission framework, the PPP determines that any country is responsible for those emissions associated with its domestic production regardless where it is going to be consumed, whereas under the UPP the country's responsibility depends on its consumption, i.e. a country is responsible for the emissions generated in order to satisfy the inside final demand regardless where they have been produced. This distinction could have an important implication in environmental international agreements as the Kyoto Protocol[3]. There is some empirical literature following this theoretical framework. Lenzen *et al.* (2004) calculate the CO_2 multipliers and trade balance for five regions (Denmark, Germany, Sweden, Norway, and the rest of the world). Other studies, combining input-output approach and the difference between the producer and/or consumer responsibility concepts, estimate the CO_2 emission embodied in trade: Munksgaard and Perdersen (2001) for Denmark; Machado *et al.*, (2001) for Brazil; and Sánchez-Chóliz and Duarte (2004) for Spain. Finally, some works address the question applying an input-output decomposition method,

[3] These national targets have been established on the basis of the emissions generated by domestic production, neglecting emissions embodied in international trade. It has been argued that, open economies which export pollutant intensive commodities have to make a considerable effort in order to carry out its national target. Therefore, in order to achieve equitable reduction targets, international trade should be taken into account (Munksgaard and Pedersen, 2001).

i.e. *Structural Decomposition Analysis* (SDA). These empirical works are Mukhopadhyay and Chakraborty (1999) for India, Jacobsen (2000) for Denmark and De Haan (2001) for the Netherlands.

Most of the above studies have incorporated the UPP in their analyses are based on input-output analysis. Wassily Leontief in the 1930s established the foundations of input-output approach, which aim was to relate general equilibrium theory to the data (Leontief, 1936). This approach provides a theoretical framework for analysing the relationship between production and consumption sectors of an economy. The capability of this approach to examine this kind of interactions opens the way for studies that deal not only with industrial production but also with other aspects such as the effects of production and consumption on the environment (Leontief, 1970) and in particular on the atmospheric pollution (Leontief and Ford, 1972). Precisely, this is the methodology which we will use in this paper.

The rest of the paper is as follows. In section 2, we develop an environmental extended input-output model, in which both a quantity and a price model have been considered. The quantity input-output model make possible to reallocate the emissions generated in the economy according to the responsibility definition considered, i.e. production vs consumption accounting principle. The price input-output model will allows us to analyse the effects of alternative environment tax policies according to the two "pay-principles" mentioned above, i.e. PPP vs UPP. In section 3, in order to evaluate the two alternative tax policies, we provide a numerical description of a hypothetical economy. And in section 4, we offer some conclusions and point out some further investigations. Finally, in appendix A the standard input-output model is described briefly.

2. The Model

As mentioned above, we develop this paper within the input-output methodological framework. This approach have the capacity of taking into account physical quantities measured in physical units, as it is the case of the atmospheric emissions. However, this potential has not been fully exploited because input-output models are typically implemented using monetary data base. Moreover, it is generally accepted that the general form of the basic input-output model is summarised in one equation measured only in money values; when in fact, it is composed by a set of three expressions: one related with the quantity model, another of the price model and an income equation[4]. The quantity model tracks flows of products throughout the economy, the price model determines

[4] A detailed description of the standard input-output model is given in appendix A.

their unit prices, and finally, the income equation assures that the value of final deliveries is equal to total value-added.

In this paper we use both the quantity and the price input-output models in a complementary way. On the one hand, the quantity model will allow us to reallocate the emissions generated in the economy according to the responsibility definition, i.e. applying the production or the consumption accounting principle (section 2.2). On the other hand, the price model will allow us to analyse the effects of two alternative environment tax policies: one based upon the PPP and the other on the UPP (section 2.3). But before going deeply in theses aspects, we will describe the basic characteristics of this simple economy in section 2.1.

2.1. The Economy

Let us considerer a small closed economy composed of *n* industries. Theses industries can be divided into two groups: the so-called type I-industries, which only produce intermediate commodities that are delivered as inputs to other industries; and type II-industries that produce final commodities, i.e. their production are exclusively addressed to the final demand components. In fact, this distinction followed the standard classification of CPA and COICOP products, and it is a very important in the model since, in fact, consumers do not purchased CPA but COICOP products[5]. Furthermore, the latter will allow us to design two alternative environmental tax policies.

The *m* type I-industries produce intermediate goods by combining intermediate inputs and primary factors, while the *p* type II-industries produce final goods only by combining intermediate goods. The intermediate commodities and the primary factors are used in fixed proportions according to a Leontief technology and therefore it is assumed that all industries operate under constant returns to scale. Thus, the production technology of this economy can be represented by the technical coefficient matrix A of dimension *nxn*, i.e. $(m+p)x(m+p)$, whose $\{a_{ij}\}$ elements represent the input delivery from sector *i*th to *j*th per unit of sector *j*'s output.

