

Dynamic Models of International Environmental Agreements: A Differential Game Approach

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Definition

A game is said to be dynamic if at least one player can use a strategy which conditions his single-period action at any instant of time on the actions taken previously in the game. Previous actions are those of the rivals but also a player's own actions.

- ▶ To analyze a dynamic game, we need to start by describing in which **order** the players take their actions and what **information** is available to a player when he takes action.
- ▶ In such a game we say that players move **simultaneously** at time t if no player - when taking his action at time t - knows about the actions that the other players take at time t .
- ▶ In what follows we confine our interest to a dynamic game in which players' actions are observable by all players. The game is said to be one of **perfect information**.

► The best-understood class of dynamic games is that of **repeated games**, in which players face the same “stage game” or “constituent game” in every period, and the player’s overall payoff is a weighted average of the payoffs in each stage.

 Barrett (1999, 2002)

 Finus and Rundshagen (1998)

► A **differential game** is a dynamic game, played in **continuous** time. Two distinguishing features of a differential game are:

- 1 The modeler introduces a set of variables to characterize the **state** of the dynamical system at any instant of time during the play of the game.
- 2 The evolution over time of the state variables is described by a set of **differential equations**.

► Feature (1) makes the dynamic game a **state space game** and feature (2) makes the game a differential games (as opposed to, for instance, a **difference game**).

A Differential Game of International Pollution Control

 Long (1992)

 van der Ploeg and de Zeeuw (1991,1992)

Presentation:

 Rubio and Casino (2001,2005)

► Suppose a pollutant is emitted by N identical countries that share a natural resource as the environment.

1. Benefits from emissions for each country

$$B_i(q_i(t)) = aq_i(t) - \frac{b}{2}q_i(t)^2, \quad a, b > 0.$$

2. Emissions accumulate into the atmosphere creating a stock that evolves according to the differential equation

$$\dot{z}(t) = \frac{dz(t)}{dt} = \sum_{i=1}^N q_i(t) - kz(t),$$

k = the rate of natural decay, $k \in (0, 1)$,

$z(t)$ = the stock of accumulated emissions (pollution).

3. Environmental damages for each country depend on accumulated emissions

$$C_i(z(t)) = \frac{c}{2}z(t)^2, \quad c > 0.$$

4. Net benefits for each country

$$\pi_i(q_i(t), z(t)) = aq_i(t) - \frac{b}{2}q_i(t)^2 - \frac{c}{2}z(t)^2.$$

$z(t)$ = the stock of accumulated emissions = pure public bad.

- All the countries contributes at the “provision” of a public bad.

- ▶ The decentralized or **noncooperative** provision of the public bad can be represented as a differential game.
- ▶ Each country chooses a path of emissions that maximizes the **discounted present value** of the stream of net benefits

$$\begin{aligned} & \max_{\{q_i(t)\}} \int_0^{\infty} \pi_i(q_i(t), z(t)) e^{-\delta t} dt \\ \text{s.t. } \dot{z}(t) &= \sum_{j=1}^N q_j(t) - kz(t), \quad z(0) = z_0 \geq 0, \end{aligned} \quad (1)$$

δ = the discount rate, $\delta \in (0, 1)$.

$z(t)$ = the state variable,
 $(q_1(t), \dots, q_N(t))$ = the control variables.

► Features:

- 1 An infinite horizon.
- 2 Time independent.

The Full Cooperation

- A public bad is an example of a multilateral or reciprocal and negative **externality**.
- Then, the noncooperative provision of the public bad is Pareto **inefficient**.
- To find the symmetric, efficient provision we have to calculate the temporal path of emissions that maximizes the **aggregate** discounted present value of the game.
- In this way, the external effects of the emissions are **internalize**.

$$\begin{aligned} \max_{\{q_i\}} \int_0^{\infty} N \left(aq_i - \frac{b}{2}q_i^2 - \frac{c}{2}z^2 \right) e^{-\delta t} dt \\ \text{s.t. } \dot{z}(t) = Nq_i - kz, \quad z(0) = z_0 \geq 0. \end{aligned}$$

► This is an **optimal control problem** with one state variable.

$$H(z, q_i, \lambda) = N \left(aq_i - \frac{b}{2}q_i^2 - \frac{c}{2}z^2 \right) + \lambda(Nq_i - kz),$$

λ = the costate variable.

