COALITION FORMATION UNDER UNCERTAINTY AND RISK: THE SUCCESS OF INTERNATIONAL ENVIRONMENTAL AGREEMENTS

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1. **Coalition Formation and Uncertainty (No Risk)**
2. **Optimal Transfer Scheme**
3. **Introducing Risk in a Simple PD-Game**
4. **Possible Extensions**

Coalition Formation and Uncertainty (No Risk)

Dellink, R. and M. Finus (2009), Uncertainty and Climate Treaties: Does Ignorance Pay? Stirling Economics Discussion Paper No. 2009-16, UK.

Dellink, R., M. Finus and N. Olieman (2008), The Stability Likelihood of an International Climate Agreement. *Environmental and Resource Economics*, **39**: 357-377.

Finus, M. and P. Pintassilgo (2010), International Environmental Agreements under Uncertainty: Does the Veil of Uncertainty Help? Economics Department Discussion Paper Series, 10/03, 2010, University of Exeter, UK.

Kolstad, C. (2007), Systematic Uncertainty in Self-enforcing International Environmental Agreements. *Journal of Environmental Economics and Management*, **53**: 68-79.

Kolstad, C. and A. Ulph (2008), Learning and International Environmental Agreements. *Climatic Change*, **89**: 125-141.

Kolstad, C. and A. Ulph (2009), Uncertainty, Learning and Heterogeneity in International Environmental Agreements.

Na, S.-L. and H.S. Shin (1998), International Environmental Agreements under Uncertainty. *Oxford Economic Papers*, **50**: 173-185.

Theme

- climate change
- uncertainty
- learning (research, Stern Report, IPCC)
- international environmental agreements (IEAs)

Motivation of Research

Result: Learning can be bad in the context of IEAs!

Ulph (1998), Na/Shin (1998), Kolstad (2007), Kolstad/Ulph (2008, 2009)

Question 1: How general is this result?

Question 2: What are the driving forces?

Question 3: Can the problem be fixed?

Dellink et al. (2008) and Dellink and Finus (2009).

Basic Setting

Stage 1: Membership

internal stability: $\enclose{circle}{\Pi_i^*(S)} \geq \boxed{\Pi_i^*(S \setminus \{i\})} \; \forall i \in S$

{{S}, {i}, …., {m}}

external stability: $\enclose{circle}{\Pi_j^*(S)} \geq \boxed{\Pi_j^*(S \cup \{j\})} \; \forall j \notin S$

Stage 2: Abatement Decision

Coalition Members: $\max_{q^S} . \sum_{i \in S} \Pi_i(q^S, q^{-S}) \; \forall i \in S$

$$\Rightarrow \boxed{q^*(S)} \Rightarrow \boxed{\pi^*(S)}$$

Singletons: $\max_{q_j} . \Pi_j(q_j, q^{-j}) \; \forall j \notin S$

Basic Setting

- symmetric uncertainty (analytical uncertainty; vs asymmetric= strategic uncertainty)
- risk neutrality
- uncertainty about the parameters of the payoff function

Assumptions

1) Number of Players: 3 ⇒ N players

2) Payoff Function: $\Pi_i = b_i \left(\sum_{k=1}^{n} y_k \right) - c_i y_i^2$

$$\Pi_i = \theta_i \left(\sum_{k=1}^{n} y_k \right) - y_i^2 \qquad \text{if } c_i = c_j = c,\ \theta_i = {}^{b_i}\!/_{c}$$

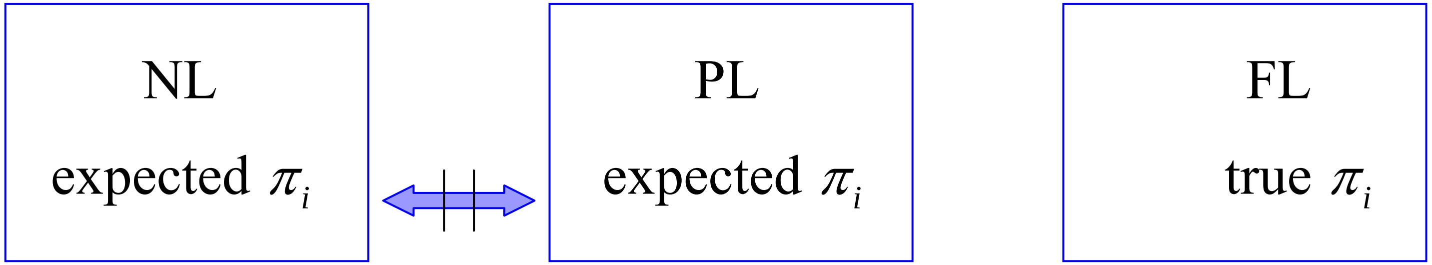
$$\Pi_i = \left(\sum_{k=1}^{n} y_k \right) - \frac{1}{\theta_i} y_i^2 \qquad \text{if } b_i = b_j = b,\ \theta_i = {}^{b}\!/_{c_i}$$