The final demand *nx1* vector of the economy includes the three major components: private consumption *nx1* vector c, public consumption *nx1* vector g, and gross fixed capital formation *nx1* vector f. For all commodities taken together, this can be represented by:

$$y = c + g + f \qquad (1)$$

[5] CPA is the acronim of *Classification of Products and Activities* and COICOP is the acronim of *Classification of Individual Consumption by Purpose.*

The domestic gross output *nx1* vector x is defined by the intermediate demand Ax and the final demand y. Thus we have:

$$x = Ax + y \tag{2}$$

In equilibrium, total supply equals total demand, thereby the balance equation for the economy can therefore be straightforward written as:

$$x = Ax + c + g + f \tag{3}$$

Given the above economic specifications, we can define the environmental characteristics of this economy. We consider that only type I-industries generate atmospheric pollution directly through their production process. So, neither type II-industries nor final demand components are direct pollutants, although they can be considered indirect pollutants since they use directly or indirectly type I-industries commodities.

Thus, we can define it in matrix terms. Let us consider that $B = \left[B^{I} \middle| B^{II} \right]$ of dimension *kxn* is the atmospheric emission matrix, whose $\{b_{lj}\}$ elements represent the amount of pollutant l emitted by industry j measured in physical units. Such as is indicated, B is made up by B^{I} *kxm* matrix and B^{II} *kxp* matrix. According to the above environmental characteristics, B^{I} represents the pollution generated by type I-industries; likewise, B^{II} is the pollution generated by type II-industries and obviously whose elements are all zero.

From this matrix B, we can specify the atmospheric emission coefficient matrix V of dimension *kxn* as:

$$V = B\hat{x}^{-1} \tag{4}$$

Where $\hat{x}^{-1}$ is the diagonal matrix gross output. Each v_{lj} th element of matrix V represent the emissions of pollutant l emitted per unit of industry *j*'s output.

2.2. The Quantity Input-Output Model

Bearing in mind expression (2) and since the final demand y is an exogenous variable, the quantity input-output model has a simple solution for output:

$$x = (I - A)^{-1} y \tag{5}$$

Where I is the identity matrix of dimension *nxn*, and $(I-A)^{-1}$ is the Leontief inverse matrix whose α_{ij}th element is the partial derivative of industry *i*'s gross output with respect to final demand on industry *j*, i.e. $\alpha_{ij} = \partial x_i / \partial y_j$. This matrix has an important economic significance, since the column sum of the Leontief inverse $\sum_i \alpha_{ij}$ shows the direct and indirect effects on the economy when the final demand of an industry increases by one unit remaining all other final demands' industries unchanged.

According to the input-output model there is no restriction in expression (5) on the choice of units for measuring output, whether physical or monetary units, nor does it require that all quantities be measured in the same unit. Therefore, each sector's output can be quantified in a unit appropriate for measuring the characteristic product of that sector, i.e. tonnes, kWh, numbers of standard units, or money's worth of sector output[6].

Thereby, bearing in mind expression (4) we can therefore define the atmospheric emissions produced in the economy as:

$$E = Vx \tag{6}$$

Where E is the total emission vector of dimension *kx1*. Replacing x with expression (5) the atmospheric emissions can also be computed depending on the final demand y. In this case we have:

$$E = \left[V(I-A)^{-1} \right] y \tag{7}$$

Now, vector E shows both direct and indirect emissions required to fulfil the final demand. The expression into brackets $V(I-A)^{-1}$ has an especial meaning: it is the total emission intensity matrix of dimension *kxn*, whose elements are the emission multipliers that measure the amount of pollutant l caused by exogenous and unitary inflows to the final demand of sector *j*.

Producer accounting principle vs Consumer accounting principle

The above quantity input-output model allows us to apply both accounting principles. That is, we can share out the total emissions E among the different industries according to the production or to the consumption accounting principles, depending on who is considered responsible for the emissions. In fact, the production accounting principle will allocate the

[6] This might be the case of some sectors, which have output mixes that are so heterogeneous as to be more usefully measured in the money value of output. Moreover, since the statistical data are not prepared in physical unit (with the notable exception of China), all input-output tables are prepared in value units but choosing quantities such that their price is unity, i.e. using the called Leontief units.

atmospheric emission to those industries that deliver their output exclusively to other industries, i.e. type I-industries. While the consumption accounting principle will allocate them to those industries that deliver their output exclusively to final demand, i.e. type II-industries.

