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Criteria for Assessing Sustainable Development: Theoretical Issues and Empirical Evidence

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Abstract

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1 Introduction

Concerns about environmental deterioration and natural resource depletion have advanced sustainable development as a key concept in policy formulation both at the national and international level. Sustainable development has been the central concept in the World Conservation Strategy (1980) and in the report of the World Commission on Environment and Development (WCED, 1987) also known as the Brundtland Report. Sustainability has also become a central concept in the policy of the European Union.

The most commonly used definition of sustainable development now is that of the Brundtland Report which defines *sustainable development* as *development which meets the needs of the present without compromising the ability of future generations to meet their own needs.* This definition stresses the aspects of intertemporal distribution and intergenerational equity but since it embeds many complex economic ideas suffers from tractability, especially when it comes to provide answers to applied questions regarding the sustainability of economies, or the design and evaluation of sustainable development policies.

In the attempt to make the definition of sustainable development operational and useful for the development of sustainability criteria or sustainability indicators and for the design of sustainable policies, some auxiliary definitions have been developed. These definitions identify conditions which, when satisfied, an economy can be regarded as following a sustainable development path. The most prevailing of these definitions, as reported by Pezzey (2004), associate sustainability with:

1. achieving constant utility (Solow 1974; Hartwick 1977).

2. avoiding any decline in utility (Pearce et al. 1989; Pezzey 1992, 1997).

3. avoiding any decline in the present value from time t and onwards of intergenerational social welfare defined in terms of a Ramsey-Koopmans social welfare functional (Riley 1980; Dasgupta and Mäler 2000; Pemberton and Ulph 2002; Arrow et al. 2003b).

The concept of *non declining social welfare* or *non declining well being* is used to interpret sustainability as maintenance of social welfare. As shown by Arrow et al., (2003b) sustainable development defined in this context implies, and is implied by, the maintenance of the economy's productive base. This means that each generation should bequeath to each successor at least as large a productive base as it inherited from its predecessors. For this to be achieved, the productive base of the economy should be preserved for the future generations. The productive base includes a list of assets such as *manufactured capital, human capital, natural capital* and *knowledge.* If genuine investment, defined as the sum of the investment in the above forms of capital, valued at accounting prices, is non-decreasing over time, then social welfare is also non-decreasing and development is sustainable. This concept of sustainability can be regarded as corresponding to the *weak sustainability* concept (Pearce and Atkinson 1992; Hediger 1999, 2000).

One of the advantages of this definition of sustainable development is that it can be extended to a very general framework which does not depend upon whether optimizing or non optimizing behavior is assumed, and which can be used to provide empirical estimations regarding the sustainability conditions for an economy. There is a clear distinction between optimizing and non-optimizing economy as illustrated by Arrow et al. (2003b).

> *A non-optimizing economy is an economy where the government whether by design or incompetence does not choose policies that maximize intergenerational welfare. The term sustainable development acquires particular bite when it is put to work in imperfect economies, that is economies suffering from weak or even bad governance.*

In such an economy firms could be profit maximizers, but we do not need to assume the existence of a social planner or the existence of optimizing behavior from the households' point of view as in the standard optimal growth framework.

If we assume that the economy can be described by a dynamical system with the state variables corresponding to assets and the control variables corresponding to choice variables or policy instruments,[1] then the paths of the assets are determined by the way that the controls are chosen. The paths of the state variables can be used to define a value function for the economy through the Ramsey-Koopmans, or felicity, functional at a given point in time.[2] The value function at any given point t in time is taken to represent social welfare from time t onwards, and it is a function of the values of the economy's assets at time t.[3] .If the value function is not declining over time that is, if its time derivative is non negative, then social welfare is also non-decreasing and development is sustainable at this point in time.

The future paths of the assets will be optimal, if controls are chosen optimally in order to maximize the social welfare functional. However, as indicated by Arrow et al. (2003 b), the economy's value function, and its time evolution is well defined for non-optimal choices of the instruments. This makes possible to define conditions for sustainable development in a general context and to provide a basis for empirical estimations. It is clear that by choosing the structure of the dynamic system describing the economy it is possible to highlight the impact of different factors on sustainability. Arrow, et al. (2003b) focus on issues such as non-convexities, natural resources, exogenous productivity growth, human capital, while Arrow et al. (2003a) and Asheim (2004) link population change with sustainable development in an optimizing framework.

The present paper follows this methodological approach and seeks to provide a well defined theoretical framework for determining sustainability criteria for non optimizing economies, which can also be used to provide a basis for empirical estimations. We believe that since, especially for developing countries, there is no reason to assume that observed data are generated by optimizing processes, the non optimizing framework, properly defined, will be very useful both for purposes of theoretical foundations of sustainability criteria under alternative hypotheses about the structure and the objectives

[1] For example in growth models, consumption is a control variable or a policy instrument, and the stock of capital is a state variable.

[2] For the definition of the felicity functional see Arrow and Kurz (1970).

[3] This is an interpretation of the value function similar to the one given in Dynamic Programming. The main difference is that no optimization needs to be assumed here.

of the economy, and for empirical estimations.

Using the non-optimizing theoretical framework, we derive the (weak) sustainability criterion when controls are chosen according to some feedback rule.[4] We also show that when controls (or policy instruments) are chosen in an arbitrary way which is independent of the stock of assets[5], the non-declining social welfare sustainability criterion, depends not only on the growth of the assets and their corresponding accounting prices, but also on the arbitrary paths of the controls. In this case the value function for the economy depends both on current stocks and current flows. These results suggest that in certain cases of non optimizing economies with arbitrary choices of controls, positive genuine investment in assets might not be entirely appropriate for characterizing sustainable development paths. In these cases genuine investment should be adjusted for the growth of the arbitrary chosen policy variables, such as for example emission limits. In this sense we extend in this paper the concept of genuine investment, an extension that could be useful in empirical applications.

This theoretical framework is then applied to data from an actual economy with the purpose of providing estimates of sustainability conditions. Thus the paper's contribution, in the long discussion about sustainability, consists of developing a systematic theoretical framework for determining value functions, accounting prices and sustainability criteria, under fairly general non-optimizing behavioral rules, and then showing that this framework can be used in applied work to estimate sustainability conditions and genuine investment.

The rest of the paper is organized as follows. Section 2, provides the framework for determining sustainability criteria in the case of a non-optimizing economy under a feedback, or an arbitrary rule of policy instruments' choice. In each case the economy is described by a dynamical system, the corresponding value function is defined, and the sustainability criterion, or indicator, for each case is presented. We also provide a definition according to which

[4] A feedback rule in this context is, for example, a behavioral rule according to which instruments are determined in some relation to the values of the state variables.

[5] This implies a non-feedback (arbitrary) way of choosing the controls.

a policy is promoting sustainability if it implies a relative higher growth of social welfare relative to another policy. In section 3, we seek to obtain exact representations and closed form solutions of the above concepts which will be useful in empirical applications. To do this a stylized dynamic economy is considered in the context of the Solow growth model, without household optimization. In this framework, domestic population growth, migration, labour augmenting technical change, environmental damages associated with pollutant flows generated by economic activities are taken into account in determining the economy's sustainability conditions. In the same context we use an arbitrary performance standard that determines an upper limit for the emissions of a pollutant and analyze the structure of the value function and accounting prices under an arbitrary environmental policy. We show how accounting prices for arbitrary chosen instruments should enter the sustainability criteria according to our previous theoretical models. In section 4, we provide the exact definitions of sustainability criteria under the feedback and the arbitrary rule respectively. In section 5, we use our theoretical model to explore the current sustainability conditions within the Greek economy. Our findings suggest that in the case where environmental considerations are not taken into account, or there is no binding environmental policy, migration, the rate of growth of capital per worker and exogenous technical change are strong positive factors supporting sustainability for the Greek economy. When we introduce potential environmental damages due to sulphur dioxide (SO_2) emissions, our empirical findings confirm the notion that environmental damages have a negative impact on sustainability. When we analyze the case of a possible arbitrary chosen performance standard for SO_2 emissions - a case corresponding to an arbitrary binding environmental policy - the accounting value of the emission limit enters the sustainability criterion as suggested by the theoretical model. The effect of the performance standard on sustainability depends on the relative strength of its effects on production and environmental conditions. The last section of the paper concludes.