Assumptions

3) Learning Scenarios: No, Full and Partial Learning (NL, FL, PL)

Three Scenarios of Learning

Stage 1: Membership



NL
expected π_i

PL
expected π_i

FL
true π_i

Stage 2: Abatement Decision

NL
expected θ_i

PL
true θ_i

FL
true θ_i

Assumptions

3) Learning Scenarios: No, Full and Partial Learning (NL, FL, PL)
4) Four Cases of Uncertainty

Case 1: Uncertainty about the Distribution of Benefits without Transfers (Na/Shin Case)

$$\Pi_i = \theta_i\left(\sum_{k=1}^{n} y_k\right) - y_i^{2} \qquad \text{if } \; c_i = c_j = c,\; \theta_i = {b_i}/{c}$$

ex-ante: symmetric; $E\left[\Theta_i\right]$ same for all players

ex-post : asymmetric; uniform probability distribution

$$f_{\Theta_i}\left(\theta_i\right) = \begin{cases} \dfrac{1}{n} & \mathit{for} \quad \theta_i = k,\; k \in N \\ 0 & \quad \mathit{otherwise} \end{cases}$$

$$\Theta_i \neq \Theta_k \;\; \forall\, i, k \in N$$

$$\bigcup_{i=1}^{n} \Theta_i = \{1, 2, \ldots, n\}$$

Lemma 1: Second Stage in Case 1

In every possible coalition structure:

Individual and Total Expected Abatement Levels: $FL = PL = NL$.

Individual and Total Expected Payoff Levels: $FL = PL \leq NL$ *with strict inequality if* $S \neq N$.

Lemma 2: First Stage in Case 1

$$E\left[m^{*PL}\right] = E\left[m^{*NL}\right] = 3 \;>\; E\left[m^{*FL}\right] = \begin{cases} 1 & if \quad n = 3 \\ 2 & if \quad n \geq 4 \end{cases}$$

Proposition 1: Outcome in Case 1

1) *Total Abatement:* $NL = PL > FL$

2) *Total Payoff:* $$\begin{cases} NL = PL > FL & if \quad n = 3 \\ NL > PL > FL & if \quad n \geq 4 \end{cases}.$$

Intuition?

$$\Pi_i = \theta_i \sum_{k=1}^{n} q_k - \frac{q_i^2}{2}$$

Simple Example

- only two players
- only 2 learning scenarios: FL and NL
- only 2 cases of uncertainty: level and distribution
- only 2 realizations of parameter

Three Driving Forces

a) information effect from learning: zero

b) strategic effect from learning: negative

c) distributional effect from learning: negative

Case	**Uncertainty about**	**Example**	**Ex-Post Realizations**
1	distribution	$(1,2), (2,1)$	asymmetric
2	level	$(1,1), (2,2)$	symmetric

$$\Pi_i = \theta_i \sum_{k=1}^{n} q_k - \frac{q_i^{\,2}}{2}$$

$$\text{Full Cooperation}: \sum MB_j = MC_i \;\Rightarrow\; \sum \theta_j = q_i$$

$$\text{No Cooperation}: \quad MB_i = MC_i \;\Rightarrow\; \theta_i = q_i$$

$$\Pi_i = \theta_i \sum_{k=1}^{n} q_k - \frac{q_i^{\,2}}{2}$$

$$\text{Full Cooperation}: \sum MB_j = MC_i \Rightarrow \sum \theta_j = q_i$$

$$\text{No Cooperation}: \quad MB_i = MC_i \Rightarrow \theta_i = q_i$$

	Full Cooperation				No Cooperation			
θ_i	Abatement		Payoffs		Abatement		Payoffs	
	FL	NL	FL	NL	FL	NL	FL	NL
Case 1	*Uncertainty about the Distribution of Benefits*							
1; 2	3; 3	3; 3	1.5; 7.5	4.5; 4.5	1; 2	1.5; 1.5	2.5; 4	3.38; 3.38
2; 1	3; 3	3; 3	7.5; 1.5	4.5; 4.5	2; 1	1.5; 1.5	4, 2.5	3.38; 3.38
$\varnothing$	3; 3	3; 3	4.5; 4.5	4.5; 4.5	1.5; 1.5	1.5; 1.5	3.25; 3.25	3.38; 3.38
Case 2	*Uncertainty about the Level of Benefits*							
1; 1	2; 2	3; 3	2; 2	4.5; 4.5	1; 1	1.5; 1.5	1.5; 1.5	3.38; 3.38
2; 2	4; 4	3; 3	8; 8	4.5; 4.5	2; 2	1.5; 1.5	6; 6	3.38; 3.38
$\varnothing$	3; 3	3; 3	5; 5	4.5; 4.5	1.5; 1.5	1.5; 1.5	3.75; 3.75	3.38; 3.38