Let us define v'_l as the row vector of dimension *1xn* of the atmospheric emission coefficient matrix V , whose elements represent the emissions of the considered pollutant l emitted per unit of industry *j*'s output. Thus, we can calculate the emission distribution matrix D_l of dimension *nxn* for each pollutant l by the following expression:

$$D_l = \left[\hat{v}(I-A)^{-1}\right]\hat{y} \tag{8}$$

Where, $\hat{y}$ is the diagonal matrix (*nxn*) of the final demand vector; and $\hat{v}$ is the diagonal matrix (*nxn*) of the emission coefficient of pollutant l vector.

For each pollutant, the column sum of this emission distribution matrix D_l gives the total emissions according to the production accounting principle vector E_l^P of dimension *nx1*:

$$E_l^P = \left[\hat{v}(I-A)^{-1}\hat{y}\right]i \tag{9}$$

Whereas the row sum of this matrix shows the total emissions according the consumption accounting principle vector E'^C_l of dimension *1xn*:

$$E'^C_l = i'\left[\hat{v}(I-A)^{-1}\hat{y}\right] \tag{10}$$

Where i is a *nx1* column vector of ones, and i' is a *1xn* row vector of ones, i.e. the transpose of i .

As in expression (4) we can calculate the emission coefficient according both principles. Thereby, given $\hat{x}^{-1}$ the diagonal matrix of gross output, the emission coefficient for each pollutant l will be defined according to the production accounting principle by the *nx1* vector e_l^P and according to the consumption accounting principle by the *1xn* vector e_l^C . Hence, we have respectively:

$$e_l^P = E_l^P \hat{x}^{-1} \tag{11}$$

$$e_l^C = E_l^C \hat{x}^{-1} \tag{12}$$

2.3. The Price Input-Output Model

As it pointed above, the basic input-output model is made up by the quantity model and the price model. When, some of the variables of the quantity model are measured in non-monetary physical units, the price model determines the unit price of products, i.e. money values per physical unit. However, the latter is widely ignored because the quantity model is usually implemented in monetary units (a special case of quantity unit). In this case, each component of the quantity model represents the monetary value, i.e. the product of a quantity and a unit price, and therefore all products' prices in the price model would be 1.0.

Under theses circumstances, the general belief is that there is no benefit from consider a specific and separate price input-output model. Nevertheless, even in extreme cases where the whole quantity model is measured in monetary units, the price model provides information about the impact on unit prices not only of changes in technical coefficients or in value-added per unit of output, but also of the introduction of taxes.

Similarly to expression (i) in Appendix A, the value of output is by definition equal to the value of inputs. For any *j*th sector it can be written as:

$$x_j p_j = x_{1j} p_1 + x_{2j} p_2 + \cdots + x_{nj} p_n + \mathrm{L}_j w_j + \prod_j \tag{13}$$

Where x_{ij} is the amount of *i*th commodity delivered by *i*th industry to *j*th industry, p_j is the price per unit of quantity, L_j is the amount of physical labour input, w_j is the wage rate and $\prod_j$ is a residual equivalent to gross operating surplus. Since $x_{ij} = a_{ij} x_j$ and dividing all the expression by x_j , (13) can be rewritten as:

$$p_j = a_{1j} p_1 + a_{2j} p_2 + \cdots + a_{nj} p_n + 1_j w_j + \pi_j \tag{14}$$

Hence, in matrix terms:

$$p = A' p + \varphi \tag{15}$$

Where the matrix A' of dimension *nxn* is the transpose of the technical coefficient matrix A, and p and φ are *nx1* vectors: p are unit prices, and φ is value-added per unit of output.

Rewriting the above equation (15) and similarly to expression (5) we obtain the solution of the price input-output model[7]:

$$p = (I - A')^{-1}\varphi \qquad (16)$$

PPP Environmental Tax vs UPP Environmental Tax

The general idea to design an environmental tax in order to reduce the atmospheric emissions of any pollutant is to provide an incentive for consumers and/or firms to substitute the consumption of those commodities that have the highest pollutant intensity.

In this paper, the environmental tax is specified as a number of monetary units per physical unit of pollutant generated by the production of each commodity, that is we use and ad-quantum environmental tax. Thereby, an environmental tax rate τ is placed on the atmospheric emissions generated by one unit of output, i.e. the emission coefficient e_l^P or e_l^C depending on the "pay-principle" applied.

Thus, if it is considered that the producer is responsible for the atmospheric pollutant emissions caused and, in consequence it should pay, the PPP environmental tax for each pollutant l can be expressed by the following *nx1* vector:

$$T_l^{PPP} = \tau_l e_l^P \qquad (17)$$

On the contrary, if it is considered that the consumer should pay because it is responsible for the atmospheric pollutant emissions caused by its demand, the UPP environmental tax *nx1* vector for each pollutant l is:

$$T_l^{UPP} = \tau_l e_l^C \qquad (18)$$

Once the price model and taxes have been defined, we can examine the effects of a PPP environmental tax or an UPP environmental tax on products prices.

