

# Fixed coefficients interregional trade in a general equilibrium model with long-haul freight transport

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## Abstract

We explore interregional trade patterns within areas with heterogeneous, by using a simplified regional trade model with three aggregate sectors, two production factors and no-intermediary consumption. We replicate this model on an aggregated Social Accounting Matrix. Heterogeneity of trading economies is captured through different elasticities to consumption and production and differences in the intercept of Cobb-Douglas functionals. The model is based on the assumption that consumers (residents) are not able to distinguish goods by their origin (producing) country and that consumption prices, including the transport rate, are leveled through trade balances. We obtain increasing in transports import demand and decreasing exports (in a context in which transport price is supported by the shipper). We also obtained that the global demand of tradeable goods is an increasing and convex function of the transport cost between regions (i.e countries). We furthermore present two simulations in order to give a

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better intuition on the impact of world prices on the transport rate, which is defined implicitly (we find no closed form for the transport cost). The fixed coefficient model represents a simplifying alternative to the CES functional forms used in general equilibrium and trade, which may be considered analytically inconvenient when the nests (levels) are numerous enough.

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*JEL Codes: R13, F11, F13*

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## **1 Introduction and literature review**

The spatial and regional applied literature states that what drives the location of firms and consumers is the accessibility to spatially dispersed markets, a fact that has been recognized for long time, both in spatial economics and regional science (Fujita and Thisse, 2002). Accessibility is itself measured by all the costs generated by the various types of spatial frictions that economic agents face in the exchange process. In the case of goods and services, such costs are called trade costs. Spulber (2007) refers to them as “the four Ts”. In this paper we only deal with transport costs, but the research may be easily extended to other types of costs.

The paper is structured as following: the first part introduces the generic model which describes in a traditional fashion the microeconomic behavior of one representative closed region. We add the simplifying hypothesis of fixed coefficients according to which exports represent fixed proportions (or fixed coefficients, FC) from the total domestic production, under the hypothesis that there is no differentiation in tastes

between domestic and foreign goods. In other words, Armington elasticities of substitution across goods from the same industry, but traded among different countries, are all equal to unity. In this sense, a working paper by Yilmazkuday (2008) has already find by using empirical data on the U.S. export data, that the elasticity of demand with respect to the destination price is equal to the sum of the elasticity of demand with respect to the source price and the elasticity of demand with respect to the trade costs (i.e transport rate) which anticipates future applications of our FC-assumption. The clearing condition for trade given the production constraints, is the balance of trade. The second part provides comparative statics. The last part concludes with simulations, based on the theoretical results obtained.

## **2 Model framework: two regions, three sectors and no intermediary consumption**

We first consider that there are only two trading partners, therefore, all the assumptions and equations derived are symmetrically applicable to the trade partner. There are three aggregated macro-sectors: agriculture, manufacturing (traded sectors) and one non-traded aggregate (services). There is no intermediary consumption and the exported goods represent a fixed coefficient out of the total production. One representative resident household is assumed to issue the global demand in the economy (a brief definition of what we mean by global demand and global supply is given below). The Social Accounting Matrix (SAM) of an economy reproducing this economy is presented in Figure 1 from the "Appendix: Figures".

Our aim is to derive conditions on price-changes when trade is per-

formed among trade partners. We analyse the trade determinants and its impact on local economies, with a particular focus on transport costs affecting trade. A higher level of complexity is obtained by adding a third trading partner in the model (see the last subsection of this chapter). The model is organised as following: the first subsection defines formally the static demand side of the model, for the representative region (for the trading partner, i.e. region  $r'$ , equations are symmetrically identical); the second subsection defines the supply side; the third subsection presents the balance of trade and the last subsection presents the possibility of introducing the third trading partner in the model.

