The Effects of the Length of the Period of Commitment on the Size of Stable International Environmental Agreements[1]

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Abstract

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This paper extends the standard model of self-enforcing dynamic international environmental agreements by allowing the length of the period of commitment of such agreements to vary as a parameter. It analyzes the pattern of behavior of the size of stable coalitions, the stock of pollutant and the emission rate as a function of the length of the period of commitment. It is shown that the length of the period of commitment can have very significant effects on the equilibrium. Three distinct intervals for the length of the period of commitment are identified, across which the equilibrium and its dynamic behavior differ considerably. Whereas for sufficiently high values of the period of commitment only self-enforcing agreements of two countries are possible, for sufficiently low such values full cooperation can be generated. Lengths of periods of commitment between those two thresholds are characterized by an inverse relationship between the length of commitment and the membership size of the agreement. This suggests that considerable attention should be given to the determination of the length of such international agreements.

JEL classification: Q5; C73; F53

1 Introduction

In many contexts, International Environmental Agreements (IEAs) necessarily involve dynamic considerations. This is because they have to deal with stock pollutants and involve interactions over time among countries. Two approaches have been adopted in modeling such agreements. One consists in assuming that membership and emission strategies of the signatories and non-signatories are determined once and for all at the outset, with each of the signatories and non-signatories committing to an infinite path of emissions. Another consists in analyzing the problem in a discrete-time framework and assuming that membership and emission decisions are revised every period.

Those two formulations correspond to two very particular assumptions about the length of the period of time for which the countries are required to commit. In reality the length of the period of commitment can be an important element of negotiation and the resulting equilibrium may well depend significantly on this length. Intuitively, one might think that a short period of commitment could favor a larger coalition size than a longer one, since the parties will then have the option of revising their membership and emission decisions more frequently, after having observed the state that results at the close of the previous agreement. The purpose of this paper is to analyze the effect of varying the length of the period of commitment on the size and stability of such IEAs.

The model used is closely related to that of Rubio and Casino [6] and, Rubio and Ulph [5]. Rubio and Casino [6] adapt to a dynamic framework the concept of IEA introduced by Barrett [1] and Carraro and Siniscalco [2]. They assume that at the initial date, given the initial stock of pollutant, countries play a two stage game. In the first stage (the membership game), anticipating the play of the game in the second stage, the countries decide non-cooperatively whether or not to join the agreement. In the second stage (the emission game), each non-signatory decides non-cooperatively the emission rate that maximizes its discounted net benefit, taking as given the emission path of the other countries. Signatory countries choose jointly their emission path, acting non-cooperatively against non-signatories in order to maximize their aggregate discounted net benefits. Signatories also take as given the strategy of non signatories. The coalition formed in the membership game cannot change in the emission game. Hence countries commit to both their membership or non membership decision and to their respective emission paths for a period of infinite length. Using numerical simulations, they find that a two-country coalition is the only self-enforcing IEA.

Rubio and Ulph [5] extend that paper to an infinite-horizon model in a discrete-time framework. At the outset of each period, given the stock of pollutant at the beginning of the period, the play of the game is as in the game described above. Countries commit to membership or non membership and to their respective emission strategies for the duration of the period, *whose length is normalized to one* as is usually the case in discrete-time modeling. The authors find that, in this context, there exists a steady-state stock of pollutant and a corresponding steady-state IEA membership size and that, in the transition towards this steady-state, the membership size and the stock of pollutant vary inversely.

In this paper, we adopt an infinite horizon continuous-time framework, but treat the length of the period of commitment as a parameter that can take any strictly positive value. It is thus possible to study the effect of exogenously varying the length of the period of commitment on the equilibrium size of the stable coalition and stock of pollution, as well as on their pattern of behavior over time. Except for the extreme case of a single period of commitment of infinite length, there will be an infinite number of periods of commitment, the length of which is exogenously given at the outset. At the begin of every period of commitment, each country decides whether or not to adhere to the agreement. The signatories then jointly decide on their emission rate for the period of commitment, while the non-signatories make that decision unilaterally.

It is first shown analytically that non-signatories always pollute more than signatories and that they always gain more than signatories from any agreement, irrespective of the length of the period of commitment. Numerical simulations are then used to show that the length of the period of commitment can have a very significant effect on the equilibrium. Two critical values of the length of commitment come out. A first critical value is shown to exist below which the model generates the cooperative equilibrium at each period. Above this critical value and below the next one, there is a negative relationship between the membership size and the length of commitment. Above this second critical value, the only equilibrium is a two members coalition.

As in Rubio and Ulph [5], simulations also indicate an inverse relation between membership size and the pollutant stock: when starting below its steady state, the stock of pollutant rises monotonically until it attains the steady-state, while the membership size decreases to its steady-state. The limiting case of a single period of commitment and the open-loop emissions strategy to which it corresponds is shown to yield, as in Rubio and Casino [6], a coalition of only two signatories. This generates a lower gain from cooperation than that which arises in the case of any finite length of commitment.

The remainder of this paper is organized as follows. Section 2 sets out the model. Section 3 resolves the second stage of the game. In addition, the outcomes of the cooperative and the non-cooperative equilibria are derived in that section. Section 4 presents the first stage of the game. In Section 5, the importance of the choice of the length of the period of commitment is investigated by simulation. Section 6 concludes.

2 The model

We now consider the formation of an infinite sequence of IEAs, in which countries can make binding commitments about their emission rates and their membership decision over a limited horizon. Define a *period* to be the interval of (continuous) time over which countries can make such commitments, and let h be the length of the period. Assume an infinite number of such periods, $[0,h],[h,2h],[2h,3h],...$, and N identical countries, $i=1,...,N$. Each country makes a membership decision and commits to a level of emission for each of the intervals $[0,h],[h,2h],[2h,3h],...,$. We will assume that one unit of production generates one unit of emissions. Let q_i denote the emissions of country i. The current aggregate emissions of the world is then $Q = \sum_{i=1}^{N} q_i$.

The current stock of pollutant is denoted $z(t)$. We assume that the amount of pollutants emitted today by the world adds to the current stock of pollutant according to the kinematic equation

$$\dot{z}(t) = Q(t) - \rho z(t), \quad \rho \in (0,1) \quad z(0) = z_0, \tag{1}$$

where ρ is the natural purification rate.

The stock of pollutant at each date generates damage costs for each country which we assume to be a quadratic function of the stock: $\frac{\gamma}{2}z^2$, with γ positive constant. The instantaneous benefit function is also assumed to be quadratic in current emissions : $aq - \frac{b}{2}q^2$, where a and b are non-negative constants. Thus the flow of net benefits to a country is given by

$$\pi(q, z) = aq - \frac{b}{2}q^2 - \frac{\gamma}{2}z^2. \tag{2}$$

At the beginning of every period, each country determines an emission strategy for that period. A country's choice will depend on the beginning-of-period stock and the length of the period h. Let $q_j^k(z_k)$ denote the emission strategy planned by country j for period k when the stock of pollutant at the outset of the period is $z(kh) = z_k$.

The model of IEA formation in each period, is a dynamic version of the model of self-enforcing IEAs introduced by Carraro and Siniscalco [2] and Barrett [1] and the continuous-time version of Rubio and Ulph [5]. In each period, given the initial stock, there is a two-stage game. In the first stage (the membership game), countries first decide whether or not to join an IEA. In the second stage (the emission game), non-signatory countries choose their emissions for the current period non-cooperatively, while signatory countries act in a cooperative fashion.

For example, for the period $[kh, (k+1)h]$, given the initial stock of pollutant of the current period z_k, countries play the two stage game at the initial date $t = kh$ of the current period. The membership decision which results from the membership game and the emission strategy q_j^k of a given country j are thus decided at the initial date of the current period.

For simplicity we will assume that it commits to a constant q_j^k for the duration of the period k.

Countries being identical, we will also assume that there is a binomial random variable whose realization at any given period determines whether a particular country will be among the members or not for the period. For any stable IEA of size n in that period, the *a priori* probability of any given country being a member of the coalition is n/N. Because of the identical countries assumption, this probability is the same for all countries and is independent of the history of membership decisions of the country. Therefore each country has the same expected present value of current and future net benefits, which will depend on the initial stock of pollutant of the next period. We will denote by $\Psi(z_k)$ the expected present value of current and future net benefits of the representative country when the stock of pollutant at the outset of the period is z_k.

In each period, the second stage of the game is solved first, taking as given the set of signatories of the membership game.