λ = the marginal value of the state variable at time t
in terms of values at t .

- The necessary conditions for an **interior** solution

$$\frac{\partial H}{\partial q_i} = a - bq_i + \lambda = 0 \rightarrow a - bq_i = -\lambda. \quad (2)$$

marginal benefit = marginal cost

- The Euler equation

$$\dot{\lambda} = \delta\lambda - \frac{\partial H}{\partial z} = (\delta + k)\lambda + Ncz. \quad (3)$$

- The transversality conditions

$$\lim_{t \rightarrow \infty} e^{-\delta t} \lambda \geq 0, \quad \lim_{t \rightarrow \infty} e^{-\delta t} \lambda z = 0. \quad (4)$$

► Using (2) to eliminate λ from the Euler equation, the dynamics of the state and control variable are given by the following system of two **linear first order** differential equations

$$\begin{aligned}\dot{q}_i &= (\delta + k)q_i + \frac{Nc}{b}z - \frac{(\delta + k)}{b}a, \\ \dot{z} &= Nq_i - kz.\end{aligned}\tag{5}$$

► A particular solution: the **steady state**

$$\dot{q}_i = \dot{z} = 0$$

$$q_{i\infty}^* = \frac{k(\delta + k)a}{k(\delta + k)b + N^2c}, \quad z_{\infty}^* = \frac{N(\delta + k)a}{k(\delta + k)b + N^2c}.\tag{6}$$

- ▶ In order to determine the **stability** features of the steady state, the determinant of the **Jacobian matrix** is calculated

$$\begin{vmatrix} \delta + k & \frac{Nc}{b} \\ N & -k \end{vmatrix} = - \left((\delta + k)k + \frac{N^2c}{b} \right) < 0.$$

- ▶ Then the two roots of the **characteristic equation** have opposite signs and the steady state is a **saddle point**.
- ▶ For these types of critical points there are two stable branches in the phase diagram and there exists an **optimal path** to reach the steady state.

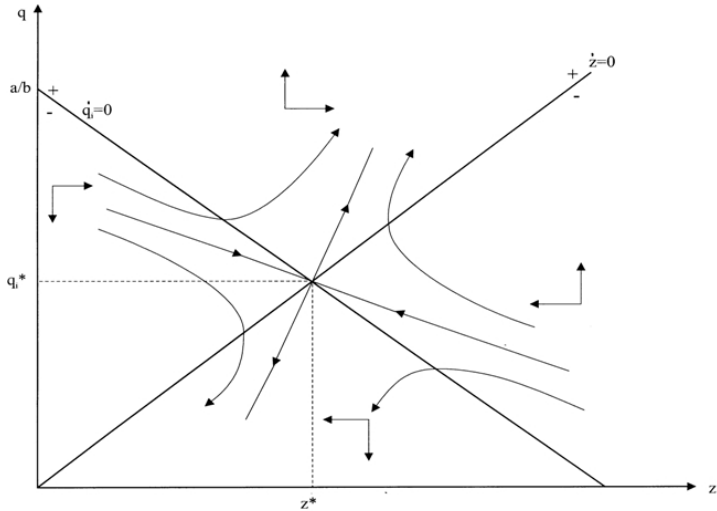
- The general solution to the system (5)

$$q_i^* = q_{i\infty}^* + (z_0 - z_\infty^*) \frac{Nc}{(r^* - \delta - k)b} e^{r^*t}, \quad (7)$$

$$z^* = z_\infty^* + (z_0 - z_\infty^*) e^{r^*t}, \quad (8)$$

r^* = the negative root of the characteristic equation.

$$r^* = \frac{1}{2} \left(\delta - \sqrt{\delta^2 + \frac{4}{b} (k(\delta + k)b + N^2c)} \right) < 0.$$



Phase diagram of the cooperative solution

► Summarizing

- 1 There exists a **unique steady state** given by (6) and the optimal path of emissions defined by (7) leads to the steady state.
- 2 The steady state is a **saddle point** equilibrium and the optimal path approaches it asymptotically.
- 3 Initial emissions are greater than the steady-state emissions and emissions are **decreasing** along the optimal path provided that z_0 is lower than z_∞^* .

