# The interplay between sustainable growth, pollution and energy resources in an oil fed economy

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## Abstract

Conditions for optimal and sustainable growth that can be reconciled with natural resource use have been discussed in the literature since the first oil shock. In the wake of the climate change debate, we generalize the standard Dasgupta-Heal model to account for the use of flow renewable resources and for the presence of stock pollution problems. We first impose a set of standard assumptions and obtain a closed form solution under a condition for strong sustainability. Here we investigate the determinants of the economic growth rate and assess the conditions that guarantee strong sustainability. We find that balanced growth is positively linked to the share of renewable energy and inversely proportional to the share of fossil fuels and that a strongly sustainable optimal balanced growth is only achieved under particular conditions. In the second part of the paper we extend our analysis by removing the strong sustainability assumption and allowing variability in energy resource substitutability. Our main conclusion supports the idea that neither changes in the energy mix nor a high pace of technical change can guarantee stabilization of pollution. It is increases in resource substitutability that plays the crucial role under this respect.

JEL classification: O41, Q01, Q42

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## 1. Introduction

The paper investigates to what extent an economy can grow sustainably when both depletable and renewable sources are available to enter the energy mix, while pollution accumulates and generates damages over time. We focus, in particular, on the role played by resource augmenting technical change and by the degree of substitutability among energy sources.

Energy production and consumption, and more specifically fossil fuel production and consumption, are the largest contributor to green house gas (GHG) emissions and, although some authors still believe differently, climate change deriving from rapidly increasing GHG emissions is now widely acknowledged as having huge environmental impacts. Furthermore, the current energy portfolio of developed and developing countries heavily relies on fossil fuels. As a result, GHGs emissions are, in the absence of substantial intervention, projected to increase[1] further over time. The related damage and the future evolution of climate related events is subject to considerable uncertainties, nevertheless it is recognized that avoiding the most detrimental impacts implies the need to stabilize $CO_2$ concentrations[2]. The need for action is urgent, and as stated in the IEA 2009 "Collective action to tackle climate change calls for the wholesale transformation of the global energy system". Low impact energy sources are already available, but they still play a limited role in meeting energy demand. Renewable energy sources[3] supply 13 percent of global energy demand today, 10.1 percent from biomass and waste, 2.3 percent from hydropower and 0.5 percent from other renewables[4]. Current policy, both in the EU and worldwide, is taking decisive steps to increase the role of renewables in the energy mix[5] but this paper argues that the measures implemented so far are not enough if increasing pollution levels need to be avoided or, at a minimum, stabilized. This paper calls for large investments in renewable energy resources that can allow for full substitution between fossil fuels and renewable energy resources.

The literature on the impacts of depletable resource scarcity on sustainability stems from the seminal article by Dassgupta and Heal (1974). Many authors have taken inspiration from this work and have added various forms of technical change (Stiglitz 1974, Valente

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[1] The IEA estimates that energy-related $CO_2$ emissions will increase by 57 percent over the 2005-2030 reference period, reaching over 1,000 part per million (ppm), equivalent to a 6 C temperature rise.

[2] Though agreement on the concentration level to be met has not yet been reached, a generally agreed temperature increase target is 2° C. In order for the world to be able to reach this target, current use of fossil fuels would need to peak by 2020 and then start reducing.

[3] These include biomass and waste, hydropower, solar, wind, geothermal, tidal and wave energy. In IEA terminology, solar, wind, geothermal, tidal and wave energy are referred to as 'other renewables'.

[4] World Energy Outlook, IEA 2009-

[5] See, among others, Directives 2001/77/CE, 2003/30/CE and 2009/28/CE.

2005) showing that technical progress is a key driver to solve the problems of scarcity. Pezzey and Withagen (1998), building on Dasgupta and Heal (1974), find that in the presence of a depletable resource, consumption behaviour over time is single peaked. Recent literature has gone a step further and added pollution as a stock (or as a flow) and/or a measure of environmental quality to the discussion (Withagen 1995, Schou 2000, Smulders 2000). Fankhauser (2005), focuses on climate change, and shows that the rise in temperature causes reduction in investment and consequently consumption.

This paper builds on this literature, but in the context of the energy debate of today, goes a step further by setting up a comprehensive model including stock pollution, fossil fuels and renewable energy sources. The focus in the paper is to determine the role played by renewable energy resource in achieving sustainability, both strong and weak. Initially a set of assumptions are included to allow for the derivation of a closed form solution. In the second part of the analysis, the assumptions are released and simulation methods are employed.

The paper is organized as follows. Section 2 presents the main features of the model, including the assumptions imposed and the model constraints. Section 3 derives results under the assumptions set. Sections 4 extends the model to test also for weak sustainability and to allow for varying degrees of substitutability among energy sources. Finally, section 5 provides some concluding remarks.

## 2. The model

### 2.1 Production and welfare

We model an aggregate economy including an energy producing sector, a depletable resource extracting sector and pollution as a stock. Renewable energy is, on the other hand, modelled as a flow, and its availability is not limited. Energy is produced with a combination of renewable, $R(t)$, and non-renewable, $Z(t)$, energy resources. Contribution to pollution, availability of the resource, technology and cost are the four differentiating features of the two available energy sources.