3 The second stage of the game

Consider some beginning of period date $t \in \{0, h, 2h, 3h, 4h, ...\}$, when the stock of pollutant is $z(t)$. Let $K(S)$ denote the set of signatories and n the number of signatories at that date. The current value function of a non-signatory is then

$$V_j(z(t)) = \max_{q_j}\{\int_t^{t+h} e^{-r(s-t)}\pi(q_j, z(s))ds + e^{-rh}\Psi(z(t+h))\}, \tag{3}$$

subject to (1) and (2), where r is the discount rate.

The *aggregate* current value function of all signatories at the same date is

$$V_S(z(t)) = \max_{q_i, i\in K(S)}\{\int_t^{t+h} e^{-r(s-t)} \sum_{i\in K(S)} \pi(q_i, z(s))ds + ne^{-rh}\Psi(z(t+h))\}, \tag{4}$$

again subject to (1) and (2).

The countries being identical, the value function of signatory i is $V_i(z(t)) = V_S(z(t))/n$, $\forall i \in K(S)$.

Definition 1 *In an infinite-period game defined by (3) and (4), with the length of period* h*, an emission strategy for a country* j *is a sequence of functions* $q_j \equiv \{q_j^k : [kh, (k+1)h] \times R_+ \rightarrow R_+\}_{k=0}^{\infty}$*, where* q_j^k *is a constant function of* $s \in [kh, kh+h]$*, for* $k = 0, 1, 2, ...$

This means that at the outset of a given period, given the coalition formed in the membership game, each country chooses and commits to use a constant emission rate in the emission game.

This continuous-time problem can be transformed into a discrete-time one. Indeed, on a given interval $[kh, (k+1)h]$ the emission strategies of the players are constant and so is the aggregate emission of the world, Q. Hence, the solution of the differential equation (1), given the initial stock of pollutant $z(kh) = z_k$, is:

$$z(t) = \frac{Q}{\rho} + (z_k - \frac{Q}{\rho})e^{-\rho(t-kh)} \quad \forall t \in [kh, (k+1)h]. \tag{5}$$

So, at time $t = (k+1)h$, the dynamic evolution of the stock of pollutant between the outset of periods k and $k+1$ is given by

$$z((k+1)h) \equiv z_{k+1} = f(\rho, h)Q + z_k e^{-\rho h}, \tag{6}$$

where $f(x, h) = (1 - e^{-hx})/x, \quad \forall x > 0$. We adopt this notation in the remainder of this paper. The following integral yields the net benefit function at each period, which depends on the length of the period:

$$\int_{kh}^{(k+1)h} e^{-r(s-kh)}\pi(q, z(s))ds = (aq - \frac{b}{2}q^2)f(r, h) + D(Q, z_k), \tag{7}$$

where $D(Q, z_k) = -\frac{\gamma}{2}[(\frac{Q}{\rho})^2 f(r, h) + (z_k - \frac{Q}{\rho})^2 f(r + 2\rho, h) + 2\frac{Q}{\rho}(z_k - \frac{Q}{\rho})f(r + \rho, h)]$.

Substituting (6) and (7) into (3), we obtain the Bellman equation for non-signatories:

$$V_j(z) = \max_{q_j}\{(aq_j - \frac{b}{2}q_j^2)f(r, h) + D(Q, z) + e^{-rh}\Psi(f(\rho, h)Q + ze^{-\rho h})\}. \tag{8}$$

Similarly, substitution of (6) and (7) into (4) yields the Bellman equation for all signatories:

$$V_S(z) = \max_{q_i, i\in K(S)}\{\sum_{i\in K(S)}(aq_i - \frac{b}{2}q_i^2)f(r, h) + nD(Q, z) + ne^{-rh}\Psi(Qf(\rho, h) + ze^{-\rho h})\}. \tag{9}$$

The first-order condition for non-signatories is

$$f(r,h)(a-bq_j) \leq \lambda_1 Q + z\lambda_2 - f(\rho,h)e^{-rh}\Psi'(Qf(\rho,h)+ze^{-\rho h}), \tag{10}$$

with equality holding if $q_j > 0$ and where $\lambda_1 = \frac{\gamma}{\rho^2}(f(r,h)+f(r+2\rho,h)-2f(r+\rho,h))$ and $\lambda_2 = \frac{\gamma}{\rho}(f(r+\rho,h)-f(r+2\rho,h))$.

The first-order condition for signatories is

$$f(r,h)(a-bq_i) \leq n[\lambda_1 Q + z\lambda_2 - f(\rho,h)e^{-rh}\Psi'(Qf(\rho,h)+ze^{-\rho h})], \tag{11}$$

with equality if $q_i > 0$. In the remainder of this paper, we assume that q_j is always interior, so that equality holds in (10), but we will allow the corner solution for q_i.

The left-hand side of (10) and (11) represents the current marginal benefit of pollution (MB), which is the same for each country. The right-hand side of (10) represents the full current marginal cost of pollution for non-signatories (MC_{ns}), while the right-hand side of (11) represents the full current marginal cost of pollution for signatories (MC_s). Hence, an interpretation of the first-order conditions (10) and (11) is that at the equilibrium, the current marginal benefit of each country must be no greater than its current marginal cost of polluting, with equality holding if emissions are positive. Notice that $MC_s = n \times MC_{ns}$.

As explained in the previous section, since the countries are identical, each has *a priori* the same probability n/N of being a signatory. Therefore the expected present value of future net benefits for each country is:

$$\begin{aligned}\Psi(z) &= \frac{n(z)}{N}V_i(z) + (1-\frac{n(z)}{N})V_j(z) \\ &= \frac{n(z)}{N}(aq_i - \frac{b}{2}q_i^2)f(r,h) + f(r,h)(1-\frac{n(z)}{N})(aq_j - \frac{b}{2}q_j^2) \\ &\quad + D(Q,z) + e^{-rh}\Psi(Qf(\rho,h)+ze^{-\rho h}).\end{aligned} \tag{12}$$

Knowing Ψ enables us to derive the value function of signatories and non-signatories. Despite the fact that n is a non-linear function of z, the quadratic form of the expression above in z suggests a quadratic functional form for Ψ.[1]

[1] This non-linearity comes from the fact that feasible values of n are integers.

Proposition 1 *The differential game defined by (8) and (9) admits a quadratic functional form* $\Psi(z,n) = \frac{A(n)}{2}z^2 + B(n)z + C(n)$ *at the second stage of the game, with* $A < 0$*,* $B < 0$*, and, at the equilibrium,* $\Psi(z) \equiv \Psi(z, n(z)) = \frac{A(n(z))}{2}z^2 + B(n(z))z + C(n(z))$*.*

Proof. See Appendix.

Summing up (10) over all subscripts j for an interior solution we get:

$$f(r,h)[(N-n)a - bQ_{ns}] = (N-n)\{\lambda_1 Q + z\lambda_2 - f(\rho,h)e^{-rh}[A(Qf(\rho,h) + ze^{-\rho h}) + B]\}, \quad (13)$$

where Q_{ns} is the aggregate emission of non-signatories.

Likewise, summing up (11) over all subscripts i for an interior solution we get:

$$f(r,h)[na - bQ_s] = n^2\{\lambda_1 Q + z\lambda_2 - f(\rho,h)e^{-rh}[A(Qf(\rho,h) + ze^{-\rho h}) + B]\}, \quad (14)$$

where Q_{ns} is the aggregate emission of signatories.

Finally, adding up side by side equations (13) and (14) and using the fact that the aggregate emission by all countries is $Q = Q_s + Q_{ns}$, a linear equation in Q results. Its solution yields:

$$Q = \frac{Naf(r,h) + Bf(\rho,h)e^{-rh}(N-n+n^2) + z(N-n+n^2)(-\lambda_2 + Af(\rho,h)e^{-h(r+\rho)})}{bf(r,h) + (N-n+n^2)\lambda_1 - (N-n+n^2)Af(\rho,h)^2e^{-rh}}. \quad (15)$$

Using again first-order conditions (10) and (11), we find that the emissions of each non-signatory and of each signatory respectively for an interior solution are given by:

$$q_j = \{f(r,h)a - \lambda_1 Q - z\lambda_2 + f(\rho,h)e^{-rh}[A(Qf(\rho,h) + ze^{-\rho h}) + B]\}/bf(r,h), \quad (16)$$

$$q_i = \{f(r,h)a - n\lambda_1 Q - nz\lambda_2 + nf(\rho,h)e^{-rh}[A(Qf(\rho,h) + ze^{-\rho h}) + B]\}/bf(r,h). \quad (17)$$

In the case of a corner solution for the signatories, $q_i = 0$ and $q_j = Q/(N-n)$, with the result that:

$$Q = \frac{(N-n)af(r,h) + Bf(\rho,h)e^{-rh}(N-n) + z(N-n)(-\lambda_2 + Af(\rho,h)e^{-h(r+\rho)})}{bf(r,h) + (N-n)\lambda_1 - (N-n)Af(\rho,h)^2e^{-rh}}. \quad (18)$$

Proposition 2 *The current emissions by signatories are always less than the current emissions by non-signatories and the resulting payoff of non-signatories is always greater than that of signatories for* $n \in [2, N-1]$.