The Non-Cooperative Equilibria

The Open-Loop Nash Equilibrium

Definition

A Nash equilibrium is an N -tuple of strategies $(\phi_1, \phi_2, \dots, \phi_N)$ such that, given the opponents' equilibrium strategies, no player has an incentive to change his own strategy.

- ▶ We assume that players use **Markovian strategies**

$$q_i(t) = \phi_i(z(t), t), \quad i = 1, \dots, N.$$

- ▶ The idea behind a Markovian strategy is that the history of the game at each moment can be summarized by the **value of the state** at that moment.

- ▶ An **open-loop strategy** is a degenerate form of a Markovian strategy in which function ϕ happens to be independent of the state z : $q_i(t) = \phi_i(t)$.
- ▶ This type of strategy implies that each player has **committed** to his **entire course of action** at the beginning of the game and will not revise it at any subsequent point in time.
- ▶ The Nash equilibrium open-loop strategies are relatively easy to determine as they involve a straightforward application of the standard **optimal control** methods.

$$H(z, q_1, \dots, q_N, \lambda_i) = aq_i - \frac{b}{2}q_i^2 - \frac{c}{2}z^2 + \lambda_i \left(\sum_{j=1}^N q_j - kz \right),$$
$$i = 1, \dots, N$$

- ▶ The necessary conditions for an **interior** solution for each country

$$\frac{\partial H}{\partial q_i} = a - bq_i + \lambda_i = 0, \quad (9)$$

$$\dot{\lambda}_i = (\delta + k)\lambda_i + cz. \quad (10)$$

- ▶ Comparing with the Euler equation of the cooperative solution

$$\dot{\lambda} = (\delta + k)\lambda + (N - 1)cz + cz.$$

- ▶ We realize that the **international** (external) **marginal damages**, $(N - 1)cz$, are not taking into account in the non-cooperative equilibrium.

- The **symmetric equilibrium** is given by the following system of two linear first order differential equations

$$\begin{aligned}\dot{q}_i &= (\delta + k)q_i + \frac{c}{b}z - \frac{(\delta + k)}{b}a, \\ \dot{z} &= Nq_i - kz.\end{aligned}\tag{11}$$

- The **steady state**

$$\begin{aligned}\dot{q}_i = \dot{z} &= 0 \\ q_{i\infty}^{ol} &= \frac{k(\delta + k)a}{k(\delta + k)b + Nc}, \quad z_{\infty}^{ol} = \frac{N(\delta + k)a}{k(\delta + k)b + Nc}.\end{aligned}\tag{12}$$

- The general solution to the system (11) is

$$q_i^{ol} = q_{i\infty}^{ol} + (z_0 - z_\infty^{ol}) \frac{c}{b(r^{ol} - \delta - k)} e^{r^{ol}t}, \quad (13)$$

$$z^{ol} = z_\infty^{ol} + (z_0 - z_\infty^{ol}) e^{r^{ol}t}, \quad (14)$$

r^{ol} = the negative root of the characteristic equation.

$$r^* = \frac{1}{2} \left(\delta - \sqrt{\delta^2 + 4 \left(k(\delta + k) + \frac{cN}{b} \right)} \right) < 0.$$

- ▶ *Comparison*: both emissions and accumulated emissions in the open-loop Nash equilibrium are **greater** than those in the cooperative solution for all t .
- ▶ The percentage variation of pollution stock at the steady state

$$\frac{z_{\infty}^* - z_{\infty}^{ol}}{z_{\infty}^{ol}} = -\frac{N(N-1)}{(b/c)k(\delta+k) + N^2}.$$

b/c is the ratio between the slope of the **marginal benefits** and the slope of the **marginal damages**.

- ▶ Thus, an **increase** in the **environmental damages** enlarges the gap between the cooperative solution and the non-cooperative Nash equilibrium.

► The open-loop Nash equilibrium has been criticized for two reasons:

- 1 As open-loop strategies are ones for which each player chooses all the values of his control variable for each point in time at the **outset** of the game, the countries get no opportunity to react strategically to the rival's actions, using incoming information on actions taken.
- 2 Although open-loop are **time consistent**, they do not have the property of being **subgame perfect**.