- Non renewable resources generate pollution, while renewable resources do not. This assumption is intended to capture the very low contribution of renewable energy to GHG pollution when compared with fossil fuels.
- By definition, depletable resources are available in a limited amount, namely the total amount of the resource is fixed. Consequently, as the resource is used, the stock of the depletable resource is exhausted. The renewable resource, on

the other hand, is by definition available in an unlimited amount so that society faces no quantity constraint when using this resource[6].

- There is a substantial cost advantage from the use of fossil fuels. This feature is captured by normalizing the extraction costs of the depletable resources to zero. The renewable resource unit costs are instead positive and labelled as *g*.
- Renewable energy, as opposed to the more traditional and well established fossil fuel technology, offers large technical progress potential due to the abundance of the resource and the little use society makes of it today. We follow, in this respect, a well established field of the literature (see Stiglitz (1974) and Valente (2005)) and introduce exogenous technical progress in the form of resource augmenting technology.

Depletable and non depletable resources are used to generate energy, *EN(t)*, through a Cobb Douglas production function[7]:

$$EN(t) = [Z(t)^{\beta}(mR(t))^{1-\beta}] \qquad (1)$$

where $\beta$ is the share of the depletable resource, while *m* is a measure of the renewable resource augmenting technical progress, $m = e^{\omega t}$ .,where $\omega$ is the rate of technical progress.

Energy, labour and physical capital are used in production. Labour is assumed to be fixed and normalized to 1. The production function is, therefore, the following:

$$F(K(t), Z(t), R(t)) = [K(t)^{1-\alpha} EN(t)^{\alpha}] = \left\{K(t)^{1-\alpha} Z(t)^{\alpha\beta}[mR(t)^{\alpha(1-\beta)}]\right\} \qquad (2)$$

Assuming Cobb Douglas production functional forms entails that all inputs are essential in production, and more specifically, that both fossil fuels and renewable energy resources are needed to produce energy, thus $F(0, Z, R) = F(K, 0, 0) = 0$ [8]. The standard Inada conditions are satisfied.

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[6] Our setting is therefore suitable to model flow renewable resources, such as solar and wind energy.

[7] In this first part of the analysis Cobb Douglas functional forms are used to obtain closed form solutions to our model. This is extended later in the paper with the use of computer simulations.

[8] We will check whether alternative assumptions on the essentiality of fossil fuels affect our results in section 5. A general discussion of whether production with no fossil fuel input in the long run is possible, namely $F(K, 0, R) > 0$ , and within what time horizon this may possible is outside the scope of this paper. This problem is extensively treated in Dasgupta and Heal (1974).

On the consumers side, utility is increasing in consumption and decreasing in pollution (Smulders (2000), Pittel (2002)):

$$u(C(t), P(t)) = \ln C(t) + \ln(\overline{P} - P(t)) \tag{3}$$

where *C(t)* is consumption and *P(t)* is the pollution stock[9], that is, the amount of pollution that accumulates in the economy through the use of *Z(t)*. Utility is concave in both its arguments, i.e. $U_{CC} < 0$ and $U_{PP} < 0$[10]. We take $\overline{P}$ to be an upper bound for pollution that we label as "doomsday" pollution[11].

## 2.2 Pollution, resource depletion and capital accumulation constraints

Society is constrained by the amount of pollution that accumulates, the limited availability of depletable resources and physical capital accumulation in its strive to maximize the welfare function of the representative agent over time. We model the depletable resource constraint as it is standard since Hotelling (1931) seminal paper:

$$\dot{S}(t) = -Z(t) \tag{4}$$

i.e. the stock of exhaustible resource, *S(t),* decreases by the amount used at time t.

Two opposing effects impact the pollution stock: on one hand the use of the non-renewable resource increases pollution, on the other, the environment is capable to absorb a portion of the pollution generated based on the environment's absorption capacity, $\gamma \in (0,1)$:

$$\dot{P}(t) = Z(t) - \gamma P(t) \tag{5}$$

Capital is accumulated through net investment, that is, output net of consumption, depreciation and renewable energy costs:

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[9] As outlined in the introduction, we consider pollution to be a stock since we refer to fossil fuels and the disutility generated through the consequent accumulation of carbon dioxide pollution. This is inline with the discussion in Withagen (1995).

[10] The assumption of separability in the utility function entails that the level of pollution has no impact on $U_C$ and viceversa that consumption has no impact on $U_P$. Separability is a standard simplifying assumption (see, for example, Pittel (2002)).

[11] We borrow the "doomsday" label from Smulders (2000).

$$\overset{\bullet}{K}(t) = F[K(t), mR(t), Z(t)] - C(t) - gR(t) - \delta K(t) \tag{6}$$

where $\delta$ is the depreciation rate of physical capital

## 2.3. Balanced growth and Strong Sustainability

Our first objective is to understand whether, in the presence of an unlimited amount of renewable resources, optimal balanced growth implies strong sustainability. As standard in the literature, we define balanced growth as the condition in which the growth rate of output, consumption and capital grow are the same and constant. Strong sustainability is defined as in Pittel (2002), whereby the stock of pollution must be constant over time as shown in (7):

$$\overset{\bullet}{P}(t) = 0 \Rightarrow \frac{\overset{\bullet}{P}(t)}{P(t)} = 0 \tag{7}$$

Clearly, this might be intended as a *minimum requirement* for a strongly sustainable path.