According to the European System of National and Regional Accounts (ESA 95) there are two sorts of taxes on production: the *taxes on products* and the *other taxes on production*. The former in fact assessed on a product, the

[7] It is important to bear in mind that if the quantity model is expressed in non-monetary physical units, p vector represents the unit price of products but if the whole quantity model is measured in monetary units, then p vector would be 1.0.

latter consists of all taxes that industry incurs as a result of engaging in production, independently of the quantity or value of the production. Taxes on pollution resulting from production activities are included in the last group (EUROSTAT, 1995: 4.23-f).

Hence, modelling the environmental tax according to the PPP and/or the UPP and considering it a tax on production, the new price vector would be calculated as:

$$p_I^{PPP} = (I - A')^{-1}(\varphi + T_l^{PPP}) \quad (19)$$

$$p_I^{UPP} = (I - A')^{-1}(\varphi + T_l^{UPP}) \quad (20)$$

On the other hand, if the environmental tax were considered a tax on products, the above equations will be modified in the next terms:

$$p_{II}^{PPP} = (I - A')^{-1}(\varphi + A'T_l^{PPP}) \quad (21)$$

$$p_{II}^{UPP} = (I - A')^{-1}(\varphi + A'T_l^{UPP}) \quad (22)$$

3. A Numerical Example

This section provides a numerical description of a hypothetical economy in order to evaluate the two alternative tax policies. The example provides a concrete illustration of the concepts described earlier; that is, the production vs consumption accounting principles and the producer and user pays principles. It also shows the potentiality of combining the quantity and price input-output models in a complementary way.

The following table 1 reproduces the main economic features: this hypothetical economy produces three intermediate goods (S1, S2 and S3) and two final goods (S4 and S5). The type I-industries use intermediate commodities (S1, S2 and S3) and primary factors (only labour); whereas type II-industries only use intermediate commodities (S1, S2 and S3) in order to produce their output. Thereby, the S4 and S5 commodities do not enter in any production process. For simplicity, the final demand of the economy only consumes final goods. Finally, in order to make this example more realistic, all the components of this economy are measured in monetary values, with the exception of emissions that are measured in tonnes of atmospheric pollutant (e.g. tonnes of CO_2).

Table 1: Standard Input-Output Table of the Economy

Units: Monetary units

	S1	S2	S3	S4	S5	FINAL DEMAND	TOTAL OUPUT
S1	35.00	40.00	50.00	35.00	25.00	0.00	185.00
S2	85.00	10.00	30.00	45.00	30.00	0.00	200.00
S3	20.00	10.00	10.00	30.00	200.00	0.00	270.00
S4	0.00	0.00	0.00	0.00	0.00	110.00	110.00
S5	0.00	0.00	0.00	0.00	0.00	255.00	255.00
Value Added	45.00	140.00	180.00	0.00	0.00		
TOTAL OUTPUT	185.00	200.00	270.00	110.00	255.00		

From this table we define matrices A and $(I-A)^{-1}$:

$$A = \begin{pmatrix} 0.19 & 0.20 & 0.19 & 0.32 & 0.10 \\ 0.46 & 0.05 & 0.11 & 0.41 & 0.12 \\ 0.11 & 0.05 & 0.04 & 0.27 & 0.78 \\ 0.00 & 0.00 & 0.00 & 0.00 & 0.00 \\ 0.00 & 0.00 & 0.00 & 0.00 & 0.00 \end{pmatrix} \quad (I-A)^{-1} = \begin{pmatrix} 1.46 & 0.32 & 0.32 & 0.68 & 0.43 \\ 0.73 & 1.22 & 0.28 & 0.81 & 0.44 \\ 0.20 & 0.10 & 1.09 & 0.40 & 0.89 \\ 0.00 & 0.00 & 0.00 & 1.00 & 0.00 \\ 0.00 & 0.00 & 0.00 & 0.00 & 1.00 \end{pmatrix}$$

