

# From Acid Dip to Thriving Waters

## The Impact of Emissions Reductions on Lake Recovery

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### Abstract

I develop an optimal control model for the recovery of a representative freshwater lake from acidification. My objective function is the sum of the disutility from an acidified lake and the cost of emissions abatement by firms. Using emissions as the control variable, the social regulator minimizes the objective function subject to state equations that describe the impact of emissions reductions on the state variables, pH and alkalinity of lake water. I estimate the state equations using a panel data set which monitors the recovery of 43 acidified lakes located in the region surrounding Sudbury, Ontario, Canada over a 24-year period. The results indicate a general upwards trend in both pH and alkalinity, with a decrease in emissions corresponding to an increase in both variables. Using the estimation results, I solve the control problem for the optimal path of emissions reductions. I find that optimal emissions reductions are very small, and require the assumption of extremely high damages from lake acidification. This is inconsistent with observed emissions reductions, and points to a number of limitations in my theoretical model.

## 1 Introduction

Acid rain, ozone depletion and climate change have been termed the three environmental crises of the twentieth century. While concern over climate change continues to increase, little discussion remains of acid rain and ozone depletion. This is due in large part to the successes of the Montreal Protocol for chlorofluorocarbons (CFCs), and the Convention on Long-Range Transboundary Air Pollution for various pollutants including sulphur dioxide ( $\text{SO}_2$ ), nitrogen oxides ( $\text{NO}_x$ ) and Volatile Organic Compounds (VOCs). These international agreements created a cooperative effort among countries implementing domestic regulations to control emissions of pollutants. While the problems of ozone depletion and acid rain are no longer intensifying, damage from excessive pollution in the past remains widespread. This damage, and the desire for recovery, are the primary focus of current regulations.

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This paper studies the emissions reductions influencing the recovery of freshwater lakes from acidification in Northern Ontario, Canada. Terrestrial and aquatic acidification in this area is primarily attributable to nickel mining activities centered in the City of Sudbury. At its peak in 1960, total annual sulphur dioxide emissions from smelter operations in Sudbury exceeded 2,560 kilotonnes (kt). This represents more than 4% of estimates of total global anthropogenic sulphur dioxide emissions at the time [7, 16]. Estimates indicate over 7,000 lakes located within a 17,000 km<sup>2</sup> area of Sudbury have been acidified to pH < 6.0, the point at which significant biological damage starts to occur [15]. Since the 1960s, a series of regulations introducing intensifying emission controls have induced reductions in SO<sub>2</sub> emissions of over 90%, and substantial water quality improvements have been observed.

This paper sets up a dynamic programming problem to determine how to optimally implement emissions reductions intended to achieve the recovery of a representative lake in the Sudbury area. A regulator minimizes a social welfare function, defined as the sum of the disutility from lake acidification and the cost of abatement by firms, subject to state equations describing how lake water quality evolves over time. My objective is to use data on water quality and emissions reductions to estimate the parameters of the state equations. I find water quality is increasing over time, and that decreases in acid depositions will have a positive effect on water quality. I use these results to solve the dynamic programming problem for the optimal path of emissions reductions. I find emissions reductions are very small, and require the assumption of extremely high damages from lake acidification. This result points to a number of limitations in my theoretical model, and I discuss improvements to the model that can be made in future work.

The remainder of this paper proceeds as follows. Section 2 provides an overview of the economic literature on dynamic ecological models and the scientific literature on acidification dynamics. Section 3 presents the theoretical model for the dynamic programming problem, and the state equations to be estimated. Section 4 describes the data, and Section 5 presents the estimation strategy and results. Section 6 discusses the numerical solution to the optimal control problem. Finally, Section 7 offers some brief concluding remarks, as well as a discussion of future work.

## 2 Literature Review

### 2.1 The Economics of Dynamic Ecological Systems

Integrated Assessment Models (IAMs) are dynamic models used in environmental economics to capture the interaction between economic objectives and constraints, and dynamic environmental systems. As ecologists recognize, the complex dynamics of environmental systems substantially affect the states of the world in which the economic system operates [1]. IAMs have therefore become an important tool for linking environmental and economic systems.

Past application of IAMs to acidification dynamics focus on the acidification of terrestrial ecosystems. The first major work was by Kaitala et al [13] who consider the optimal regulatory policies of Finland and the Soviet Union within a game-theoretic framework. The objective of both countries is to maximize the net benefits of emissions abatement, which Kaitala et al define as the value of forest growth minus the costs of abatement, subject to an environmental state equation. The environmental state equation describes how soil quality changes over time. It is a function of current soil quality, and current sulphur depositions which Kaitala et al estimate as a linear function of emissions in each region using a sulphur transportation model. Parameters for the equations are either estimated, or drawn from previously published work. With the appropriate identification of the equation parameters, Kaitala et al solve the maximization problem for the optimal cooperative and noncooperative emission rates in Finland and Russia.

Schmieman and van Ierland [22] develop an optimal control model to identify cost effective European abatement policies for the combined reduction of  $\text{SO}_2$  and  $\text{NO}_x$ . The objective of each country is to minimize the cost of abatement, subject to an environmental state equation and a minimum standard for soil quality. The environmental state equation is based on Kaitala et al, and uses a generalized linear transportation model for estimation of current sulphur depositions which allows for emissions from more than two countries. They solve the optimal control model for a single country and find the optimal path of emissions reductions is higher than those required under the current European protocol scenario.

Schmieman et al [23] expand upon previous work by considering the interaction between the problems of acidification and tropospheric ozone pollution. They generalize the previous optimal control problem to include damages from both problems as part of the objective function, and introduce an additional

environmental state equation describing the change in ozone levels over time. The theoretical model is used to calculate efficient abatement strategies for  $\text{SO}_2$ ,  $\text{NO}_x$ , and Volatile Organic Compounds (VOCs).

Past applications of IAMs to lake dynamics focus primarily on the problem of eutrophication. Eutrophication occurs when there is excess phosphorous and nitrogen deposition in a lake, typically as a result of runoff from fertilizers used in agriculture. A eutrophic lake can cause significant economic consequences attributable to a decrease in the recreational value of the lake, a decrease in the value of fisheries, and a deterioration of water quality [19], and may be either reversible, hysteretic or irreversible.<sup>1</sup> Acidified lakes cause similar economic consequences and display similar recovery patterns. Therefore, models of eutrophication are useful in understanding how models of lake acidification can be developed.

Carpenter et al [2] provide a general model of eutrophication. They set up an optimal control model where a regulator maximizes the net benefits from polluting activities and ecosystem services, subject to an environmental state equation which describes how phosphorus levels in the lake evolve over time. They estimate the state equation by an observation error procedure, using data from a single lake, and the control model is solved for the optimal phosphorus input rates under various assumptions regarding certainty over the reversibility of the lake, and the value of the discount rate. While the model is based on deterministic lake dynamics, an important observation of Carpenter et al is that in a more realistic situation where sources of variability exist, reductions in phosphorus input levels should be below the optimal rates the model describes. They note this result extends to other situations where pollution causes nonlinear changes in an ecosystem state, such as acid deposition.

Nævdal [19] focuses on the varying reversibility of lakes and the threshold effects of eutrophication. He sets up an optimal control problem similar to those above, and solves the problem for lakes on either side of the threshold value which determines whether a lake is currently in a state of eutrophy. He finds the optimal path for the reduction of nutrients into the lake is dependent on whether the lake is eutrophying in the initial time period, the number of times it is optimal for the lake to cross the threshold, and whether a eutrophying lake is reversible.

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<sup>1</sup>A reversible lake is one in which eutrophication can be reversed by the reduction of pollution input controls alone. A hysteretic lake is one that can be reversed from its eutrophic state, but requires a perturbation to the lower phosphorous steady state using interventions such as aluminum sulphate treatment or biomanipulation. An irreversible lake is one in which no feasible reduction of pollution input controls or chemical intervention can bring the lake out of its eutrophic state [2].

Hein [10] develops a eutrophication model with threshold effects and two steady states, one corresponding to a eutrophic lake and a second corresponding to an oligotrophic lake.<sup>2</sup> He uses an explicit ecological-economic model in which lake dynamics are modeled by a set of equations obtained through regression analysis of long-term water quality data for a shallow lake ecosystem. He combines this with information on the supply of ecosystem services and the costs of different control measures to determine the optimal control policy. The existence of two steady states has a significant impact on the cost-effectiveness of different policy options, essentially creating two points of maximum efficiency, each corresponding to one of the steady states.

This paper draws upon a number of the above studies in developing an economic-ecological model of lake acidification. I use the acidification models to understand the dynamics of acid deposition, and the eutrophication models to understand the non-linearities of lake response to changes in chemical depositions, and the process of recovery from a polluted state. While I use previous work as a basis for developing my model, this paper makes a number of contributions to the literature. First, I model the dynamics of lake acidification, and set up an optimal control framework for evaluating the implementation of emissions reductions for the recovery of acidified lakes. In addition, while I develop the theoretical model for a representative lake, I estimate the state equations using a panel data set which tracks the water quality of 43 lakes over a 25-year period. Finally, whereas previous work on acidification focuses on preventative measures for the deterioration of soil quality, I develop a model which emphasizes recovery from acidification, with the ultimate goal of returning a lake to its natural state.