2 Sustainability Criteria in Non-optimizing Economies

Following Arrow et al. (2003b) we assume that social welfare at any given time t is defined by the felicity functional:

$$V_t = \int_t^{\infty} e^{-\delta(\tau - t)} U(\mathbf{x}(\tau), \mathbf{u}(\tau)) \, d\tau \ , \tau \geq t \tag{1}$$

where $\mathbf{x} = (x_1, ..., x_n)$ denotes a vector of state variables, which can be interpreted as stocks of assets and $\mathbf{u} = (u_1, ..., u_m)$ denotes a vector of control variables, which can be interpreted as policy instruments. The function $U(\mathbf{x}(\tau), \mathbf{u}(\tau))$ can be interpreted as the welfare of the generation living at time τ, under appropriate assumptions about the growth of the population, as it will become clear in the following sections.

The evolution of the economy is described by a system of transition equations linking the state and the control variables.

$$\dot{\mathbf{x}}_\tau = \mathbf{f}(\mathbf{u}(\tau), \mathbf{x}(\tau)) \ , \ \mathbf{x}(t) = \mathbf{x}_t \ , \tau \geq t \tag{2}$$

In an optimizing economy the control paths $u(\tau)$ are chosen to maximize (1) subject to the constraints imposed by the transition equations (2). In a non optimizing economy the choice of the controls could be determined by a feedback rule $u(\tau) = g(\mathbf{x}(\tau))$ which might reflect behavioral characteristics of the economy, such as learning rules or imitation rules, or some other intentional but not optimal feedback policy rule.[6] For example, in the Solow model of economic growth consumption, which is interpreted as a control variable, is a constant fraction of output. Output is determined, through the aggregate production function, by the capital stock which is the state variable. This constant fraction is a behavioral parameter. Thus in Solow's model consumption is determined by a feedback rule. In addition, feedback controls can be chosen to stabilize the economic system around some desirable

[6] These types of controls could be also called *closed loop* controls.

steady state,[7] or can be chosen to steer the system to a certain state vector in finite time.[8]

Alternatively the choice of controls can be determined in a completely *arbitrary way*, by exogenous factors, such as domestic political conditions, historic trends or international conditions. In this case the control paths will be $\mathbf{u}(\tau) = \bar{\mathbf{u}}(\tau)$. An arbitrary control path could be for example a path for which consumption increases $x\%$ per year or CO_2 emissions are reduced $z\%$ per year, without any relation to the evolution of a state variable. [9]. To put it in another way that is closer to the recent discussion about the Kyoto protocol: choosing CO_2 emissions as a proportion of global CO_2 stock is a feedback (closed loop) rule, while keeping emissions at the 1990 levels is an arbitrary (open loop) rule. It seems that in actual economies most policy rules are arbitrary rather than feedback or optimal rules.

Consider the system of transition equations (2) under the feedback rule, or the arbitrary rule respectively:

$$\dot{\mathbf{x}}_\tau = \mathbf{f}(\mathbf{g}(\mathbf{x}(\tau)), \mathbf{x}(\tau), \mathbf{b}) \ , \ \mathbf{x}(t) = \mathbf{x}_t \tag{3}$$

$$\dot{\mathbf{x}}_\tau = \mathbf{f}(\bar{\mathbf{u}}(\tau), \mathbf{x}(\tau), \mathbf{b}) \ , \ \mathbf{x}(t) = \mathbf{x}_t \tag{4}$$

where $\mathbf{b}$ is a vector of exogenous parameters. Solutions to these systems, provided they exist, will determine the paths of the state variables as functions of their initial values, the exogenous parameters and possible the paths of the arbitrary (or open loop) controls. In general these solutions will be of the form:

$$\mathbf{x}_\tau = \boldsymbol{\phi}(\tau - t, \mathbf{x}_t, \mathbf{b}), \tag{5}$$

$$\mathbf{x}_\tau = \boldsymbol{\psi}(\tau - t, \mathbf{x}_t, \bar{\mathbf{u}}(\tau), \mathbf{b}) \tag{6}$$

Substituting the solutions (5) or (6) into (1) we obtain the value function of

[7] In ths case the feedback function is chosen so that the steady state is stable in the Lyapunov sense.

[8] In this case the feedback function is chosen so that the system starting from the initial point $\mathbf{x}_0$, reaches the terminal state $\mathbf{x}_T$, at finite time T. It is assumed in this case that the rank conditions for controllability are satisfied.

[9] Such types of controls could be also called *open loop* controls.

the system as a function of the initial state vector $\mathbf{x}_t$, and possibly the vector of arbitrary controls $\bar{\mathbf{u}}(\tau)$. If the arbitrary (or open loop) control path can be written as: $\bar{\mathbf{u}}_0(\tau) = \bar{\mathbf{u}}(\tau - t, \bar{\mathbf{u}}_t)$, [10] then the *value function* for the economy can be written for the feedback and the open loop rules respectively, as:

$$V_t(\mathbf{x}_t; \mathbf{b}) = \int_t^\infty e^{-\delta(\tau - t)} U(\mathbf{g}(\boldsymbol{\phi}(\tau - t, \mathbf{x}_t, \mathbf{b})), \boldsymbol{\phi}(\tau - t, \mathbf{x}_t, \mathbf{b})) \, d\tau \quad (7)$$

$$V_t(\mathbf{x}_t, \bar{\mathbf{u}}_t; \mathbf{b}) = \int_t^\infty e^{-\delta(\tau - t)} U(\boldsymbol{\psi}(\tau - t, \mathbf{x}_t, \bar{\mathbf{u}}_0(\tau), \mathbf{b}), \bar{\mathbf{u}}_0(\tau)) \, d\tau \quad (8)$$

Accounting prices for asset x_i or control (instrument) $\bar{u}_j$ at time t, are defined as:

$$p_{tx_{it}} = \frac{\partial V_t}{\partial x_{it}}, \; p_{tu_{jt}} = \frac{\partial V_t}{\partial \bar{u}_{jt}}, \quad (9)$$

respectively.

If we use the non-declining social welfare definition of sustainable development which requires that $\frac{dV_t}{dt} \geq 0$ we can obtain the following result:

Proposition 1 *Consider a non-optimizing economy with x_i, $i = 1, ...n$ assets and u_j, $j = 1, ..., m$ policy instruments. (i) If policy instruments are chosen following feedback rules associated with the assets of the economy, then sustainability depends on the assets' growth and their corresponding accounting prices. (ii) If policy instruments are chosen arbitrarily then sustainability depends both on the assets' and the policy instruments' growth and their corresponding accounting prices.*

Proof. (i) Differentiating (7) totally with respect to time we obtain that along a sustainable development path:

$$S_t^F \equiv \frac{dV_t}{dt} = \sum_{i=1}^{n} \frac{\partial V_t}{\partial x_{it}} \frac{dx_{it}}{dt} + \frac{\partial V_t}{\partial t} \geq 0 \quad (10)$$

(ii) Differentiating (8) totally with respect to time we obtain that along a

[10] This implies that the control is chosen according to some arbitrary time dependent rule, for example $z\%$ change relative to the previous year.

sustainable development path:

$$S_t^A \equiv \frac{dV_t}{dt} = \sum_{i=1}^{n} \frac{\partial V_t}{\partial x_{it}} \frac{dx_{it}}{dt} + \sum_{j=1}^{m} \frac{\partial V_t}{\partial \bar{u}_{jt}} \frac{d\bar{u}_{jt}}{dt} + \frac{\partial V_t}{\partial t} \geq 0 \tag{11}$$

■

It should be noticed that part (*ii*) of the above proposition shows that in *arbitrary (open loop) non optimizing economies* - that is economies where instruments are chosen without any relationship to assets - sustainability depends on the growth of these instruments too. Thus the growth of the instruments affects sustainability in addition to the growth of the assets. Since the term $\sum_{i=1}^{n} \frac{\partial V_t}{\partial x_{it}} \frac{dx_{it}}{dt}$ represents genuine investment, our results implies that in time autonomous economies, where $\frac{\partial V_t}{\partial t} = 0$, positive genuine investment does not imply that development is sustainable. To fully assess sustainability in such an economy, the impacts of instrument should be also taken into account. In this sense part (*ii*) of Proposition 1 extends previous results about non optimizing economies, where sustainable development depended on genuine investment alone. This result can be associated for example with the introduction of environmental policy, which in actual economies can be regarded most of the times as arbitrary. For example, let $\bar{u}_j$ denote an arbitrary upper limit on emissions, then $\frac{\partial V_t}{\partial \bar{u}_{jt}}$ can be interpreted as the *accounting price for this limit* and the term $\frac{\partial V_t}{\partial \bar{u}_{jt}} \frac{d\bar{u}_{jt}}{dt}$ can be interpreted as the *contribution* of a changing emission limit to the sustainability criterion.