Proof. Consider first the case of the interior solutions. Proposition 1 states that A and B are negative numbers. Using (16) and (17), $q_j - q_i = \{(n-1)\lambda_1 Q + (n-1)z\lambda_2 - (n-1)f(\rho,h)e^{-rh}[A(Qf(\rho,h)+ze^{-\rho h})+B]\}/bf(r,h)$. Because of $A, B < 0$, $\lambda_1, \lambda_2 > 0$ and $n \geq 2$, it is an easy matter to verify that $q_j > q_i$. Now substitute (17) in (9) and (16) in (8), to derive

$$V_j - V_i = (1+n)(q_j - q_i)\{\lambda_1 Q + z\lambda_2 - f(\rho,h)e^{-rh}[A(Qf(\rho,h)+ze^{-\rho h})+B]\}/2.$$

Since $q_j > q_i$, $A, B < 0$, $\lambda_1, \lambda_2 > 0$, it follows that $V_j > V_i$.

In the case of a corner solution, we have $q_i = 0$, so that $V_j - V_i = (aq_j - \frac{b}{2}q_j^2)f(r,h)$, which is positive since from (16) we must have $0 < q_j < a/b$. ■

Before solving the first stage of the game, it is useful to study the particular cases of the non-cooperative equilibrium ($n = 0$) and the fully-cooperative equilibrium ($n = N$).

3.1 The non-cooperative equilibrium

Assume that all N countries decide non-cooperatively the emission strategy of the current period that maximizes their discounted net benefit, taking as given the current emission strategy of the other countries. The Bellman equation is then the special case of (8) for which $n = 0$ and $\Psi = V_j$, and we have the following result.

Proposition 3 *In the non-cooperative equilibrium, the sequence of pollutant stocks at the outset of each period,* $\{z_k\}_{k=0}^{\infty}$*, converges to a steady state*

$$\tilde{z} = \frac{Naf(r,h)f(\rho,h) + \tilde{B}Ne^{-rh}f(\rho,h)^2}{N(\lambda_2 - \tilde{A}f(\rho,h)e^{-rh}) + (1-e^{-\rho h})[bf(r,h) + N\lambda_1 - N\tilde{A}f(\rho,h)^2e^{-rh}]},$$

if and only if

$$R_0 = \frac{N[\lambda_1 e^{-\rho h} - \lambda_2 f(\rho,h)] + bf(r,h)e^{-\rho h}}{bf(r,h) + N\lambda_1 - N\tilde{A}f(\rho,h)^2e^{-rh}} > -1.$$

This convergence is monotone if and only if $R_0 > 0$. The corresponding steady-state level of emissions is given by

$$\tilde{q}_j = \frac{af(r,h) + \tilde{B}f(\rho,h)e^{-rh} + \tilde{z}[-\lambda_2 + \tilde{A}f(\rho,h)e^{-h(r+\rho)}]}{bf(r,h) + N\lambda_1 - N\tilde{A}f(\rho,h)^2 e^{-rh}},$$

where $\tilde{A}$ and $\tilde{B}$ are particular values of A and B from the Appendix obtained by setting $n = 0$.

Proof. See Appendix.

3.2 The cooperative equilibrium

Suppose now that all the countries decide cooperatively the emission strategies of the current period that maximizes their aggregate discounted net benefit. The Bellman equation is then the particular case of (9) for which $n = N$ and $\Psi = V_S/N = V_i$. Again, the quadratic nature of the net benefit function in each period (see equation (7)) suggests that a logical guess for the value function is:

$$V_i(z) = \frac{\bar{A}}{2}z^2 + \bar{B}z + \bar{C}, \tag{19}$$

and we have the following result.

Proposition 4 *In the fully-cooperative equilibrium, the sequence of pollutant stocks at the outset of each period, $\{z_k\}_{k=0}^{\infty}$, converges to a steady state*

$$\bar{z} = \frac{Naf(r,h)f(\rho,h) + \bar{B}N^2 e^{-rh} f(\rho,h)^2}{N^2(\lambda_2 - \bar{A}f(\rho,h)e^{-rh}) + (1 - e^{-\rho h})[bf(r,h) + N^2\lambda_1 - N^2\bar{A}f(\rho,h)^2 e^{-rh}]}.$$

if and only if

$$R_N = \frac{N^2[\lambda_1 e^{-\rho h} - \lambda_2 f(\rho,h)] + bf(r,h)e^{-\rho h}}{bf(r,h) + N^2\lambda_1 - N^2\bar{A}f(\rho,h)^2 e^{-rh}} > -1.$$

This convergence is monotone if and only if $R_N > 0$. The corresponding steady-state level of emissions is

$$\bar{q}_i = \frac{af(r,h) + \bar{B}f(\rho,h)e^{-rh}N + \bar{z}N[-\lambda_2 + \bar{A}f(\rho,h)e^{-h(r+\rho)}]}{bf(r,h) + N^2\lambda_1 - N^2\bar{A}f(\rho,h)^2 e^{-rh}},$$

where $\bar{A}$ and $\bar{B}$ are given in Appendix.

Proof. See Appendix.

Let us denote respectively by q_c, q_i, q_j and q_{nc} the current emissions respectively of the cooperative equilibrium, the signatories, the non-signatories and of the non-cooperative equilibrium.

Proposition 5 *Assuming interior solutions we have: (i) $q_c \leq q_i < q_j \leq q_{nc}$; (ii) the current aggregate emissions by all countries from any IEA (Q_{iea}) lies between that of the cooperative (Q_c) and the non-cooperative (Q_{nc}) equilibria for any $n \in [2, N-1]$.*

Proof. To prove (i), denote by MC_{nc} the current marginal cost of pollution of the non-cooperative equilibrium, which, from (10), is given by

$$MC_{nc}(q) = \lambda_1 Q + z\lambda_2 - f(\rho, h)e^{-rh}\Psi'(Qf(\rho, h) + ze^{-\rho h}),$$

and denote by MC_c the current marginal-cost of pollution of the cooperative equilibrium, which, from (11) is given by $MC_c(q) = N \times MC_{nc}(q) \quad \forall q$.[2] With each country emitting the same quantity q, we then have the following inequalities: $MC_c(q) \geq MC_s(q) \geq MC_{ns}(q)$ and $MC_c(q) \geq MC_s(q) \geq MC_{nc}(q)$, as illustrated in Figure 1. Invoking the fact that for interior solutions, at the equilibrium, the marginal benefit of each country must equal its full marginal cost and that the marginal benefit function is the same for all and is decreasing in q, it follows that $q_c \leq q_i < q_j$ and $q_c \leq q_i \leq q_{nc}$ as shown in Figure 1. Therefore, at equilibrium, we have $q_j, q_{nc} \geq q_i$. But $MC_{ns}(q) \geq MC_{nc}(q)$ $\forall q \geq q_i$,[3] which implies that $q_j \leq q_{nc}$.

The proof of (ii) follows from the fact that, because of the above inequalities, $Q_{iea} - Q_c = n(q_i - q_c) + (N-n)(q_j - q_c) > 0$ and $Q_{nc} - Q_{iea} = n(q_{nc} - q_i) + (N-n)(q_{nc} - q_j) > 0$. Thus we have $Q_c \leq Q_{iea} \leq Q_{nc}$ ■

The results of Proposition 2 and Proposition 5 are known in the literature. They have been shown, among others, by Rubio and Ulph [5] in their discrete-time model, with the

[2] Note that MC_c is the particular case of MC_s obtained by setting $n = N$.

[3] This inequality holds since $MC_{nc}(q)$ and $MC_{ns}(q)$ are increasing in q, due to the fact that $\Psi' < 0$.