## 2.2 Acidification Dynamics

The primary pollutants responsible for acidification are  $\text{SO}_2$  and  $\text{NO}_x$ .<sup>3</sup> These pollutants are derived from a number of sources, with the largest contributors being power stations, industrial plants, and vehicle emissions. Acidification of ecosystems is a result of excessive wet and dry depositions of acid. Wet depositions occur when  $\text{SO}_2$  and  $\text{NO}_x$  reach the atmosphere, where they react with the moisture and undergo oxidation, resulting in the formation of sulphuric and nitric acids. These acids exist primarily in the clouds and are

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<sup>2</sup>An oligotrophic lake is characterized by low nutrient content and low plant growth. The lake waters are clear and oxygen levels are high, particularly in the bottom waters. Oligotrophic lakes are therefore a good environment for many fish species, and generally provide high recreational benefits.

<sup>3</sup>The science in this section is based on the discussion of acidification in Chapter 5 of Mason [17] and Chapter 1 of Charles [3]

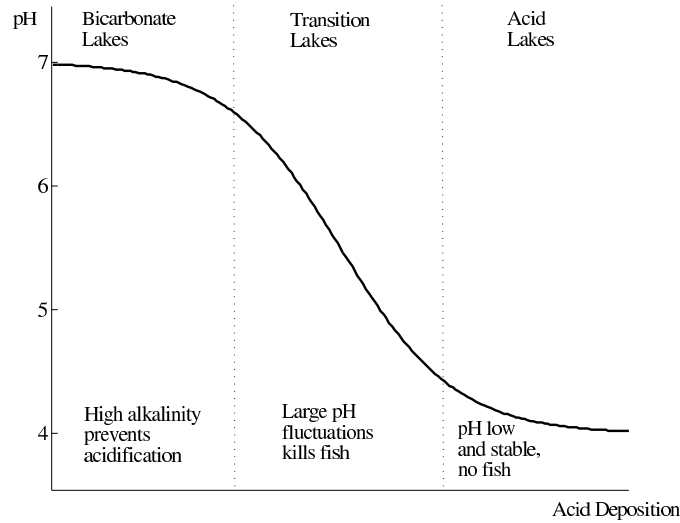


Figure 1: Acidification process of lakes

transported to the ground through rain or snow. Dry depositions, alternatively, occur in a dry atmosphere through a series of complex photochemical reactions in which highly reactive oxidizing agents such as ozone produce sulphuric and nitric acids. Acids from these reactions are transported to the ground in gaseous or particulate form. Wet deposition is relatively easy to quantify, while quantification of dry deposition is more difficult since gases and particulates are more widespread – they enter surface and groundwater basins, are absorbed by vegetation, and are dissolved by precipitation.

In addition to the wet and dry depositions from the atmosphere, freshwater is affected by acidic inputs through indirect atmospheric depositions via run-off in the catchment, and from the generation of acidity within the catchment. Given these two additional sources, acidification of freshwater lakes is most likely to occur in areas with thin soil where there are insufficient base cations freely available to neutralize the deposition of acid to the soil. Similarly, land use also influences the rate of acidification, with acid deposition generally increasing in forested areas.

The process of acidification can be divided into three stages, as shown in Figure 1 (adapted from Mason [17]). It begins with the increasing deposition of sulphate and nitrate ions, which have a negative charge.

The lake water responds by an increase in the positive charge,  $H^+$ , which measures acidity.<sup>4</sup> However, this

<sup>4</sup>pH is defined as the negative logarithm of the concentration of  $H^+$  ions in the water,  $pH = -\log [H^+]$ . As the concentration of  $H^+$  ions in the lake water increases, the water becomes acidified, and pH falls.

can be matched by a decrease in one of the other negative charges in the water. This is what occurs in the first stage of Figure 1 when the alkalinity of the water, which has a negative charge, is positive. In this stage, the positive alkalinity acts as a buffer against increases in acid deposition and the concentration of  $H^+$  ions.<sup>5</sup> The end result is a decrease in the negative charge (alkalinity) and no change in the positive charge ( $H^+$ ), so that pH remains at its natural level, and communities of aquatic life remain stable. Some lakes with a high buffering capacity will never move beyond this stage. However, more generally, a lake will move into the second stage where the alkalinity buffer is lost, and continued acid deposition results in increases in  $H^+$ , large decreases in pH, and the beginning of damages to the biological ecosystem. In the third and final stage, the loss of alkalinity is complete and the pH stabilizes at some low level, typically below 5. In this state the lake is acidic and there are typically increasing levels of metals with positive charges, particularly aluminum. This results in the extermination of fish populations and a decrease in the diversity of other aquatic life.

There are two methods for reversing the acidification of freshwater lakes. The first is a reduction in emissions, which is typically accomplished by a switch to cleaner production technologies, such as the installation of scrubbers at emitting sources. There has been a significant reduction in acidifying emissions since the 1970s, however, this has yet to lead to recovery of all lakes. While sulphuric and nitric acids in precipitation have declined, a large amount of acid remains deposited in soils and wetlands. Therefore, while direct deposition of acidic inputs is decreasing, depositions via run-off in the catchment and from the generation of acidity within the catchment remain. A second method for reversing acidification is the liming of waters. In liming either pulverized limestone, hydrated lime or quicklime is added to the water to neutralize the acid. This method of recovery has more immediate results, but its effectiveness depends on the retention time of water in the lakes. Lakes with short retention times must be relimed either annually or biannually, and those with longer retention times will generally re-acidify 5 to 10 years after liming. While liming is effective in restoring water chemistry, ecological recovery is not guaranteed, and it is typically an expensive alternative with localized results. Since acidification is a widespread problem, the general consensus is that the causes and not the symptoms of acidification must be addressed, and reductions in emissions should be the main tool used in reversing acidification.

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<sup>5</sup>The major component of alkalinity in most surface waters is bicarbonate,  $HCO_3^-$ . When alkalinity is positive, bicarbonate is positive, and will combine with the hydrogen ion to form aqueous carbon dioxide and water:  $HCO_3^- + H^+ \rightarrow CO_2 + H_2O$ , so that the increased acid deposition has no effect on the acidity of the lake water.

### 3 Theoretical Model of Lake Recovery

I develop an optimal control model for the recovery of a representative lake in the Sudbury area from acidification. Following Nævdal [19], I assume there is a regulator who is concerned about the state of the lake, and define preferences such that any deviation from the lake’s “natural state” causes increasing disutility. I define the state of the lake by the observed pH and alkalinity levels, and assume the natural state is exogenously defined to represent the necessary conditions for biological recovery from acidification. A convenient definition for the natural state of a lake is a return to its predisturbance state. However, this may not be an accurate definition since recovery will typically take several decades during which time the lake in a healthy state may naturally evolve due to either internal chemical processes or external factors such as climate change. Therefore, a more accurate definition for the natural state is the reference data approach that defines recovery as a return to a state that is typical of the least-disturbed lakes in the area [9]. In a given period, the disutility from the degree of acidification is thus given by:

$$U(P_t) = \frac{A}{2} (P_t - \bar{P})^2 + \frac{B}{2} (L_t - \bar{L})^2 \quad (1)$$

where  $P_t$  and  $L_t$  are the observed pH and alkalinity of the lake in period  $t$ ,  $\bar{P}$  and  $\bar{L}$  are the pH and alkalinity of the lake in its natural state, and  $A$  and  $B$  are positive parameters. I assume  $P_0 < \bar{P}$  and  $L_0 < \bar{L}$ , i.e., the starting point for all lakes is in a state of acidification. Since the utility function provides a measure of disutility it is convex, and at the starting point ( $t = 0$ ),  $U'(P_t, L_t) < 0$ ,  $U''(P_t, L_t) > 0$ , so the degree of disutility is decreasing as  $P_t$  and  $L_t$  increase.

The regulator must balance the desire for returning a lake to its natural state with the costs of doing so. I consider only the cost of abatement undertaken by local firms emitting  $\text{SO}_2$ .<sup>6,7</sup> Following Kaitala et al [13], I assume the cost function defines the minimum cost envelope of the entire range of sulphur abatement options for firm  $j$  in a given time period. In any given period, however, I observe actual emissions by a firm, and not abatement. Therefore, I define the abatement cost function by the following quadratic equation:

$$C^j(E_t^j) = \psi_1^j + \psi_2^j (\bar{E}^j - E_t^j) + \psi_3^j (\bar{E}^j - E_t^j)^2 \quad (2)$$

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<sup>6</sup>I define local firms as those that are subject to regulation.

<sup>7</sup>From this point forward, I consider only the effects of changes in  $\text{SO}_2$  emissions on the acidification and recovery of lakes. I ignore the effects of  $\text{NO}_x$  since the local firms under consideration are all industrial plants where the primary pollutant is  $\text{SO}_2$  and emissions of  $\text{NO}_x$  are minimal.

where  $\bar{E}^j$  are the SO<sub>2</sub> emissions of firm  $j$  in the absence of regulation,  $E_t^j$  are the observed emissions of firm  $j$  in period  $t$  given regulation, and  $\psi_1^j, \psi_2^j, \psi_3^j$  are positive parameters. Actual abatement is therefore given by  $(\bar{E}^j - E_t^j)$ . The cost function is convex in emissions, and I require that abatement be non-negative in all periods ( $\bar{E}^j - E_t^j \geq 0$ ) so abatement costs are increasing as actual emissions decrease,  $C'(E_t^j) < 0$ ,  $C''(E_t^j) > 0$ .