If in the arbitrary instrument choice case, instruments are constant so that , $\frac{d\bar{u}_{jt}}{dt} = 0$, the value function (8) depends on the vector of parameters $\bar{\mathbf{u}}$ and is written as $V_t(\mathbf{x}_t; \bar{\mathbf{u}})$. In this case we can still define the accounting price for the instrument, although the sustainability criterion does not depend directly on $\bar{\mathbf{u}}$ but indirectly, through the accounting prices for the assets, which can be written as: $p_{tx_i}(\bar{\mathbf{u}}) = \partial V_t(\mathbf{x}_t; \bar{\mathbf{u}}) / \partial x_{it}$.

It should be noticed that the sustainability criteria (or indicators) (10), and (11) are defined for the *current period* t. A positive value for S_t^F or

S_t^A implies that the economy is *currently* sustainable. The economy will be sustainable for the entire future horizon if:

$$S_\tau^l \geq 0, \text{ for all } \tau \geq t \text{ }, l = F \text{ or } A \tag{12}$$

Given however the arbitrary choice of instruments, and the implied arbitrary paths for the state variables, it might not be feasible to satisfy (11) for the entire future time horizon. For example suppose that an arbitrary policy leads to the exhaustion of an essential asset (resource). Then estimates of sustainability indicators (10) or (11) obtained by using the current changes in assets, dx_{it}/dt, could very well be positive indicating that the economy is currently sustainable. The economy will not however be sustainable according to (12) for the entire future path, since the exhaustion of the essential asset will eventually force the value function to decline over time. So results based on (10) or (11) should be interpreted as "local" with respect to time especially in the presence of essential resources. On the other hand sustainability criteria based on (10) or (11) can be used to check whether sustainability conditions are satisfied at least in the medium run. The definition (12) should be used when it is possible supplementary to provide information about whether an economy is on a long run sustainable path. This requires information about the expected long run growth rates of assets especially the essential ones. These observations suggest that for empirical purposes, where *current* estimates of the assets' growth rates are used in order to obtain an estimate of the criterion (10) or (11), it might be more appropriate to define a "time bounded" sustainability criterion (or indicator), which is defined for a finite time horizon, within which there is a more confidence in the estimates of the assets' growth rates obtained from the economy's data. The criterion should be *reestimated* as time goes by, so that possible unexpected effects of the arbitrary policy rules are realized.

Using the definitions (7) and (8) and the above argument for a bounded sustainability criterion, accounting prices could be defined more precisely as

:

$$p_{tx_{it}} = \frac{\partial V_t}{\partial x_{it}} = \int_t^T e^{-\delta(\tau-t)} \frac{\partial}{\partial x_{it}} \left[U \left(\mathbf{g} \left(\boldsymbol{\phi} \left(\tau - t, \mathbf{x}_t, \mathbf{b} \right) \right), \boldsymbol{\phi} \left(\tau - t, \mathbf{x}_t, \mathbf{b} \right) \right) \right] d\tau \tag{13}$$

$$p_{t\bar{u}_{jt}} = \frac{\partial V_t}{\partial \bar{u}_{jt}} = \int_t^T e^{-\delta(\tau-t)} \frac{\partial}{\partial \bar{u}_{jt}} \left[U \left(\boldsymbol{\psi} \left(\tau - t, \mathbf{x}_t, \bar{\mathbf{u}}_0 \left(\tau \right), \mathbf{b} \right), \bar{\mathbf{u}}_0 \left(\tau \right) \right) \right] d\tau \tag{14}$$

where $T \leq \infty$. Furthermore the impact from changes in a parameter b_v on accounting prices is defined as:

$$\frac{\partial p_{tx_{it}}}{\partial b_v} = \frac{\partial^2 V_t}{\partial b_v \partial x_{it}} = \frac{\partial}{\partial b_v} \left(\frac{\partial V_t}{\partial x_{it}} \right), \quad \frac{\partial p_{t\bar{u}_{it}}}{\partial b_v} = \frac{\partial^2 V_t}{\partial b_v \partial \bar{u}_{jt}} = \frac{\partial}{\partial b_v} \left(\frac{\partial V_t}{\partial \bar{u}_{jt}} \right)$$

The sustainability criteria (10), (11) along with the definitions of accounting prices (13) and (14) can be used to define a rule for evaluating current policies according to their impact on sustainable development. Consider two alternative *feedback rules* $(\mathbf{g}_1 (\mathbf{x} (\tau)), \mathbf{g}_2 (\mathbf{x} (\tau)))$, or two *arbitrary policies* $(\bar{\mathbf{u}}_1 (\tau), \bar{\mathbf{u}}_2 (\tau))$. Then the corresponding sustainability criteria will be defined through (10) or (11), as (S_1^F, S_2^F) or (S_1^A, S_2^A).

Definition 1: *A policy either in a feedback form $g_1 (\mathbf{x} (\tau))$ or in an arbitrary form $\bar{u}_1 (\tau)$ is said to promote sustainable development, relative to the corresponding policies $g_2 (\mathbf{x} (\tau))$ or $\bar{u}_2 (\tau)$ if:*

$$S_1^F \geq S_2^F, S_1^A \geq S_2^A \tag{15}$$

According to this definition a policy is promoting current sustainability if it implies a relative higher growth of social welfare. If policy 2 is the status quo then (15) it can be used to evaluate new policies with respect to their impact on sustainable development. Definition 1 can also be used to assess whether a change in an exogenous parameter promotes sustainable development or not.[11]

[11] If for example, parameter b_m changes from b_{m1} to b_{m2}, the change promotes sustainability if $S^F (b_{m2}) \geq S^F (b_{m1})$.

3 Defining Value Functions and Accounting Prices in a non Optimizing Economy

Having defined the value functions, the accounting prices and the sustainability criteria, we proceed to consider a structured model of an economy with the purpose of providing exact and whenever possible closed form representations of these concepts. These representations will provide more insight into sustainability criteria as well as a solid basis for empirical estimations.

We consider therefore an aggregate model of a growing economy where output is produced by capital and labor. The production processes is affected by exogenous labor augmenting technical change, while the total labor force is determined by domestic population growth and migration inflows (or outflows). Output is divided among consumption and investment and consumption generates utility. On the other hand output production generates emissions which affect utility negatively. Thus, although we are dealing with a stylized model, important characteristics of modern economies such as *technical change, environmental pollution* and *migration* are taken into account in exploring sustainability conditions.[12]

Capital accumulation in our stylized economy is described by using the standard Solow model. We assume that exogenous technical change of labour augmenting type is present. This means that the aggregate production function can be written as $Y = F(K, AN)$, where as usual Y is aggregate output, K is capital stock, L is labour input, $\frac{\dot{A}}{A} = g$, is the rate of exogenous technical change and $\hat{L} = AN$ is effective labour. The standard Cobb-Douglas production function $Y = K^a (AN)^{1-a}$ can then be expressed in *per effective worker* terms as $\hat{y} = \hat{k}^a$, where $\hat{y} = \frac{Y}{AN}$, $\hat{k} = \frac{K}{AN}$.[13]

[12] In our stylized economy we do not consider natural resources and their contribution to production. This is because we want to keep the model relative simple in order to obtain the representations of value functions and accounting prices which will help to provide some insights into the structure and the determinants of these concepts. The introduction of natural resources in this context is undoubtedly an area for further research.