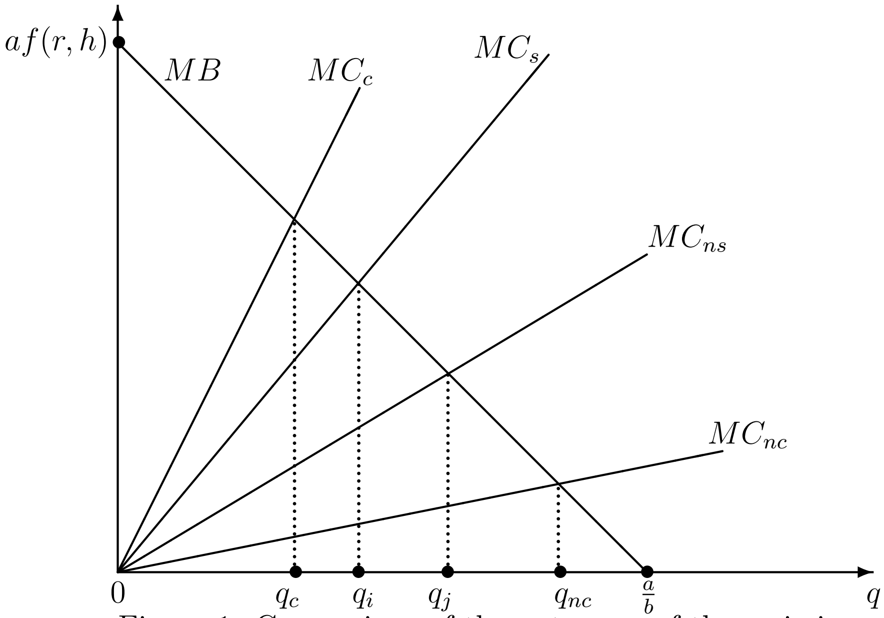

Figure 1: Comparison of the outcomes of the emission game.

length of the period of commitment set equal to one, and by Rubio and Casino [6], with a period of commitment of infinite length. The two propositions show that those results hold irrespective of the length of the period of commitment.

4 The first stage of the game

To resolve the membership game, we use the notion of stability introduced by D'Aspremont et al. [3].

Definition 2 *At the beginning of a period, if the current stock of pollutant is z, a coalition of signatories $K(S)$ of size n is said to be stable, or self-enforcing, if and only if*

$$V_i(n,z) \geq V_j(n-1,z)$$

$$V_j(n,z) \geq V_i(n+1,z).$$

The first inequality of Definition 2 is the internal stability condition. Its interpretation is that a signatory country cannot be better off by leaving the coalition, given that the other countries maintain their membership decision. The second inequality is the external stability condition. It means that a non-signatory cannot be better off by joining the coalition, given that the other countries maintain their membership decision.

Unfortunately, it is not possible to check analytically the stability conditions, nor is it possible to determine analytically the effect of the length of commitment on the size of the stable coalitions. For this reason, we proceed by numerical simulation to illustrate the level of cooperation that can be reached and how can it vary regarding to the length of commitment.

5 Numerical simulations: the effects of the length of commitment

In this section we present the outcome of the numerical analysis on a set of $N = 20$ identical countries. We use the same parameter values as Rubio and Casino [6].[4] These are $b = 1650$; $a = 100000$, the scale parameter;[5] $\gamma = 0.001$; $r = 0.025$; $\rho = 0.005$.[6] The model captures the result of Rubio and Casino [6] and Rubio and Ulph [5] for some values of the length of commitment and yields different results for others.

The simulations have been carried out for more than two thousand values of the length of commitment.[7] What first appears clearly is that the current value function of signatories as well as of non-signatories increases with the number of signatories, whereas the aggregate emissions by all countries decrease as the membership size increases. Of greater interest is the effect of the length of commitment on the equilibrium number of signatories and on the gains from cooperation, to which we now turn.

5.1 The length of commitment and the size of self-enforcing coalitions

As concerns the size of self-enforcing coalitions, our simulations highlight two critical values of the length of commitment, which distinguish three possible cases for the dynamic behavior of the model. The first case corresponds to short lengths of commitment ($h < 7.352$).

[4]We do this simulation for a group of 20 countries while Rubio and Casino [6] did it for a group of 10, but this is of no qualitative consequence for the results.

[5]It is a scale parameter in the sense that, first there is a positive relation between a and the stock of pollutant, second the coalition size is not affected by its feasible values.

[6]For convenience, the simulations presented are performed for $z_0 = 0$, as in Rubio and Casino [6]. The simulations have been done also for numerous positive values of z_0 with no impact on the results.

[7]These values are obtained from the arithmetic sequence $h_p = h_{p-1} + 0.1$; $h_0 = 0.1$, $p = 1, ..., 2499$.

It is characterized by full cooperation at each period, and the steady-state stock of pollutant and emission level are then given by Proposition 4 which characterizes the cooperative equilibrium.[8] This is illustrated in Figure 3 for $h = 1$.

The second case is for lengths of commitment in the semi-open interval $[7.352, 226)$. In that case, simulations suggest two main results. First, there is a negative relation between the length of the period of commitment and the number of signatories at each period. Second, as in Rubio and Ulph [5], the stock of pollutant rises asymptotically to its steady state while the coalition size is non-decreasing over time and converges after a finite number of periods to its steady state. This is illustrated in Figure 3 for the case where the length of the period of commitment is equal to $h = 10$. The stock rises from $z_0 = 0$ to $z_{34} = 1.44959 \times 10^5$ after 35 periods. After that, it continues to rise following the dynamic relation $z_{k+1} = 0.992 z_k + 1137$ and converges to its steady-state $\bar{z} = 1.69213 \times 10^5$ asymptotically. The coalition begins with 18 signatories at the initial period then decreases and reaches its steady state after 35 periods, remaining at 10 signatories.

The third case corresponds to very large lengths of the period of commitment ($h \geq 226$). It is illustrated in Figure 3 for $h = 250$. The size of the coalition is $n = 2$ at each period of the game and the stock of pollutant converges to its steady state asymptotically. This is consistent with the result obtained by Rubio and Casino [6]. Those authors analyze an IEA in the case of an open-loop emission strategy, allowing the emission strategy of countries to vary over time, and they find that an IEA can be sustained only by a coalition of two signatories. We have assumed that each country commits to a constant emission strategy in each period. But we also find a two-signatories coalition in the case of the open-loop strategy, which is obtained by equating h to the infinite planning horizon at the initial date.

Another striking outcome of the simulations is that reducing the length of commitment

[8] For some lengths of periods of commitment smaller than 7.352, we may have more than one self-enforcing IEA, the fully-cooperative equilibrium being always one of them. When this occurs we focus on the fully-cooperative equilibrium, which is the coalition that sustains both the greatest gain to signatories as well as to non-signatories and the lowest aggregate emissions. For lengths of commitment greater than 7.352, we do not find more than one stable coalition.

increases the number of signatories and decreases the stock of pollutant at every instant, as illustrated in Figure 3.[9] At every instant, the stock of pollutant resulting from the open-loop infinite-horizon equilibrium is always greater than the stock of pollutant that result from the adoption of a finite length of commitment.

These results can be summarized as follows. Simulation suggests two thresholds, $\underline{h}$ and $\overline{h}$, of the length of the period of commitment, which define the three possibilities for the dynamic behavior. If $h < \underline{h}$, the model exhibits a cooperative equilibrium at each period. If the length of the period of commitment is greater than $\underline{h}$, there is a negative relation between the length of commitment and the number of signatories over time. Furthermore, as in Rubio and Ulph [5], the model captures a negative relation between the membership size and the stock of pollutant. The stock of pollutant rises and converges asymptotically to its steady-state, while the membership size decreases and reaches its steady-state after a finite number of periods. Lengths of commitment greater than $\overline{h}$ result in a coalition of only two signatories as the outcome of the membership game at each period. Furthermore, the stock of pollutant increases and converges asymptotically to its steady-state. In the case of the open-loop emission path, as in Rubio and Casino [6], a two-members coalition is the outcome.

The fact that the membership size increases as the length of the period of commitment is shortened can be rationalized as follows. In a coalition of size n, the gain for an individual insider of being an insider rather than unilaterally becoming an outsider is given by:

$$\Omega(n, h) \equiv V_i(n, h) - V_j(n-1, h), \quad \forall n = 1, 2, \ldots, N,$$

whereas the gain for an outsider of unilaterally becoming an insider is:

$$\Lambda(n, h) \equiv -[V_i(n+1, h) - V_j(n, h)] = -\Omega(n+1, h), \quad \forall n = 1, 2, \ldots, N-1.$$

But

$$\text{sign}\frac{\partial \Omega(n+1, h)}{\partial h} = \text{sign}\frac{\partial \Omega(n, h)}{\partial h},$$

[9] Even though Figure 3 presents this result only for $h \in \{1, 10, 250\}$, we obtain similar results for all the 2500 values of h used in the simulation.

which means that

$$\text{sign}\frac{\partial\Lambda(n+1,h)}{\partial h} = -\text{sign}\frac{\partial\Omega(n,h)}{\partial h}.$$

Hence, if, for a given n, decreasing the length of the period of commitment makes it more attractive to be part of the coalition ($\frac{\partial\Omega(n,h)}{\partial h} < 0$) it also makes it less attractive to be outside the coalition.