Case 2: Uncertainty about the Level of Benefits (Kolstad/Ulph Case)

$$\Pi_i = \theta_i\left(\sum_{k=1}^{n} y_k\right) - y_i^{2} \qquad \text{if } c_i = c_j = c,\ \theta_i = {}^{b_i}\!/_{c}$$

Kolstad (2007) and Kolstad and Ulph (2008, 2009): systematic uncertainty

ex-ante: symmetric; same expectations

ex-post: symmetric; once uncertainty is resolved, $\Theta_i = \Theta_k \ \forall\, i, k \in N$

no assumption about probability distribution f_{Θ_i} is required

Proposition 2: Outcome in Case 2

1) *Total Abatement:* $FL = PL = NL$

2) *Total Payoff:* $FL = PL > NL$.

Three Driving Forces

a) information effect from learning: positive

b) strategic effect from learning: positive

c) distributional effect from learning: zero

$$\Pi_i = \theta_i \sum_{k=1}^{n} q_k - \frac{q_i^{\,2}}{2}$$

$$\textit{Full Cooperation}: \sum MB_j = MC_i \Rightarrow \sum \theta_j = q_i$$

$$\textit{No Cooperation}: \quad MB_i = MC_i \Rightarrow \theta_i = q_i$$

	Full Cooperation				No Cooperation			
θ_i	Abatement		Payoffs		Abatement		Payoffs	
	FL	NL	FL	NL	FL	NL	FL	NL
Case 1	*Uncertainty about the Distribution of Benefits*							
1; 2	3; 3	3; 3	1.5; 7.5	4.5; 4.5	1; 2	1.5; 1.5	2.5; 4	3.38; 3.38
2; 1	3; 3	3; 3	7.5; 1.5	4.5; 4.5	2; 1	1.5; 1.5	4, 2.5	3.38; 3.38
$\varnothing$	3; 3	3; 3	4.5; 4.5	4.5; 4.5	1.5; 1.5	1.5; 1.5	3.25; 3.25	3.38; 3.38
Case 2	*Uncertainty about the Level of Benefits*							
1; 1	2; 2	3; 3	2; 2	4.5; 4.5	1; 1	1.5; 1.5	1.5; 1.5	3.38; 3.38
2; 2	4; 4	3; 3	8; 8	4.5; 4.5	2; 2	1.5; 1.5	6; 6	3.38; 3.38
$\varnothing$	3; 3	3; 3	5; 5	4.5; 4.5	1.5; 1.5	1.5; 1.5	3.75; 3.75	3.38; 3.38

Case 3: Uncertainty about the Distribution of Benefits with Transfers

- almost ideal transfer scheme
- cannot influence negative strategic effect (FL, PL $\prec$ NL)
- counterbalances negative distribution effect from learning and even turns it into positive effect (improves FL over PL and NL)

$$E\left[m^{*PL}\right]=E\left[m^{*NL}\right]=3 \;\leq\; E\left[m^{*FL}\right]=\begin{cases}3 & if \quad n\leq 8\\ f\left(n\right)>3 & if \quad n\geq 9\end{cases}$$

Total Payoff:

$$\begin{cases}FL=PL=NL & if \quad n=3\\ NL>FL=PL & if \quad 4\leq n\leq 8\\ NL>FL>PL & if \quad n=9\\ FL>NL>PL & if \quad n\geq 10\end{cases}$$