[13] An alternative approach would be to specify the production function in the context of an endogenous growth model, by using an AK function or more general a production function with knowledge externalities or human capital. This approach is another

In our stylized economy we seek to incorporate the impact of migration into the change of the total labour force. Given the importance that migration flows have played in the history of economic development, it is interesting to determine the contribution of migration to the sustainability conditions of an economy, along with technical change and environmental pollution. Migration is a phenomenon that affects an economy's population and labor supply. It represents gains in population for the destination economy and at the same time losses for the source economy. The movement of a person could also entail the movement of human capital and that is the reason why migration also implies some degree of capital mobility[14].

Let $M(t)$ be the flow of migrants into the domestic economy. If $N_l(t)$ is the local population then the migration rate m is defined as $m = \dfrac{M}{N_l}$. The overall growth of domestic population, or equivalently labor force, is $\dfrac{\dot{N}}{N} = \tilde{n}$, with $\tilde{n} = n + m$, where n is the rate of growth of the domestic labour force and m is the migration rate. Then, the evolution of the total labour force in the country is determined by:

$$N_\tau = N_t e^{\tilde{n}(\tau - t)}, \tau \geq t \tag{16}$$

If $m > 0$, this means that there is an inflow of immigrants in the destination economy whereas if $m < 0$, then there is an outflow. Let z be the capital defined in the broad sense of each person, immigrant or emigrant. If $z = 0$, this means that the immigrants or the emigrants do not come with human capital such as special skills or education, or any other type of capital, and this can be interpret as migration which does not support any type of capital movement. In this case there is only labour force change and not human or physical capital mobility. If $z \neq 0$ that means that migration also includes some kind capital mobility.

Under these assumptions the accumulation of capital, measured in per

area of further research, once the structure of the value function and accounting prices is understood in the context of the traditional Solow model.

[14] See for example Barro and Sala-i-Martin (2004).

effective worker terms, is given by (Barro and Sala-i-Martin, 2004):

$$\dot{\hat{k}}_t = s\hat{k}_t^a - (\eta + \delta + g)\,\hat{k}_t - m\hat{k}_t + z$$

where s is the saving rate and δ is the depreciation rate. Assuming no capital mobility due to migration, that is, setting $z = 0$ we obtain:

$$\dot{\hat{k}}_t + (\eta + \delta + m + g)\,\hat{k}_t = s\hat{k}_t^a \tag{17}$$

Thus capital accumulation in our economy is described by a Bernoulli differential equation which can be solved to obtain:[15]

$$\hat{k}_\tau = \left[\left(\hat{k}_t^{1-a} - \frac{s}{\omega}\right) e^{-(1-a)\omega(\tau - t)} + \frac{s}{\omega}\right]^{\frac{1}{1-a}} \;, \tau \geq t \;\;, \omega = (\eta + \delta + m + g) \tag{18}$$

Since in the Solow model consumption is a fixed proportion of output,[16] we have, in per effective worker terms:

$$\hat{c}_\tau = (1 - s)\,\hat{k}_\tau^a \tag{19}$$

or

$$\hat{c}_\tau = (1 - s)\left[\left(\hat{k}_t^{1-a} - \frac{s}{\omega}\right) e^{-(1-a)\omega(\tau - t)} + \frac{s}{\omega}\right]^{\frac{a}{1-a}} \tag{20}$$

Environment is introduced into the model by assuming that pollution, denoted by P, which is a by-product production affects utility in a negative way. Then the utility function becomes a function of per capita consumption c_τ and total pollution P_τ and is assumed, as it is common in this type of analysis, to have the following separable specification:

$$U(c_\tau, P_\tau) = -c_\tau^{-(\sigma - 1)} - D(P_\tau) \tag{21}$$

In (21) $-\sigma$ is the elasticity of marginal utility, with $\sigma > 1$, and $D(P_\tau)$ can

[15] For the solution see the Appendix 1.

[16] In the terminology of the previous section, consumption is a feedback control.

be interpreted as a damage function assumed strictly increasing and convex. We specify the damage function as $D(P_\tau) = \theta P_\tau^\gamma$ with $\theta > 0$ and $\gamma \geq 1$. Since the production structure is determined in per effective worker terms, we need to specify the utility function (21) in per effective worker terms. If we define consumption in per effective worker as $\hat{c} = \frac{C}{AN}$, from the definition of per capita consumption we have:

$$\frac{C_\tau}{N_\tau} \equiv c_\tau = \hat{c}_\tau A_t e^{g(\tau - t)} \ , \ u(c_\tau) = -c_\tau^{-(\sigma-1)} = -\left(\hat{c}_\tau A_t e^{g(\tau-t)}\right)^{-(\sigma-1)}$$

and the utility function (21) becomes:

$$U(c_\tau, P_\tau) = -\left(\hat{c}_\tau A_t e^{g(\tau-t)}\right)^{-(\sigma-1)} - \theta P_\tau^\gamma \tag{22}$$

We assume that pollution is of the flow type and that the flow of emissions, since it is a by-product of production, is related to output by a strictly increasing function $P_\tau = \mu(Y_\tau)$. In terms of the discussion in section 2, pollution can be regarded as a form of a feedback control, since by using the production function to substitute for output, emissions can be written as a function of the capital stock. This feedback rule can be associated with technical conditions and production practices which determine completely, in the absence of environmental policy, the evolution of emissions.[17] The $\mu(\cdot)$ function can be further specified as:

$$P_\tau = \mu Y_\tau^\beta e^{xt}, \mu > 0, \beta > 0 \tag{23}$$

where x reflects technical change in pollution generation.[18] A negative x reflects pollution reducing technical change. Since in per effective worker terms, $Y_\tau = \hat{y}_\tau A_\tau N_\tau = \hat{k}_\tau^a A_t N_t e^{\left(g+\tilde{n}\right)(\tau-t)}$, $\tilde{n} = n + m$, by substituting Y_τ in (23) and using (18), (20), and (22) the utility flow in per effective worker

[17]For example in the absence of environmental policy or any other environmental constraint, a firm will emit as much as possible for a given level of output and technical conditions, since emissions can be regarded as an unpaid factor.

[18]This type of technical change can be induced by environmental policy. We do not model this process here, but in the empirical application we try to make inference about the existence of this type of technical change from data.

terms is specified as:

$$U\left(\hat{k}_t, N_t, A_t\right) = -\left(\hat{c}_\tau A_t e^{g(\tau-t)}\right)^{-(\sigma-1)} - \theta\left[\mu\left(\hat{k}_\tau^a A_t N_t e^{\left(g+\tilde{n}\right)(\tau-t)}\right)^\beta e^{x(\tau-t)}\right]^\gamma \tag{24}$$

The flow of total utility in the economy is $N_\tau U(c_\tau, P_\tau)$ and therefore *the value function* for the economy, using (24) becomes:

$$V_t = \int_t^T e^{-\rho(\tau-t)} N_\tau U(\hat{k}_\tau, N_\tau, A_\tau)dt \ , T \leq \infty \ \ , N_\tau = N_t e^{\tilde{n}(\tau-t)} \tag{25}$$

It should be noted that the value function depends only on the current values of state variables of the problem $\left(\hat{k}_t, N_t, A_t\right)$ and the parameters describing the structure of the economy.

Following (13) of the previous section the current accounting prices are defined as:

$$p_{t\hat{k}_t} = \frac{\partial V_t}{\partial \hat{k}_t}, p_{tN_t} = \frac{\partial V_t}{\partial N_t}, p_{tA_t} = \frac{\partial V_t}{\partial A_t} \tag{26}$$

Since $\hat{k} = \dfrac{k}{A} = \dfrac{K}{AN}$, $k = \dfrac{K}{N}$ the accounting price of capital in physical units and per capita units is defined respectively as:

$$p_{tK_t} = \frac{\partial V_t}{\partial \hat{k}_t}\frac{\partial \hat{k}_t}{\partial K_t} = \frac{1}{A_t N_t} p_{t\hat{k}_t} \tag{27}$$

$$p_{tk_t} = \frac{\partial V_t}{\partial \hat{k}_t}\frac{\partial \hat{k}_t}{\partial k_t} = \frac{1}{A_t} p_{t\hat{k}_t} \tag{28}$$

It should be noted that in this case there is no specific accounting price for pollution since pollution is not a stock, but the impact of pollution is realized through the accounting price of capital $p_{t\hat{k}_t} = \partial V_t / \partial \hat{k}_t$ which depends on the parameters of the damage function.