Following the previous literature on acidification ([13], [22], [23]), I assume there is a linear relationship between SO<sub>2</sub> emissions and acid deposition at a lake site. Acid deposition will be affected by several stochastic factors beyond actual SO<sub>2</sub> emissions, most notably the amount of precipitation a lake receives and the buffering capacity of the land surrounding the lake. Precipitation is exogenous to the model, and monitoring buffering capacity for the land over time requires the introduction of an additional state equation. For simplicity, I disregard these factors and assume the acid deposition equation is a linear transformation of all current emissions impacting the environmental state of the lake. The equation for acid depositions is thus given by:

$$D_t = \sum_m c^m E_t^m \quad (3)$$

where  $c^m$  is a weighting factor relating emissions from firm  $m$  to acid depositions at the lake,  $m$  is the total number of firms with emissions impacting the lakes,  $m \geq j$ , and  $(m - j)$  is the number of firms not under control of the regulator. I further assume the acid deposition equation is deterministic and the regulator knows the emissions of the  $(m - j)$  firms not subject to regulation.<sup>8</sup> With this assumption, the regulator can accurately predict how changes in the emissions of regulated firms will impact the environmental state of the lake.

The regulator's control variable is emissions of local firms,  $E_t^j$ . Following the previous literature, I assume the regulator specifies an emissions cap for each firm in every period, and that observed emissions of firms are exactly equal to their regulated amount. The regulator can therefore precisely determine acid depositions from local firms at the lake site in each period. These acid depositions will impact water quality

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<sup>8</sup>A more complicated model is found in Kaitala et al [13] who sets up the optimal control problem within a game theoretic framework. He solves the problem for the cooperative solution, where regulators jointly maximize their social welfare functions, and the non-cooperative solution, where regulators consider only the acidification dynamics in their region given a fixed emissions strategy of their opponent. In Kaitala's framework, my model corresponds to the derivation of a non-cooperative solution to the optimal control problem.

at the lake site, which I measure by the state variables pH,  $P_t$ , and alkalinity,  $L_t$ .

The state equations for alkalinity and pH describe changes in the environmental state of the lake. Starting from a state of acidification, the desired chemical response to a reduction in acid depositions is an increase in alkalinity and pH. If the lake water follows this desired response path then alkalinity and pH will increase together until the lake returns to its natural state. As alkalinity increases, the effect of changes in acid deposition on pH will begin to decrease. This is the mechanism through which the pH of the water stabilizes to the level corresponding to its natural state. In this state, the lake is able to resist reasonable changes in acid deposition since the restored alkalinity provides a buffering capacity which allows the water to neutralize itself. However, if the acid deposition exceeds some critical threshold then the lake will move out of its natural state and again begin to decline into an acidified state.

The chemical recovery of a lake is a dynamic process, and is dependent on the lake's chemical history. Therefore, I include the lagged value of pH in its state equation. With the inclusion of lagged pH I capture the entire history of the effects of acid deposition on pH. Any measured influence of acid deposition in the current period is therefore conditioned on this history, and will represent only the effect of new information [8]. The next term in the state equation for pH is the lagged value of acid deposition, which captures the direct effect of acid deposition from the previous period on the current period's pH. I also include an interaction term between the lagged values of alkalinity and depositions. This term captures the non-linearity of pH response to changes in emissions. In particular, I expect the coefficient on depositions to be negative, and the coefficient on the interaction term to be positive so that as alkalinity increases, the effect of further changes in acid depositions on pH goes to zero. Finally, I include an error term with mean zero and variance  $\sigma^2$ . This term accounts for the stochastic factors impacting changes in pH each period. The state equation for pH is therefore given by:

$$P_{t+1} = \beta_0 + \beta_1 P_t + \beta_2 \left( \sum_m c^m E_t^m \right) + \beta_3 L_t \left( \sum_m c^m E_t^m \right) + \epsilon_{t+1} \quad (4)$$

I define the state equation for alkalinity analogously to the state equation for pH. In this case I do not include the last interaction term since the level of pH does not affect how alkalinity responds to changes in

acid deposition. Therefore, the state equation for alkalinity is given by:

$$L_{t+1} = \gamma_0 + \gamma_1 L_t + \gamma_2 \left( \sum_m c^m E_t^m \right) + \nu_{t+1} \quad (5)$$

Combining the above equations, I get the following discrete time optimal control problem:

$$\min_{E_t^j} \sum_{t=0}^{\infty} \delta^t \left( \frac{A}{2} (P_t - \bar{P})^2 + \frac{B}{2} (L_t - \bar{L})^2 + \sum_j \left( \psi_1^j + \psi_2^j (\bar{E}^j - E_t^j) + \psi_3^j (\bar{E}^j - E_t^j)^2 \right) \right) \quad (6)$$

$$\text{s.t.} \quad P_{t+1} = \beta_0 + \beta_1 P_t + \beta_2 \left( \sum_m c^m E_t^m \right) + \beta_3 L_t \left( \sum_m c^m E_t^m \right) \quad (7)$$

$$L_{t+1} = \gamma_0 + \gamma_1 L_t + \gamma_2 \left( \sum_m c^m E_t^m \right) \quad (8)$$

$$E_t^j \geq 0 \quad \forall j \quad (9)$$

where  $\delta$  is the appropriate discount factor.

The discrete time dynamic programming equation corresponding to (6) is:

$$J(P_t, L_t) = \max_{E_t^j} - \left( \frac{A}{2} (P_t - \bar{P})^2 + \frac{B}{2} (L_t - \bar{L})^2 + \sum_j \left( \psi_1^j + \psi_2^j (\bar{E}^j - E_t^j) + \psi_3^j (\bar{E}^j - E_t^j)^2 \right) \right) + \dots \\ \dots + \delta J(P_{t+1}, L_{t+1}) \quad (10)$$

s.t. Equations (7), (8), (9)

For simplicity in notation, from this point forward I express the utility function as  $U(P_t, L_t)$ , the cost function as  $C(E_t^j)$ , the state equation for pH as  $f(P_t, L_t, E_t^m)$ , and the state equation for alkalinity as  $g(P_t, L_t, E_t^m)$ . With this formulation of the problem, I cannot exclude the possibility that the non-negativity constraint on emissions will be binding in certain states. Therefore, along the optimal solution path, emissions in each period must be chosen so that the following Euler equilibrium conditions are satisfied:

$$-C_{E_t^j}(t) + \delta [J_P(t+1)f_{E^j}(t) + J_L(t+1)g_{E^j}(t)] = \mu_t^j \quad (11)$$

$$J_P(t) = -U_P(t) + \delta [J_P(t+1)f_P(t)] \quad (12)$$

$$J_L(t) = -U_L(t) + \delta [J_P(t+1)f_L(t) + J_L(t+1)g_L(t)] \quad (13)$$

$$E_t^j \geq 0, \quad \mu_t^j \geq 0, \quad E_t^j \cdot \mu_t^j = 0 \quad (14)$$

where  $\mu_t^j$  measures the current and expected future reward from a marginal decrease in emissions by firm  $j$  in period  $t$  [18]. The conditions along the optimal path require that in every period, each local firm

reduces its emissions until either the long-run marginal reward from further decreasing emissions, or emissions themselves, are zero. In addition to satisfying the above Euler conditions, the steady state to the problem must also satisfy the following state stationarity conditions:

$$P^* = f(P^*, L^*, E^{j*}) \quad (15)$$

$$L^* = g(P^*, L^*, E^{j*}) \quad (16)$$

So in the steady state, pH, alkalinity and regulated emissions are all constant from one period to the next.

## 4 Data Summary

The data used in this paper are from the Sudbury Environmental Study (SES) Extensive Monitoring Programme. The SES Programme is conducted by the Ontario Ministry of the Environment, through the Freshwater Ecology Unit at Laurentian University in Sudbury, Ontario. It began as a chemistry survey of 209 lakes from 1974–1976. This survey revealed significant acidification, and loss and depression of fish populations in a 5,300 km<sup>2</sup> area around Sudbury, which included 650 km<sup>2</sup> of lake surface area [4]. A second chemistry survey of 250 lakes was subsequently conducted from 1981–1983. In 1983, 44 lakes which had an observed pH of less than 5.5 in at least one of the previous surveys were chosen for continued monitoring. These lakes have been sampled once per year, during the summer stratified period, from 1981–2006. This paper uses annual observations of pH and alkalinity recorded from 1981–2004 for 43 of these lakes.<sup>9</sup> Summary statistics for these data are provided in Table 1, and Figure 2 shows the location of the lakes in relation to Sudbury.<sup>10</sup>

Lake water samples are taken either from a location near the lake centre, or near the centre of a main basin on a very large lake. From 1981 to 1994 lake water samples were collected as non-volume weighted tygon tube composites through the two, upper stratified layers of the lake. If the lake was too shallow for thermal stratification then the sample was collected to 1 metre above the lake bottom. Beginning in 1995,

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<sup>9</sup>For the majority of years in the sample, the laboratory value of alkalinity is reported as the total inflection point. The total inflection point is routinely measured by titration of the water with strong acid or base until the inflection point is reached. At this point, the acid neutralizing capacity of the water is zero. A positive alkalinity indicates a net strong base in the water, and a negative alkalinity indicates a net strong acid. The exception to measuring by total inflection point is 1995 and 1996 where the laboratory value of alkalinity is the fixed-endpoint alkalinity value. For these years, I use inflection point alkalinity values calculated by the Freshwater Ecology Unit, and reported in their 2006 data report [14].