And its corresponding transposes A' and $(I-A')^{-1}$:

$$A' = \begin{pmatrix} 0.19 & 0.46 & 0.11 & 0.00 & 0.00 \\ 0.20 & 0.05 & 0.05 & 0.00 & 0.00 \\ 0.19 & 0.11 & 0.04 & 0.00 & 0.00 \\ 0.32 & 0.41 & 0.27 & 0.00 & 0.00 \\ 0.10 & 0.12 & 0.78 & 0.00 & 0.00 \end{pmatrix} \quad (I-A')^{-1} = \begin{pmatrix} 1.46 & 0.73 & 0.20 & 0.00 & 0.00 \\ 0.32 & 1.22 & 0.10 & 0.00 & 0.00 \\ 0.32 & 0.28 & 1.09 & 0.00 & 0.00 \\ 0.68 & 0.81 & 0.40 & 1.00 & 0.00 \\ 0.43 & 0.44 & 0.89 & 0.00 & 1.00 \end{pmatrix}$$

The gross output x, the final demand y, and the value added per unit of output φ vectors are obtained straightforward:

$$x = \begin{pmatrix} 185.00 \\ 200.00 \\ 270.00 \\ 110.00 \\ 255.00 \end{pmatrix} \quad y = \begin{pmatrix} 0.00 \\ 0.00 \\ 0.00 \\ 110.00 \\ 255.00 \end{pmatrix} \quad \varphi = \begin{pmatrix} 0.24 \\ 0.70 \\ 0.67 \\ 0.00 \\ 0.00 \end{pmatrix}$$

Given the emission coefficient of one atmospheric pollutant generated by type I-industries by:

$$v'_l = (0.14 \quad 0.05 \quad 0.02 \quad 0.00 \quad 0.00)$$

We can calculate the emission distribution matrix D_l and straight afterwards the total emissions according to both the production accounting principle E_l^P and the consumption accounting principle E_l^P. So, the economic table 2 can be extended in order to gather the environmental information. Notice that in this case we combine monetary and physical units:

Table 2: Environmental Extended Input-Output Table of the Economy

Units: Monetary units and tonnes of pollutant.

	S1	S2	S3	S4	S5	FINAL DEMAND	TOTAL OUPUT
S1	35.00	40.00	50.00	35.00	25.00	0.00	185.00
S2	85.00	10.00	30.00	45.00	30.00	0.00	200.00
S3	20.00	10.00	10.00	30.00	200.00	0.00	270.00
S4	0.00	0.00	0.00	0.00	0.00	110.00	110.00
S5	0.00	0.00	0.00	0.00	0.00	255.00	255.00
Value Added	45.00	140.00	180.00	0.00	0.00		
TOTAL OUTPUT	185.00	200.00	270.00	110.00	255.00		
Emissions:							
Producer Principle	**25.00**	**10.00**	**5.00**	**0.00**	**0.00**	-	**40.00**
Consumer Principle	**0.00**	**0.00**	**0.00**	**15.43**	**24.57**	-	**40.00**

The emission coefficient vector e_l^P according to the production accounting principle is:

$$e_l^P = \begin{pmatrix} 0.14 \\ 0.05 \\ 0.02 \\ 0.00 \\ 0.00 \end{pmatrix}$$

Whereas the corresponding to the consumption accounting principle vector of emission coefficient e_l^C is:

$$e_l^C = \begin{pmatrix} 0.00 \\ 0.00 \\ 0.00 \\ 0.14 \\ 0.10 \end{pmatrix}$$

Given an environmental tax rate $\tau = 1$ and applying expressions (17) and (18), the PPP environmental tax T_l^{PPP} and the UPP environmental tax T_l^{UPP} can be represented straightforward by the above emission coefficients. That is:

$$T_l^{PPP} = \tau e_l^P = 1 \begin{pmatrix} 0.14 \\ 0.05 \\ 0.02 \\ 0.00 \\ 0.00 \end{pmatrix} = \begin{pmatrix} 0.14 \\ 0.05 \\ 0.02 \\ 0.00 \\ 0.00 \end{pmatrix} \qquad T_l^{UPP} = \tau e_l^C = 1 \begin{pmatrix} 0.00 \\ 0.00 \\ 0.00 \\ 0.14 \\ 0.10 \end{pmatrix} = \begin{pmatrix} 0.00 \\ 0.00 \\ 0.00 \\ 0.14 \\ 0.10 \end{pmatrix}$$

As already mentioned in section 2.3, since the whole quantity model is measured in monetary units, the price vector would be 1.0:

$$p = (I - A')^{-1} \varphi = \begin{pmatrix} 1.46 & 0.73 & 0.20 & 0.00 & 0.00 \\ 0.32 & 1.22 & 0.10 & 0.00 & 0.00 \\ 0.32 & 0.28 & 1.09 & 0.00 & 0.00 \\ 0.68 & 0.81 & 0.40 & 1.00 & 0.00 \\ 0.43 & 0.44 & 0.89 & 0.00 & 1.00 \end{pmatrix} \begin{pmatrix} 0.24 \\ 0.70 \\ 0.67 \\ 0.00 \\ 0.00 \end{pmatrix} = \begin{pmatrix} 1.00 \\ 1.00 \\ 1.00 \\ 1.00 \\ 1.00 \end{pmatrix}$$

Now, we have all the necessary elements to analyse the effects on the price vector of the two alternative environmental taxes: one based on the PPP and the other on the UPP. We suggest distinguishing two scenarios: one more realistic in which the environmental tax is considered a tax on production; and another more hypothetical in which it were considered a tax on product.