3.1 Sustainability in the presence of environmental policy.

In the previous section, emissions were considered as a by-product of output production determined by technical conditions alone. In this section we explicitly introduce environmental policy which is expressed through a *performance standard* that determines an upper limit for the emissions of the firms. Since the emission function of the representative firm can be written as:

$$P_\tau = \mu Y_\tau^\beta e^{x(\tau-t)} = \mu\left(\hat{y}_\tau AN\right)^\beta e^{x(\tau-t)} = \phi\left(f\left(\hat{k}_\tau\right)AN\right)e^{x(\tau-t)} \;, \phi' > 0 \quad (29)$$

the emission limit will take the form:

$$P_\tau \leq \bar{P} \quad (30)$$

The *profit function* of the representative firm can be written in per effective worker terms as:

$$AN\left[f\left(\hat{k}_\tau\right) - (r+\delta)\hat{k}_\tau - we^{-g(\tau-t)}\right] \quad (31)$$

The firm considers the interest rate r and the wage rate w as fixed and chooses capital, for any fixed level of effective labour AN to maximize (31) subject to (30). The Lagrangian for the problem is:

$$\mathcal{L} = AN\left[f\left(\hat{k}_\tau\right) - (r+\delta)\hat{k}_\tau - we^{-g(\tau-t)}\right] + \lambda\left[\bar{P} - \phi\left(f\left(\hat{k}_\tau\right)AN\right)e^{x(\tau-t)}\right] \quad (32)$$

The Kuhn-Tucker conditions for an interior solution to the problem imply:

$$f'\left(\hat{k}_\tau^*\right)\left[1 - \lambda\phi' e^{x(\tau-t)}\right] = r+\delta \;\;, \hat{k}_\tau^* > 0 \quad (33)$$

$$\lambda\left[\bar{P} - \phi\left(f\left(\hat{k}_\tau^*\right)AN\right)e^{x(\tau-t)}\right] = 0 \;, \lambda \geq 0 \quad (34)$$

If the emission constraint is not binding then $\lambda = 0$ and the solution $\hat{k}_\tau^*$ is obtained by the usual condition $f'\left(\hat{k}_\tau\right) = r + \delta$.[19] Under concavity of the production function and Inada conditions a unique solution always exists.[20] If $\lambda > 0$ then the constraint is binding and the capital stock is determined as a function of the emission limit by the solution of:

$$\bar{P} = \phi\left(f\left(\hat{k}_\tau\right)AN\right)e^{x(\tau-t)}\text{, as} \tag{35}$$

$$\hat{k}_\tau^* = \psi\left(\bar{P}; AN, e^{x(\tau-t)}\right) = \hat{k}_\tau^*\left(\bar{P}\right)\text{, with } \frac{d\hat{k}_\tau}{d\bar{P}} > 0 \tag{36}$$

Thus a more stringent emission limit will reduce the equilibrium stock of capital. This can be also seen from (33). A positive λ shifts the marginal product curve $f'\left(\hat{k}_\tau\right)$ to the left. As a result $\hat{k}_\tau^* < \hat{k}_\tau$ and the binding performance standard reduces the equilibrium stock of capital. It can be also noticed that if $x < 0$ so that we have emission saving technical change then the reduction of the equilibrium stock of capital, under the performance standard will be smaller, the larger this type of technical change is. Since capital stock is reduced from a binding performance standard or equivalently from a more stringent performance standard, output is also reduced ceteris paribus. This reduction is determined as $f\left(\hat{k}_\tau\right) - f\left(\hat{k}_\tau^*\left(\bar{P}\right)\right)$, where $f\left(\hat{k}_\tau\right)$ is the output of the economy without the performance standard, and $f\left(\hat{k}_\tau^*\left(\bar{P}\right)\right)$ is the output of the economy under the binding performance standard $\bar{P}$. Consumption in per effective worker terms is defined as $\hat{c}_\tau = (1-s)\,y_t$, and since $y = f\left(\hat{k}_\tau^*\left(\bar{P}\right)\right)$,we have:

$$\hat{c}_\tau = (1-s)\,f\left(\hat{k}_\tau^*\left(\bar{P}\right)\right) = \hat{c}_\tau(\bar{P}) \tag{37}$$

[19] Zero profits for any given wage w, require that

$$\left[f\left(\hat{k}_\tau\right) - \hat{k}_\tau f'\left(\hat{k}_\tau\right) + \lambda\phi' f'\left(\hat{k}_\tau\right)e^{x(\tau-t)}\right]e^{-g(\tau-t)} = w$$

[20] Inada conditions state that $lim_{k\to 0} f'(k) = +\infty$ and $lim_{k\to\infty} f'(k) = 0$. When they are combined with a concave production function then $f'(k)$ is monotonically declining and intersects $r + \delta$ only once providing a unique solution.

then the per capita utility flow in the economy under the performance standard will be:

$$U\left(\hat{c}_\tau, \bar{P}\right) = \left[- \left(\hat{c}_\tau A_t e^{g(\tau - t)}\right)^{-(\sigma-1)} - \theta \bar{P}^\gamma \right] \tag{38}$$

with $\hat{c}_\tau$ determined by (37). In empirical applications, where the main purpose is to examine the impact of a performance standard on the sustainability of the economy a reliable estimate of $f\left(\hat{k}^*_\tau\left(\bar{P}\right)\right)$ is unlikely due to data limitations. In this case an approach could be to assume that the reduced output under the binding standard is approximately proportional to the output obtained without a limit on emissions. This means that we set:

$$f\left(\hat{k}^*_\tau\left(\bar{P}\right)\right) \approx \left(1 - z_{\bar{P}}\right) f\left(\hat{k}_\tau\right) \tag{39}$$

which implies that $f\left(\hat{k}_\tau\right)$ can be interpreted as full capacity output, without environmental constraints.[21] Under the Cobb-Douglas assumption, we have:

$$\hat{y} = \left(1 - z_{\bar{P}}\right) \hat{k}^a$$

In this case the accumulation of capital equation in per effective worker terms, setting as before $z = 0$ for the capital of the immigrants, implies:

$$\dot{\hat{k}}_t + (\eta + \delta + m + g)\, \hat{k}_t = s(1 - z_{\bar{P}})\hat{k}^a_t \tag{40}$$

The solution of this Bernoulli equation is[22]:

$$\hat{k}_\tau = \left[\left(\hat{k}_t^{1-a} - \frac{s(1 - z_{\bar{P}})}{\omega} \right) e^{-(1-a)\omega(\tau - t)} + \frac{s(1 - z_{\bar{P}})}{\omega} \right]^{\frac{1}{1-a}} \tag{41}$$

$$\omega = \eta + \delta + m + g \tag{42}$$

Therefore, $\hat{c}_\tau = (1 - s)\left(1 - z_{\bar{P}}\right)\hat{k}^a_\tau = \hat{c}_\tau\left(\hat{k}_\tau; z_{\bar{P}}\right)$, and the value function for the economy becomes:

[21] For an estimate of the proportion of output loss due to environmetal regulation in the US economy see Jorgenson and Wilcoxen (1998)

[22] See Appendix1 for details

$$V_t = \int_t^T e^{-\rho(\tau-t)} N_t U(\hat{k}_\tau, N_\tau, A_\tau, \bar{P}; z_{\bar{P}}) dt \ , T \leq \infty \text{ or} \tag{43}$$

$$V_t = -\int_t^T e^{-\rho(\tau-t)} N_t \left[\left(\hat{c}_\tau \left(\hat{k}_\tau; z_{\bar{P}} \right) A_t e^{g(\tau-t)} \right)^{-(\sigma-1)} - \theta \bar{P}^\gamma \right] dt \ , T \leq \infty$$

The current accounting price for the performance standard $\bar{P}$ can be calculated as:

$$p_{t\bar{P}} = \frac{\partial V_t}{\partial \bar{P}} = \int_t^T e^{-\rho(\tau-t)} \frac{\partial}{\partial \bar{P}} U(\hat{k}_\tau, N_\tau, A_\tau, \bar{P}; z_{\bar{P}}) dt$$

Thus, there is a specific accounting price for the arbitrary control $\bar{P}$ as was anticipated by Proposition 1.

