Integrating Bottom-Up into Top-Down: A Mixed Complementarity Approach

Christoph Böhringer

Centre for European Economic Research (ZEW), Mannheim, Germany
Department of Economics, University of Heidelberg, Germany

(based on past and ongoing work with Thomas F. Rutherford)

- Motivation
- Mixed Complementarity
- From Bottom-up to Top-Down
- Illustration
- Conclusion

Overview

Impact Assessment of Energy Policies

- ➢ **Motivation**
- **Mixed Complementarity**
- **From Bottom-up to Top-Down**
- **Illustration**
- **Conclusion**

Complementary (hybrid) modeling framework:

- Comprehensive coverage of markets:
  – interactions, distortions, imperfections
- Incorporation of income flows:
  – origination and spending of income (endowments and preferences)

↓

Top-down (general equilibrium)

- Technological foundation:
  – discrete technological options

↓

Bottom-up (partial equilibrium)

↓

Synthesis of Top-down and Bottom-up

Dichotomy of Top-down and Bottom-Up

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Policy focus and availability of solvers ⇒ mathematical format

- **Top-down:** system of equations
  - \+ equilibrium constraints in prices and quantities
  - – activity analysis, weak inequalities
- **Bottom-up:** mathematical programs
  - \+ activity analysis, weak inequalities
  - – integrability restrictions

Excursus: Integrability

- Equivalence of first order conditions with equilibrium conditions:
  - coincidence of shadow price of mathematical programming constraints with market prices
  - restrictive symmetry and efficiency properties of mathematical programs:
    - symmetry of (cross-price) demand elasticities
    - omission of multiple agents (income effects)
    - efficient allocation <==> taxes, price caps, spillover externalities
  - sophisticated sequential joint maximization (SJM) techniques to overcome „non-integrabilities" in optimization approach

Framework for Synthesis: Mixed Complementarity Problem (MCP) Format (Rutherford 1995, JEDC)

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Mixed Complementarity Problem (MCP):

$$\begin{aligned}
&Given: && f: R^N \to R^N, \quad l,u \in R^N \\
&Find: && z,w,v \in R^N \\
&s.t.: && F(z) - w + v = 0 \\
& && l \le z \le u, \quad w \ge 0, \quad v \ge 0, \\
& && w^T(z-l) = 0, \quad v^T(u-z) = 0
\end{aligned}$$

Mixed: Mixture of equalities and inequalities

Complementarity: Complementarity between system variables and system conditions

+ coverage of system of equations and mathematical programs as subcases
+ equilibrium constraints in prices and quantities (no integrability restrictions)
+ activity analysis, weak inequalities
+ availability of large-scale robust solvers (PATH)

The Arrow-Debreu-Model as MCP

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p := a non-negative n-vector of prices for all goods and factors (I={1,...,n})

y := a non-negative m-vector of activity levels for CRTS production sectors (J={1,...,m})

M := a non-negative k-vector of incomes (H={1,...,k})

Zero profit condition for CRTS producers:

$$-\Pi_j(p) = C_j(p) - R_j(p) \ge 0 \quad \forall j$$

Market clearance for all goods and factors:

$$\sum_j y_j \frac{\partial \Pi_j(p)}{\partial p_i} + \sum_h b_{ih} \ge \sum_h d_{ih} \quad \forall i$$

Budget constraints for households:

$$\sum_h p_i b_{ih} = M_h \ge \sum_h p_i d_{ih} \quad \forall h \qquad d_{ih}(p, M_h) \equiv \arg\max \left\{ U_h(x) \,\middle|\, \sum_i p_i x_i = M_h \right\}$$