Under the *scenario I* and according to expressions (19) and (20) the new prices after the environmental taxation are respectively:

$$p_I^{PPP} = (I - A')^{-1} (\varphi + T_l^{PPP}) = \begin{pmatrix} 1.237 \\ 1.107 \\ 1.077 \\ 1.140 \\ 1.096 \end{pmatrix} \qquad p_I^{UPP} = (I - A')^{-1} (\varphi + T_l^{UPP}) = \begin{pmatrix} 1.000 \\ 1.000 \\ 1.000 \\ 1.140 \\ 1.096 \end{pmatrix}$$

The PPP environmental tax affects both intermediate and final product prices, whereas the UPP environmental tax only has effects on the final product price. It should stress that with both taxes the price of S4 and S5 final products are increased in 14.0% and 9.6% respectively.

These results might have important economic implications, since the PPP environmental tax would provide an incentive for firms to substitute their old technology with a new one less pollutant, whenever the cost of the new technology was lesser than the cost of paying the environmental tax. However, this situation does not exist with the UPP environmental tax. Likewise, both PPP and UPP environmental taxes would give an incentive to consumers for changing their consumption pattern substituting the consumption of those commodities that have highest pollutant intensity.

Although the *scenario II* is a hypothetical situation, it seems important evaluate which would be their results. According to expressions (21) and (22), we have:

$$p_{II}^{PPP} = (I - A')^{-1}(\varphi + A'T_l^{PPP}) = \begin{pmatrix} 1.102 \\ 1.057 \\ 1.059 \\ 1.140 \\ 1.096 \end{pmatrix} \qquad p_{II}^{UPP} = (I - A')^{-1}(\varphi + A'T_l^{UPP}) = \begin{pmatrix} 1.000 \\ 1.000 \\ 1.000 \\ 1.000 \\ 1.000 \end{pmatrix}$$

If the environmental tax were modelled as a tax on products the new prices after taxation will be rather different. Firstly, it is quite surprising that under the UPP the environmental tax would not have any effect on prices, neither intermediate nor final product prices. Then, under the PPP the environmental tax has the same impact on the final product prices as the both taxes in *scenario I*. On the other hand, although the prices of the intermediate commodities (S1, S2 and S3) would increase, the increasing would be lesser than in the previous situation. However, in relative terms, the product S3 would be more damage with the *scenario II* than with *scenario I*.

4. Final Remarks

This paper, adopting an input-output approach, evaluates two alternative tax policies aimed at reducing atmospheric pollutant emissions. One based upon an environmental tax that burdens directly firms' emissions, and the other one that burdens both directly and indirectly household consumption's emissions. The difference between these two environmental tax systems is due to the existence of two methodologically accounting principles: the production principle and the consumption principle. According to the production accounting principle, the producer is responsible for the atmospheric pollutant emissions caused by the production of energy, goods and services. In that way, emissions are allocated to those processes actually emitting pollutants to the atmosphere (i.e. industrial production, energy production or household fuels consumption). In contrast, according to the consumption accounting principle, the responsible for the emissions is not the economic agent who produces energy, goods or services, but who demands them.

In order to achieve this aim, we combine both the expression of the quantity input-output model and the expression of the price input-output model. Unlike others input-output works, we think that the analytical benefit from using the quantity and price input-output models in a complementary way are greater than using them separately. In fact, in this paper by applying the quantity model we reallocate the emissions generated in the economy according to the responsibility definition, i.e. applying the production or the consumption accounting principle. This step is necessary in order to evaluate the effects of two different environmental tax policies. This evaluation is carried out by using the price input-output model. Concretely, we analyse the effects on the products prices of implementing an ad-quantum environmental tax based on the PPP or based on the UPP.

Moreover, in this paper we provide a concrete illustration of the concepts described earlier. For evaluating the two alternative tax policies mentioned above, it is presented a numerical example of a hypothetical economy. The results obtained show that both a PPP environmental tax and an UPP environmental tax has the same effect on the final products prices, i.e. they raise its prices in the same percentage. However, the price of the intermediate products is only affected by the PPP environmental tax, whereas the UPP environmental tax keeps the prices unchanged.