► The **subgame perfection** requires that the strategies $\phi_i, i \in \{1, 2, \dots, N\}$ should represent optimal behavior not only along the equilibrium state trajectory but also off this trajectory.

The Stationary Markov Perfect Nash Equilibrium

- ▶ We assume that players use **stationary** Markovian strategies

$$q_i(t) = \phi_i(z(t)), \quad i = 1, \dots, N.$$

- ▶ In the case of the nondegenerate Markovian strategy, the commitment is not so strong because the players can react to different states with different values for the control variable.
- ▶ Now the players commits to the feedback rule, that is, to the Markovian strategy.
- ▶ In this case, the Markovian strategies must satisfy the principle of optimality of dynamic programming or the **Hamilton-Jacobi-Bellman equation**

$$\delta W_i(z) = \max_{\{q_i\}} \left\{ aq_i - \frac{b}{2}q_i^2 - \frac{c}{2}z^2 + W_i'(z) \left(\sum_{j=1}^N q_j - kz \right) \right\},$$
$$i = 1, \dots, N$$

where $W_i(z)$ stands for the optimal control **value function** associated with the optimization problem (1).

► The **necessary condition** for the maximization of the right-hand side of the HJB equation

$$a - bq_i + W_i'(z) = 0 \rightarrow q_i = \frac{1}{b}(a + W_i'(z)) = \phi_i(z). \quad (15)$$

- ▶ By incorporating this **optimal strategy** into the Bellman equation, we eliminate the maximization and obtain a **nonlinear differential equation**

$$\delta W_i(z) = \frac{1}{2b}(a + W_i'(z))^2 + \frac{N-1}{b}W_i'(z)(a + W_i'(z)) - W_i'(z)kz - \frac{c}{2}z^2. \quad (16)$$

- ▶ **Linear-quadratic differential games**
- ▶ We guess a **quadratic representation** for the value function

$$W_i(z) = \frac{1}{2}\alpha_i z^2 + \beta_i z + \mu_i.$$

- Substituting into (16) and equating coefficients yields the **Riccati equations**

$$(2N - 1)\alpha_j^2 - 2b \left(k + \frac{\delta}{2} \right) \alpha_j - bc = 0, \quad (17)$$

$$Na\alpha_j + ((2N - 1)\alpha_j - b(k + \delta))\beta_j = 0, \quad (18)$$

$$(a + \beta_j)(a + (2N - 1)\beta_j) - 2b\delta\mu_j = 0. \quad (19)$$

- The solution to this system can be obtained in a recursive way beginning with the solution to (17)

$$\alpha_j = \frac{b}{2N - 1} \left(k + \frac{\delta}{2} \pm \sqrt{\left(k + \frac{\delta}{2} \right)^2 + \frac{2N - 1}{b/c}} \right). \quad (20)$$

- Thus, equation (17) has two roots, one positive and one negative.

- Then, (15) yields a **linear** stationary Markovian strategy

$$q_i = \frac{1}{b} (\mathbf{a} + \beta_i + \alpha_i \mathbf{z}). \quad (21)$$

- So that the dynamics of the state is given by

$$\dot{\mathbf{z}} = \frac{N}{b} (\mathbf{a} + \beta_i + \alpha_i \mathbf{z}) - k \mathbf{z}$$

where β_i can be eliminated using (18) to get

$$\dot{\mathbf{z}} = \frac{N}{b} \frac{b(k + \delta) - (N - 1)\alpha_i}{b(k + \delta) - (2N - 1)\alpha_i} \mathbf{a} - \left(k - \frac{N}{b} \alpha_i \right) \mathbf{z}. \quad (22)$$

► The **stability condition**

$$\frac{d\dot{z}}{dz} = - \left(k - \frac{N}{b} \alpha_j \right) < 0.$$

► Only the **negative root** of (20) satisfies this condition.

► The stationary Markov perfect Nash equilibrium is given by the solution to first order differential equation (22) where α_j is the negative root of equation (17).