Complementarity Features of Economic Equilibria

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Walras' law („Non-satiation") yields:

$$\sum_j y_j \Pi_j(p) = 0 \qquad \text{resp.} \qquad y_j \Pi_j(p) = 0 \quad \forall j$$

$$p_i \left(\sum_j y_j \frac{\partial \Pi_j(p)}{\partial p_i} + \sum_h b_{ih} - \sum_h d_{ih} \right) = p_i \xi_i = 0 \quad \forall i$$

$$M_h \left(\sum_h p_i b_{ih} - \sum_h p_i d_{ih} \right) = 0 \quad \forall h$$

Ergo: The problem of solving the economic equilibrium corresponds to a MCP where:

$$z = [y, p, M] \qquad \text{resp.} \qquad f(z) = \left[\Pi_j(p), \xi_i, \left(\sum_h p_i b_{ih} - \sum_h p_i d_{ih} \right) \right]$$

Economic Equilibrium Problem as MCP

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Equivalence of market equilibrium problem with complementarity problem:

$$Given: f : R^n \rightarrow R^n$$
$$Find: z \in R^n$$
$$subject\ to: f(z) \geq 0,\ z \geq 0,\ z^T f(z) = 0$$

$$l = 0,\ u = +\infty, \quad z = [y, p, M],\ f(z) = \left[\Pi_j(p), \xi_i, \left(\sum_h p_i b_{ih} - \sum_h p_i d_{ih} \right) \right]$$

Likewise: Mathematical Programs as a special case of MCP!

From Top-down towards Bottom-up:

- write equations as weak inequalities
- specify complementarity
- add activity analysis/weak inequalities for energy sectors (replacing smooth production function representation)

From Bottom-up towards Top-down:

- re-cast NLP as an MCP
- add multiple markets
- add income constraints

The 2x2x1 - Model

- Motivation
- ➤ Mixed Complementarity
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Equilibrium conditions for competitive 2x2x1-economy:

Zero profit:	$p_i = r K_i^y\,(r,w) + wL_i^y\,(r,w)$	$i=1,2$
Capital demand:	$K_i = K_i^y(r,w)Y_i = \frac{\partial p_i}{\partial r} Y_i$	$i=1,2$
Labor demand:	$L_i = L_i^y(r,w)Y_i = \frac{\partial p_i}{\partial w} Y_i$	$i=1,2$
Market clearance:	$Y_i = X_i$	$i=1,2$
Goods markets:	$X_i = X_i\,(p_1, p_2, M)$	$i=1,2$
Capital market:	$\sum_{i=1}^{2} K_i^y(r,w)Y_i = \bar{K}$	
Income definition:	$M = r\bar{K} + w\,\bar{L}$	
Numéraire:	$w = 1$	

➡ **System of 12 nonlinear equations in 12 variables**

N.B.: implicit variables ⇒ ***K_i, L_i, X_i, M***

Coefficient Form versus Calibrated Share Form

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	CES coefficient form:	CES calibrated share form:
Production:	$y = \gamma\left(\sum_i \alpha_i x_i^{\rho}\right)^{1/\rho}$	$y = \bar{y}\cdot\left[\sum_i\left(\theta_i\cdot\left(\frac{x_i}{\bar{x}_i}\right)^{\rho}\right)\right]^{1/\rho}$
Cost:	$C = \gamma^{-1/\sigma}\left[\sum_i \alpha_i^{\sigma}\cdot\gamma^{(\sigma-1)\cdot\rho}\cdot w_i^{1-\sigma}\right]^{1/(1-\sigma)}\cdot y$	$C = \bar{C}\cdot\left[\sum_i \theta_i\cdot\left(\frac{w_i}{\bar{w}_i}\right)^{1-\sigma}\right]^{1/(1-\sigma)}\cdot\frac{y}{\bar{y}}$
Demand:	$x_i = \gamma^{\sigma-1}\cdot\left(\frac{\alpha_i p}{w_i}\right)^{\sigma}\cdot y$	$x_i = \bar{x}_i\cdot\frac{y}{\bar{y}}\cdot\left(\frac{c}{\bar{c}}\cdot\frac{\bar{w}_i}{w_i}\right)^{\sigma}$

Advantage of *calibrated share form*:

No messy inverting:

➡ Direct calibration from benchmark values

Calibration - The Basics

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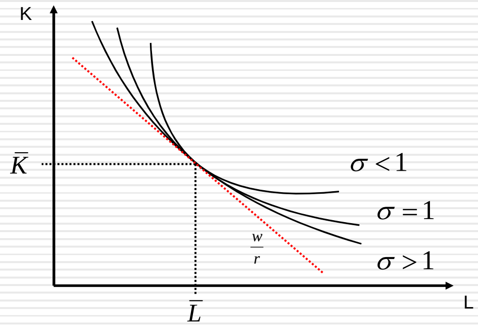
CES function is determined by:

- Quantities (Zeroth order approximation - anchor point)
- Prices (First order approximation - slope)
- Elasticity (Second order approximation - curvature)



Calibration - Microconsistent Dataset

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Benchmark equilibrium:

Price convention: $p_1 = p_2 = r = w = 1$

	Y_1	Y_2	Household	Σ
Y_1	40	–	-40	0
Y_2	–	40	-40	0
$\bar{K}$	-20	-30	50	0
$\bar{L}$	-20	-10	30	0
Σ	0	0	0	

- Zero profit: column sum
- Market clearance: row sum

} input-output table

- Budget constraint

→ Social Accounting Matrix (SAM)

MCP-Implementation of 2x2x1 - Model

- Motivation
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Equilibrium conditions	Variables	Complementarity features
Zero profit	**Activity variables**	
$r^{0.5} w^{0.5} \ge p_1$	$y_1 \ge 0$	$\left(r^{0.5} w^{0.5} - p_1\right) y_1 = 0$
$r^{0.75} w^{0.25} \ge p_2$	$y_2 \ge 0$	$\left(r^{0.75} w^{0.25} - p_2\right) y_2 = 0$
Market clearance	**Price variable**	
$40\, y_1 \ge 40 \frac{M}{80} \frac{1}{p_1}$	$p_1 \ge 0$	$\left(40\, y_1 - 40 \frac{M}{80} \frac{1}{p_1}\right) p_1 = 0$
$40\, y_2 \ge 40 \frac{M}{80} \frac{1}{p_2}$	$p_2 \ge 0$	$\left(40\, y_2 - 40 \frac{M}{80} \frac{1}{p_2}\right) p_2 = 0$
$30 \ge 20\, y_1 \frac{p_1}{w} + 10\, y_2 \frac{p_2}{w}$	$w \ge 0$	$\left(30 - \left(20\, y_1 \frac{p_1}{w} + 10\, y_2 \frac{p_2}{w}\right)\right) w = 0$
$50 \ge 20\, y_1 \frac{p_1}{r} + 30\, y_2 \frac{p_2}{r}$	$r \ge 0$	$\left(50 - \left(20\, y_1 \frac{p_1}{r} + 30\, y_2 \frac{p_2}{r}\right)\right) r = 0$
Budget constraint	**Income variable**	
$30w + 50r \ge M$	$M \ge 0$	$\left(\left(30w + 50r\right) - M\right) M = 0$

From Bottom-Up to Top-Down (1)

- Motivation
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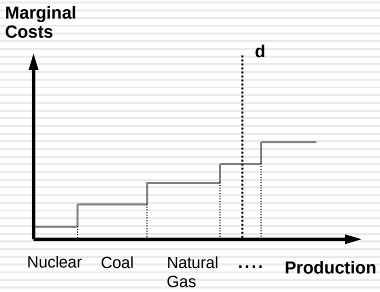
Least-cost energy supply planning problem:

$$\min \sum_i c_i x_i$$

$$s.t. \sum_i x_i \ge d \qquad \perp p_E$$

$$a_i x_i \le b_i \qquad \perp r_i$$

x_i := activity level of technology *i*,
c_i := unit cost coefficient (Leontief) of technology *i*,
a_i := unit capacity requirement (Leontief) of technology *i*,
b_i := capacity constraint for technology *i*,
d := exogenous energy demand
p_E := shadow price of energy market constraint
r_i := shadow price of capacity constraint for technology *i*



From Bottom-Up to Top-Down (2)