$$\Pi_i = \theta_i \sum_{k=1}^{n} q_k - \frac{q_i^{2}}{2}$$

$$\text{Full Cooperation}: \sum MB_j = MC_i \Rightarrow \sum \theta_j = q_i$$

$$\text{No Cooperation}: \quad MB_i = MC_i \Rightarrow \theta_i = q_i$$

	Full Cooperation				No Cooperation			
θ_i	Abatement		Payoffs		Abatement		Payoffs	
	FL	NL	FL	NL	FL	NL	FL	NL
Case 1	*Uncertainty about the Distribution of Benefits*							
1; 2	3; 3	3; 3	1.5; 7.5	4.5; 4.5	1; 2	1.5; 1.5	2.5; 4	3.38; 3.38
2; 1	3; 3	3; 3	7.5; 1.5	4.5; 4.5	2; 1	1.5; 1.5	4, 2.5	3.38; 3.38
∅	3; 3	3; 3	4.5; 4.5	4.5; 4.5	1.5; 1.5	1.5; 1.5	3.25; 3.25	3.38; 3.38
Case 2	*Uncertainty about the Level of Benefits*							
1; 1	2; 2	3; 3	2; 2	4.5; 4.5	1; 1	1.5; 1.5	1.5; 1.5	3.38; 3.38
2; 2	4; 4	3; 3	8; 8	4.5; 4.5	2; 2	1.5; 1.5	6; 6	3.38; 3.38
∅	3; 3	3; 3	5; 5	4.5; 4.5	1.5; 1.5	1.5; 1.5	3.75; 3.75	3.38; 3.38

Two effects from forming a coalition:

1) Interalization of an externality (non-exclusive to coalition)

2) Equalizing marginal abatement costs (exclusive to coalition)

Case 4: Uncertainty about the Distribution of Costs

- information and strategic effect from learning positive (FL, PL $\succ$ NL)
- without transfers: distributional effect from learning negative for FL
- transfers: mitigate distributional effect, but does not turn it into positive effect
- with transfers: Total Payoffs and Abatement: FL=PL>NL

Summary

Only if there is much uncertainty about the distribution of the benefits

from cooperation (relative to the level of the benefits or the distribution

of abatement costs) and no hedging strategy is used

will the veil of uncertainty be good.

Optimal Transfer Scheme

Eckmans, J. and M. Finus (2009), An Almost Ideal Sharing Scheme for Coalition Games with Externalities. Stirling Economics Discussion Paper No. 2009-10, UK.

Fuentes-Albero, C. and S.J. Rubio (2009), Can the International Environmental Cooperation Be Bought? Forthcoming European Journal of Operation Research.

McGinty, M. (2007), International Environmental Agreements among Asymmetric Nations. “Oxford Economic Papers”, vol. 59(1), pp. 45-62.

Weikard, H.-P. (2009). Cartel Stability under Optimal Sharing Rule. The Manchester School, vol. 77(5), 575-593.

→ 1. **Motivation**

2. **Definitions**

3. **Illustration and Preliminaries**

4. **Results**

positive versus negative externalities

- output and price cartel
- R&D-joint ventures with high spillovers
- public goods (environment, health and anti-terrorism)
- monetary policy

- R&D-joint ventures with low spillovers
- customs union

- sharing the gains from cooperation
  - fixed sharing rule
    - symmetry
    - asymmetry

1) existence

2) robustness

3) optimality

- research items
  - positive approach (Maskin 2003, Ray/Vohra 1999)
  - normative approach

→ optimal transfer scheme

→ cartel formation game (d´Aspremont et al. 1983)

1. **Motivation**

2. → **Definitions**

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4. **Results**

Definitions

- coalition game: $\Gamma(N, \pi)$
- players: $N = \{1, \ldots, n\}$, $n \geq 2$
- coalition: $S \subseteq N$; non-coalition members are singletons
- partition function

$$[1] \quad \pi : S \mapsto \pi(S) = (\enclose{circle}{\pi_S(S)}, \enclose{circle}{\pi_j(S)}) \in \mathbb{R}^{1+(n-s)}, \; j \in N \setminus S.$$

- valuation function

$v : S \mapsto v(S)$, such that

$$[2] \quad \begin{cases} \boxed{\sum_{i \in S} v_i(S) = \pi_S(S)} \\ \enclose{circle}{v_j(S) = \pi_j(S)} \quad \forall \, j \in N \setminus S. \end{cases}$$

- **stable coalition**

Let v be a valuation function for coalition game $\Gamma(N, \pi)$ and $v(S) \in \mathbb{R}^n$ the vector of valuations for the players in N when coalition S forms. Coalition S is stable with respect to the valuation v(S) if and only if:

- internal stability: $v_i(S) \geq v_i(S \setminus \{i\}) \quad \forall\, i \in S$
- external stability : $v_j(S) > v_j(S \cup \{j\}) \quad \forall\, j \in N \setminus S.$

1. **Motivation**

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Partition function (superadditivity, positive externality)

Shapley-Value

$v_i^{SV}(S)$

Nash-Bargaining Solution

$v_i^{NB}(S)$

Optimal Transfer Scheme

$v_i^{OP}(S)$

Nash-Bargaining Solution

$$v_i^{NB}(S) = \pi_i(\{i\}) + \lambda_i \boxed{\left[\pi_S(S) - \sum_{i\in S} \pi_i(\{i\})\right]},\ \sum_{j\in S} \lambda_j = 1,\ \lambda_j \geq 0$$