► The **steady state**

$$\dot{z} = 0 \rightarrow z_{\infty}^f = \frac{aN(b(k + \delta) - (N - 1)\alpha_j)}{b(k(b(k + \delta) - (N - 1)\alpha_j) + cN)}. \quad (23)$$

- The **general solution** to differential equation (22) is

$$z^f = z_{\infty}^f + (z_0 - z_{\infty}^f)e^{-(k - \frac{N}{b}\alpha_i)t}. \quad (24)$$

- Then by substitution in the linear strategy, the **optimal path for emissions** is obtained

$$q_i = q_{\infty}^f + \frac{\alpha_i}{b}(z_0 - z_{\infty}^f)e^{-(k - \frac{N}{b}\alpha_i)t}. \quad (25)$$

$\alpha_i =$ the negative root of equation (17).

$$\alpha_i = \frac{b}{2N-1} \left(k + \frac{\delta}{2} - \sqrt{\left(k + \frac{\delta}{2}\right)^2 + \frac{2N-1}{b/c}} \right) < 0.$$

► Summarizing

- 1 There exists a **unique linear equilibrium strategy** given by

$$q_i = \frac{1}{b} (a + \beta_i + \alpha_i z)$$

where α_i is the **negative root** of equation (17).

- 2 The **steady state** is also **unique** and the optimal path given by (24) and (25) approaches it asymptotically.
- 3 The steady state is **globally stable**.
- 4 Initial emissions are greater than the steady-state emissions and emissions are **decreasing** along the optimal path provided that z_0 is lower than z_∞^f .

- ▶ *Comparison*: both emissions and accumulated emissions in the feedback Nash equilibrium in linear strategies are **greater** than those in the open-loop Nash equilibrium for all t .
- ▶ The percentage variation of accumulated emissions at the steady state

$$\frac{z_{\infty}^{ol} - z_{\infty}^f}{z_{\infty}^f} = \frac{N(N-1)(\alpha_j/b)}{(k(\delta+k)(b/c) + N)(\delta+k - (N-1)(\alpha_j/b))} < 0,$$
$$\frac{\alpha_j}{b} = \frac{1}{2N-1} \left(k + \frac{\delta}{2} - \sqrt{\left(k + \frac{\delta}{2}\right)^2 + \frac{2N-1}{b/c}} \right) < 0.$$

- ▶ Thus, it is easy to check that an **increase** in the **environmental damages** enlarges the gap between the open-loop Nash equilibrium and the feedback Nash equilibrium in **linear** strategies.

Dockner and Long (1993)

- 1 **Two** symmetric countries.
 - 2 **Nonlinear** Markovian strategies.
- ▶ They find that there exist an **uncountable** number of nonlinear strategies that support different levels of steady-state pollution stocks.
- ▶ In the limiting case when the discount rate tends to zero, the fully cooperative steady-state pollution stock can be supported as an asymptotically stable steady state of nonlinear strategies (\approx **Folk Theorem**).

Rubio and Casino (2002)

- ▶ We show that, given the **local nature** of nonlinear strategies, Dockner and Long's result requires that the initial value of the stock of pollution is greater than the Pareto-efficient pollution stock.
- ▶ In this case, the equilibrium path of emissions involves a **decreasing** stock of pollution.

Kossioris et al. (2008)

- ▶ They develop a method to obtain numerically non-linear feedback Nash equilibrium for a class of differential games with nonlinear-quadratic structure.
- ▶ They show that the value of the best feedback Nash equilibrium is in general worse than the open-loop and fully cooperative solutions.
- ▶ Example: the shallow lake pollution control.

Self-Enforcing IEAs with a Stock Pollutant and a Fixed Membership

- ▶ Two-stage agreement formation game

First stage (the *membership game*): each country decides whether or not to join an IEA.

Second stage (the *emissions differential game*): each country determines its emissions.

The emissions differential game

- ▶ Suppose that, as the outcome of the first-stage game, there are n **signatory** countries (a representative signatory being denote by s) and $N - n$ **nonsignatory** countries (a representative nonsignatory being denoted by fr).

- ▶ **Nonsignatories** choose emissions acting noncooperatively in order to maximize the discounted present value of the stream of net benefits taking as given the strategy of the other countries.
- ▶ **Signatories** choose emissions acting noncooperatively against nonsignatories in order to maximize the discounted present value of the stream of **agreement** net benefits.
- ▶ Signatories also take as **given** the strategy of nonsignatories.
- ▶ For these assumptions the optimal emissions and accumulated emissions are given by the **partial agreement Nash equilibrium** of a differential game between $N - n + 1$ players.
- ▶ The solution to this stage yields:

$$W_i^S(n), W_j^{fr}(n), i = 1, \dots, n \text{ and } j = 1, \dots, N - n.$$

The membership game

- ▶ Countries play at the initial moment a **simultaneous open membership game with commitment**.
- ▶ In a simultaneous open membership game, the **strategies** for each country are to sign or not to sign and any player is **free** to join the agreement.
- ▶ The **signature** of the agreement is binding on signatories so that they acquire a commitment to stay into the agreement during the second stage of the game playing cooperatively.