## 2.4 The maximization problem

Social welfare maximization requires the maximization of the present value of utility, as a function of consumption and pollution, subject to fossil fuels, pollution and physical capital constraints (4), (5) and (6):

$$\max_{\{C(t),R(t),Z(t)\}} \int_0^T u(C(t), P(t)) \cdot e^{-\rho t} dt \tag{8}$$

subject to

$$\overset{\bullet}{P}(t) = Z(t) - \gamma P(t)$$

$$\overset{\bullet}{S}(t) = -Z(t)$$

$$\overset{\bullet}{K}(t) = F[K(t), mR(t), Z(t)] - C(t) - gR(t) - \delta K(t)$$

where $m=e^{\omega t}$, is the resource augmenting technology, $\rho$ is the rate of time preference and $T$ is the end of the time horizon[12]. All quantities are positive and the initial conditions are as follows:

$C(0) = C_0, K(0) = K_0, R(0) = R_0, P(0) = P_0, Z(0) = Z_0$

$C(t) \geq 0, K(t) \geq 0, R(t) \geq 0, P(t) \geq 0, Z(t) \geq 0$

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[12] We thank C. Withagen and C. Roseta-Palma for pointing out the need to limit out attention to a finite time horizon in our framework. This is to allow consolidation across time path between renewable and non-renewable resources.

Necessary conditions with respect to control variables *C,R* and *Z* are:

$$1/C = \mu \tag{9}$$

$$F_R[K,R,Z] = g \tag{10}$$

$$F_z[k,R,Z] = \frac{\xi - \lambda}{\mu} \tag{11}$$

where $\xi(t)$ is the shadow price of the exhaustible resource, $\lambda(t)$ is the shadow price of pollution, and $\mu(t)$ the shadow price of physical capital.

Condition (9) is the standard condition on utility: the shadow price of consumption has to equal the marginal utility. Condition (10) requires that, along the optimal path, the marginal benefit accruing from production thanks to an additional unit of renewable resource equal its cost. Interpretation of (11) is less standard. This condition implies that the marginal cost of the depletable resource due both to its exhaustibility and to its contribution to pollution must equal the marginal (shadow) benefits related to the use of Z in production.

Loglinearizing and time differentiating (9) we obtain, along a balanced growth path:

$$\frac{\dot{C}}{C} = \frac{\dot{Y}}{Y} = \frac{\dot{K}}{K} = -\frac{\dot{\mu}}{\mu} \tag{12}$$

so that along the optimal path, consumption, production and physical capital all grow at the same rate, ultimately equal to the opposite of the growth rate of the shadow price of consumption.

Following the same procedure with respect to (10) we find that in the long run the output and renewable energy resources' growth rates coincide[13]:

$$\frac{\dot{Y}}{Y} = \frac{\dot{R}}{R} \tag{13}$$

Finally, differentiating (11) with respect to time we obtain the growth rate of the amount of fossil fuels extracted, *Z*:

$$\frac{\dot{Z}}{Z} = \frac{\dot{\lambda} - \dot{\xi}}{\lambda - \xi} \tag{14}$$

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[13] Notice that, departing from results in Smulders (2000) and due to the distinction between renewable and non renewable resources accounted for in our paper, renewable resources grow at the same rate as output, which is not necessarily zero.

Co-state equations corresponding to $P$, $S$ and $K$ are:

$$\frac{\dot{\lambda}}{\lambda} = \rho - (\frac{u_P}{\lambda} - \gamma) \tag{15}$$

$$\frac{\dot{\xi}}{\xi} = \rho \tag{16}$$

$$\frac{\dot{\mu}}{\mu} = \rho - (F_K(\cdot) - \delta) \tag{17}$$

Condition (15) shows that the growth rate of the shadow price of pollution along the optimal path must equal the social discount rate net of a (negative) “pollution factor”. The pollution factor is equal to the ratio of the marginal (dis)utility of pollution divided by the benefit from pollution net of the environment’s absorption capacity. It is clear form (15) that the growth rate of the shadow price of pollution is larger than the social discount rate ($\rho$). Condition (16) is the standard Hotelling (1931) rule whereby the optimal use of the non-renewable resource is defined as the path along which the growth rate of the resource’s shadow price is equal to the rate of time preference. Finally, according to (17), and as it is standard, along the optimal path, the rate of growth of the capital’s shadow price is equal to the difference between the discount rate and output’s productivity net of its depreciation which, as standard, leads to the Ramsey Golden Rule for consumption.

The set of optimum conditions is completed transversality conditions:

$$e^{-\rho T}\xi(T)S(T) = 0 \tag{18}$$

$$e^{-\rho T}\mu(T)K(T) = 0 \tag{19}$$

$$e^{-\rho T}\lambda(T)P(T) = 0 \tag{20}$$

Condition (18) requires that, at time T, the stock of depletable resource is either exhausted or worthless (i.e. the shadow price is zero). As, however, (18) implies that $\xi$ can only be zero if its initial value is zero, (18) implies that in $t = T$ the stock $S$ has to be exhausted. Condition (19) is the standard condition for physical capital. Condition (20) requires that the discounted value of the stock of pollution has to be equal to zero at the end of the time horizon must be 0, either due to a zero shadow price or to $P(T) = 0$ .