Assumption Example:

$$\omega_A = 1,\ \omega_B = 2,\ \omega_C = 3 \text{ and } \omega_D = 4 \text{ with } \lambda_i = \frac{\omega_i}{\sum_{j\in S} \omega_j}$$

Optimal Transfer Scheme

$$v_i^{OP}(S) = \pi_i(S\setminus\{i\}) + \lambda_i \boxed{\left[\pi_S(S) - \sum_{i\in S} \pi_i(S\setminus\{i\})\right]},\ \sum_{j\in S} \lambda_j = 1,\ \lambda_j \geq 0$$

Assumption Example: equal weights

Shapley-Value

coalition	v_a	v_b	v_c	v_d	$\sum v_i$	IS	ES	S
singletons	0	0	0	0	0	1	0	0
{a,b}	0.5	0.5	1	1	3	1	0	0
{a,c}	0.5	2.33	0.5	3.33	6.67	1	0	0
{a,d}	1.17	1	2	1.17	5.33	1	0	0
{b,c}	3.33	1.5	1.5	1	7.33	1	0	0
{b,d}	3.33	0.5	2	0.5	6.33	1	0	0
{c,d}	1	6	0.5	0.5	8	1	0	0
{a,b,c}	2.33	3.33	3.33	4.33	13.33	0	1	0
{a,b,d}	2	1.33	3	2	8.33	0	0	0
{a,c,d}	2	7	1.33	2	12.33	0	1	0
{b,c,d}	4.33	3	3	2	12.33	0	1	0
{a,b,c,d}	4	5	5	3.67	17.67	0	1	0

Shapley-Value

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Definition: Superadditivity

A coalition game $\Gamma(N,\pi)$ is superadditive if and only if its partition function π satisfies:

$$\forall\, S \subseteq N, \forall\, i \in S:\ \pi_S(S) \geq \pi_{S\setminus\{i\}}(S\setminus\{i\}) + \pi_i(S\setminus\{i\}).$$

Definition: Positive Externalities

A coalition game $\Gamma(N,\pi)$ exhibits positive externalities if and only if its partition function π satisfies:

$\forall\, S \subseteq N, \forall\ j \neq i,\ j \notin S:\ \pi_j(S) \geq \pi_j(S\setminus\{i\})$ and

$\exists\, k \neq i,\ k \notin S:\ \pi_k(S) > \pi_k(S\setminus\{i\}).$

Shapley-Value

coalition	v_a	v_b	v_c	v_d	$\sum v_i$	IS	ES	S
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{a,b,d}	2	1.33	3	2	8.33	0	0	0
{a,c,d}	2	7	1.33	2	12.33	0	1	0
{b,c,d}	4.33	3	3	2	12.33	0	1	0
{a,b,c,d}	4	5	5	3.67	17.67	0	1	0

Shapley-Value

coalition	v_a	v_b	v_c	v_d	$\sum v_i$	IS	ES	S
singletons	0	0	0	0	0	1	0	0
{a,b}	0.5	0.5	1	1	3	1	0	0
{a,c}	0.5	2.33	0.5	3.33	6.67	1	0	0
{a,d}	1.17	1	2	1.17	5.33	1	0	0
{b,c}	3.33	1.5	0.5	1	7.33	1	0	0
{b,d}	3.33	0.5	2	0.5	6.33	1	0	0
{c,d}	1	6	0.5	0.5	8	1	0	0
{a,b,c}	2.33	3.33	3.33	4.33	13.33	0	1	0
{a,b,d}	2	1.33	3	2	8.33	0	0	0
{a,c,d}	2	7	1.33	2	12.33	0	1	0
{b,c,d}	4.33	3	3	2	12.33	0	1	0
{a,b,c,d}	4	5	5	3.67	17.67	0	1	0

Shapley-Value

coalition	v_a	v_b	v_c	v_d	$\sum v_i$	IS	ES	S
singletons	0	0	0	0	0	1	0	0
{a,b}	0.5	0.5	1	1	3	1	0	0
{a,c}	0.5	2.33	0.5	3.33	6.67	1	0	0
{a,d}	1.17	1	2	1.17	5.33	1	0	0
{b,c}	3.33	1.5	0.5	1	7.33	1	0	0
{b,d}	3.33	0.5	2	0.5	6.33	1	0	0
{c,d}	1	6	0.5	0.5	8	1	0	0
{a,b,c}	2.33	3.33	3.33	4.33	13.33	0	1	0
{a,b,d}	2	1.33	3	2	8.33	0	0	0
{a,c,d}	2	7	1.33	2	12.33	0	1	0
{b,c,d}	4.33	3	3	2	12.33	0	1	0
{a,b,c,d}	4	5	5	3.67	17.67	0	1	0