4 The Sustainability Criterion in a Non Optimizing Economy

The previous section obtained representations of value functions and accounting prices, combining these representations with Proposition 1 it follows that our stylized economy follows a weakly sustainable path at time t if:

$$\dot{V}_t = p_{K_t} \dot{K} + p_{N_t} \dot{N} + p_{A_t} \dot{A} \ \geq 0$$

Dividing by Nk, where $k = \frac{K}{N}$, using the fact that $\dot{k} = \frac{d(K/N)}{dt} = \frac{\dot{K}}{N} - \frac{\dot{N}}{N} k$, and that the accounting price for capital in physical terns is related to the accounting price of capital in per effective worker terms by (27) we obtain:

$$S_{1t} = \frac{\dot{V}_t}{N_t k_t} = \frac{p_{t\hat{k}_t}}{A_t N_t} \left(\frac{\dot{k}}{k} + \frac{\dot{N}}{N} \right) + p_{tN_t} \frac{\dot{N}}{N} \frac{1}{k_t} + p_{tA_t} \frac{\dot{A}}{A} \frac{A_t}{N_t k_t}$$

where S_{1t} measures the change in the value of the economy per unit of produced capital stock at time t. Thus S_{1t} could be interpreted as the rate of return on produced capital measured in terms of social welfare. It is clear that by multiplying S_{1t} by the current stock of capital we obtain a measure of current genuine investment. Using as before $\frac{\dot{A}}{A} = g$, $\frac{\dot{N}}{N} = \tilde{n} = n + m$, with $m \lesseqgtr 0$ depending on the migration rate, and denoting the rate of growth of per capital per worker $\frac{\dot{k}}{k} = v$, we have that development is currently sustainable, at time t, if:

$$S_t^F = \frac{p_{t\hat{k}_t}}{A_t N_t}(v + \tilde{n}) + p_{t_{N_t}} \tilde{n} \frac{1}{k_t} + p_{t_{A_t}} g \frac{1}{k_t} \frac{A_t}{N_t} \geq 0 \tag{44}$$

When an arbitrary environmental policy in the form of the emission limit $\bar{P}$ is present the criterion becomes:

$$\dot{V}_t = p_{K_t} \dot{K} + p_{N_t} \dot{N} + p_{A_t} \dot{A} + p_{\bar{P}_t} \frac{d\bar{P}}{dt} \geq 0 \text{ or} \tag{45}$$

$$S_{2t} = \frac{p_{\hat{k}_t}}{A_t N_t}(v + \tilde{n}) + p_{Nt} \tilde{n} \frac{1}{k_t} + p_{A_t} g \frac{1}{k_t} \frac{A_t}{N_t} + p_{\bar{P}t} \pi \frac{1}{k_t} \frac{\bar{P}_t}{N_t} \geq 0 \tag{46}$$

where π is *the rate of growth of the* emission limit, with $\pi < 0$ indicating that environmental policy becomes gradually more stringent and $\pi > 0$ indicating that environmental policy is gradually becoming more lax. As before, by multiplying S_{2t} by the current stock of capital we obtain a measure of current genuine investment. In this case genuine investment is adjusted for the changes in environmental policy, a required adjustment that has not been noticed in earlier literature.

Sustainability criteria (or indicators) (44) and (46) are basically short-term measures since they reflect sustainability conditions at time t. Sustainability conditions will change if basic parameters, such as growth rates of assets or choices of instruments, change. Since the economy is not on an optimal path these changes, especially in the case of arbitrary choice of controls, might actually take place. Therefore, if the basic parameters are likely to change then recalculations and updating of (44) or (46) are necessary. We believe that this observation is important especially for applied work.

5 Exploring Sustainability Conditions within the Greek Economy

The stylized model developed above is used to explore the current sustainability conditions within the Greek economy. To apply the model we need estimates of the parameters required to define value functions like those defined in (25) or (43).

Our approach was to estimate, using econometric estimations, the parameters that correspond to structural relations and to assign plausible values to those parameters that econometric estimation was not possible. For these parameters we used sensitivity analysis to explore the robustness of our results.

The parameters required in order to estimate criterion (44), (46) are: n the rate of growth of domestic labour force and m the migration rate; v the rate of growth of capital per worker; g the rate of growth of labour augmenting technological change; s which expresses savings as proportion of Greek GDP in the period analyzed; a which is the parameter of the production function reflecting the elasticity of capital input; ρ which represents the discount rate; σ the elasticity of marginal utility the value of which reflects preferences towards equality in income distribution; δ which is the depreciation rate; μ and γ which are the parameters of the postulated damage function $D(P_\tau) = \theta P_\tau^\gamma$; μ, β and x which are the parameters of the emission function $P_\tau = \mu Y_\tau^\beta e^{xt}$; and finally, when we need to examine the impact of an emission limit, the potential reduction in GDP due to this emission limit is required, which is the parameter $z_{\bar{P}}$. It was assumed in the absence of any data that z, the capital brought in Greece by migrants, was zero.

The fundamental data for the Greek economy were *GDP*, *Capital*, and *Labour*, measured in million 1985 US$ and thousands of workers respectively, taken from the Penn World Table (Mark 5.6) for the period 1965-1990. We obtain the average annual growth rates of these variables in physical units and in per capita terms during the sample period by estimating the relationship $\ln x_t = a_o + a_1 t$, where x_t is the variable of interest and t takes values $t =$

$1, ..., T$ during the sample period.[23]

The estimates of the growth rates for the variables of interest in physical and in per worker terms are shown in the table below

Table 1: Average growth rates of capital, output and labour in the Greek economy, 1965-1990.

Physical units	% per year	Per worker terms	% per year
Capital (K)	5.55	Capital (k)	4.95
GDP (Y)	3.64	GDP (y)	3.035
Labour (N)	0.6		

The basic structural relationship for the Greek economy is the aggregate production function (23), since estimates from the production function will be used to determine the elasticity of capital with respect to output, which is the parameter a, and the rate of labour augmenting technical change g. For this estimation we assume the existence of a constant returns to scale Cobb-Douglas long run aggregate production function for the Greek economy, defined over man made capital and effective labour input, which takes the form:

$$Y_t = BK^a \left(Le^{gt}\right)^{1-a}$$

or in per worker terms:[24]

$$y_t = Bk_t^a e^{qt} \ , q = g\left(1-a\right))$$

The statistical model can be written as:

$$\ln y_t = \ln B + a \ln k_t + qt + \varepsilon_t \ , t = 1, ..., T \tag{47}$$

where ε_t is the usual error term. The production function (47) can be interpreted as a long run equilibrium relationship that shifts in time as it is affected by technical change. To test for the existence of such equilibrium

[23] Relationship $\ln x_t = a_o + a_1 t$ corresponds to the standard exponential growth model $x_t = A_o e^{a_1 t}$.

[24] It is clear the in per worker terms this function becomes $\hat{y}_t = B\hat{k}_t^a$, which is the function used in the previous sections.with $B \equiv 1$

relationship we test for the existence of a cointegrating relationship. The Johansen cointegration test suggests that both the trace and the maximum eigenvalue tests indicate one cointegrating relationship with constant and deterministic trend at 5% level. When a cointegrating relationship exists ordinary least square (OLS) estimation is superconsistent, that is the estimated coefficients are consistent and asymptotically normal (Stock 1987). Using therefore OLS to estimate (47) we obtain that the elasticity of capital input is $a = 0.4025$, [25] while the rate of labour augmenting technical change is $g = \frac{q}{1-a} = 0.009$ or 0.9% annually[26]. The details of the cointegration test, and the OLS estimation results are presented in the Appendix 2.[27]

To model environmental pollution we consider as the main flow pollutant, sulfur dioxide emissions (SO_2). Sulfur dioxide emissions in Greece are mainly localized because the majority of them are created in the processes of power generation.[28] These emissions were related to output, assuming an emission function of the constant elasticity form (23), which was regarded as a technological relationship and was estimated using data of annual emissions in kilotons covering the period $1980 - 1999$.[29] The estimated elasticity of SO_2 emission with respect to aggregate output was 0.225. A trend term which could indicate technical change associated with SO_2 emissions was highly insignificant.[30]

To complete the set of required parameters we require the migration flow m, the marginal propensity to save s, the discount rate ρ, the depreciation rate δ, the elasticity of marginal utility σ, the parameters of the damage

[25] We did not include human capital in our production function. However, the value of estimated a can be regarded, under certain assumption as incorporating human capital effects. (Barro and Sala-i-Martin 2004).