**Definition 1** *The **global demand** of the economy is exogeneously defined as the sum of the domestic demand and the demand for imports for each and every tradeable good (i.e. the global demand for agriculture, at optimum, is  $XAGR^* = XAGRIMP + XAGRDOM$ ).*

**Definition 2** *The **global supply** of the economy is endogeneously defined as the sum of local sales and exports, for each and every tradeable good (i.e. the global supply for agriculture, at optimum, is  $PRODAGR^* = XAGRDOM + EXPAGR$ ).*

**Definition 3** ***Tradeable goods** are goods which may be transported from one region to another, therefore their price at consumption is formed of the production price (at the world level, since our economies are small and they do not have enough power to impose their own prices on the world market) plus the transport rate, which is characteristic to each industry (in our case,  $T_a$  and  $T_m$ )*

## 2.1 Demand side

In the simplest case that we may consider, the global demand for each good is derived from well-known Cobb-Douglas indifference curves.

**Proposition 4** *Cobb-Douglas optimal demands for the three aggregates (agriculture, manufacturing and services) are given by the following:*

$$\begin{aligned} XAGR^* &= \alpha_a \frac{Inc}{PAGR} \\ XMF^* &= \alpha_m \frac{Inc}{PMF} \\ XSERV^* &= (1 - \alpha_a - \alpha_m) \frac{Inc}{PSERV} \end{aligned}$$

in which *Inc* is the aggregated income of residents,

**Proof.** is provided below. ■

The utility-maximizing agent solves the following program:

$$\begin{aligned} \text{MAX. } \textit{Utility} &= \alpha_a \log(XAGR) + \alpha_m \log(XMF) + (1 - \alpha_a - \alpha_m) \log(XSERV) \\ \text{s.t. } BC &= XAGR \cdot PAGR + XMF \cdot PMF + PSERV \cdot XSERV \end{aligned}$$

The Lagrangian of this maximization is:

$$\mathcal{L} = \textit{Utility} + \lambda \cdot (Inc - BC)$$

which gives the following system of first order conditions:

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial XAGR} &= -\lambda \cdot PAGR + \frac{\alpha_a}{XAGR} = 0 \\ \frac{\partial \mathcal{L}}{\partial XMF} &= -\lambda \cdot PMF + \frac{\alpha_m}{XMF} = 0 \\ \frac{\partial \mathcal{L}}{\partial XSERV} &= -\lambda \cdot PSERV + \frac{1 - \alpha_a - \alpha_m}{XSERV} = 0 \end{aligned}$$

Solving this system of equations and replacing in the budget constraint, one obtains the well-known optimal demand functions:

$$\begin{aligned} XAGR^* &= \alpha_a \frac{Inc}{PAGR} \\ XMF^* &= \alpha_m \frac{Inc}{PMF} \\ XSERV^* &= (1 - \alpha_a - \alpha_m) \frac{Inc}{PSERV} \end{aligned}$$

**Proposition 5** *Domestic demands and demand for imports for both tradeable goods, under the assumption of undifferentiated consumption (uniform prices for domestic and imported goods) are:*

$$\begin{aligned}
XAGRIMP &= \psi_a \alpha_a \frac{Inc}{PAGR} \\
XAGRDOM &= (1 - \psi_a) \alpha_a \frac{Inc}{PAGR} \\
XMFIMP &= \psi_m \alpha_m \frac{Inc}{PMF} \\
XMFDOM &= (1 - \psi_m) \alpha_m \frac{Inc}{PMF} \\
XSERV^* &= (1 - \alpha_a - \alpha_m) \frac{Inc}{PSERV}
\end{aligned}$$

**Proof.** is provided below. ■

In the following we disaggregate the global demand in demand of imported goods and demand of locally produced goods, with the help of a fixed-coefficient assumption. This assumption is based on the fact that individuals may not distinguish between imported and domestic goods at the moment of their consumption. Due to competition (added to open-frontiers trade) among producers and due to non-arbitrage conditions which usually exist among neighbouring trade partners, consumers perceive the same price for the same type of good, for example manufactured goods (either locally produced or imported). The only "distortion" that influences trade is the transport rate. For the time being we ignore other transaction costs, our interest being to derive prices as function of the transport cost.