Manipulating the pollution constraint (5) and imposing strong sustainability condition (7) implies that the ratio of $Z$ to $P$ must be constant and equal to $\gamma$. As a consequence:

$$\frac{\dot{P}}{P} = \frac{\dot{Z}}{Z} = 0 \tag{21}$$

Combining (21) and (14) we get:

$$\frac{\dot{\lambda} - \dot{\xi}}{\lambda - \xi} = 0 \Rightarrow \dot{\lambda} = \dot{\xi} \text{ if } \lambda \neq \xi \tag{22}$$

Log-linearizing the production function, assuming that indeed $\lambda \neq \xi$ and using (12), (17) and (21), we obtain the growth rate of our strongly sustainable economy under the assumption of exogenous, renewable resource augmenting technical progress:

$$\frac{\dot{Y}}{Y} = \frac{(1-\beta)}{\beta}\omega \tag{23}$$

which leads to the first result of our paper.

**Proposition 1***: In the case of an economy that dislikes pollution and uses both depletable and renewable energy resources, the long run strongly sustainable growth rate of the economy will be higher the larger the share of renewable energy resources and the larger the rate of exogenous technical progress. The growth rate of the economy will decrease the larger the share of exhaustible resources.*

Proposition 1 is in line with the received literature. The emphasis on technology is standard, see Solow (1953) and Stiglitz (1974). Proposition 1, however, explicitly accounts for the role played by renewable energy in contributing to output growth. Three factors influence the growth rate of the economy, namely the share of renewable resources, the share of exhaustible resources and the exogenous technical progress rate. As the share of the renewable (depletable) resources increases (decreases), the economy growth rate increases. Higher rates of technical change also have a positive influence on the economy’s growth rate, as expected. Furthermore, in the absence of technical change the growth rate of the economy would be constant and equal to zero. This is in line with the classical results of the Solow Model in which exogenous technical progress is the driver of economic growth. Result summed up in Proposition 1 suggests an interesting policy implication: for a given rate of technical improvement, strongly sustainable growth can be increased by increasing the share of renewable resources in production. This is interesting, because it suggests new channels through which the economy can grow once a sustainable path has been achieved.

Conditions (12), (21) and (23) define the sustainable balanced growth path: the longer the time horizon the smaller the amount of non-renewable resource available per period, and the stronger the substitution requirements between natural resources. This implies, intuitively, that in an economy with renewable and non-renewable resources, assuming an exogenous technical progress parameter, strongly sustainable balanced growth in the long run is only possible if the two resources are "almost perfectly" substitutable (i.e. it is possible to produce output using "almost" no depletable resources). We can prove an even stronger result:

**Proposition 2**: *In an economy characterized by renewable and non renewable energy resources, Cobb Douglas technology and an exogenous technical progress parameter on the renewable resource, strongly sustainable balanced growth implies the following level of "terminal" (i.e. time T) pollution level:*

$$P(T) = \overline{P} - \frac{1}{\rho \xi_0 e^{\rho T}} \tag{24}$$

*Proof.* Manipulation of (15), (16) and (22) yields:

$$\lambda(t) = \frac{u_p + \rho \xi_0 e^{\rho t}}{\gamma + \rho} \tag{25}$$

Evaluating (25) in $t = T$ and substituting it in (20) we obtain:

$$\left( \frac{u_p(T) + \rho \xi_0 e^{\rho T}}{\gamma + \rho} \right) e^{-\rho T} P(T) = 0 \tag{26}$$

Using the definition of utility in (3) we get, after some simple manipulation,

$$P(T) = \overline{P} - \frac{1}{\rho \xi_0 e^{\rho T}}$$

□

Since $\gamma + \rho > 0$[14], condition (24) states that the terminal stock of pollution, and thus, following from the definition of strong sustainability, the level of pollution in the economy along a balanced growth path, is lower than the maximum level society can withstand. The optimal "terminal" (i.e. time *T*) pollution level under the strong sustainability constraint will be higher the larger are the depletable resource time 0 shadow price, the social discount rate $\rho$ and/or the terminal time *T*. As it is reasonable, when individuals are more impatient, namely with a higher $\rho$, the sustainable level of pollution is higher. Further, the longer the time horizon the closer the economy can get to the threshold level of pollution. The intuition of our

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[14] If $\gamma + \rho = 0$, this would mean that either $\gamma$ or $\rho$ would have to negative but neither $\gamma$ nor $\rho$ can ever take on negative values.

result with respect to the shadow price of the depletable resource is more tricky. A relatively large initial value of $\xi_0$ implies that the depletable resource is more valuable, so that the economy will use more of that from time *0* to time $T$. This implies a larger "terminal" level of pollution.

The result summed up in Proposition 2 leads us to the following straightforward Corollary.

**Corollary 1** *If the time horizon were to extend to infinity, the optimal path would imply pollution reaching "doomsday" level.*

*Proof.* From (24), as $T \to \infty$ we get $P(T) = \overline{P}$ □

Corollary 1, is a quite surprising and pessimistic result. If we extend the planning horizon enough, then the mere existence of a renewable energy source, do not make strong sustainability optimal. This is even more surprising as 1) the renewable resource is a flow resource and is available in an unlimited amount, and 2) Corollary 1 holds for any (however large) rate of technical progress.

Corollary 1 has indeed strong policy implications: the mere existence of the renewable resource and an exogenous technical change process are not sufficient for the economy to be strongly sustainable in the long run. In other words, boosting technical change and the role of renewable energy is not enough to stabilize pollution concentration.