<sup>10</sup>The map of the location of lakes is provided by the Freshwater Ecology Unit. Note that Whitson Lake, immediately North-East of Sudbury, is the 44<sup>th</sup> lake in the SES study, and the only one for which I do not have data.



Figure 2: Location of SES Study Lakes in relation to Sudbury. The area within the dotted lines identifies the zone of lakes affected by Sudbury emissions.

sampling methods changed to the use of a four-litre plastic jug immersed by hand to completely below the lake surface. In its 2006 data report [14], the Freshwater Ecology Unit conducts an analysis of the difference between water samples collected using the collected tube composite and surface grab sampling methods.<sup>11</sup> Of 22 chemical variables, they find significant differences in results for 8 of the variables, including pH, which is significantly lower in tube composite samples. Since I am interested in determining how emissions reductions influence the increase of pH over time, I recognize this change in sampling methods may cause an upwards bias in my results.<sup>12</sup>

Due to outliers in the measured chemical values, and some years in which alkalinity and pH were not measured, there are 13 lakes for which one or both of the observations on pH and alkalinity are missing for

<sup>11</sup>The analysis of water samples is conducted using data from 15 Ontario Ministry of Environment long-term monitoring lakes in Northeastern Ontario where the two collection methods were simultaneously used on sampling dates in the summer stratified period. Comparisons between results from the mean grab and mean tube composite samples were conducted using paired t-tests.

<sup>12</sup>While the change in sampling methods does create a potential bias, I hope it will be somewhat mitigated by the fact that the majority of emissions reductions are observed prior to 1995. Total Sudbury emissions are reduced by 66.4% between 1981 and 1995, whereas the reduction between 1995 and 2004 is a more modest 14.6%.

Table 1: Summary Statistics

	Mean	Std Dev	Min	Max	N
<b>Water Quality Variables: 1981</b>					
pH	4.87	0.35	4.13	5.76	43
Alkalinity	-0.91	0.89	-4.40	0.46	42
<b>Water Quality Variables: 2004</b>					
pH	5.57	0.52	4.66	6.47	43
Alkalinity	0.29	0.75	-1.06	2.14	43
<b>Environmental Variables: 1981</b>					
Current Regulatory Cap (kt of SO <sub>2</sub> )	1082.23	-	-	-	-
Future Regulatory Cap (kt of SO <sub>2</sub> )	882	-	-	-	-
Total Emissions (kt of SO <sub>2</sub> )	837	-	-	-	-
Lake Site Acid Deposition	21.4	20.6	6.54	104.63	43
<b>Environmental Variables: 2004</b>					
Current Regulatory Cap (kt of SO <sub>2</sub> )	365	-	-	-	-
Future Regulatory Cap (kt of SO <sub>2</sub> )	241	-	-	-	-
Total Emissions (kt of SO <sub>2</sub> )	240	-	-	-	-
Lake Site Acid Deposition	6.11	5.85	1.88	30.0	43
<b>Lake Characteristics</b>					
Distance from Sudbury (km)	59.6	30	8	128	43
Direction from Sudbury (=1 Downwind)	0.58	0.5	0	1	43
Lake Area (hectares)	273.7	305.8	14.54	1316.45	43
Elevation (m)	300.4	73.5	189	486	43
Shoreline Length (km)	19.8	20.8	2.6	89.3	43
Maximum Depth (m)	31.1	17.4	8.0	90.3	40
Mean Depth (m)	9.3	4.8	3.8	24.1	39
Volume (x 10 <sup>4</sup> m <sup>3</sup> )	3125.3	4032.4	83.0	17,621.0	36
Road Access (=1 Access)	0.35	0.48	0	1	43
No. of Observations	1019				

a single year. For each lake I choose to drop these years from the data set, thereby creating an unbalanced panel. Since my estimation equations are dynamic, in addition to losing the observations from the year in which the measurements are not taken, I must also drop from my sample the observations from the year immediately following. This is because for those years I do not have data for the value of the lagged dependent variable in the state equations. The result is that I drop from my sample 25 periods of observations.<sup>13</sup>

As noted previously, the acidification of Sudbury area lakes occurred primarily as a result of sulphur dioxide emissions from smelter operations in the Sudbury area. The goal of the SES Programme is to assess

<sup>13</sup>I drop only 25 observations because for one lake, the data is missing for the first period of observation, which is not included in my estimation for any of the lakes. Therefore, for this lake, I only drop the observation for the following year.

the impacts of emissions reductions from these operations on lake water quality, and to provide ongoing documentation of the recovery of lakes from acidification. Damage from acidification typically begins to occur when the pH of a lake drops below 6.0. However, an observed pH of 5.5 or less was used to identify lakes for inclusion in the SES Programme as this is the approximate threshold at which damage to acid-sensitive sport fish begins to occur [14]. None of the lakes included in the SES Programme were part of the region's Experimental Liming Program [25], so observed improvements in lake water quality over the study period can be strictly attributed to emissions reductions. While not a random sample of all lakes impacted by SO<sub>2</sub> emissions, the data are representative of those lakes that suffered significant biological damage, and which are the primary targets of emissions reductions introduced to aid in the recovery of lakes from acidification.

The two major mining facilities in the Sudbury area are INCO and Falconbridge. Historically, INCO has been the largest point source of SO<sub>2</sub> emissions in North America, with peak emissions in the 1960s of over 2,500 kt of SO<sub>2</sub> per year [24]. While substantially smaller than INCO, Falconbridge is still one of the main pollution sources in Ontario, with peak emissions in the 1960s of around 300 kt per year. The first environmental regulations introducing caps on the emissions of INCO and Falconbridge came into effect in the early 1970s. From 1970–1980, INCO's emissions were reduced by 59% and Falconbridge by 64%. The major program introduced during the study period, however, was the Countdown Acid Rain Program, which began in 1985. Relative to 1980 emission levels, it required both INCO and Falconbridge to achieve a 60% reduction in their emission levels by 1994 [24]. These 1994 emission levels remained the standard through to the end of 2005. Regulation in 2000, 2002 and 2004 set new caps that came into effect in 2006, and provides annual emissions limits for both firms through 2015 ([21],[6],[11]). A summary of the current and future emissions caps faced by INCO and Falconbridge at the start and end of the study period is provided in Table 1.<sup>14</sup>

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<sup>14</sup>My theoretical model assumes the regulator sets an annual emissions cap for each firm and that firms' emissions are exactly equal to this cap. This does not accurately reflect the regulatory environment in Sudbury where INCO and Falconbridge face a single, current emissions cap extending between 2 and 10 years, and in most cases, a future emissions cap which they work towards meeting during this time period. As a result, I often observe actual emissions that are significantly lower than the current cap. To accurately model this scenario emissions from each firm must be defined as a state variable, where the state equation describes observed emissions as a function of the regulator's control variables, the current and future emissions caps. I choose to follow previous literature in assuming emissions of firms are the control variable for the regulator. This is for simplicity in solving the optimal control model, and is also due to the endogeneity problem that arises when estimating observed emissions as a function of the current and future emissions caps.

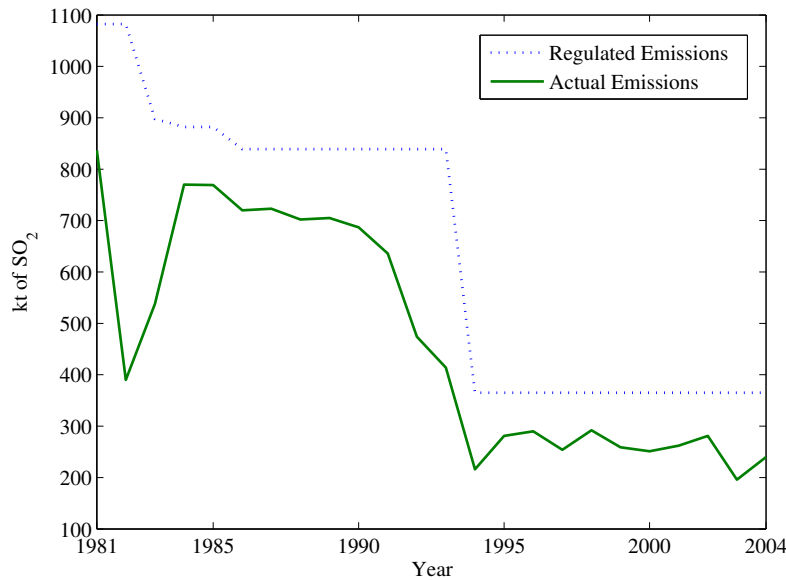


Figure 3: Regulated and Actual SO<sub>2</sub> Emissions for INCO and Falconbridge, 1981 – 2004

Currently I only have data on annual sulphur dioxide emissions for INCO and Falconbridge.<sup>15</sup> These data are available from the Ontario Ministry of the Environment, and were provided by the Freshwater Ecology Unit. While INCO is much larger than Falconbridge, both firms had comparable emissions reductions of approximately 73% between 1981 and 2004. I do not have specific information on the annual acid deposition at each lake site resulting from sulphur dioxide emissions. To estimate depositions, I weight the annual total emissions from INCO and Falconbridge by the inverse of the lake’s distance from Sudbury. Relating this estimation method to the equation for depositions, equation 3, I am assuming the local regulated firms are the only firms with emissions impacting the lakes,  $j = m$ .<sup>16</sup> I also assume  $c^m$  is the same for both my local firms, and is equal to the inverse of the distance from each lake to Sudbury. The distance measurement is provided by the Freshwater Ecology Unit. Summary statistics for actual sulphur dioxide emissions, and for the estimates of depositions at each lake site, are provided in Table 1. Figure 3 provides a comparison

<sup>15</sup>I have data on SO<sub>2</sub> emissions from other major emitting firms in Ontario for 1985, 1990, 1995 and then 2001 forward only. On average, Inco and Falconbridge are responsible for over 55% of emissions in the province. In addition, other major emitting firms are between 260 and 430 kilometres away from Sudbury. This suggests Inco and Falconbridge are the two dominant sources of acid depositions for lakes in the Sudbury area.