Applying Definition 2, a self-enforcing coalition of size $n(h)$ is therefore characterized by $\Omega(n(h),h) \geq 0$ and $-\Omega(n(h)+1,h) = \Lambda(n,h) \geq 0$. Since $\Omega(m,h)$ is a smooth function and $\Omega(n(h),h)\Omega(n(h)+1,h) \leq 0$, there exists a real number $m(h) \in [2,N]$ that solves the equation $\Omega(m(h),h) = 0$. Let $I[m(h)]$ denote the largest integer no larger than $m(h)$. Then, for any given h, the equilibrium size of the coalition is $n(h) = I[m(h)]$. If $m(h)$ is unique, it follows that $\Omega(n(h),h) \geq 0$ and $\Omega(n(h)+1,h) = -\Lambda(n(h),h) < 0$ and therefore the function $\Omega(n,h)$ has a strictly negative slope with respect to n at a neighborhood of $m(h)$, or $\frac{\partial\Omega(m(h),h)}{\partial n} < 0$.[10]

Differentiating totally the equation $\Omega(m(h),h) = 0$ with respect to h, we have that:

$$\frac{\partial m(h)}{\partial h} = -\frac{\frac{\partial\Omega(m(h),h)}{\partial h}}{\frac{\partial\Omega(m(h),h)}{\partial n}}.$$

It follows that in the case where $m(h)$ is unique, if $\frac{\partial\Omega(m(h),h)}{\partial h} < 0$, then $\frac{\partial m(h)}{\partial h} < 0$. But since $m(h)$ is decreasing in h, so is its integer part. Therefore $n(h) = I[m(h)]$ is decreasing in h.

That $\frac{\partial\Omega(n,h)}{\partial h}$ is negative is verified by numerical simulation for all values of n, as illustrated in Figure 2. This means that for any given n, reducing the period of commitment makes it more profitable to become part of the coalition and less attractive to be an outsider. Or, put another way, reducing the period of commitment reduces the cost of committing, the reason being that the shorter the period of commitment, the earlier the decision can be revised.

[10] As noted earlier, the solution for $m(h)$ may in some circumstances not be unique. For the sake of the argument, we neglect those cases here.

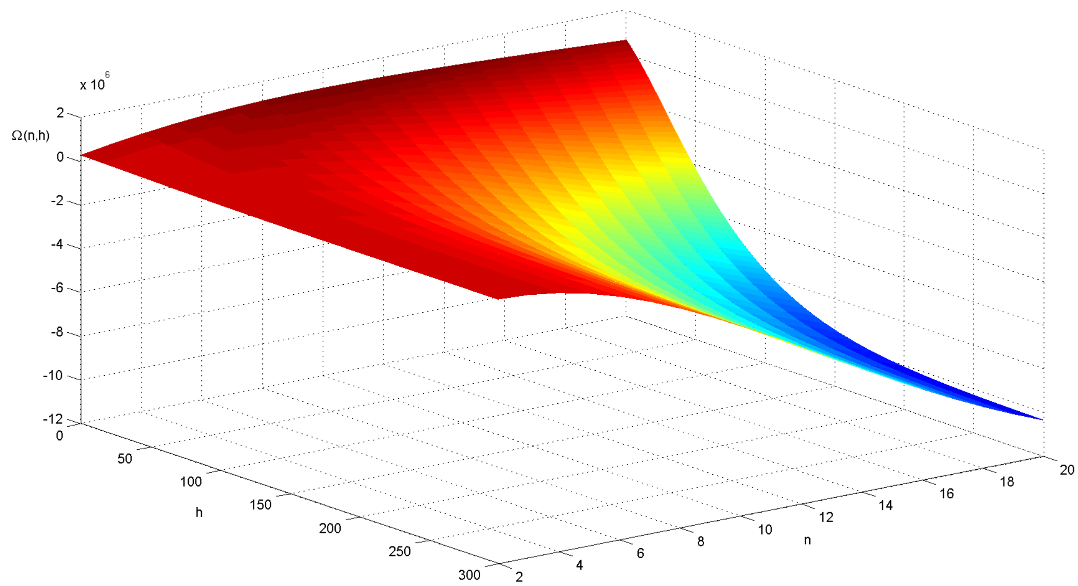

Figure 2: The function $\Omega(n, h)$.

5.2 The length of commitment and the gains from cooperation

We now consider the effect of the length of commitment on the gain from cooperation.[11] It is useful to distinguish different concepts of gain. The potential gain from cooperation (POC) is defined as the difference between the sum of the discounted net benefits from cooperative and non-cooperative equilibrium. It is given by:

$$POC = V_i(N, z_0, h) - V_j(0, z_0, h).$$

The other concept of gain compares the partial cooperative equilibrium to the non-cooperative equilibrium. Assume a coalition of $n^*(z_0, h)$ signatories. As seen in Section 3, this results in a decrease in the emission level of signatories and in the aggregate emissions by all countries, as compared to the non-cooperative equilibrium. But both signatories as well as non-signatories gain from the reduction of the global emissions. The partial gain from cooperation by signatories (PAC_s), defined as the difference between the sum of the

[11]The simulations are done at $t = 0$ over h for the initial period only. But since the rates of emission at each period depend only on the stock of pollution and not explicitly on calendar time, the qualitative results are the same for each subsequent period.

discounted net benefits by signatories in a stable IEA and in the non-cooperative equilibrium, is given by:

$$PAC_s = V_i(n^*(z_0, h), z_0, h) - V_j(0, z_0, h).$$

The partial gain from cooperation by non-signatories (PAC_{ns}) is defined as the difference between the sum of the discounted net benefits by non-signatories given a stable IEA and that in the non-cooperative equilibrium. It is given by:

$$PAC_{ns} = V_j(n^*(z_0, h), z_0, h) - V_j(0, z_0, h).$$

The average partial gain from cooperation by all countries (PAC) is defined as the mean of the PAC_s and the PAC_{ns} with the respective weights $n^*(z_0, h)/N$ and $1 - n^*(z_0, h)/N$. Formally, it is given by:

$$PAC = \frac{n^*(z_0, h)}{N} PAC_s + (1 - \frac{n^*(z_0, h)}{N}) PAC_{ns}.$$

By the definition of external stability, if $n^*(z_0, h) = 0$ no country can be made better off by cooperating. Hence, in that case, we set $PAC_s = PAC_{ns} = PAC = 0$.

In Figure 4, the top graph illustrates that POC is a decreasing function of the length of commitment and that it has a limit which is its open-loop outcome. The bottom graph illustrates that the cooperative equilibrium can be attained for $h \in (0, 7.352)$, while a negative relation holds between the coalition size and the length of commitment for $h \in [7.352, 226)$. It also illustrates that the coalition size remains at two signatories for all $h \geq 226$. In particular, taking the limit of $n^*(z_0, h)$ as h goes to infinity, we obtain a two-signatories coalition as the outcome of the open-loop coalition size.

Notice that since we have the cooperative equilibrium for $h \in (0, 7.352)$, the POC and PAC are equal and decreasing in h over this interval, as illustrated by the top two graphs of Figure 4.[12]

It is interesting to note that while in the static model of Barrett [1] an IEA may result in a significant level of cooperation only if the POC is very small, in our model this pessimistic

[12]Because of a scale effect, this decrease as a function of h is not very apparent in the graphs, but it is real.

result need not hold for some values of the length of commitment. Indeed, as shown in Figure 4, the length of commitment that maximizes the POC also sustains the cooperative equilibrium.

For $h > 7.352$, the PAC_s, PAC_{ns} and the PAC are all decreasing function in h as illustrated in Figure 5. They decrease, along with the coalition size, and each of them has a limit which is its open-loop outcome. Because non-signatories gain more than signatories from any cooperation, it follows that $PAC_s \geq PAC_{ns}$ for all values of the length of commitment as shown in the bottom graph of Figure 5.

In spite of the fact that we cannot claim any general result, the above suggests that for some lengths of commitment $h \neq 1$ the gain from cooperation is higher than for $h = 1$, and that any finite length of commitment can, for each of POC, PAC, PAC_s and PAC_{ns}, sustain a higher value than their open-loop outcomes obtained by letting the length of commitment go to infinity. It is clear that the length of commitment significantly affects the size of stable coalitions and the gains from cooperation.