Definition

An IEA consisting of n signatories is self-enforcing if $W_i^s(n) \geq W_i^{fr}(n-1)$ and $W_j^{fr}(n) \geq W_j^s(n+1)$, where $i = 1, \dots, n$ and $j = 1, \dots, N-n$.

The emissions differential game

► Signatories

$$\max_{\{q_i^s\}} \int_0^{\infty} n \left(aq_i^s - \frac{b}{2}(q_i^s)^2 - \frac{c}{2}z^2 \right) e^{-\delta t} dt, \quad i = 1, \dots, n.$$

► Nonsignatories

$$\max_{\{q_j^{fr}\}} \int_0^{\infty} \left(aq_j^{fr} - \frac{b}{2}(q_j^{fr})^2 - \frac{c}{2}z^2 \right) e^{-\delta t} dt, \quad j = 1, \dots, N - n.$$

► The state equation

$$\dot{z}(t) = \sum_{i=1}^n q_i^s + \sum_{j=1}^{N-n} q_j^{fr} - kz(t), \quad z(0) = z_0 \geq 0.$$

The Open-Loop Nash Equilibrium

The steady state

► Signatories

$$q_{i\infty}^s = \frac{a(k(\delta + k)b - (N - n)(n - 1)c)}{b(k(\delta + k)b + (N + n^2 - n)c)}.$$

► Nonsignatories

$$q_{j\infty}^{fr} = \frac{a(k(\delta + k)b + n(n - 1)c)}{b(k(\delta + k)b + (N + n^2 - n)c)}.$$

► The stock pollution

$$z_{\infty} = \frac{aN(\delta + k)}{k(\delta + k)b + (N + n^2 - n)c}.$$

► Effects of cooperation on the steady-state values

- 1 The steady-state accumulated emissions **decrease** as the number of countries that sign the agreement increases.
- 2 The steady-state emissions of nonsignatories **increase** with the number of signatories.
- 3 The steady-state emissions of signatories can increase or decrease depending on the value of $\gamma = b/c$.
- 4 Nevertheless, the **aggregate** steady-state emissions always **decrease** with respect to the number of signatories.

The dynamics

► Signatories

$$q_i^s = q_{i\infty}^s(n) + (z_0 - z_\infty(n)) \frac{nc}{b(r - \delta - k)} e^{rt}.$$

► Nonsignatories

$$q_i^{fr} = q_{i\infty}^{fr}(n) + (z_0 - z_\infty(n)) \frac{c}{b(r - \delta - k)} e^{rt}.$$

► The pollution stock

$$z = z_\infty(n) + (z_0 - z_\infty(n)) e^{rt}.$$

$$r = \frac{1}{2} \left(\delta - \sqrt{\delta^2 + 4 \left(k(\delta + k) + (N + n^2 - n) \frac{c}{b} \right)} \right) < 0$$

Proposition

The emissions of nonsignatories are greater than the emissions of signatories for all t .

Corollary

The discounted present value of nonsignatories is greater than the discounted present value of signatories for all n in the interval $[2, N - 1]$: $W_j^{fr}(n) > W_i^s(n)$.