<sup>16</sup>In addition to the other major emitting firms in Ontario, there are also firms in Michigan and the Ohio Valley with SO<sub>2</sub> emissions that will have some impact on the lakes around Sudbury. As a result of a number cross-border agreements for the reduction of acid depositions, many of these firms will have undertaken similar emissions reductions to INCO and Falconbridge during my study period. The exclusion of these firms’ emissions from my dataset will therefore result in an upwards bias in my estimation of the impact of a decrease in Inco and Falconbridge emissions on pH.

between actual sulphur dioxide emissions and their regulated amount.<sup>17</sup>

Data on time-invariant physical characteristics of the lakes are also provided by the Freshwater Ecology Unit. Characteristics available for all lakes are the direction of the lake from Sudbury (upwind or downwind), lake area, elevation, shoreline length, and whether there is road access. For a subset of lakes, data is also available for mean depth, maximum depth, and volume. The lake area and shoreline length are calculated by the Freshwater Ecology Unit using the mapping software MapInfo, while the other data are collected from a variety of available sources [14]. Summary statistics for data describing the physical characteristics of the lakes are provided in Table 1.

## 5 Estimation & Results

My objective is to estimate the parameters of the state equations for pH and alkalinity. I begin by assuming the error components of each state equation are made up of two terms;  $\epsilon_{it}$  and  $\nu_{it}$ , which are iid over  $i$  and  $t$ , and  $\alpha_i$  and  $\eta_i$ , which are random variables that capture unobserved heterogeneity among the lakes. I further assume strict exogeneity of the error terms,  $E[\epsilon_{it}|\alpha_i, \mathbf{x}_{i1}, \dots, \mathbf{x}_{iT}] = 0$ ,  $E[\nu_{it}|\eta_i, \mathbf{y}_{i1}, \dots, \mathbf{y}_{iT}] = 0$  where I let  $\mathbf{x}_{it}$  and  $\mathbf{y}_{it}$  be the vectors of right hand side variables in the state equations for pH and alkalinity respectively. I expect the random variables,  $\alpha_i$  and  $\eta_i$ , will be correlated with the observed regressors  $\mathbf{x}_{it}$  and  $\mathbf{y}_{it}$ , particularly the lagged values of the dependent variables in each state equation. This is because certain time invariant characteristics of the lake, such as the types of soil or vegetation found on surrounding land, may impact both the natural state of the lake, and the rate at which it recovers from acidification. Therefore, I use the least squares dummy variable estimator to estimate a fixed effects model of the following form:

$$P_{it} = \alpha_i + \beta_0 + \beta_1 P_{i,t-1} + \beta_2 D_{i,t-1} + \beta_3 L_{i,t-1} D_{i,t-1} + \epsilon_{it} \quad (17)$$

$$L_{it} = \eta_i + \gamma_0 + \gamma_1 L_{i,t-1} + \gamma_2 D_{i,t-1} + \nu_{it} \quad (18)$$

Results from the estimation of equations 17 and 18, using the fixed effects model are provided in column

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<sup>17</sup>The downwards spike in emissions observed in 1982 is the result of a prolonged shutdown of the INCO and Falconbridge smelters from June 1982 until March 1983. This was the combined result of a labour dispute at INCO, high energy prices and low nickel prices.

Table 2: State Equation for pH, Estimation Results

	(1) OLS	(2) OLS	(3) OLS	(4) OLS	(5) OLS	(6) OLS
$P_{i,t-1}$	0.6840 (12.64)***	0.6806 (12.71)***	0.6451 (11.02)***	0.6984 (12.79)***	0.6945 (12.87)***	0.6368 (11.01)***
$D_{i,t-1}$	-0.0080 (4.92)***	-0.0126 (5.17)***	-0.0249 (3.69)***	-0.0737 (2.65)**	-0.1212 (3.08)***	-0.6953 (3.69)***
$D_{i,t-1} \cdot L_{i,t-1}$	-3.93E-04 (2.10)*	-4.0E-04 (2.27)**	-7.42E-05 -0.36	-0.0062 -1.53	-0.0065 (1.80)*	0.0034 0.90
$D_{i,t-1} \cdot \text{Precipitation}_{i,t-1}$		5.13E-06 (3.79)***			5.19E-05 (3.28)***	
$D_{i,t-1} \cdot \text{Lake Area}_i$			-2.37E-05 -1.58			-0.0013 (2.74)**
$D_{i,t-1} \cdot \text{Direction}_i$			-0.0085 (2.58)**			-0.3571 (3.44)***
$D_{i,t-1} \cdot \text{Elevation}_i$			8.84E-05 (3.90)***			0.0029 (3.48)***
$D_{i,t-1} \cdot \text{Shoreline Length}_i$			2.89E-04 1.26			0.0133 (1.77)*
$D_{i,t-1} \cdot \text{Road Access}_i$			-0.0063 (2.23)**			-0.1442 (1.91)*
Constant	1.7644 (5.92)***	1.7885 (6.10)***	1.9565 (5.88)***	1.6536 (5.53)***	1.6784 (5.70)***	2.0142 (6.59)***
Year Dummies	Yes	Yes	Yes	Yes	Yes	Yes
Emissions Weighting Factor	$\frac{1}{\text{Distance}_i}$	$\frac{1}{\text{Distance}_i}$	$\frac{1}{\text{Distance}_i}$	$\frac{1}{\text{Distance}_i}$	$\frac{1}{\text{Distance}_i}$	$\frac{1}{\text{Distance}_i}$
Observations	964	964	964	964	964	964
Number of Lakes	43	43	43	43	43	43
R-squared	0.85	0.85	0.85	0.85	0.85	0.85

Robust t statistics in parentheses, standard errors are clustered by watershed

\* significant at 10%; \*\* significant at 5%; \*\*\* significant at 1%

(1) of Tables 2 and 3 respectively.<sup>18</sup> The results from the estimation of both equations are mostly as expected. The coefficients on the lagged dependent variables are positive, significant and less than 1, indicating the state equations for pH and alkalinity are stable, and there is a general trend of improving water quality over time. The coefficients on depositions are negative and significant, indicating that current values of pH and alkalinity are increasing as lagged depositions decrease. Finally, the coefficient on the interaction between depositions and alkalinity in the state equation for pH is negative and significant, although only at the 10% level. The low level of significance suggests the interactive relationship between pH, alkalinity and

<sup>18</sup>Given the lagged dependent variable in both state equations, using the least squares dummy variable estimator leads to a violation of the strict exogeneity assumption. The typical correction for this is to use the Arellano-Bond estimator, which is an IV variant of the first differences estimator. Judson and Owen [12] show that the Arellano-Bond estimator should be used when  $T = 20$ , but when  $T = 30$  the least squares dummy variable estimator performs just as well or better than the alternatives that correct for endogeneity. In my sample  $T = 24$ , suggesting it would be best to use the Arellano-Bond estimator. However, as this is a first differences estimator it eliminates the constant from the estimation. With this correction, I therefore cannot solve the optimal control problem as my theoretical model relies on the constant in order to achieve increasing pH and alkalinity over time. As solving the control problem is my main objective I instead use the least squares dummy variable estimator, recognizing this introduces a bias to my results. However, the bias should be small relative to that from a shorter panel. In addition, the SES Programme is ongoing and I will continue to obtain data on lake recovery so that with another few years of observations I will be close to the recommended  $T = 30$  at which the least squares dummy variable estimator is preferred to the Arellano-Bond.