**Definition 6** *Import of products (import demand) is made according to a fixed coefficient from the global demand of tradeables:*

$$\begin{aligned}
XAGRIMP &= \psi_a \cdot XAGR^* \\
XMFIMP &= \psi_m \cdot XMF^*
\end{aligned}$$

it implies that domestic demands of tradeables are equal to the remaining:

$$XAGRDOM = (1 - \psi_a) \cdot XAGR^*$$

$$XMFDOM = (1 - \psi_m) \cdot XMF^*$$

Values of  $\psi$  may be calibrated from usual bilateral trade databases, for each trading partners (countries, regions, etc.). In other words, we used the well-known form of total demands,  $XAGR^* = XAGRDOM + XAGRIMP$  and  $XMF^* = XMFDOM + XMFIMP$  and split it according to a fixed coefficient rule which we will calibrate with the help of real data.

We apply the definition to the previous optimal demand functions, and obtain the postulated result:

$$\begin{aligned} XAGRIMP &= \psi_a \alpha_a \frac{Inc}{PAGR} \\ XAGRDOM &= (1 - \psi_a) \alpha_a \frac{Inc}{PAGR} \\ XMFIMP &= \psi_m \alpha_m \frac{Inc}{PMF} \\ XMFDOM &= (1 - \psi_m) \alpha_m \frac{Inc}{PMF} \\ XSERV^* &= (1 - \alpha_a - \alpha_m) \frac{Inc}{PSERV} \end{aligned}$$

Remark that services are considered not to be tradeable, they are produced and consumed locally.

## 2.2 Supply side

As for the demand side, we consider the simplest case here, in which production functions are constant-elasticity Cobb-Douglas frontiers.

**Proposition 7** *Optimal production factor demands in the three sectors*

of the economy (agriculture, manufacturing and service) are:

$$\begin{aligned}
LAGR^* &= \beta_a \frac{PPAGR}{w} \\
KAGR^* &= (1 - \beta_a) \frac{PPAGR}{r} \\
LMF^* &= \beta_m \frac{PPMF}{w} \\
KMF^* &= (1 - \beta_m) \frac{PPMF}{r} \\
LSERV^* &= \beta_s \frac{PPSERV}{w} \\
KSERV^* &= (1 - \beta_s) \frac{PPSERV}{r}
\end{aligned}$$

in which  $PP^{**}$  represent production prices,  $r > 0$  is the unit price of capital,  $w > 0$  is the unit price of labor,  $\beta_* \in ]0, 1[$  are production elasticities.

**Proof.** is provided below. ■

Global production functions (containing exports and local consumption) are constant-elasticity Cobb-Douglas curves:

$$\begin{aligned}
PRODAGR &= \log(CTAGR) + \beta_a \cdot \log(LAGR) + (1 - \beta_a) \cdot \log(KAGR) \\
PRODMF &= \log(CTMF) + \beta_m \cdot \log(LMF) + (1 - \beta_m) \cdot \log(KMF) \\
PRODSERV &= \log(CTSERV) + \beta_s \cdot \log(LSERV) + (1 - \beta_s) \cdot \log(KSERV)
\end{aligned}$$

Total revenues in each industry are:

$$\begin{aligned}
REVAGR &= PPAGR \cdot PRODAGR - w \cdot LAGR - r \cdot KAGR \\
REVMF &= PPMF \cdot PRODMF - w \cdot LMF - r \cdot KMF \\
REVSERV &= PPSERV \cdot PRODSERV - w \cdot LSERV - r \cdot KSERV
\end{aligned}$$

in which  $PPAGR$ ,  $PPMF$ , and  $PPSERV$  are production prices for the three goods (they are prices at the factory gate, which do not include transport and trade costs), while  $PRODAGR$  is the agriculture output,  $LAGR$  and  $KAGR$  are labor and respectively, capital demanded by agriculture. Similar are the definitions for the manufacturing sector.