As long as theses results could be generalised, they might have important implications not only in the economy level but also in the political sphere. Whenever policy makers fix an environmental tax rate high enough, the PPP environmental tax would provide an incentive for firms to substitute their old technology with a new one less pollutant. On the contrary, the UPP environmental tax does not have the same effect on the firms costs.

On the other hand, the final price paid by the consumers, or other final demand components, is the same whatever the environmental tax applied. Therefore, both the PPP and the UPP environmental taxes would give an incentive to consumers for changing their consumption pattern substituting the consumption of those commodities that have highest pollutant intensity.

The outcomes obtained in this paper suggest that further investigation in this direction is needed. In fact, within the input-output approach, it might be two possibilities to extend this study. On one the hand, it would be interesting to analyse the after tax price effects on the quantity input-output model. That is, to measure the impact of the two alternatives environmental taxes on the amount of atmospheric pollutant emitted (Stone, 1972). On the other hand, since most of the results are due to the perfect competition of the economy, the second possibility would be to introduce imperfect competition in some of the sectors.

However, due to the complexity involved by the implementation of tax environmental policies, we also think that it is required to deal with them adopting an approach more capable of handling such complex interrelation. Accordingly, in a future we are interested in evaluating the applied general equilibrium models as a suitable approach to cope with tax environmental policy analysis. In general, we can state some advantages of following this methodology. First of all, these kind of models allow us to take into account the simultaneous effects produced by implementing a particular policy. In addition, these models permit not only to determine the sign of such effects, but also to specify their magnitude. On the other hand, it can be said that applied general equilibrium models allow for the analysis of income distribution effects caused by the execution of certain policy. Finally, as Shoven and Whalley (1984) has shown, applied general equilibrium models turn out to be particular useful for evaluating tax policy reforms.

Concretely, we think it will be interesting to evaluate the effects on economy performance derived from the two alternative environmental tax reforms described in this paper in a real economy, such Spain. In future works, we will intent to compute a static and multi-sector applied general equilibrium model by considering the Spanish setting. For doing so, we will use the Spanish Social Accounting Matrix (SAM) corresponding to 2000 (Lucena and Serrano, 2005). In addition, for modelling the Spanish environmental characteristics, we also will use the 2000 Spanish National Accounting Matrix including Environmental Accounts (NAMEA) (Roca and Serrano, 2006). This fact will allow us to go from the theoretical structure involved in a general equilibrium model to the empirical information gathered in the "System of National Accounts", SNA. In that way, it will be possible to both determine and quantify the effects caused by the set of environmental tax policies analysed in this research.

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Appendix A: The open static input-output model

Wassily Leontief in the 1930s established the foundations of input-output analysis, which aim was to relate general equilibrium theory to the data (Leontief, 1936). This approach provides a theoretical framework for analysing the relationship between production and consumption sectors of an economy. In the well known open static input-output model, the economic structure is defined by a set of *n* industries, which deliver goods and services to each other and to final consumers:

$$\begin{aligned} x_1 &= x_{11} + x_{12} + \cdots + x_{1n} + y_1 \\ x_2 &= x_{21} + x_{22} + \cdots + x_{2n} + y_2 \\ &\vdots \\ x_n &= x_{n1} + x_{n2} + \cdots + x_{nn} + y_n \end{aligned} \quad \text{(i)}$$

Where x_i is the output of the *i*th sector, x_{ij} represents the quantity of product delivers form the *i*th sector to the *j*th sector and y_i is final demand for the *i*th sector. In the standard open static input-output model[8], y_i comprises private consumption (c_i), public consumption (g_i) and capital formation (f_i). Assuming an open economy, y_i also includes net exports –i.e. exports (e_i) of the *i*th commodity as a positive entry and imports (m_i) of the *i*th commodity as a negative entry-; thus x_{ij} is total absorptions of the *i*th commodity by the *j*th sector inclusive both domestic and imported.

In input-output work, a fundamental assumption is that the relationship between inputs and outputs is assumed to be linear proportional, i.e. constant returns to scale, and we can write the relationship as:

$$x_{ij} = a_{ij} x_j \quad \text{(ii)}$$

Where a_{ij} is the technical or input-output coefficient. Then, the specification of the production function that is embodied in the input-output model is:

$$x_j = \min\left(\frac{x_{1j}}{a_{1j}}, \frac{x_{2j}}{a_{2j}}, \cdots, \frac{x_{nj}}{a_{nj}} \right) \quad \text{(iii)}$$

[8] The term "open" is used to differentiate this model from the "close" version, in which all variables of the model are endogenous.