Table 3: State Equation for Alkalinity, Estimation Results

	(1) OLS	(2) OLS	(3) OLS	(4) OLS	(5) OLS	(6) OLS
$L_{i,t-1}$	0.5037 (5.15)***	0.4936 (5.12)***	0.4953 (4.81)***	0.4885 (5.09)***	0.4756 (5.16)***	0.4766 (4.80)***
$D_{i,t-1}$	-0.0208 (7.06)***	-0.0480 (6.82)***	-0.0485 (1.89)*	-0.1831 (5.76)***	-0.4385 (7.54)***	-0.8778 (2.21)**
$D_{i,t-1} \cdot \text{Precipitation}_{i,t-1}$		3.00E-05 (6.16)***			2.85E-04 (8.66)***	
$D_{i,t-1} \cdot \text{Lake Area}_i$			-3.86E-05 -0.71			-5.62E-04 -0.69
$D_{i,t-1} \cdot \text{Direction}_i$			-0.0091 -0.56			0.0119 0.02
$D_{i,t-1} \cdot \text{Elevation}_i$			1.28E-04 1.43			0.0031 (2.09)*
$D_{i,t-1} \cdot \text{Shoreline Length}_i$			5.60E-04 0.68			0.0032 0.27
$D_{i,t-1} \cdot \text{Road Access}_i$			-0.0070 -1.32			-0.1560 -1.70
Constant	0.1428 (2.21)**	0.3522 (4.51)***	0.1034 0.57	-0.1271 (1.90)*	-0.1334 (2.13)**	-0.1271 -1.46
Year Dummies	Yes	Yes	Yes	Yes	Yes	Yes
Emissions Weighting Factor	$\frac{1}{\text{Distance}_i}$	$\frac{1}{\text{Distance}_i}$	$\frac{1}{\text{Distance}_i}$	$\frac{1}{\text{Distance}_i}$	$\frac{1}{\text{Distance}_i}$	$\frac{1}{\text{Distance}_i}$
Observations	964	964	964	964	964	964
Number of Lakes	43	43	43	43	43	43
R-squared	0.76	0.76	0.76	0.76	0.77	0.76

Robust t statistics in parentheses, standard errors are clustered by watershed  
 \* significant at 10%; \*\* significant at 5%; \*\*\* significant at 1%

depositions is not well defined in my data. A contributing factor to this is likely that the recovery of lakes from acidification will often take much longer than 24 years, and the data captures only an interim period of recovery. In addition, the lakes are surveyed relatively early in the recovery process, during which time alkalinity will generally be more responsive than pH to changes in depositions. In this case, pH only starts responding to changes in deposition after alkalinity has already been increasing. This is consistent with the negative sign on the interaction between depositions and alkalinity.

When previously deriving my theoretical model, I choose to ignore certain variable factors, such as precipitation, in order to simplify the estimates of acid deposition, and thereby the solution to the optimal control model. I now check the impact of this simplifying assumption by adding to both regression equations an interaction term for lagged precipitation and depositions. I use data on annual precipitation which includes total rain and snowfall. Due to the remoteness of many of the lakes, I only have data from two weather stations, one which is representative of the lakes located South (upwind) of Sudbury, and a second that is representative of the lakes located North (downwind) of Sudbury. For each lake, I interact the lagged annual

precipitation from the appropriate monitoring site with the estimate of lagged depositions at the lake. The results from estimating my regressions with this added term are given in column (2) of Tables 2 and 3. In the estimation of both state equations, the direct effect of precipitation on the impact of depositions is negligible, with the coefficient on the interaction term virtually equal to zero. The inclusion of the additional term, however, does alter the previous results. In the state equation for pH, the coefficient on lagged pH slightly decreases, while the coefficient on depositions decreases by just over 50%. I observe the same results in the state equation for alkalinity, although the effects are larger, with the coefficient on depositions decreasing by more than 100% with the inclusion of the precipitation term.

Similar to lakes that are recovering from eutrophication, a lake that is recovering from acidification may be either reversible, hysteretic, or irreversible. In addition, some lakes may be acidified in their natural state, in which case I would not expect to observe a significant recovery over time. I do not directly observe which of the study lakes fall into each of these categories, however, there are large differences in the recovery rates of lakes. The pH of the most improved lake increases by 1.97 between 1981 and 2004, while the pH of the least improved increases by only 0.14. To better understand these differences, and the impact of specific lake characteristics on water quality recovery, I add to both regressions interaction terms for lagged depositions and a set of time-invariant lake characteristics.<sup>19</sup> In the results presented in column (3) of Tables 2 and 3, I include characteristics available for all lakes – lake area, direction from Sudbury, elevation, shoreline length, and whether there is road access. In the state equation for alkalinity, addition of these variables has only a small effect. None are significant, and the impact on estimation of the variables from the original state equation is almost identical to that from adding the precipitation term. I observe more significant results in the state equation for pH. The coefficient on direction from Sudbury is negative and significant, indicating that as expected, a decrease in depositions has a greater effect on the pH of lakes that are down wind of Sudbury. The coefficient on elevation is positive, suggesting lakes at higher elevations are slightly less responsive to changes in depositions than lakes at lower elevations. Finally, the coefficient on road access is negative and significant, indicating a decrease in acid depositions will have a greater effect on lakes that

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<sup>19</sup>With eutrophication of lakes, a significant variable in this context is depth. Often referred to as the shallow lakes problem, shallow lakes are more likely to be either hysteretic or irreversible as they tend to have higher rates of phosphorous recycling, making them unresponsive to phosphorous input controls [2]. I am interested in seeing if there are any similar characteristics of acidified lakes that are significant in determining how and whether they recover from acidification.

are closer to population areas. Inclusion of these extra variables also has a large effect on the coefficients on lagged pH and depositions. Relative to the results with no interaction terms, the coefficient on lagged pH decreases by almost 0.04, and the coefficient on depositions more than triples.

I also estimate the equations using the full set of characteristics available for only a subset of the lakes. However, I do not report these results here as they are not readily interpretable. In the state equation for pH, the only significant variable is the coefficient on lagged pH. In the state equation for alkalinity the coefficient on lagged alkalinity, weighted emissions, and the interaction terms for lake area, volume, mean depth and maximum depth are significant, but many of the coefficients have unexpected signs. I suspect the lack of results in this estimation may be due to the reduced sample size since I use observations for only 36 lakes. To check this, I re-estimate the regressions reported in column (3) of Tables 2 and 3 using only data from the 36 lakes for which all characteristics are available. For each state equation I find the only significant coefficients are the lagged dependent variables. This suggests the smaller sample size is driving the lack of results when I include the additional lake characteristics as dependent variables.

As discussed in my data section, I do not have accurate information on annual acid depositions at each lake site, and therefore do not have a good estimate of  $c^m$ . I use as a rough estimate the inverse of the lake's distance from Sudbury. To see how my results change using an alternative estimate for  $c^m$ , I re-estimate my three fixed effects regressions using the inverse of the lake's squared distance from Sudbury as the estimate for  $c^m$ . The results from these regressions are reported in columns (4) through (6) of Tables 2 and 3, and are qualitatively similar to those previously found. The coefficient on the lagged dependent variable is positive, significant, and less than one in all three specifications. The coefficient on depositions is negative and significant, and its absolute value increases as additional terms are added to the state equations. The coefficient on the interaction term in the state equation for pH remains negative, although is no longer significant. I again find the coefficient on the interaction term of lagged depositions and precipitation is virtually zero and significant. There are some differences, however, in the final regressions which include the interaction terms of lagged depositions and time-invariant lake characteristics. In the state equation for alkalinity the coefficient on elevation is significant, and in the state equation for pH, the coefficients on all the interaction terms are now significant.

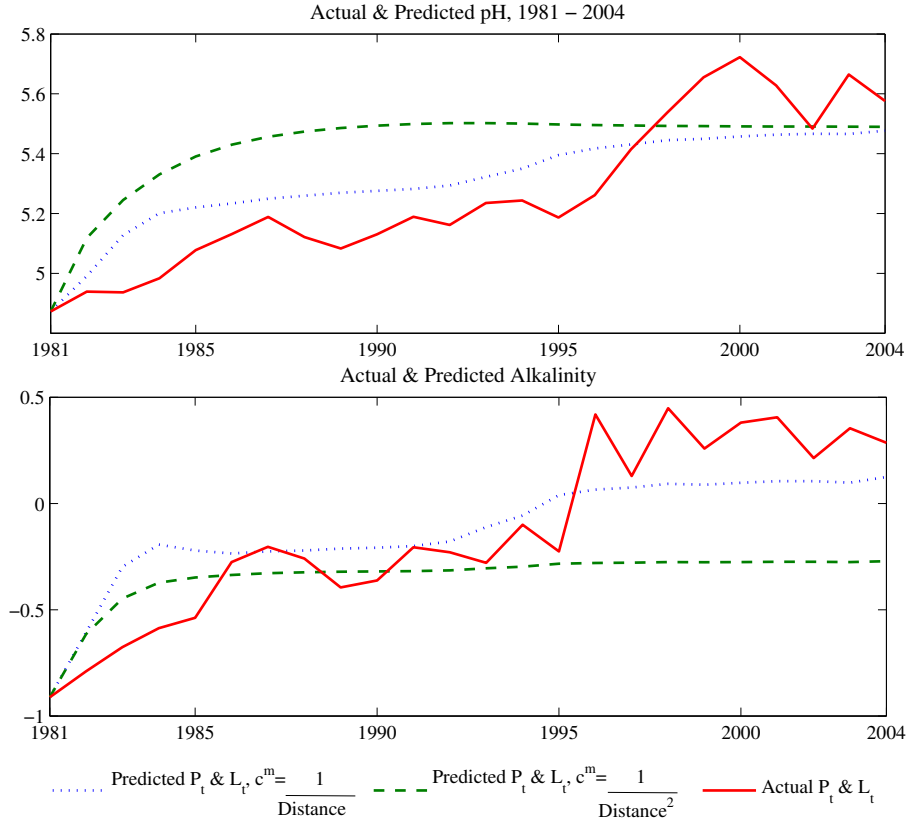


Figure 4: Actual & Predicted Values of pH and Alkalinity, 1981 - 2004

With the alternative estimate of  $c^m$  I do find a large difference in the estimated marginal effect of a decrease in emissions on pH and alkalinity. Using the inverse of the distance from Sudbury as the estimate of  $c^m$ , for a lake that is located 60 km from Sudbury, the original state equation indicates a 100 kt decrease in emissions will increase pH by 0.0127, and alkalinity by 0.0347. With the inverse of the squared distance from Sudbury as the estimate of  $c^m$ , however, the estimate of the increase in pH is only 0.0019, and for alkalinity is 0.0051.<sup>20</sup> On average, for the three specifications of the state equations, defining  $c^m$  as the inverse of distance rather than distance squared increases the estimate of the marginal effect of depositions on improvements in pH and alkalinity by almost seven times.