- ▶ Finally, substituting the optimal paths of emissions and pollution stock the discounted present value of net benefits for signatories and nonsignatories are obtained.
- ▶ Unfortunately, it is not possible to check analytically the stability conditions.

n	$q_{i\infty}$	$q_{j\infty}$	z_{∞}	W_i	W_j	W
1		26.087	52173.913		42494805	424948050
⇒ 2	23.687	26.129	51282.051	42535812	42673667	426460960
3	21.488	26.210	49586.777	42662880	43020919	429135073
4	19.573	26.322	47244.094	42871247	43516977	432586850
5	17.989	26.455	44444.444	43153388	44135677	436445325
6	16.749	26.601	41379.310	43499700	44847225	440387100
7	15.833	26.752	38216.561	43899272	45621198	444158498
8	15.205	26.901	35087.719	44340702	46429025	447583666
9	14.821	27.044	32085.561	44812801	47245665	450560874
10	14.634		29268.293	45305098		453050980

* Assumes $k=0.005$, $N=10$, $a=100000$, $b=3500$, $c=0.005$ and $\delta = 0.025$

Table 1. Stability analysis (open-loop strategies)

n	$q_{i\infty}$	$q_{j\infty}$	z_{∞}	W_i	W_j	W
1		26.114	52227.699		42482548	424825480
2	23.732	26.161	51351.010	42522777	42657598	426306338
3	21.540	26.252	49676.318	42647897	42999455	428939876
4	19.622	26.377	47350.254	42854182	43490779	432361402
5	18.030	26.527	44556.940	43135027	44107150	436210885
6	16.777	26.691	41485.569	43481491	44819793	440168118
7	15.850	26.859	38305.125	43883036	45598495	443976737
8	15.213	27.025	35150.249	44328318	46414217	447454978
9	14.823	27.183	32117.445	44805945	47241061	450494566
10	14.634		29268.293	45305098		453050980

*Assumes $k=0.005$, $N=10$, $a=100000$, $b=3500$, $c=0.005$ and $\delta = 0.025$

Table 2. Stability analysis (feedback strategies)

► Many of the attempts to model **empirically** how many countries might join an IEA to deal with climate change have the same feature that countries are assumed to make a **once-for-all** decision whether to join an IEA, with the dynamics of the stock pollutant affecting only emissions paths and hence present-value payoffs.



[Eyckmans and Tulkens \(2003\)](#)

- Are these dynamic models substantially **different** from the static models?
- Not really.
- The reason: the level of cooperation and strategies are established in the initial moment **forever**.

Self-Enforcing IEAS with a Stock Pollutant and a Variable Membership

 [Rubio and Ulph \(2007\)](#)

► A Difference Game of International Pollution Control

Definition

A difference game is a dynamic game where the evolution over time of the state variables is described by a set of difference equations.

1. Benefits from emissions for each country

$$B_i(q_{it}) = q_{it}, \quad q_{it} \in [0, 1].$$

We assume that there is an **upper bound** on emissions, which we normalize to 1.

2. Emissions accumulate into the atmosphere creating a stock that evolves according to the difference equation

$$z_{t+1} - z_t = \sum_{i=1}^N q_{it} - kz_t,$$

$$z_{t+1} = \sum_{i=1}^N q_{it} + (1 - k)z_t = \sum_{i=1}^N q_{it} + \rho z_t, \quad \rho = 1 - k \in (0, 1).$$

3. Environmental damages for each country depend on pollution stock

$$C_i(z_t) = cz_t^2, \quad c > 0.$$

4. Net benefits for each country

$$\pi_i(q_{it}, z_t) = q_{it} - cz_t^2.$$

- Each country chooses a path of emissions that maximizes the **discounted present value** of the stream of net benefits.

$$\max_{\{q_{it}\}_0^\infty} \sum_{t=0}^{\infty} (q_{it} - cz_t^2) \delta^t$$

$$\text{s.t. } z_{t+1} = \sum_{i=1}^N q_{it} + \rho z_t, \quad z_0 \geq 0$$

and $q_{it} \in [0, 1]$.

$\delta =$ the discount factor, $\delta \in (0, 1)$.

► The Agreement Formation Game with a Variable Membership

- We consider the formation of an infinite sequence of IEAs, in which, in each period, countries are free to join or leave the agreement.
- Thus, **in each period**, given the pollution stock we solve, using dynamic programming, a two-stage game by backward induction where in the second stage only the emissions of the period are determined as a function of the stock and the number of signatories.
- However, since we have assumed that all countries are **identical**, all we can determine in the first stage is the **number** of signatories of the self-enforcing IEA in each period as a function of the pollution stock.