Finally, it is useful to examine the relation between the initial stock of any given period of commitment (z_k), the length of commitment which maximizes the PAC_s (denote it h^*) and the length of commitment which can sustain the minimum aggregate emissions (denote it $\hat{h}$). To do this, we have first simulated 8001 values of the current stock of pollutant following the replication $z_k = z_{k-1} + 50, k = 1, ..., 8000;\ z_0 = 0$. For each of those values, we calculate h^* and $\hat{h}$. The top graph of Figure 6 illustrates the fact that $\hat{h}$ is a decreasing function of the initial stock of the period of commitment. The bottom graph shows that h^* is independent of the initial stock of pollutant of the period of commitment. Furthermore, $\hat{h}$ is always greater than h^*. These results highlight the difficulties of reconciling the private gain from cooperation and the best protection of the environment.

6 Conclusion

The existing literature on dynamic International Environmental Agreements has relied on one of two approaches. The first consists in assuming that membership and emission strategies are determined once and for all, as a function of time, at the outset of an infinite horizon. The other consists in analyzing the problem in a discrete-time framework and assuming that membership and emission decisions are revised at the beginning of each period, whose length has been arbitrarily set equal to one. This paper has explored the middle ground by treating the length of the period of commitment as a positive parameter and studying the effect of varying this parameter on the size of stable International Environmental Agreements. It has been shown that the length of the period of commitment can have considerable impact on the size of stable International Environmental Agreements. The results suggest that for very large lengths of commitment, only very small stable coalitions can be sustained. But, below some threshold, as the length of the period of commitment is decreased, the size of the stable coalition tends to increase. It does so until, if this length is sufficiently small, the full cooperation equilibrium might be attained.

Since our results rest on particular functional forms and on numerical simulations, there is no claim to generality. But they do show clearly that the length of the period of commitment can have very significant effects on the outcome of International Environmental Agreements. This suggests that considerable attention should be devoted to the determination of the length of the period of commitment in discussions of this type of international treaties.

For the purpose of this paper, it has been sufficient to treat the length of commitment as a parameter. However, how best to determine the length of commitment is another matter, which is clearly worthy of further research.

Appendix

Proof of proposition 1

Let us recall that the state space is R_+ and the set of feasible coalitions is $\mathcal{N} = \{0, 2, 3, ..., N-1, N\}$. It is after observing the length of the period of commitment and the stock of pollutant at the outset of the period that countries make their membership decision. Thus the decision rule is the correspondence:

$$\begin{aligned} n :& R_+ \times R_+ \to \mathcal{N} \\ & (z, h) \mapsto n(z, h). \end{aligned}$$

For a given $i \in \mathcal{N}$, let us denote by $\mathfrak{O}_i(h) = \{z \in R_+ / n(z, h) = i\}$, the subset of all z such that coalition size i occurs.

Let z denote the stock of pollutant at the outset of the current period and n denote the current coalition size. By definition $n(z, h) = n$ is independent of z on the subset of the state space $\mathfrak{O}_n(h)$. Q, q_i, q_j are hence linear functions of z if Ψ can take the special functional form

$$\Psi(z, n) = \frac{A(n)}{2} z^2 + B(n) z + C(n), \quad \forall z \in \mathfrak{O}_n(h), \tag{20}$$

with $A(n) < 0$ and $B(n) < 0$.

On the subset $\mathfrak{O}_n(h)$, equation (12) becomes, $\forall z \in \mathfrak{O}_n(h)$:

$$\begin{aligned} \Psi(z) \;&=\; \frac{n}{N}(aq_i - \frac{b}{2}q_i^2) f(r, h) + f(r, h)(1 - \frac{n}{N})(aq_j - \frac{b}{2}q_j^2) \\ &+\; D(Q, z) + e^{-rh}\Psi(Qf(\rho, h) + ze^{-\rho h}). \end{aligned} \tag{21}$$

Because $n(z, h) = n$ is independent of z on the subset of state space $\mathfrak{O}_n(h)$, both the RHS and the LHS of (21) are second degree polynomial in z. Equating their coefficients yields the following third degree polynomial in A, the coefficient of the quadratic term in (20):[13]

$$a_3 A^3 + a_2 A^2 + a_1 A + a_0 = 0,$$

[13] To do this, we use the well-known fact that equating the coefficients of two polynomials of degree two is equivalent to equating their first three derivatives evaluated at zero.

where

$$
\begin{aligned}
a_3 =&(N-n+n^2)^2 f(\rho,h)^4 e^{-2rh} \\
a_2 =&-2(N-n+n^2) f(\rho,h)^2 e^{-rh}(bf(r,h)+\lambda_1(N-n+n^2)) - \alpha f(\rho,h)^2 e^{-2h(r+\rho)} \\
&-2\lambda_2(N-n+n^2)^2 f(\rho,h)^3 e^{-h(\rho+2r)} + \gamma f(r+2\rho,h)(N-n+n^2)^2 f(\rho,h)^4 e^{-2rh} \\
a_1 =&(bf(r,h)+\lambda_1(N-n+n^2))^2 + 2\alpha\lambda_2 f(\rho,h) e^{-h(r+\rho)} \\
&+2\lambda_2(N-n+n^2)[\lambda_2 f(\rho,h)^2 e^{-rh}(N-n+n^2) \\
&+f(\rho,h)e^{-h(r+\rho)}(bf(r,h)+\lambda_1(N-n+n^2))] \\
&-2\gamma f(r+2\rho,h)(N-n+n^2) f(\rho,h)^2 e^{-rh}(bf(r,h)+\lambda_1(N-n+n^2)) \\
&-e^{-rh}[bf(r,h)e^{-\rho h} + (N-n+n^2)(\lambda_1 e^{-\rho h} - \lambda_2 f(\rho,h))]^2 \\
a_0 =&-\alpha\lambda_2^2 - 2\lambda_2^2(N-n+n^2)(bf(r,h)+\lambda_1(N-n+n^2)) \\
&+\gamma f(r+2\rho,h)(bf(r,h)+\lambda_1(N-n+n^2))^2 \\
\alpha =&-(N-n+n^3)bf(r,h)/N - \lambda_1(N-n+n^2)^2.
\end{aligned}
$$

Pick A to be the negative root of this polynomial.[14] For any given value of A, we can compute:

$$Q_p = \frac{(N-n+n^2)(-\lambda_2 + Af(\rho,h)e^{-h(r+\rho)})}{bf(r,h)+(N-n+n^2)(\lambda_1 - Af(\rho,h)^2 e^{-rh})} \tag{22}$$

$$q_{ip} = \frac{nQp}{N-n+n^2} \tag{23}$$

$$q_{jp} = \frac{Q_p}{N-n+n^2} \tag{24}$$

$$
\begin{aligned}
u =&(\frac{n^2}{N}q_{ip} + (1-\frac{n}{N})q_{jp})(\lambda_1 - f(\rho,h)^2 e^{-rh}A) - \lambda_1 Q_p - \lambda_2 \\
&+e^{-rh}(f(\rho,h)Q_p + e^{-\rho h})Af(\rho,h)
\end{aligned} \tag{25}
$$

$$d = bf(r,h) + (N-n+n^2)(\lambda_1 - Af(\rho,h)^2 e^{-rh}). \tag{26}$$

[14]Using the parameter values of section 5, for all values of h, simulation yields one negative root, the remaining roots being either both real and positive or complex.

Let B denote the solution of the linear equation $b_1 B = b_0$, where

$$\begin{aligned} b_1 =& 1 + f(\rho,h)e^{-rh}(\frac{n^2}{N}q_{ip} + (1-\frac{n}{N})q_{jp}) - e^{-rh}(f(\rho,h)Q_p + e^{-\rho h}) \\ & - e^{-rh}f(\rho,h)(N-n+n^2)\frac{u}{d} \\ b_0 =& Naf(r,h)u/d. \end{aligned}$$

Proof of $B < 0$

Using (23), (24), (25) and (26) the expressions of b_1 and u above become

$$\begin{aligned} b_1 = \left(1 - e^{-h(r+\rho)}\right) &+ e^{-rh}f(\rho,h)\left(N-n+n^2\right)\left(\lambda_2 - Af(\rho,h)e^{-h(r+\rho)}\right)/d \\ &+ \frac{bQ_p f(r,h)f(\rho,h)e^{-rh}}{dN(N-n+n^2)}\left[\frac{N-n+n^3}{N(N-n+n^2)} - 1\right] \end{aligned} \quad (27)$$

and

$$u = -\left[\lambda_2 - Af(\rho,h)e^{-h(r+\rho)}\right]\left[bf(r,h) + \frac{(N-n+n^3)(\lambda_1 - Af(\rho,h)^2e^{-rh})}{N}\right]/d \quad (28)$$

Since $A < 0$, $\lambda_1, \lambda_2 > 0$ and $n \leq N$, we have $u, Q_p < 0, d > 0$ and hence $b_1 > 0$, due to the fact that each of the three terms constituting the expression (27) is positive.