To determine which of the estimates of  $c^m$  is more accurate I calculate the predicted paths for pH and alkalinity over the period 1981–2004, given observed emissions over this time period and using the parameter estimates from the basic state equations with no interaction terms. I use the average value of pH

<sup>20</sup>The calculation of the marginal effect of a change in depositions on pH is done assuming alkalinity equals -0.9, the average value of alkalinity across all lakes in 1981.

and alkalinity across all 43 lakes in 1981 as the starting value for both paths. The comparison of the predicted paths with the actual average values of pH and alkalinity over the time period are provided in Figure 4. Neither predicted path is able to capture the volatility of the average values of pH and alkalinity over the period. Both are relatively smooth, with larger than expected increases in pH and alkalinity towards the start of the period (likely due to the large decrease in emissions in 1982 as a result of operation shutdowns), and smaller increases towards the end of period as both pH and alkalinity appear to approach steady state values. However, for both pH and alkalinity, the paths generated using inverse distance as the estimate of  $c^m$  are closest to the actual paths. I will therefore use the parameter estimates from these regressions when solving for the optimal path of emissions reductions in the next section.<sup>21</sup>

## 6 Numerical Solution to Optimal Control Problem

In order to solve the optimal control problem I must first parameterize the social welfare function. In past work, this has been done through either formal estimation or simple assumption of the parameter values. I currently do not have any data on abatement costs, or on measures of disutility from an acidified lake. Therefore, I rely primarily on the latter approach. As this is my first time solving the optimal control problem, I also assume for simplicity that there is only one emitting firm. I therefore parameterize only a single abatement cost function, and consider optimal abatement of total Sudbury emissions.

To obtain a rough estimate of the parameters for the abatement cost function, I use values from a Government of Ontario Ministry of the Environment report which provides estimates of the marginal costs of abatement at Inco and Falconbridge under different abatement control strategies [21]. The average annual operating and maintenance cost of the different abatement control strategies is 6.67 million dollars per year, while the average marginal cost of each kilotonne reduction in emissions is 0.362 million dollars.<sup>22</sup>

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<sup>21</sup>I also look at the predicted paths for pH and alkalinity using the state equations with the interaction term for precipitation, and the interaction terms for the time-invariant lake characteristics. In the precipitation equations I use average precipitation from the two weather stations as the measure of precipitation in each year, and for the lake-specific characteristics I use average values across all lakes. In all cases I find the predicted paths for pH and alkalinity from the state equations with no interaction terms and using inverse distance as the estimate of  $c^m$  provide the closest approximation to the actual paths.

<sup>22</sup>The three abatement control strategies I include in the reported averages are a Fluid Bed Roaster Off-Gas Control at Inco, a Continuous Converting Technology at Inco, and Acid Gas Scrubbing at Falconbridge. The annual operating and maintenance costs for these three options range from 3 to 9 millions dollars, while the marginal costs of abatement range from 0.13 to 0.50 million dollars per kt. The report also provides an estimate of the capital cost for each of these technologies which I do not consider here. However, this cost should not affect the optimal emissions decision as it is a sunk cost and is independent of the level of emissions. A fourth abatement control technology for which the report also provides cost estimates are Secondary Capture Hoods at Inco. I do not consider this option because the report notes that only one of the Secondary Capture Hoods and

Table 4: Parameter Estimates for Social Welfare Function

Parameter	Value	Unit of Measurement
$\psi_1$	6.67	million \$
$\psi_2$	0.362	million \$ per SO <sub>2</sub> kt abated
$\psi_3$	0	million \$ per (SO <sub>2</sub> kt abated) <sup>2</sup>
$\bar{E}$	837	kt of SO <sub>2</sub>
$A$	10,000 - 100,000	million \$ per (1 unit deviation from P*) <sup>2</sup>
$P^*$	7.0	Optimal pH
$B$	100 - 1,000	million \$ per (1 unit deviation from L*) <sup>2</sup>
$L^*$	3.0	Optimal Alkalinity

My theoretical model assumes abatement costs are quadratic, however, this particular report only provides information on linear abatement costs. As I do not have any good information on the value of the coefficient on quadratic abatement, I assume it is equal to zero and use a linear abatement cost function for the current numerical solution. The final parameter in the abatement cost function is  $\bar{E}$ , the SO<sub>2</sub> emissions of the firm in the absence of regulation. I assume this value is equal to 837, the total emissions in Sudbury in 1981. The full set of parameter estimates for the abatement cost function is provided in Table 4.

Freshwater lakes in their natural state will typically vary in the biological populations they support, although most will have a pH between 6.0 and 8.0. In parameterizing the disutility function I therefore choose the optimal level of pH to be 7.0, the level at which lake water is neutral. Defining an optimal level of alkalinity is more difficult as there is no set standard for its value, and it tends to be more variable across lakes. Using the assumptions that it is optimal to have some buffering capacity in the lake water, and that no lake in the study has recovered to an optimal alkalinity level (given that no lake has recovered to its optimal pH), I choose the optimal level of alkalinity to be 3.0.

Parameterizing the remainder of the disutility function is more challenging as the acidification of lakes can have numerous effects on welfare, many of which relate to non-use value and are difficult to measure. As I currently do not have any good measures of these welfare effects I will solve the optimal control problem using a range of values for the coefficients on the deviation of pH and alkalinity from their optimal values. I choose the range of coefficients on pH to be larger than the range of coefficients on alkalinity for two reasons.

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the Continuous Converting Technology may be implemented as an abatement control strategy, and the costs of the Secondary Capture Hoods are strictly larger than those for the Continuous Converting Technology.

First is that alkalinity has a much wider range of values that it can reasonably take. A single unit deviation from its optimal value is therefore much less consequential than a single unit deviation from the optimal value of pH, which represents a 10-fold increase in acidity of the lake water. Second is that low levels of alkalinity do not directly affect fish populations and other ecosystems in the lake. Therefore, deviations in alkalinity from its optimal value should have only a secondary effect on decreases in welfare from lake acidification.

A second challenge in parameterizing the disutility function is that I formulate my control problem for the recovery of a representative lake. Actual emissions reductions, however, will have an impact on thousands of heterogeneous lakes, and the optimal emissions path must reflect this. Introduction of individual state equations to separately describe the changes in water quality for any significant number of lakes will make the solution of the control problem intractable. Instead I follow previous literature ([13],[22],[23]), and assume that all lakes in the area are homogeneous. To reflect this assumption I inflate the parameters of the disutility function so they are representative of the disutility from widespread acidification. The final range of parameter values I use are reported in Table 4. I solve the problem at intervals of 10,000 for the coefficient on pH, and at intervals of 100 for the coefficient on alkalinity.

I solve the optimal control problem using the discrete time, continuous state dynamic programming toolbox from Miranda & Fackler [18]. I specify the stochastic component in the state equations for pH and alkalinity as a nine-node discretization of the bivariate normal shock with mean zero and three standard univariate nodes in each direction. Using the solution to the optimal control problem, and starting from the average values of pH and alkalinity in 1981, I simulate 5,000 optimal paths for pH, alkalinity and emissions over the time period of my data (1981 – 2004). The mean paths from these simulations, for the two lower and upper values of the parameters  $A$  and  $B$ , are shown in Figure 5.<sup>23</sup>

The numerical solution to the optimal control problem differs from expected results in a number of respects. First, for the lowest values of  $A$  and  $B$  there is no reduction in optimal emissions below observed levels in 1981. As  $A$  and  $B$  increase the level of optimal emissions does fall, but even at their highest values the optimal level of emissions is well above the observed level. Second, in cases where there is a reduction

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<sup>23</sup>The simulated paths for all other combinations of parameter values for  $A$  and  $B$  fall in between the two paths given in Figure 5. Virtually all the variation in the paths, however, is a result of changes in the value of  $A$ . Holding  $A$  constant and varying only  $B$  causes only minimal changes in the paths for pH, alkalinity, and optimal emissions.