[26] This method of estimating the labour augmenting technical change from a Cobb-Douglas production function.is similar to what is proposed by Barro and Sala-i-Martin (2004).

[27] All estimations were performed by using the sofware package EViews 5.0.

[28] Lignite fired power plans in Greece produced 63% of total electricity in 2003, and are concentrated mainly in two locations in the Northern and in the Southern part of the country.

[29] The source of the data was the European Environment Agency (Copenhagen).

[30] Estimates were corrected for first order serial correlation, which turned out to be highly significant. Details are presented in Appendix 2.

function, and the parameter $z_{\bar{P}}$ when we examine the impact of an emission limit.

For the migration rate a recent study (Lianos 2003) indicates that between 1991 and 2001 the number of immigrants who entered the Greek economy were around 630.000. Assuming an average annual flow of $M(t) = 60.000$ and dividing by the average number of workers in the same period of $N(t) = 4000000$ we arrive at an estimate of $m = \frac{M}{L} = 0.015$. For the marginal propensity to save we use the average value for the period $1970 - 1990$ of savings as a proportion of GDP, with $s = 0.21.$[31]. The depreciation rate was taken at $\delta = 3\%$ following Mankiw et al. (1992); the discount rate at $\rho = 3\%$; and the elasticity of marginal utility at $\sigma = 3$ which reflects relatively strong preferences towards equal income distribution. The parameter γ of the damage function was set at $\gamma = 1$. This implies a linear damage function in which θ reflects marginal damages. Since the units of output and consumption were million US \$, θ reflects the environmental damages in Greece, in million $US\$$, from the emissions of one kiloton of sulphur dioxide in a year. Beacuse, as mentioned above, SO_2 emissions are mainly localized, the value of θ in our model can be interpreted as capturing marginal damages averaged over the whole population. There are no estimates of environmental damages due to SO_2 emissions even at the local level. In the absence of any information the value of θ was taken in the interval $[10^{-6}, 10^{-3}]$ indicating damages from 1 US\$ to 1000 US\$ per kiloton of sulphur dioxide a year. For the parameter $z_{\bar{P}}$ there is also no information for the Greek economy. Jorgenson and Wilcoxen (1998), using a computable general equilibrium approach, estimated the cost of all environmental restrictions for the US economy, to be 2.592% of real GNP, so we set $z_{\bar{P}}$ at a conservative value of 1%.

The parameter values used are summarized in the following table:

Parameter	n	m	v	g	s	a	ρ	δ	σ
Value	0.006	0.015	0.0495	0.009	0.21	0.4025	0.03	0.03	3
Parameter	β	μ	x	γ	θ	$z_{\bar{P}}$			
Value	0.225	4.146	0	1	$[10^{-6}, 10^{-3}]$	0.01			

Using the above parameters accounting prices were calculated with nu-

[31]Data were taken from: "The Greek Economy in Figures, 2002". page 105.

merical integration of the derivatives of the value function[32] for a time horizon of 100 years.[33] *Two set of results were obtained*, one set corresponding to emissions determined by a feedback rule through the emission function and using the criterion (44) and another one regarding the 1999 sulphur dioxide emissions as an upper emission limit and using (46). Table 2 below shows accounting prices and the sustainability criterion for different marginal damages in Greece in 1990.

Table2: Accounting Prices and the Sustainability Criterion in Greece in 1990

m	θ	p_K	p_N	p_A	S_1
0	0	0.0011216	-0.0464493	315.511	0.00007839
0.015	0	0.00238486	-0.125464	852.225	0.00142968
0.015	10^{-6}	0.00238326	-0.126324	851.689	0.0001411
0.015	10^{-5}	0.00237046	-0.134064	846.860	0.00013461
0.015	10^{-4}	0.00224252	-0.21147	798.574	0.00005950
0.015	10^{-3}	0.00096316	-0.985523	315.712	-0.0006916

We can observe from the table above that for marginal environmental damages below 1000 US$ per kiloton of SO_2 the Greek economy was on a sustainable path in 1990. Since the basic structure of the economy has not changed, it might be claimed that the Greek economy is currently on a (weak) sustainable path. Furthermore, it is clear that migration has played an important role in the current sustainability conditions of the Greek economy, since the criterion is reduced substantially when we set $m = 0$. In addition, the accounting prices have the expected signs and the sustainability criterion is declining in environmental damages as expected. For sufficiently high marginal environmental damages the criterion becomes negative. Thus, as anticipated, the sustainability conditions for the Greek economy are sensitive to environmental damages.

In table 3 we present sustainability conditions under a binding environ-

[32] Mumerical results were obtained by using *Mathematica*.

[33] The fundamental parameters of the Greek economy imply that convergence to a steady state will take place in approximately 70 years. Thus, the time horizon chosen extends well into the steady state period. The results are robust to changes in the time horizon. Of course as noted above accounting prices need to be recalculated, if the fundamental growth rates used in the estimations change.

mental policy. Thus table 3 shows accounting prices and the sustainability criterion *as if* the emission limit for sulphur dioxide has been set at the 1999 emission level, which was 541 kilotons. Values have been calculated for $m = 0.015$ and $z = 0.01$

Table 3: Accounting Prices and the Sustainability Criterion in Greece in 1990 under an Emission Limit.

θ	p_K	p_N	p_A	$p_{\bar{P}}$	$p_{z_{\bar{P}}}$	S_2
10^{-6}	0.002445	−0.134	837.99	−0.03396	−1107.63	0.0001412
10^{-5}	0.002445	−0.183	837.99	−0.33963	−1107.63	0.0001047
10^{-4}	0.002445	−0.669	837.99	−3.33963	−1107.63	−0.000266
10^{-3}	0.002445	−5.559	837.99	−33.963	−1107.63	−0.003962

In the above table the column $p_{\bar{P}}$ refers to $\dfrac{\partial V}{\partial \bar{P}}$ which is the accounting price for the emission standard. This price is negative as expected, since an increase in $\bar{P}$ that is a more lax environmental policy, is expected to reduce the economy's value, *when* $z_{\bar{P}}$ remains constant. The column $p_{z_{\bar{P}}}$ refers to $\dfrac{\partial V}{\partial z_{\bar{P}}}$ which is negative as expected. This means that if the cost of the standard in terms of output foregone increases then the economy's value is reduced ceteris paribus. Since a lax standard is expected to reduce $z_{\bar{P}}$ the final outcome from a change in the performance standard on the value of the economy depends on the expression $\frac{\partial V}{\partial \bar{P}}d\bar{P} + \frac{\partial V}{\partial z_{\bar{P}}}dz_{\bar{P}}$. Again as expected the sustainability criterion is declining in marginal environmental damages.

6 Concluding Remarks

This paper aimed in formulating a concept of (weak) sustainable development under a non-optimizing framework. The main purpose was to help in providing an applicable and operational definition of sustainability, given today's needs for such a definition, and to contribute to the development of a framework for the evaluation of sustainability policies.