## 4. Sustainability with technical progress and substitutability

The closed form solution and the results obtained so far rely on the strong sustainability condition as defined in (7) and a Cobb-Douglas energy production functional form. We now remove these assumptions. At first, we drop the strong sustainability assumption and investigate how the exclusion of this assumption affects the results and whether a less stringent condition of sustainability, namely weak sustainability, can be achieved under the constructs of the model analyzed. Coherently with the existing literature (see, among others Neumayer (2003)), we define weak sustainability as *non declining utility* over time. Under these conditions we investigate whether resource augmenting technical progress and an increasing share of renewable resources can lead to a sustainable economy.

Secondly, we introduce a Constant Elasticity of Substitution (CES) energy production function in which the substitutability between renewable and non-renewable resources varies. Again we investigate the impact on the sustainability of the economy. Removing the assumptions listed adds considerably to the complexity of the model and does not allow to obtain a closed form solution. Thus, results in this section are obtained through computer simulations.

Initially we set up the baseline model that represents the counterfactual for the analysis. We then proceed in steps. At first we analyze the impact of increasing resource use and the inclusion of resource augmenting technology on sustainability. Secondly, we assess the consequences of a varying degree of substitutability amongst renewable and non-renewable resources. .

### 4.1 The baseline

Starting from the model outlined above, we simulate an economy that can use both renewable and non renewable energy resources and that is neither weakly nor strongly sustainable. This condition represents the counterfactual for the set of simulations presented. As shown in Figure 1a and 1b, this means that as time progresses and the economy grows, the stock of pollution increases over time, while utility decreases (Figure 1a and 1b).

We start by setting the share of renewable energy resources to a level of 6 percent[15], as can be found in the EU today. We also include for two slightly more optimistic scenarios in which the renewable energy share reaches 20 percent, as envisaged by the 2020 objectives in the EU, and a 30 percent share. Figure 1a and 1b illustrate how, for all values of $\beta$, the economy described is both weakly and strongly unsustainable

### 4.2 Allowing for renewable resource augmenting technical progress

We run two sets of simulations. In the first set we let the rate of technical progress vary. In the second set, we define a minimum and maximum rate of technical progress, and let the share of the renewable resources in the economy vary.

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[15] This is the share of renewables in energy consumption in the EU-25 today based on the European Environment-State and Outlook 2005 report of the EU. This share encompasses all renewables. The target rate for 2030 is that this share increases to 20 percent, equivalent to a beta of 0.8 in the analysis presented here. Worldwide the current share of renewable energy is 0.5 percent, projected to increase under a business as usual scenario to 1.7 percent by 2030 (IEA 2006).

We start by setting the rate of exogenous technical progress at a very low level and then gradually increase it[16]. Results are reported in Figure 1c and 1d. We find that increasing the rate of technical progress has no significant effect on the optimal pollution trajectory over time. On the other hand, when technical progress is comparatively fast, utility increases, at least after a certain time period. Thus faster exogenous technical progress, at low levels of renewable resource use, does not allow to achieve strong sustainability, but can lead to weak sustainability in the long run.

The second set of simulations adds changes in the resource share to changes in the rate of technological progress, see Figure 1d. We select two cases, one of relatively slow and one of relatively rapid technical progress, namely by setting $\omega=0.1$ and $\omega=0.9$. We start from the base case share of non-renewable resource, 0.94, in which renewables hold a small share of the market, and decrease it to 0.8 and 0.7, allowing for a more prominent role of the renewable energy resource.

We find that in all cases the economy is not strongly sustainable and that pollution always increases over time. For low levels of technological progress, as the share of the renewables increases, the level of pollution decreases, but when technological progress is fast, the contrary is true. This is because $\beta$ plays a double role: on the one hand, as $\beta$ decreases the share of renewable resources increases, while on the other, a decline in $\beta$ also coincides with an increase in the rate of technical progress which in turns makes all resources more productive. When the rate of technological progress is fast enough, it seems that the second effect overcomes the first, so that overall more pollution is optimal for the economy. For what concerns utility, the economy manages to reach a weakly sustainable path when technological progress is fast and the share of the depletable resource decreases. These results are in line with the conclusion achieved in the theoretical framework of the preceding sections. Weak sustainability is possible because of the exponential growth rate of technical progress that generates rapid output and consumption growth, counteracting the negative effects of increasing pollution stock in terms of utility. On the other hand, as shown in Corollary 1, strong sustainability cannot be achieved under a sufficiently long term planning horizon.

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[16] We initially set $\omega$ at 0.1 and then increase it up to 0.5 and 0.9

### 4.3. Varying the degree of substitutability.

In order to allow for varying degrees of substitution between renewable and non renewable energy sources we introduce a more general CES functional form for the energy production function as shown in (27)

$$EN(t) = \left(\beta Z(t)^{\sigma} + (1-\beta)\left[mR(t)\right]^{\sigma}\right)^{\frac{1}{\sigma}} \tag{27}$$

where *σ* is the elasticity of substitution between renewable and non renewable resources and β represents the share of non-renewable resources in energy generation as in the case of the Cobb-Douglas energy production function.

Introducing a CES energy production functional form allows us to move away from the assumption of essentiality of both resources in energy production. With a CES energy production renewable and non-renewable resources are no longer both essential inputs but become substitute inputs in energy production, where substitution possibilities vary based on the value of *σ*. The extreme case, when the elasticity of substitution is equal to 1, entails that *Z* and *R* can be fully substituted one for another, while $\sigma = 0$ is a limit case in which the CES production function tends to the Cobb Douglas function (this case has been analyzed above).