Since $u < 0$ and $d > 0$, we have $b_0 < 0$. Finally $B < 0$, since it is the quotient of b_0 and b_1, two numbers of different signs.

Given A and B, let C denote the solution of the linear equation $c_1 C = c_0$, where,

$$\begin{aligned} c_1 =& \frac{n}{N}f(r,h)(aq_{io} - \frac{b}{2}q_{io}^2) + f(r,h)(1-\frac{n}{N})(aq_{jo} - \frac{b}{2}q_{jo}^2) - \frac{1}{2}\lambda_1 Q_o^2 \\ & + e^{-rh}f(\rho,h)Q_o(B + \frac{A}{2}f(\rho,h)Q_o) \\ c_0 =& 1 - e^{-rh} \\ Q_o =& [Naf(r,h) + Bf(\rho,h)e^{-rh}(N-n+n^2)]/d \\ q_{jo} =& [Q_o + n(n-1)\frac{a}{b}]/(N-n+n^2) \\ q_{io} =& -(n-1)\frac{a}{b} + nq_{jo} \end{aligned}$$

In the case of a corner solution, we have $q_i = 0$, and the above proof still holds by replacing: n^m by zero for $m = 2, 3$; q_{ip} and q_{io} by zero; Na by $(N-n)a$ and q_{jo} by $Q_o/(N-n)$.

The non-cooperative equilibrium.

By combining (6) and (15) in the particular case $n = 0$, we obtain the dynamic evolution of the stock in the non-cooperative equilibrium:

$$\begin{aligned} z_{k+1} &= f(\rho,h)\frac{Naf(r,h) + \tilde{B}f(\rho,h)e^{-rh}N + z_k N[-\lambda_2 + \tilde{A}f(\rho,h)e^{-h(r+\rho)}]}{bf(r,h) + N\lambda_1 - N\tilde{A}f(\rho,h)^2 e^{-rh}} + z_k e^{-\rho h} \\ &\equiv \varphi_0(z_k) \end{aligned}$$

The unique solution of the equation $x = \varphi_0(x)$ is:

$$\tilde{z} = \frac{Naf(r,h)f(\rho,h) + \tilde{B}Ne^{-rh}f(\rho,h)^2}{N(\lambda_2 - \tilde{A}f(\rho,h)e^{-rh}) + (1 - e^{-\rho h})[bf(r,h) + N\lambda_1 - N\tilde{A}f(\rho,h)^2 e^{-rh}]}$$

Hence $z_{k+1} - \tilde{z} \equiv \varphi_0(z_k) - \tilde{z} = R_0(z_k - \tilde{z})\ \forall\ k = 0, 1, 2, 3, ...$, so that $z_k = \tilde{z} + (R_0)^k(z_0 - \tilde{z})$ $\forall\ k = 0, 1, 2, 3, ...$, where:

$$R_0 = \frac{N[\lambda_1 e^{-\rho h} - \lambda_2 f(\rho,h)] + bf(r,h)e^{-\rho h}}{bf(r,h) + N\lambda_1 - N\tilde{A}f(\rho,h)^2 e^{-rh}}.$$

We have $1 > R_0$. Indeed, because all the parameters are non-negative and $\tilde{A} < 0$, we have the following inequality:

$$bf(r,h)(1 - e^{-\rho h}) + N\lambda_1(1 - e^{-\rho h}) + N\lambda_2 f(\rho,h) - N\tilde{A}f(\rho,h)^2 e^{-rh} > 0.$$

Rearranging the terms of this inequality, one gets:

$$bf(r,h) + N\lambda_1 - N\tilde{A}f(\rho,h)^2 e^{-rh} > N[\lambda_1 e^{-\rho h} - \lambda_2 f(\rho,h)] + bf(r,h)e^{-\rho h}.$$

Dividing the LHS and the RHS of the last inequality by its LHS, one obtains $1 > R_0$.

The sequence $z_k - \tilde{z}$ being a geometric progression, a necessary and sufficient condition for z_k to converge is $1 > R_0 > -1$. It has already been established that $1 > R_0$. Therefore, if and only if $R_0 > -1$, z_k converges and its limit is $\tilde{z}$. It converges monotonically if $R_0 > 0$.

The steady-state emission rate exists if and only $R_0 > -1$ and is given by:

$$\tilde{q}_j = \frac{af(r,h) + \tilde{B}f(\rho,h)e^{-rh} + \tilde{z}[-\lambda_2 + \tilde{A}f(\rho,h)e^{-h(r+\rho)}]}{bf(r,h) + N\lambda_1 - N\tilde{A}f(\rho,h)^2 e^{-rh}},$$

where $\tilde{A}$ and $\tilde{B}$ are the particular values of A and B for $n = 0$.

The fully-cooperative equilibrium.

Using the guess (19), each country must emit at the equilibrium

$$q_i(z) = \frac{af(r,h) + BNf(\rho,h)e^{-rh} + zN(-\lambda_2 + Af(\rho,h)e^{-h(r+\rho)})}{bf(r,h) + N^2\lambda_1 - N^2 Af(\rho,h)^2 e^{-rh}}. \quad (29)$$

Substituting this quantity in the current value function and using the envelope theorem, one obtains:

$$V_i'(z) = -z\gamma f(r+2\rho,h) - N\lambda_2 q_i(z) + e^{-h(r+\rho)}V_i'(Nf(\rho,h)q_i(z) + ze^{-\rho h}), \forall z \geq 0.$$

Equating the coefficients of the LHS and RHS of this first-degree polynomial in z, we find that

$$\bar{A} = \left[-\bar{a}_1 - \sqrt{\bar{a}_1^2 - 4\bar{a}_2\bar{a}_0}\right]/2\bar{a}_2,$$

which is the negative root of the second degree polynomial $\bar{a}_2 A^2 + \bar{a}_1 A + \bar{a}_0 = 0$, where,

$$\bar{a}_2 = N^2 f(\rho,h)^2 e^{-rh}$$
$$\bar{a}_1 = -(bf(r,h) + \lambda_1 N^2)(1 - e^{-h(r+2\rho)}) - 2N^2\lambda_2 f(\rho,h) + \gamma f(r+2\rho,h)N^2 f(\rho,h)^2 e^{-rh}$$
$$\bar{a}_0 = -b\gamma f(r,h)f(r+2\rho,h) - N^2\lambda_2^2.$$

Given $\bar{A}$, we find:

$$\bar{B} = \frac{aNf(r,h)(-\lambda_2 + \bar{A}f(\rho,h)e^{-h(r+\rho)})}{(bf(r,h) + N^2\lambda_1 - N^2\bar{A}f(\rho,h)^2 e^{-rh})(1 - e^{-h(r+\rho)}) - N^2 f(\rho,h)e^{-rh}(-\lambda_2 + \bar{A}f(\rho,h)e^{-h(r+\rho)})}.$$

The dynamic evolution of the stock of pollutant is then

$$\begin{aligned} z_{k+1} &= f(\rho,h)\frac{Naf(r,h) + \bar{B}f(\rho,h)e^{-rh}N^2 + z_k N^2[-\lambda_2 + \bar{A}f(\rho,h)e^{-h(r+\rho)}]}{bf(r,h) + N^2\lambda_1 - N^2\bar{A}f(\rho,h)^2 e^{-rh}} + z_k e^{-\rho h} \\ &\equiv \varphi_N(z_k). \end{aligned}$$

The unique solution to the equation $x = \varphi_N(x)$ is:

$$\bar{z} = \frac{Naf(r,h)f(\rho,h) + \bar{B}N^2 e^{-rh} f(\rho,h)^2}{N^2(\lambda_2 - \bar{A}f(\rho,h)e^{-rh}) + (1 - e^{-\rho h})[bf(r,h) + N^2\lambda_1 - N^2\bar{A}f(\rho,h)^2 e^{-rh}]}.$$

Hence, $\forall\ k = 0, 1, 2, 3, ...,$

$$z_{k+1} - \bar{z} \equiv \varphi_N(z_k) - \bar{z} = R_N(z_k - \bar{z})$$

so that

$$z_k = \bar{z} + (R_N)^k(z_0 - \bar{z}), \quad \forall \quad k = 0, 1, 2, 3, ...,$$

where,

$$R_N = \frac{N^2[\lambda_1 e^{-\rho h} - \lambda_2 f(\rho, h)] + bf(r, h)e^{-\rho h}}{bf(r, h) + N^2\lambda_1 - N^2\bar{A}f(\rho, h)^2 e^{-rh}}.$$

By a similar argument as for the $1 > R_0$, we can show that $1 > R_N$. Therefore a necessary and sufficient condition for z_k to converge is $R_N > -1$. When this condition holds, the sequence z_k converges $\bar{z}$. It does so monotonically if $R_N > 0$.