- Motivation
- Mixed Complementarity
- ➢ **From Bottom-up to Top-Down**
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MCP formulation of supply planning problem:

Equilibrium conditions	Variables	Complementarity features
Zero profit	**Activity variable**	
$c_i + a_i r_i \geq p_E,$	$x_i \geq 0$	$x_i\left(c_i + a_i r_i - p_E\right) = 0$
Market clearance	**Price variable**	
$\sum_i x_i \geq d$	$p_E \geq 0$	$p_E\left(\sum_i x_i - d\right) = 0$
$a_i x_i \leq b_i$	$r_i \geq 0$	$r_i\left(a_i x_i - b\right) = 0$

From Bottom-Up to Top-Down (3)

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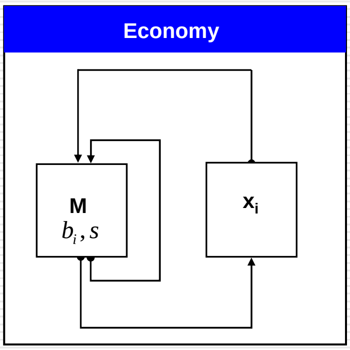
Simplistic CGE extension:

- additional macro-good as endowment (input to energy production and final consumption)
- only energy production activities
- Cobb-Douglas preferences in energy and the macro-good



From Bottom-Up to Top-Down (4)

- Motivation
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- ➢ **From Bottom-up to Top-Down**
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MCP formulation of simplistic CGE-extension:

Equilibrium conditions	Variables	Complementarity features
Zero profit	**Activity variable**	
$c_i p + a_i r_i \ge p_E,$	$x_i \ge 0$	$x_i \left(c_i p + a_i r_i - p_E \right) = 0$
Market clearance	**Price variable**	
$\sum_i x_i \ge \alpha M / p_E$	$p_E \ge 0$	$p_E \left(\sum_i x_i - \alpha M / p_E \right) = 0$
$a_i x_i \le b_i$	$r_i \ge 0$	$r_i \left(a_i x_i - b \right) = 0$
$\sum_i c_i x_i + (1-\alpha) M / p \le s$	$p \ge 0$	$p \left(\sum_i c_i x_i + (1-\alpha) M / p - s \right) = 0$
Budget constraint	**Income variable**	
$sp + \sum_i r_i b_i \ge M$	$M \ge 0$	$M \left(sp + \sum_i r_i b_i - M \right) = 0$

p := market price of the macro-good,
M := income of the representative agent,
s := endowment with macro-good,
α := share parameter for energy in Cobb-Douglas utility function

GAMS Implementation (1)

```
$TITLE    A Simple Maquette for a Hybrid Energy-Economy Model in MCP Formulation

$ontext
Here we demonstrate the MCP implementation of a CGE model featuring discrete
activity analysis across various technologies that supply the same good.
In our concrete example, the only good that is produced is energy E which requires
inputs of a MACRO good X and technology-specific capital R. We do no start from a
balanced benchmark. Instead we have the following information on technologies,
preferences and initial endowments:

(i) Energy supply technologies are Leontief with inputs R and X.
The per-unit-output demands in (X,R) are as follows:
e(T) = min(C(T)/4, 3/4)

(ii) Endowments are given as
X: 95; R(T) = 3;

(iii) Preferences are Cobb-Douglas in X and E:
value share of X: (95/100); value share of E: (5/100)

We use this information to compute a consistent equilibrium where technologies are
active or inactive depending on their profitability.
$offtext
```

GAMS Implementation (2)

```
SET          T      TECHNOLOGIES /T0, T1, T2/;
PARAMETER    C(T)   COSTS        /T0 1,T1 4, T2 5/;

POSITIVE VARIABLES
             E(T)         Energy supply by technology T
             PE           Price of energy
             PX           Price index of other goods
             PR(T)        Rental rate on capital

             RA           Income level of representative consumer;

EQUATIONS
             ZPRF_E(T)    Zero profit condition

             MKT_PE       Market clearance for energy
             MKT_PR       Market clearance for technology specific capital
             MKT_PX       Market clearance for macro good

             INCOME       Income constraint;
```