Substituting the equation (ii) into expression (i) it can be rewritten the output of each sector depending on the final demand for that sector and the output of all other sectors:

$$\begin{aligned} x_1 &= a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n + y_1 \\ x_2 &= a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n + y_2 \\ \vdots \;\; & \;\; \vdots \quad\;\; \vdots \qquad\quad\;\; \vdots \quad\;\; \vdots \\ x_n &= a_{n1}x_1 + a_{n2}x_2 + \cdots + a_{nn}x_n + y_n \end{aligned} \qquad \text{(iv)}$$

And in matrix terms:

$$x = Ax + y \qquad \text{(v)}$$

Where A_{nxn} is the technical coefficient matrix, whose $\{a_{ij}\}$ elements are defined as the delivery from sector *i* to *j* per unit of sector *j*'s output. x_{nx1} is the gross output vector and y_{nx1} is the final demand vector. Given y, we can solve for the vector of gross outputs from expression (v):

$$x = (I - A)^{-1} y \qquad \text{(vi)}$$

Where I_{nxn} is the identity matrix and $(I - A)^{-1}$ the Leontief inverse matrix. This matrix has an important economic significance; let us define this inverse as:

$$(I - A)^{-1} = L = \{a_{ij}\} = \{\partial x_i / \partial y_j\} \qquad \text{(vii)}$$

Where α_{ij} is the *ij*th element of the Leontief inverse. Because of the strict linearity of equation (vi), the *ij*th element of the Leontief inverse is the partial derivative of x_i with respect to y_j. If, therefore, y_j increases by one unit –all other final demands unchanged- the total effect on the productive system, or the increase in the gross output of al sectors, is captured by the expression $\sum_i \alpha_{ij}$. Thus, the column sum of the Leontief inverse shows the direct and indirect effects on the economy of a unit change in final demand for the sector shown at the head of column. Similarly, $\sum_j \alpha_{ij}$, i.e. the row sum of the Leontief inverse, shows the total effect on the *i*th sector when each final demand increases by unit.

In the derivation of the input-output model, we have made only one assumption, namely that there are fixed coefficients of production; if constant returns to scale are also assumed, we get the relationship $x_{ij} = a_{ij}x_j$, although this additional assumption is not strictly necessary. However, by interpreting the Leontief assumption in the way suggested above, we have in fact made two further assumptions, which warrant discussion.

First, we have assumed that when the *j*th final demand increases, only the *j*th sector's output is affected in the first instance. This will only be the case if there are no joint products, so that the principal product of the *j*th sector must not be produced elsewhere as a secondary or by-product; in fact, such cases of joint production are quite common in the real world in order to bring the input-output table into harmony with the assumptions of the model some changes in the data are needed.

Secondly, we have to assume that there are no external economies or diseconomies associated with the production process. The production function for the *j*th sector is therefore of the form:

$$x_j = F\left(x_{1j}, x_{2j}, \cdots, x_{nj}, w_{1j}, w_{2j}, \cdots, w_{nj}\right) \qquad \text{(viii)}$$

Where w_{ij} is the input of the *i*th primary factor of production. If, on the other hand, technological external (dis)economies are present such that the output of the *j*th sector depends also upon the output and input utilization of the *k*th sector, then the simple linear system of equations breaks down.

The above model is often treated as comprising the entire analytic core of input-output economics. In fact, the familiar equation (vi) in only an abbreviation form of the basic input-output model. The full model created by Leontief for an economy described in terms of *n* sectors requires two more equations:

$$x = (I - A)^{-1} y \qquad \text{(vi)}$$

$$p = (I - A')^{-1} \varphi \qquad \text{(ix)}$$

$$p' y = \varphi' x \qquad \text{(x)}$$

Where A' of dimension *nxn* is the transpose matrix of the technical coefficients matrix A; p is the unit price *nx1* vector and φ is also a *nx1* vector which represents the value added per unit of output.

An input-output model places no restriction on the choice of units for measuring output, whether physical or monetary units, nor does it require that all quantities be measured in the same unit. The resulting table, and the coefficient matrix derived from it, can be constructed with no conceptual difficulty in a mix of units.

Equation (vi) is called a quantity input-output model. If variables are measured in physical quantities, the corresponding technical coefficients are ratios of physical units (e.g. tonnes of iron per machine). If y is given, the solution vector x represents the quantities of sectoral outputs. Equation (ix) is the input-output price model, and the components of the vector of unit prices are prices per unit of product (e.g. price per tonnes of iron or price per machine). For a sector whose output is measured in monetary units in equation (vi), for example business services, the corresponding unit price is simply 1.0. Finally equation (x), called the income equation, is derived form the first two: this identity assures that the value of final deliveries is equal to total value-added, not only in the actual base-year situation for which the data have been collected but also under scenarios where values of parameters and exogenous variables are changed.