- ▶ To make progress, we assume that in each period there is a **random process** for determining which countries become signatories, such that the probability of any country being a signatory in that period is simply the membership of the stable IEA in that period divided by the total number of countries.
- ▶ This probability is clearly the **same** for all countries, and is independent of whether a country was a signatory or nonsignatory in previous periods.
- ▶ The result is that the **expected value function** is also the same for all the countries independently of the type.
- ▶ Although the model becomes nonlinear, we develop an **algorithm** that allows us to obtain a quadratic **numerical approximation** of the expected value function.

- ▶ An important finding from our numerical simulations is that for all 2160 combinations of parameter values and for all values of the stock in the first 151 periods, there always exists a **unique** self-enforcing IEA.
- ▶ We also find that the number of signatories can be greater than **two**.
- ▶ However, in the comparison with the previous results it must be taken into account that the **net benefits functions** have different properties.
- ▶ In particular, in the previous model we assumed that marginal benefits from emissions were **decreasing** whereas in the present model we assume that they are **constant**.

► With **constant** marginal benefits, there exists a **threshold** for the number of signatories that depends on the stock such that if the number of signatories fell below this threshold, the optimal policy for signatories is to select the maximum level of emissions as the nonsignatories and this implies a serious threat that deters the signatories from leaving the agreement.

Dynamics of IEA

► We find that the pollution stock is always increasing, that it converges to a steady state and that the profile of the number of signatories of a self-enforcing IEA is a **non-increasing** function of the stock.

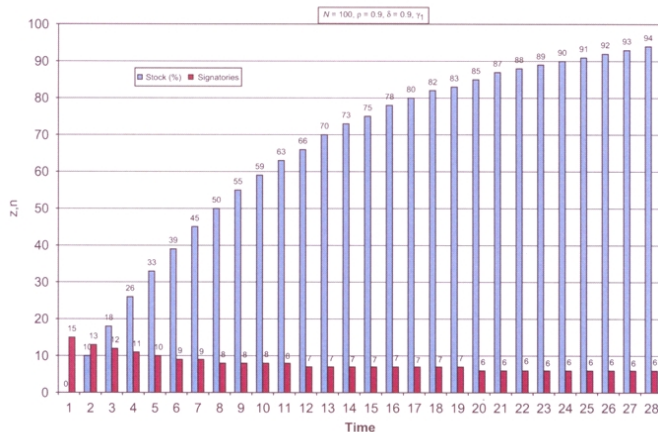


Fig. 1. Dynamics of stock and number of signatories.

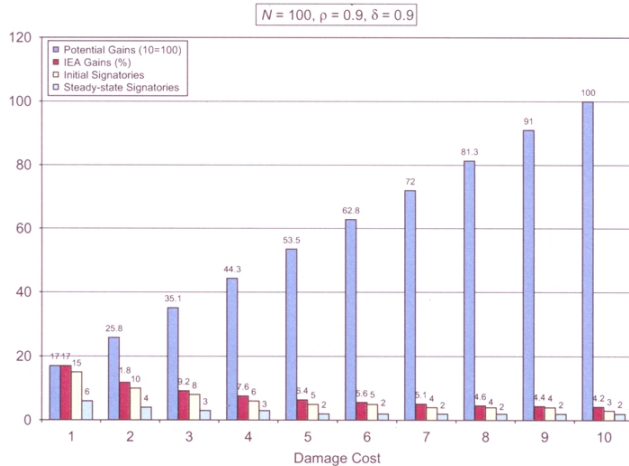


Fig. 2. Gains from co-operation and number of signatories.

- ▶ We note that the potential gains from full cooperation rise sharply as the damage cost parameter rises whereas the **partial gains** achieved by dynamic IEAs **fall** equally sharply.
- ▶ We find that the average partial gains achieved by dynamic IEAs for all the combinations of the parameter values analyzed in the paper is **7.5%** of potential gains whereas the maximum gains is 22.5%.
- ▶ Increasing the damage cost parameter causes both the initial and steady-state membership of the IEA to fall.
- ▶ These relationships are a **dynamic generalization** of the pessimistic results of **Barrett (1994)**, but apply both to a variation across models due to changes in the damage cost, and within a model, so that for a given damage cost IEA membership falls as the pollutant stock increases.

► More recently, this issue has been addressed by



de Zeeuw (2008)

► Dynamic farsighted stability.



Breton et al. (2010)





► Replicator dynamics.



Germain et al. (2010)

► Dynamic core solution.

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