Shapley-Value

coalition	v_a	v_b	v_c	v_d	$\sum v_i$	IS	ES	S
singletons	0	0	0	0	0	1	0	0
{a,b}	0.5	0.5	1	1	3	1	0	0
{a,c}	0.5	2.33	0.5	3.33	6.67	1	0	0
{a,d}	1.17	1	2	1.17	5.33	1	0	0
{b,c}	3.33	1.5	0.5	1	7.33	1	0	0
{b,d}	3.33	0.5	2	0.5	6.33	1	0	0
{c,d}	1	6	0.5	0.5	8	1	0	0
{a,b,c}	2.33	3.33	3.33	4.33	13.33	0	1	0
{a,b,d}	2	1.33	3	2	8.33	0	0	0
{a,c,d}	2	7	1.33	2	12.33	0	1	0
{b,c,d}	4.33	3	3	2	12.33	0	1	0
{a,b,c,d}	4	5	5	3.67	17.67	0	1	0

Nash-Bargaining Solution

coalition	v_a	v_b	v_c	v_d	$\sum v_i$	IS	ES	S
singletons	0	0	0	0	0	1	0	0
{a,b}	0.33	0.67	1	1	3	1	0	0
{a,c}	0.25	2.33	0.75	3.33	6.67	1	0	0
{a,d}	0.2	1	2	0.8	5.33	1	0	0
{b,c}	3.33	0.4	0.6	1	7.33	1	0	0
{b,d}	3.33	0.33	2	0.66	6.33	1	0	0
{c,d}	1	6	0.43	0.57	8	1	1	1
{a,b,c}	1.5	3	4.5	4.33	13.33	0	0	0
{a,b,d}	0.76	1.52	3	3.05	8.33	0	0	0
{a,c,d}	0.67	7	2	2.67	12.33	0	1	0
{b,c,d}	4.33	1.78	2.67	3.56	12.33	0	1	0
{a,b,c,d}	1.77	3.53	5.3	7.01	17.67	0	1	0

Preliminary Conclusion

A stable coalition may not exist.

Different transfer rules imply different equilibria.

Question

Is there a transfer rule that guarantees existence of an equilibrium?

Is there a transfer rule that leads to an equilibrium with higher global welfare?

Is there an optimal transfer rule?

Potentially Internally Stable Coalitions

A coalition S is called *potentially internally stable* (PIS) for partition function π if: $\boxed{\pi_S(S)} \geq \boxed{\sum_{i\in S} \pi_i(S\setminus\{i\})}$.

Lemma 1

Every coalition $S \subseteq N$ is potentially internally stable if and only if S is internally stable for any transfer rule belonging to the AISS (irrespective of weights).

Optimal Transfer Scheme, $\lambda_i = 1/s$

coalition	v_a	v_b	v_c	v_d	$\sum v_i$	IS	ES	S
singletons	0	0	0	0	0	1	0	0
{a,b}	0.5	0.5	1	1	3	1	0	0
{a,c}	0.5	2.33	0.5	3.33	6.67	1	0	0
{a,d}	1.17	1	2	1.17	5.33	1	0	0
{b,c}	3.33	1.5	1.5	1	7.33	1	0	0
{b,d}	3.33	0.5	2	0.5	6.33	1	0	0
{c,d}	1	6	0.5	0.5	8	1	1	1
{a,b,c}	4.11	3.11	1.78	4.33	13.33	1	1	1
{a,b,d}	3.33	1	3	1	8.33	1	1	1
{a,c,d}	0.67	7	1.67	3	12.33	0	1	0
{b,c,d}	4.33	5.67	1.67	0.67	12.33	0	1	0
{a,b,c,d}	4.08	6.75	2.75	4.08	17.67	0	1	0

Optimal Transfer Scheme, $\lambda_i = 1/s$

coalition	v_a	v_b	v_c	v_d	$\sum v_i$	IS	ES	S
singletons	0	0	0	0	0	1	0	0
{a,b}	0.5	0.5	1	1	3	1	0	0
{a,c}	0.5	2.33	0.5	3.33	6.67	1	0	0
{a,d}	1.17	1	2	1.17	5.33	1	0	0
{b,c}	3.33	1.5	1.5	1	7.33	1	0	0
{b,d}	3.33	0.5	2	0.5	6.33	1	0	1
{c,d}	1	6	0.5	0.5	8	1	1	1
{a,b,c}	4.11	3.11	1.78	4.33	13.33	1	1	1
{a,b,d}	3.33	1	3	1	8.33	1	1	1
{a,c,d}	0.67	7	1.67	3	12.33	0	1	0
{b,c,d}	4.33	5.67	1.67	0.67	12.33	0	1	0
{a,b,c,d}	4.08	6.75	2.75	4.08	17.67	0	1	0