Thus the steady-state emission rate exists if and only if $R_N > -1$, in which case its expression is given by:

$$\bar{q}_i = \frac{af(r, h) + \bar{B}f(\rho, h)e^{-rh}N + \bar{z}N[-\lambda_2 + \bar{A}f(\rho, h)e^{-h(r+\rho)}]}{bf(r, h) + N^2\lambda_1 - N^2\bar{A}f(\rho, h)^2 e^{-rh}}.$$

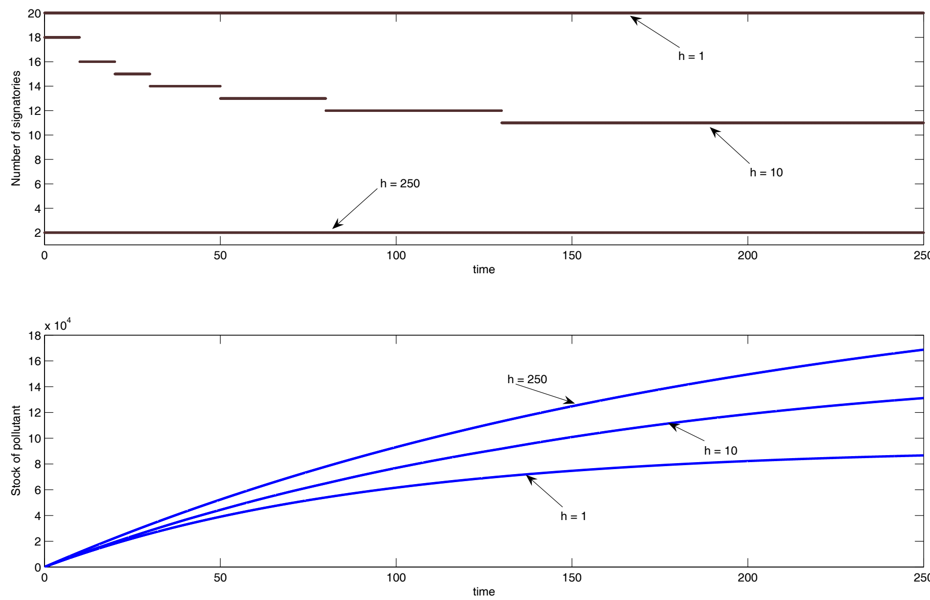

Figure 3: Length of commitment, coalition size and stock of pollutant over time.

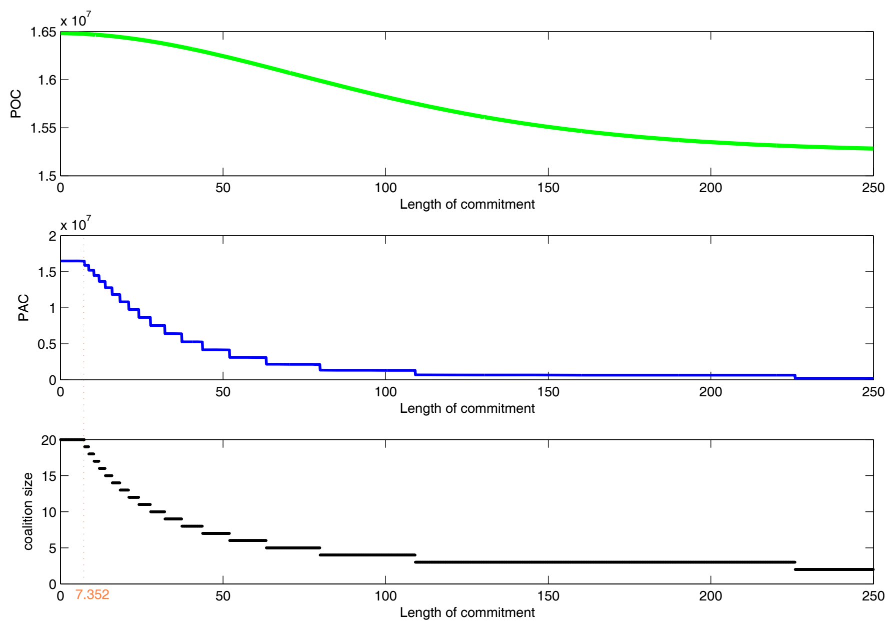

Figure 4: POC, PAC and coalition size as functions of h.

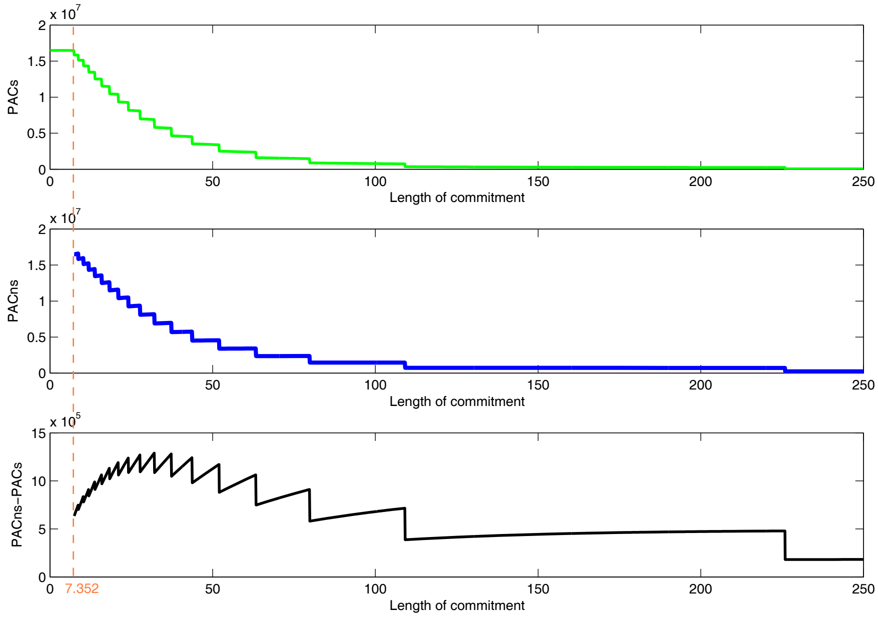

Figure 5: Gain from cooperation by signatories and by non-signatories as functions of h.

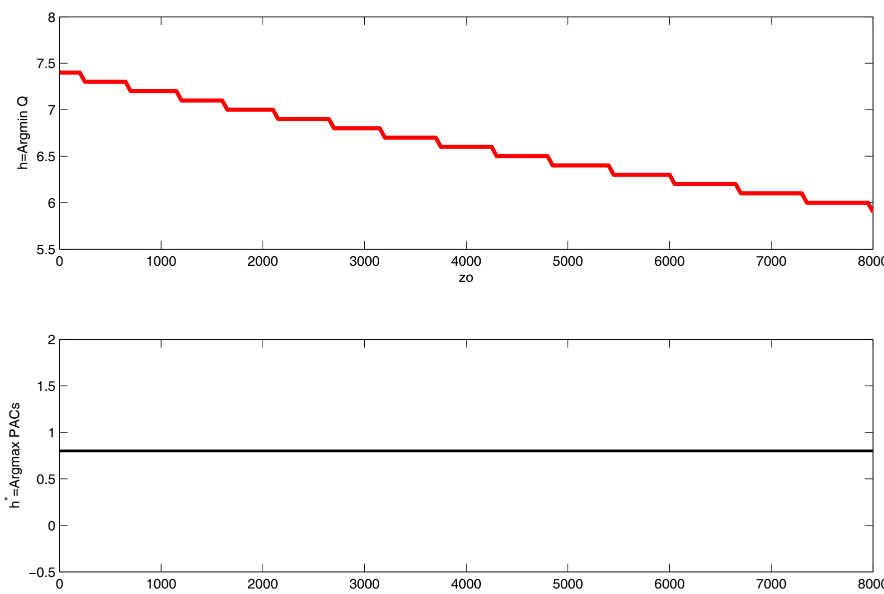

Figure 6: $\hat{h} = \arg max_h PAC_s$ versus $h^* = \arg min_h Q$.

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