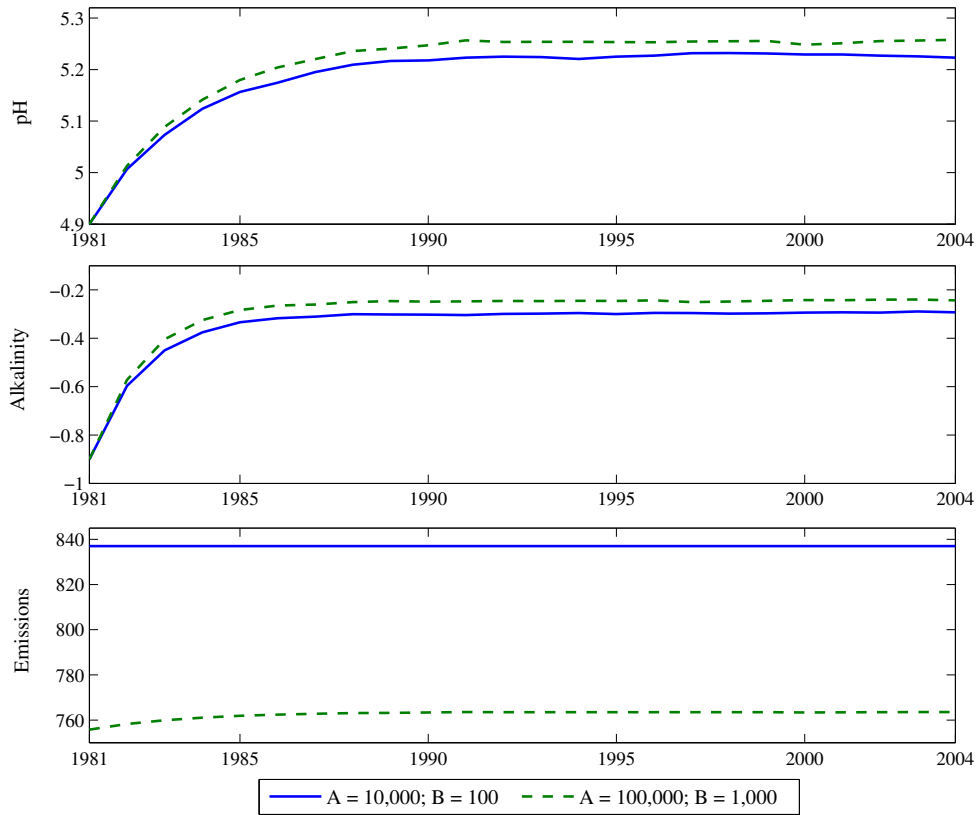


Figure 5: Simulated Paths for pH, Alkalinity and Optimal Emissions

in optimal emissions, the optimal emissions path is always increasing over time. Intuitively this result make sense as it recommends that firms incur large abatement costs up front in order to achieve faster recovery. Once pH and alkalinity start to increase firms can then begin to increase emissions to a level that will maintain the recovery path. In actuality, however, it is not reasonable to regulate firms in this fashion as it would require costly, short-term over investments in abatement capital. Finally, even for the emissions paths that are increasing, there is never a particularly large change in the optimal level of emissions over time. The maximum change, shown in Figure 5, is an increase of less than 10 kt over the 24-year period. This is in stark contrast to actual emissions, which decrease by almost 600 kt. The lack of similarity between the numerical solution and actual emissions point to a number of limitations in my empirical and theoretical model, which I will discuss in the final section.

## 7 Conclusions & Future Work

The acidification of freshwater lakes continues to be a significant environmental problem in many areas of the world. The problem of how to optimally implement emissions reductions that will lead to their recovery therefore remains a relevant policy question. I develop a framework for an optimal control problem that seeks to answer this question. I define a social welfare function that is maximized by a social regulator who must balance the cost of imposing emissions reductions on firms against the social disutility resulting from the existence of acidified lakes that are unable to support aquatic life. The change in water quality over time is described by two state equations which measure how the pH and alkalinity of the lake water respond to emissions reductions by firms. The parameters of the state equations are estimated using a fixed effects model, and the results are mostly consistent with the theoretical model of lake recovery. When I solve the optimal control problem, however, the results deviate from what I expect. The marginal effect of decreases in emissions on lake recovery is very small and I must assume significantly large damages from lake acidification before it is optimal to implement any emissions reductions. In addition, when found, optimal emissions reductions are substantially smaller than observed reductions over the period, and the path of emissions is increasing, rather than decreasing, over time.

These deviations from the expected results point to a number of shortcomings in my current theoretical model. First is that the marginal effect of emissions reductions on the pH and alkalinity of lake water is very small, making it difficult for even the largest reductions in emissions to generate any significant change in the recovery process. This stems from the result that in the estimated state equations, the recovery paths for pH and alkalinity are generated mostly by the constant term and the coefficients on the lagged dependent variables. Varying depositions will have only a small change on this path. In some respects this is a relatively accurate characterization – reductions in emissions do not have immediate, significant effects on the pH and alkalinity of lake water. This is due in part to the lag between reductions in emissions at the source and the generation of acid depositions from the catchment, for which I currently do not have a good measure. However, what the equations do not capture is that the recovery paths being generated by the constant and lagged dependent variable are a result of emissions reductions of approximately 1700 kt of  $\text{SO}_2$  between the late 1960s and the start of my data collection period.  $\text{SO}_2$  emissions data from Inco and Falconbridge

are available from 1960 forward so one option is to include several lagged deposition estimates in the state transition equations. However, without data on lake water quality from earlier than 1980, particularly during the period when emissions were high and pH and alkalinity were both declining, I don't know how well this will address the problem.

Also potentially contributing to the small marginal effect of emissions reductions on lake recovery is that I do not have a good estimate of sulphur depositions at each lake site. I am considering a couple of options to improve this estimate. First is to use data from SO<sub>2</sub> air monitoring stations, and to look at how ambient air concentrations of SO<sub>2</sub> at various distances from the smelters change in response to emissions reductions. A second option is to use lake sulphate concentrations in place of an estimate of depositions in the state transition functions. I have done some preliminary regressions in this case, and find that the recovery path of pH and alkalinity are more sensitive to changes in sulphate concentrations. In addition, the simulated recovery paths for both pH and alkalinity, using observed emissions, converge to higher steady state values which in the final period are closer to observed values of pH and alkalinity. In this case, however, I still need to link source emissions with the level of sulphates in each lake. I am looking at estimating a spline function for this, with the kinks in the function chosen based on the distance of lakes from the emissions source. A final option is to calibrate the state equations to match simulated scientific models of lake recovery, and not to use the SES Programme data. The benefit of this approach is that it provides data on sulphate depositions, pH and alkalinity over a period of increasing emissions and then into a full period of recovery. In addition, it does not involve the annual volatility and measurement challenges inherent to using real lake data. The downside, however, is that I currently only have graphical simulation results for three lakes, and only one which includes the simulations of acid depositions. I have not yet found a source for the simulation data, and without this information I likely cannot obtain an accurate calibration.

The second problem with the theoretical model is that it focuses only on lake acidification, and when considering the level of optimal emissions reductions, it does not account for the other benefits associated with these reductions. In my current solution to the optimal control model the emissions reductions are very small, and I must assume a seemingly unrealistic level of damages (greater than \$10 billion per one unit deviation of pH from its optimal level) before any reduction in emissions is optimal. In addition, even at the

highest level of damages optimal emissions never fall below 754 kt. Observed emissions, on the other hand, fall from 837 kt in 1981 to 240 kt in 2004. It is difficult to argue that observed emissions reductions were over seven times larger than optimal. Rather, the likely reason for the observed reductions of this magnitude is that regulations are taking into account damages of SO<sub>2</sub> emissions in other areas including mortality and morbidity effects, corrosion of materials, soiling of property, and deterioration of landscapes [20]. In many of these areas the marginal effects of emissions reductions will not be as small as those for lake recovery, and large reductions therefore become optimal. I am currently trying to determine how, and whether I can introduce these other damages into a model where my main interest is lake recovery.

The final problem with the model is that it does not accurately reflect the emissions reduction costs of firms, nor the paths they will take to achieve these reductions. The model currently assumes that the marginal cost of abating each unit of emissions is constant across all units. A more likely scenario, however, is that as firms invest in different abatement technologies the marginal cost for new units of abatement will change. For example, Ontario's Ministry of Environment report on abatement [21] discusses two abatement strategies for INCO, one for abatement of up to 90 kt of SO<sub>2</sub> at a cost of \$130/tonne, and a second for abatement of up to 110 kt of SO<sub>2</sub> at a cost of \$456/tonne. A reasonable expectation is that Inco will invest in the first technology for initial abatement, and then introduce the second technology as regulations become stricter, resulting in both an initial investment cost and an increase in the marginal cost of the additional units abated. To capture this effect, I plan to make the level of emissions an additional state variable in my model. The initial level of emissions will be  $\bar{E}$ , and in each period, firms will choose how many additional units of emissions to abate,  $A_t$ . This value may be positive or negative, depending on whether firms choose to decrease or increase emissions relative to the previous period. The state transition function is then  $E_t = E_{t-1} - A_t$ . The abatement cost function will consist of two components; a marginal cost for the current level of abatement,  $(\bar{E} - E_{t-1})$ , and an adjustment cost for the change in abatement in the current period. The total adjustment cost will be increasing in  $A_t$ , and for the first period, will therefore accurately capture the fact that a large initial decrease in emissions is very costly to firms. With this specification of the abatement costs I can also capture the potential irreversibilities of investment in abatement capital by changing the adjustment cost when  $A_t$  is negative. Previous work on the introduction of adjustment costs

to dynamic models has shown they can lead to a number of interesting results including multiple equilibria and cycling towards a steady state [5].

Currently the main contribution of this paper is in the development of a basic model that considers the acidification dynamics of freshwater lakes, and the role of emissions reductions in the recovery of lakes from acidification. As outlined above, however, there are numerous limitations in my theoretical model which I must address before I can obtain a more realistic solution to the optimal control model. While I do not attempt to fully address these problems here, I do provide a reasonable starting framework for further work.

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