Optimal Transfer Scheme, $\lambda_i = 1/s$

coalition	v_a	v_b	v_c	v_d	$\sum v_i$	IS	ES	S
singletons	0	0	0	0	0	1	0	0
{a,b}	0.5	0.5	1	1	3	1	0	0
{a,c}	0.5	2.33	0.5	3.33	6.67	1	0	0
{a,d}	1.17	1	2	1.17	5.33	1	0	0
{b,c}	3.33	1.5	1.5	1	7.33	1	0	0
{b,d}	3.33	0.5	2	0.5	6.33	1	0	1
{c,d}	1	6	0.5	0.5	8	1	1	1
{a,b,c}	4.11	3.11	1.78	4.33	13.33	1	1	1
{a,b,d}	3.33	1	3	1	8.33	1	1	1
{a,c,d}	0.67	7	1.67	3	12.33	0	1	0
{b,c,d}	4.33	5.67	1.67	0.67	12.33	0	1	0
{a,b,c,d}	4.08	6.75	2.75	4.08	17.67	0	1	0

Optimal Transfer Scheme, $\lambda_i = 1/s$

coalition	v_a	v_b	v_c	v_d	$\sum v_i$	IS	ES	S
singletons	0	0	0	0	0	1	0	0
{a,b}	0.5	0.5	1	1	3	1	0	0
{a,c}	0.5	2.33	0.5	3.33	6.67	1	0	0
{a,d}	1.17	1	2	1.17	5.33	1	0	0
{b,c}	3.33	1.5	1.5	1	7.33	1	0	0
{b,d}	3.33	0.5	2	0.5	6.33	1	0	1
{c,d}	1	6	0.5	0.5	8	1	1	1
{a,b,c}	4.11	3.11	1.78	4.33	13.33	1	1	1
{a,b,d}	3.33	1	3	1	8.33	1	1	1
{a,c,d}	0.67	7	1.67	3	12.33	0	1	0
{b,c,d}	4.33	5.67	1.67	0.67	12.33	0	1	0
{a,b,c,d}	4.08	6.75	2.75	4.08	17.67	0	1	0

Optimal Transfer Scheme, $\lambda_i = 1/s$

coalition	v_a	v_b	v_c	v_d	$\sum v_i$	IS	ES	S	
singletons	0	0	0	0	0	1	0	0	
{a,b}	0.5	0.5	1	1	3	1	0	0	
{a,c}	0.5	2.33	0.5	3.33	6.67	1	0	0	
{a,d}	1.17	1	2	1.17	5.33	1	0	0	
→ {b,c}	3.33	1.5	1.5	1	7.33	1	0	0	
→ {b,d}	3.33	0.5	2	0.5	6.33	1	0	1	9
→ {c,d}	1	6	0.5	0.5	8	1	1	1	
{a,b,c}	4.11	3.11	1.78	4.33	13.33	1	1	1	
{a,b,d}	3.33	1	3	1	8.33	1	1	1	
{a,c,d}	0.67	7	1.67	3	12.33	0	1	0	
→ {b,c,d}	4.33	5.67	1.67	0.67	12.33	0	1	0	8
{a,b,c,d}	4.08	6.75	2.75	4.08	17.67	0	1	0	

Optimal Transfer Scheme, $\lambda_i = 1/s$

coalition	v_a	v_b	v_c	v_d	$\sum v_i$	IS	ES	S
singletons	0	0	0	0	0	1	0	0
{a,b}	0.5	0.5	1	1	3	1	0	0
{a,c}	0.5	2.33	0.5	3.33	6.67	1	0	0
{a,d}	1.17	1	2	1.17	5.33	1	0	0
{b,c}	3.33	1.5	1.5	1	7.33	1	0	0
{b,d}	3.33	0.5	2	0.5	6.33	1	0	1
{c,d}	1	6	0.5	0.5	8	1	1	1
{a,b,c}	4.11	3.11	1.78	4.33	13.33	1	1	1
{a,b,d}	3.33	1	3	1	8.33	1	1	1
{a,c,d}	0.67	7	1.67	3	12.33	0	1	0
{b,c,d}	4.33	5.67	1.67	0.67	12.33	0	1	0
{a,b,c,d}	4.08	6.75	2.75	4.08	17.67	0	1	0

Lemma 2

If coalition S is not externally stable, then there exists a $j \in N \setminus S$ such that a coalition $S \cup \{j\}$ is potentially internally stable (and hence internally stable).