In order to test for the impact of substitutability we use three values of *σ,* namely *a* low substitutability value, a good substitutability case and the perfect substitutability condition. In addition to substitutability, we also account for technological improvement and consider low, high and intermediate rates of technical progress. All parameter values are listed in Table 2 and the main results are presented in Figure 2.

Initially we consider the case of low substitutability and low technical progress (CES 2). In this case we find that neither strong nor weak sustainability can be achieved: consumption and utility decrease over time and while pollution increases. We then vary the rate of technical progress. At first we increase the rate of technical progress to an intermediate or “realist” rate. We find that the optimal paths more or less coincide with the results for CES 2. When the rate of technical progress is increased to a very high rate, which might be unrealistic, we find that under rapid technical progress optimality leads to an increasing use of the renewable resource. Over time, consumption increases as does utility allowing for weak sustainability in the long run. The optimal path of pollution is bell shaped which implies that the current generation is more heavily affected than generations in the future. This result is compatible with the optimistic views underlying how technical change can lead to increases in the standard of living coupled with environmental improvements (at least in the future) even under imperfect substitutability. However, the rate of technical change assumed is probably unrealistic. Thus, strong sustainability seems not to be reachable if the degree of substitutability is low, while the conclusion with respect to weak sustainability depends critically on the pace of technical change.

When imperfect (but still relatively high) substitutability is considered, eak sustainability is achievable and a relatively optimistic conclusion can be drawn, see results for CES 4 and CES 5. Of course, a higher degree of technical change brings about higher standards of living as the simulation results in CES 5 illustrate compared to CES 4. The CES 5 simulation set assumes high pace technical change and results in a higher utility and consumption level with respect to CES 4. Nevertheless, when focusing on strong sustainability, the conclusion is not that optimistic. In fact, strong sustainability is not obtained even under good substitutability *stricto sensu*. As in the high technical change rate – low substitutability case, we get a bell shaped relationship, and pollution starts decreasing only after a few years. Again strong sustainability seems not to be reachable if the degree of substitutability is imperfect, while improved substitutability leads to weak sustainability.

Turning to the case of perfect substitution (CES 3, with $\sigma = 1$), as expected, both weak and strong sustainability are achieved *via* the use of the renewable resource, although at a slower rate than under very rapid technical change. In this case, no role is played by technical change itself if we assume that a renewable resource is available and it can fully substitute for the depletable one. This is, however, a rather extreme scenario.

Summing up, although rapid technical progress can lead to weak sustainability, it does not seem to solve pollution related problems. Reasonable rates of technical change require a sufficiently high degree of substitutability among energy sources. The only solution that allows to achieve a strongly sustainable optimal path is perfect substitutability between the renewable and depletable energy resource.

## 5. Concluding Remarks

The paper has focused on the links between energy, growth and sustainable development, both weak and strong. We started from a Dasgupta-Heal framework, enriching it to account for renewable energy resources as well as stock pollution.

Initially, our model relies on specific assumptions on functional forms including the substitutability between the renewable and depletable resource and on sustainability in order to obtain a closed form solution. We then generalize the model and through simulation obtain a second set of results where initial set of assumptions, including varying the degree of resource substitutability, are released.

Our main results lead us to the broad conclusion that policies targeted to increasing the substitutability of renewable and depletable energy resources are crucial to guarantee that economic growth is indeed sustainable. More specifically, no stabilization of emissions and pollution accumulation is possible if we keep limiting our attention to improvements in existing technologies and marginal changes in the energy mix, unless the rate of technical change were extremely high.

We conclude that the implications of our results are relevant for the current energy related debate. Indeed, we suggest that the mere existence of renewable resources, as well as technical change, can be considered as a necessary, yet not at all sufficient, condition to guarantee that the economy would evolve along a strongly sustainable development path. This entails that even a benevolent social planner would lead economic growth to be unsustainable if the role of depletable resources in energy production is not substantially reduced, while immediate pollution stabilization can only be achieved if “almost perfect” substitutability were possible.

**Table 1: Key parameters for the baseline and Cobb Douglas energy production simulation sets**

| **Baseline simulation set** | | |
|---|---|---|
| **Baseline simulation set** | **Share of non renewable resource ($\beta$)** | **Rate of technical progress ($\omega$)** |
| Base 1 | $\beta=0.94$ | - |
| Base 2 | $\beta=0.8$ | - |
| Base 3 | $\beta=0.7$ | - |
| **Cobb Douglas simulation set with varying resource share and technical progress rate** | | |
| **Cobb Douglas simulation set** | **Resource augmenting technical progess simulation set** | **Technical progress with increasing use of renewable resources** |
| CD 1 | 0.94 | 0.1 |
| CD 2 | 0.94 | 0.5 |
| CD 3 | 0.94 | 0.9 |
| CD 4 | 0.8 | 0.1 |
| CD 5 | 0.8 | 0.9 |
| CD 6 | 0.7 | 0.1 |
| CD 7 | 0.7 | 0.9 |

Note: $\beta$ is the share of renewable energy in total energy production and $\omega$ is the rate of exogenous technical progress