GAMS Implementation (3)

```
ZPRF_E(T)..   (3/4)*PR(T) + (C(T)/4)*PX =E= PE;

MKT_PE..      SUM(T, E(T))  =E= 5* (RA/100)/PE;
MKT_PR(T)..   3             =E= (3/4)*E(T);
MKT_PX..      95            =E= SUM(T, (C(T)/4)*E(T))+ 95*(RA/100)/PX ;

INCOME..      95*PX + SUM(T, PR(T)*3) + PE    =E= RA;


PX.FX = 1; PE.LO = 1.e-5;
RA.L = 100; PE.L = 1; PX.L = 1; PR.L(T) = 1; E.L(T) = 1;

MODEL         ACTIVITY /ZPRF_E.E, MKT_PE.PE, MKT_PR.PR, MKT_PX.PX, INCOME.RA /;

ACTIVITY.ITERLIM = 1000;
SOLVE ACTIVITY USING MCP;
```

Benchmark Data of Stylized Economy

(Böhringer & Rutherford 2005, ZEW DP-05-28)

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Table 1: Base Year Equilibrium

	ROI	*COA*	*GAS*	*OIL*	*ELE*	*RA*
ROI	200	-5	-5	-10	-180	
COA		15			-15	
GAS			15		-15	
OIL				30		-30
ELE	-10				60	-50
Capital	-80				-20	100
Labor	-110	-5	-5	-10		130
Rent		-5	-5	-10		20

Key	
ROI:	rest of industry
COA:	coal
GAS:	gas
OIL:	oil
ELE:	electricity
RA:	representative agent

Embodied least-cost energy supply problem:

$$\min \sum_i \sum_t p_i a_{ijt} y_{it}$$

$$\text{s.t.} \quad \sum_t y_{jt} + \sum_{i \neq j} a_{ji} \bar{y}_i + \sum_h w_{jh} \geq \sum_h \bar{d}_{jh}$$

$$y_{jt} \leq \sum_h w_{hjt}$$

Here:

Supply of demand for energy good *j* (electricity) by alternative technologies *t* subject to capacity constraints!

Technologies for Electricity Generation

- Motivation
- Mixed Complementarity
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Table 2: Cost Structure of Active Technologies (Base Year)

	coal	gas	nuclear	hydro
ELE	20	20	12	8
ROI	-1	-1	-8	
GAS		-15		
COA	-15			
Capital	-4	-4	-4	-8

Table 3: Cost Structure of Inactive Technologies (Base Year)

	wind	solar	biomass
ELE	1	1	1
ROI	-0.2	-0.3	-0.4
Capital	-0.9	-0.8	-0.7
wind	-1		
sun		-1	
trees			-1

Policy Simulation: Nuclear Phase-Out

- Motivation
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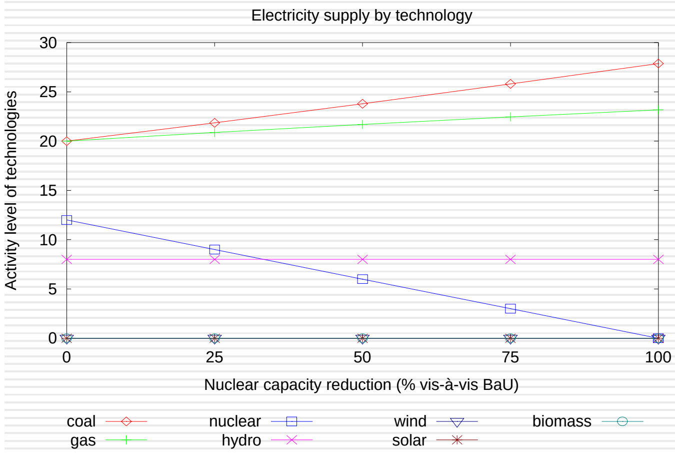
Gradual reduction in permissible nuclear power capacity:



Policy Simulation: Green Quota

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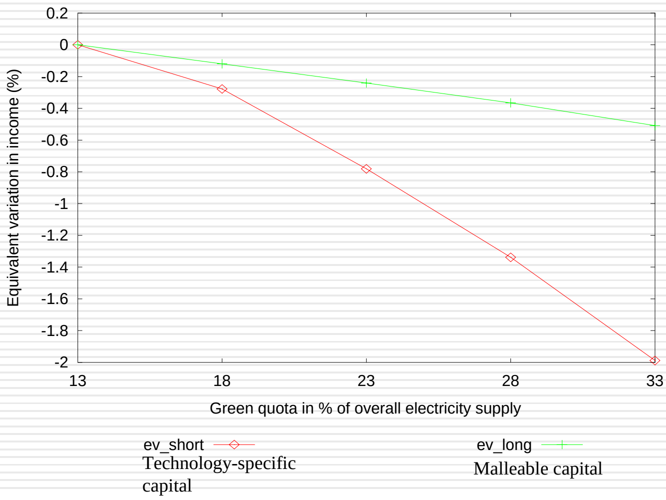
Subsidized increased of renewable electricity production:



Policy Simulation: Environmental Tax Reform

- Motivation
- Mixed Complementarity
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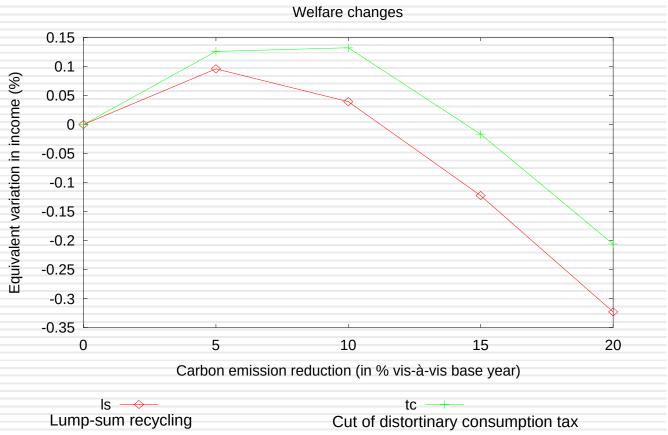
- ➤ **Illustration**
- Conclusion

Imposition and recycling of carbon taxes:

– initial partial consumption tax on non-energy commodities

– fixed level of public good provision



Summary

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- ➤ **Conclusion**

- **Perceived Dichotomy: Bottom-up versus Top-Down**
  - special (restricted) cases of general equilibrium conditions
  - policy focus and availability of efficient/robust algorithms
- **MCP framework for synthesis (hybrid models) :**
  - economic richness of top-down (CGE) models
  - technological foundation of bottom-up models
  - availability of solution algorithms for “large-scale” problems

Outlook: Decomposition of Large-Scale Hybrid Models

(Böhringer & Rutherford 2006, ZEW DP-06-07)

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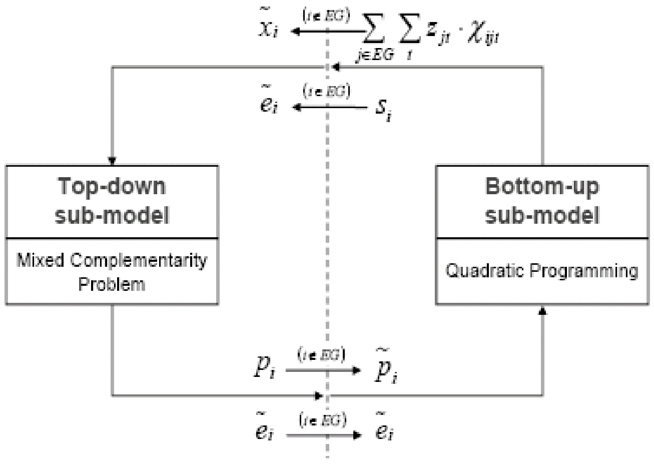

Figure 2: *Iterative Decomposition Algorithm*