For this purpose we tried to determine an operational and measurable criterion for sustainable development that would fit into a *non-optimizing economic framework,* since we consider such a framework as adequately representing current economic conditions at least in developing countries. By considering two different approaches for choosing policy instruments, a *feedback rule* and an *arbitrary rule*, we determined two criteria for sustainable development which can provide measurable results in actual economies and could be applied in empirical work. Furthermore, we extended current results by showing that when policy rules are chosen in an arbitrary way, then sustainability criteria and genuine investment should be adjusted accordingly. Given the arbitrary nature of most government policies in practice - environmental policies included - this observation might have important implications for empirical applications. We provide exact representations and closed form solutions for value functions and accounting prices, by considering a "Solow" economy, where domestic population growth, migration, labour augmenting technical change, environmental damages associated with pollutant flows generated by economic activities are taken into account in determining the sustainability conditions.

The developed sustainability criteria were further applied to the case of the Greek economy and empirical estimates were obtained. Our findings confirmed that our theoretical framework can be used for empirical purposes. In particular our results show that migration inflows, exogenous technical change, growth of capital per worker and SO_2 emissions are important factors characterizing the sustainability conditions for the Greek economy. Our approach allows the estimations of the contribution of these factors in the achievement of (weak) sustainability conditions, information which is undoubtedly useful for the design and evaluation of sustainable development policies. The empirical finding is that although the Greek economy seems to be currently sustainable if no environmental considerations are taken into account, considering such damages has undoubtedly negative effects on the sustainability conditions. High marginal emissions damages could lead to non sustainable development paths. Thus our empirical results for the case of Greece come to reinforce the perception that pollution - in this case SO_2 emissions - is an important factor which affects the sustainability conditions of the economy. Our approach not only provides an empirical conformation of this result but can be used to quantify, at least approximately, these environmental impacts on sustainability indicators, another important information for policy design. A more precise quantification of these effects, is an open research area.

Admittedly sustainable development as a general definition does not provide a systematic framework for policy design. The present paper is an attempt to make the definition operational and capable of providing empirical estimates of sustainability conditions with a firm foundation on the structure of the economy. Thus important fundamentals, such as the elasticity of the production function, the rate of technical change, migration, environmental damages, assets' rates of growth, play a key role in estimating the sustainability conditions. The model developed in this paper can be extended and become more realistic, by including transition equations for stocks of pollutants, or natural resources (depletable or renewable) human capital, or uncertainty in the evolution of the economy. These extensions will provide better insights regarding the sustainability conditions of economies and will enhance our ability to provide meaningful estimates of the sustainability concept.

Appendix 1

Solutions of the Bernoulli equations for the capital stock.

Solution of Equation (17)

The Bernoulli equation is solved in the following way: Multiplying by $\hat{k}_t^{-a}$ we have:

$$\dot{\hat{k}}_t\hat{k}_t^{-a} + (\eta + \delta + m + g)\,\hat{k}_t\hat{k}_t^{-a} = s\hat{k}_t^{-a}\hat{k}_t^{a} \tag{48}$$

$$\dot{\hat{k}}_t\hat{k}_t^{-a} + (\eta + \delta + m + g)\,\hat{k}_t^{1-a} = s \tag{49}$$

If $\gamma = \hat{k}_t^{1-a}$ and $\dot{\gamma} = (1-a)\left(\dot{\hat{k}}_t\hat{k}_t^{-a}\right)$, then we have:

$$\dot{\gamma} + (\eta + \delta + m + g)\,\gamma(1-a) = (1-a)s, \text{ which is linear in } \gamma \tag{50}$$

with solution:

$$\gamma_t = \left(\gamma_o - \frac{s}{\eta + \delta + m + g}\right)e^{-(1-a)(\eta+\delta+m+g)t} + \frac{s}{\eta + \delta + m + g} \tag{51}$$

setting $\gamma_t = \hat{k}_t^{1-a}$, we have:

$$\hat{k}_t = \left[\left(\hat{k}_o^{1-a} - \frac{s}{\eta + \delta + m + g}\right)e^{-(1-a)(\eta+\delta+m+g)t} + \frac{s}{\eta + \delta + m + g}\right]^{\frac{1}{1-a}}$$

$$\hat{k}_\tau = \left[\left(\hat{k}_t^{1-a} - \frac{s}{\eta + \delta + m + g}\right)e^{-(1-a)(\eta+\delta+m+g)(\tau-t)} + \frac{s}{\eta + \delta + m + g}\right]^{\frac{1}{1-a}}$$

Solution of Equation (40)

Following the procedure above and by using instead of $\hat{y} = \hat{k}^a$, the function $\hat{y} = (1 - z_{\bar{P}})\,\hat{k}^a$ the accumulation of capital in per effective worker terms becomes:

$$\dot{\hat{k}}_t = s(1 - z_{\bar{P}})\hat{k}_t^a - (\eta + \delta + g)\,\hat{k}_t - m\hat{k}_t + z$$

Multiplying by $\hat{k}_t^{-a}$ we have:

$$\dot{\hat{k}}_t\hat{k}_t^{-a} + (\eta + \delta + m + g)\,\hat{k}_t\hat{k}_t^{-a} = s(1 - z_{\bar{P}})\hat{k}_t^{-a}\hat{k}_t^{a} \tag{52}$$

$$\dot{\hat{k}}_t \hat{k}_t^{-a} + (\eta + \delta + m + g)\,\hat{k}_t^{1-a} = s(1 - z_{\bar{P}}) \tag{53}$$

If $\gamma = \hat{k}_t^{1-a}$ and $\dot{\gamma} = (1-a)\left(\dot{\hat{k}}_t \hat{k}_t^{-a}\right)$, then we have:

$$\dot{\gamma} + (\eta + \delta + m + g)\,\gamma(1-a) = (1-a)s(1 - z_{\bar{P}}),\ \text{which is linear in } \gamma \tag{54}$$

with solution:

$$\gamma_t = \left(\gamma_o - \frac{s(1 - z_{\bar{P}})}{\eta + \delta + m + g}\right) e^{-(1-a)(\eta+\delta+m+g)t} + \frac{s(1 - z_{\bar{P}})}{\eta + \delta + m + g} \tag{55}$$

replacing $\gamma_t = \hat{k}_t^{1-a}$, we have:

$$\hat{k}_t = \left[\left(\hat{k}_o^{1-a} - \frac{s(1 - z_{\bar{P}})}{\eta + \delta + m + g}\right) e^{-(1-a)(\eta+\delta+m+g)t} + \frac{s(1 - z_{\bar{P}})}{\eta + \delta + m + g}\right]^{\frac{1}{1-a}}$$

Appendix 2

Johansen Cointegration Test

Trend assumption: Linear deterministic trend (restricted)

Series: $lny_t, \ln k_t$

Lags interval (in first differences): 1:1

Unrestricted Cointegration Rank Test (Trace)

Hypothesized No of CE(s)	Eigenvalue	Trace Statistic	0.05 Critical Value	Prob**
None*	0.649227	30.97507	25.87211	0.0106
At most 1	0.347789	8.975136	12.51798	0.1819

Trace test indicates 1 cointegrating eqn(s) at he 0.05 level

*denotes rejection of the hypothesis at the 0.05 level

**MacKinnon-Haug-Michelis (1999) p-values

Unrestricted Cointegration Rank Test (maximum Eigenvalue)

Hypothesized No of CE(s)	Eigenvalue	Trace Statistic	0.05 Critical Value	Prob**
None*	0.649227	21.99993	19.38704	0.0204
At most 1	0.347789	8.975136	12.51798	0.1819

Max-Eigenvalue test indicates 1 cointegrating eqn(s) at he 0.05 level

*denotes rejection of the hypothesis at the 0.05 level

**MacKinnon-Haug-Michelis (1999) p-values

Econometric Estimations

The Production Function

Variable	**Coefficient**	**Std. Error**	**t-Statistic**
$\ln B$	1.438115	0.187444	7.672226
$\ln k$	0.402501	0.080129	5.023150
t	0.005392	0.003080	1.750943

R-squared	0.957372
Adjusted R-squared	0.952636
Durbin-Watson stat	1.175040

The Emission Function

Variable	Coefficient	Std. Error	t-Statistic
constant	4.156018	2.289511	1.815243
$\ln Y$	0.225308	0.241803	1.931786
AR(1)	0.745520	0.084558	8.816671
R-squared	0.906529		
Adjusted R-squared	0.894845		
Durbin-Watson stat	2.176287		

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