Figure 1: Base line and Cobb Douglas simulation results

Figure 1a Utility optimal paths for the baseline simulation

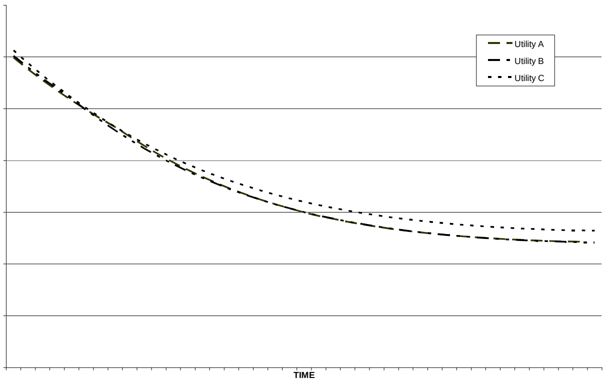


Figure 1b Pollution optimal paths for the baseline simulation

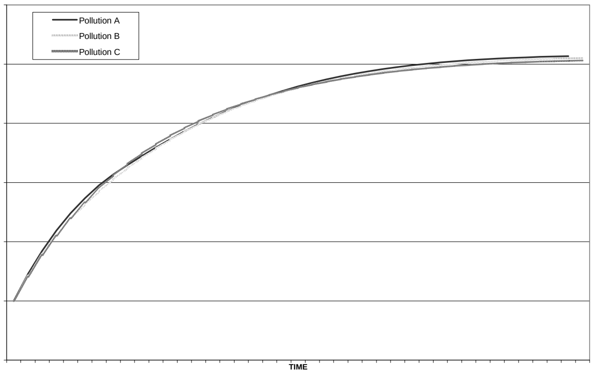


Figure 1c Utility optimal paths with varying rate of technological innovation, and constant beta

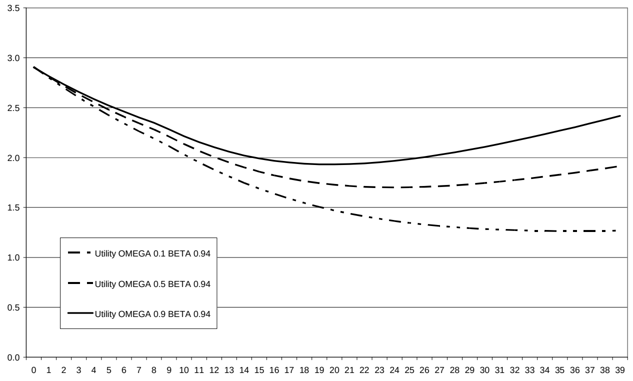


Figure 1d Pollution optimal paths with varying rate of technological innovation, and constant beta.

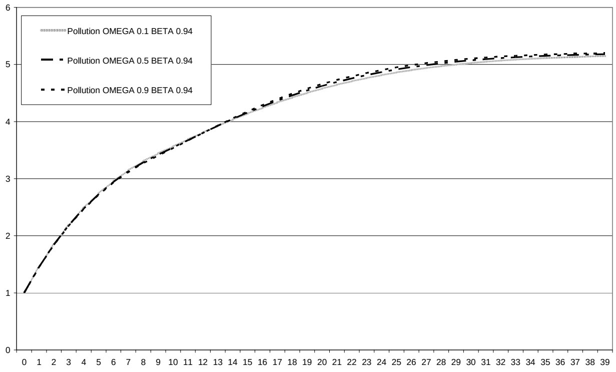


Figure 1e Optimal pollution trajectory upon increases in the exogenous growth rate of technological innovation and varying beta.

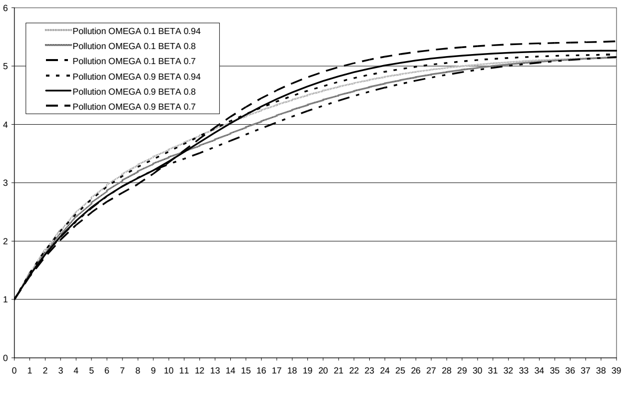


Figure 1f Optimal utility trajectory upon increases in the exogenous growth rate of technological innovation and varying beta.

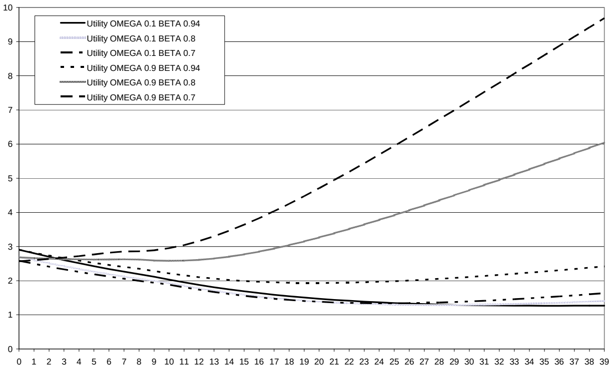


**Table 2: Key parameters for the Constant Elasticity of Substitution energy production simulation set**

| **Constant Elasticity of Substitution (CES) simulation set** | **Substitutability between renewable and non renewable energy resource (σ)** | **Rate of technical progress (ω)** |
|---|---|---|
| CES 0 | Low, σ= 0.4 | Intermediate, ω= 0.01 |
| CES 1 | Low, σ= 0.4 | High, , ω= 0.1 |
| CES 2 | Low, σ= 0.4 | Low, ω=0.0000001 |
| CES 3 | Perfect, σ= 1 | Low, ω= 0.0000001 |
| CES 4 | Good, σ= 0.8 | Low, ω= 0.0000001 |
| CES 5 | Good, σ= 0.8 | High, , ω= 0.1 |

Figure 2: Constant Elasticity of Substitution simulation results

Figure 2a: Consumption.