1. **Motivation**

2. **Definitions**

3. **Illustration and Preliminaries**

→ 4. **Results**

Result 1: Existence

In a coalition game $\Gamma(N,\pi)$, there exists an equilibrium coalition for an optimal transfer scheme (irrespective of weights).

Result 2: Robustness

In a coalition game $\Gamma(N,\pi)$, the set of equilibria is always the same for an optimal transfer scheme irrespective weights.

Result 3: Optimality

Let $\Sigma^{PIS}(\pi)$ be the set of potential internally stable coalitions in a coalition game $\Gamma(N,\pi)$ and let $S^* \in \Sigma^{PIS}(\pi)$ denote the coalition that generates the highest global welfare among them, then for an optimal transfer scheme (irrespective of weights) S^* is an equilibrium if the game $\Gamma(N,\pi)$ exhibits positive externalities.

Result 1: Existence

In a coalition game $\Gamma(N, \pi)$, there exists an equilibrium coalition for an optimal transfer scheme (irrespective of weights).

If the coalition game is superadditive, then there exists a non-trivial equilibrium coalition.

Result 1+3: Existence and Optimality

In a coalition game $\Gamma(N,\pi)$, there exists an equilibrium coalition for an optimal transfer scheme (irrespective of weights).

If the coalition game is superadditive, then there exists a non-trivial equilibrium coalition.

If the coalition game exhibits negative externalities, then there exist a unique equilibrium, which is the grand coalition.

Introducing Risk

Endres, A. and C. Ohl (2003), International Environmental Cooperation with Risk Aversion. "International Journal of Sustainable Development", vol. 6, no. 3, pp. 378-392.

Alternative:

Endres, A. and C. Ohl (2000), Taxes versus Quotas to Limit Global Environmental Risks: New Insights in an Old Affair. "Environmental Economics and Policy Studies", vol. 3, pp. 399-423.

Welfare: $W_{DC} > W_{CC} > W_{DD} > W_{CD}$

Mean Welfare: $\mu_{DC} > \mu_{CC} > \mu_{DD} > \mu_{CD}$

Standard deviation: $\sigma_{DD} > \sigma_{CD} = \sigma_{DC} > \sigma_{CC}$

Subjective Risk Assessment: $\phi = \mu - \alpha\sigma$

linear version of the mean-variance principle/criterion

Risk-neutrality: $\alpha = 0$

Risk-aversion: $\alpha > 0$

Risk-loving: $\alpha < 0$

Alternative (Endres and Ohl 2000): $\phi = \mu - \alpha\left(\mu^2 + \sigma^2\right)$

1) Given that the foreign country defects, the home country has an incentive to cooperate

$$\phi(CD) = \mu_{CD} - \alpha\sigma_{CD} > \mu_{DD} - \alpha\sigma_{DD} = \phi(DD) \Rightarrow \alpha \geq \frac{(\mu_{DD} - \mu_{CD})}{(\sigma_{DD} - \sigma_{CD})} \equiv \alpha_{I}^{min} > 0$$

2) Given the foreign country cooperates, the home country has an incentive to cooperate:

$$\phi(CC) = \mu_{CC} - \alpha\sigma_{CC} > \mu_{DC} - \alpha\sigma_{DC} = \phi(DC) \Rightarrow \alpha \geq \frac{(\mu_{DC} - \mu_{CC})}{(\sigma_{DC} - \sigma_{CC})} \equiv \alpha_{II}^{min} > 0$$

Prisoner’s dilemma game: $\alpha < \alpha_{I}^{min}, \alpha_{II}^{min}$

Chicken game: $\alpha_{I}^{min} < \alpha < \alpha_{II}^{min}$

Stag hunt game: $\alpha_{II}^{min} < \alpha < \alpha_{I}^{min}$

No conflict game: $\alpha > \alpha_{I}^{min}, \alpha_{II}^{min}$

Extension in Endres and Ohl (2000):

the effect of policy instruments on risk-evaluated welfare

Cost-effectiveness: $\mu_{TC} > \mu_{QC}$ and $\mu_{TD} > \mu_{QD}$ (own country)

Ecological accuracy: a) $\sigma_{QD} < \sigma_{TD}$ and $\sigma_{QC} < \sigma_{TC}$

b) $\sigma_{QQ} < \sigma_{TQ} = \sigma_{QT} < \sigma_{TT}$

Extensions

Boucher, V. and Y. Bramoullé (2009), Providing Global Public Goods under Uncertainty Mimeo.

Finus, M., P. Pintassilgo and A. Ulph (2009), International Environmental Agreements with Uncertainty, Learning and Risk Aversion. Mimeo, University of Stirling, UK, 2009.

- learning-by-doing
- learning-by-research
- different ex-ante expectations about parameters
- uncertainty about membership