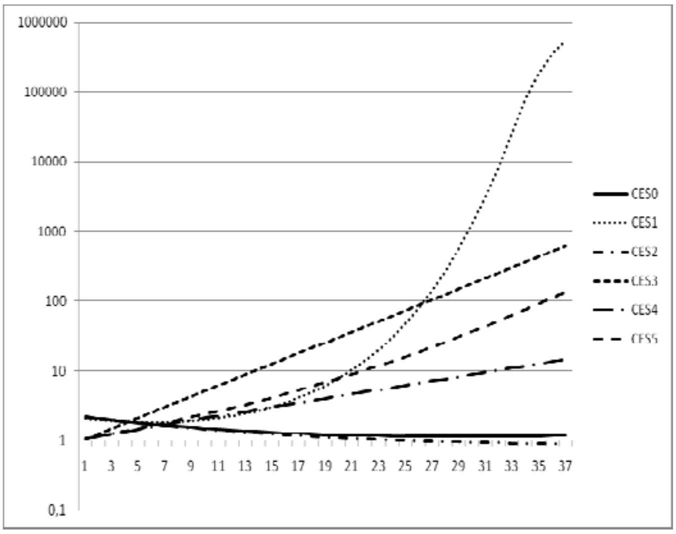


Figure 2b: Utility

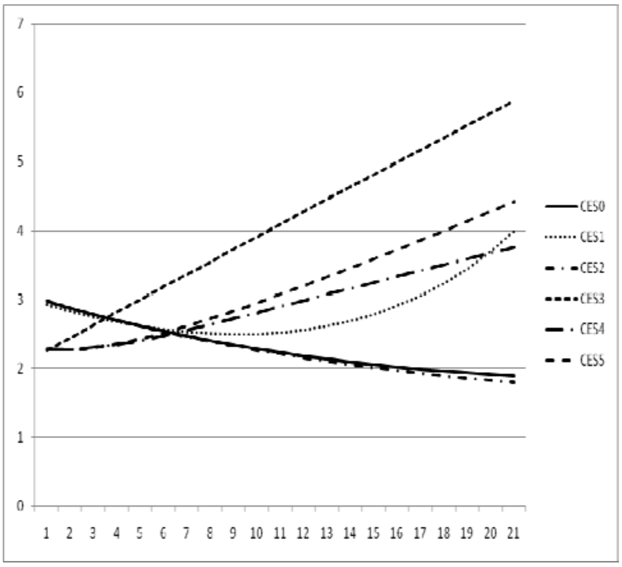


Figure 2c: Pollution

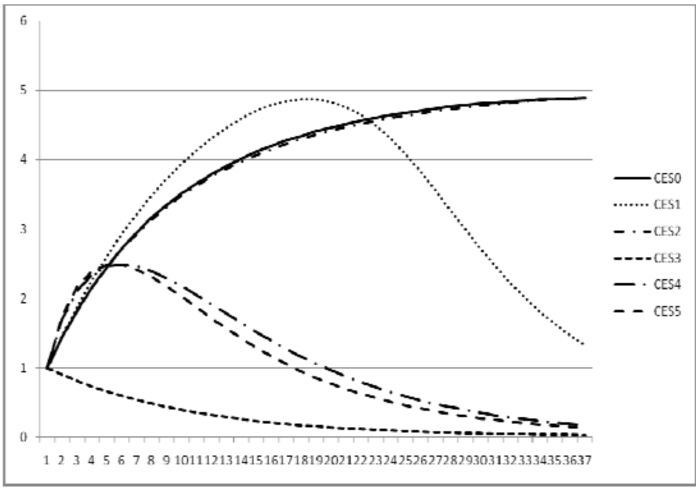
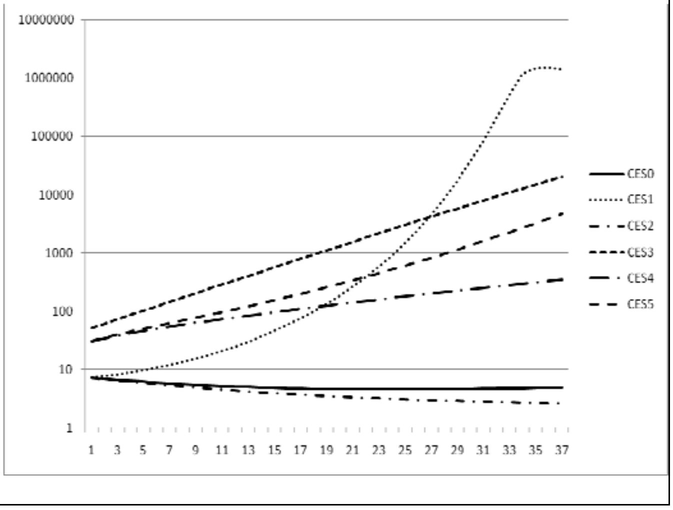


Figure 2d: Renewable resource.



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