The first order conditions associated to revenue maximizing programs are:

$$\begin{aligned}
\beta_a \frac{PPAGR}{LAGR} - w &= 0 \\
(1 - \beta_a) \frac{PPAGR}{KAGR} - r &= 0 \\
\beta_m \frac{PPMF}{LMF} - w &= 0 \\
(1 - \beta_m) \frac{PPMF}{KMF} - r &= 0 \\
\beta_s \frac{PPSERV}{LSERV} - w &= 0 \\
(1 - \beta_s) \frac{PPSERV}{KSERV} - r &= 0
\end{aligned}$$

First order conditions from the revenue maximization programs together with factor market clearing conditions lead us to the optimal factor demands in the three sectors of the economy:

$$\begin{aligned}
LAGR^* &= \beta_a \frac{PPAGR}{w} \\
KAGR^* &= (1 - \beta_a) \frac{PPAGR}{r} \\
LMF^* &= \beta_m \frac{PPMF}{w} \\
KMF^* &= (1 - \beta_m) \frac{PPMF}{r} \\
LSERV^* &= \beta_s \frac{PPSERV}{w} \\
KSERV^* &= (1 - \beta_s) \frac{PPSERV}{r}
\end{aligned}$$

**Proposition 8** *Optimal factor remunerations at equilibrium, are given by the following:*

$$\begin{aligned}
w^* &= \frac{\beta_a \cdot PPAGR + \beta_m \cdot PPMF + \beta_s \cdot PPSEV}{LSUPTOT} \\
r^* &= \frac{\beta_a \cdot PPAGR + \beta_m \cdot PPMF + \beta_s \cdot PPSEV}{KSUPTOT}
\end{aligned}$$

*in which LSUPTOT is the total supply of labor, and KSUPTOT represents the available capital in the economy.*

**Proof.** is provided below. ■

From the clearing conditions on the factors markets,  $LSUPTOT = LDEMTOT$  and  $KSUPTOT = KDEMTOT$ , we obtain the equilibrium factor prices:

$$\begin{aligned}
LSUPTOT &= LAGR + LMF + LSERV \\
LSUPTOT &= \beta_a \frac{PPAGR}{w} + \beta_m \frac{PPMF}{w} + \beta_s \frac{PPSERV}{w} \\
w^* &= \frac{\beta_a \cdot PPAGR + \beta_m \cdot PPMF + \beta_s \cdot PPSE RV}{LSUPTOT}
\end{aligned}$$

$$\begin{aligned}
KSUPTOT &= KAGR + KMF + KSERV \\
KSUPTOT &= (1 - \beta_a) \frac{PPAGR}{r} + (1 - \beta_m) \frac{PPMF}{r} + (1 - \beta_s) \frac{PPSERV}{r} \\
r^* &= \frac{\beta_a \cdot PPAGR + \beta_m \cdot PPMF + \beta_s \cdot PPSE RV}{KSUPTOT}
\end{aligned}$$

By replacing the factor remunerations in the factor demand functionals, we obtain (in order to simplify notation, we use the following abbreviation:  $LSUPTOT = \overline{L}_s$  and  $KSUPTOT = \overline{K}_s$ ):

$$LAGR^* = \frac{\beta_a \cdot PPAGR}{\beta_a \cdot PPAGR + \beta_m \cdot PPMF + \beta_s \cdot PPSE RV} \overline{L}_s$$

$$KAGR^* = \frac{(1 - \beta_a) \cdot PPAGR}{\beta_a \cdot PPAGR + \beta_m \cdot PPMF + \beta_s \cdot PPSE RV} \overline{K}_s$$

$$LMF^* = \frac{\beta_m \cdot PPMF}{\beta_a \cdot PPAGR + \beta_m \cdot PPMF + \beta_s \cdot PPSE RV} \overline{L}_s$$

$$KMF^* = \frac{(1 - \beta_m) \cdot PPMF}{\beta_a \cdot PPAGR + \beta_m \cdot PPMF + \beta_s \cdot PPSE RV} \overline{K}_s$$

$$LSERV^* = \frac{\beta_s \cdot PPSE RV}{\beta_a \cdot PPAGR + \beta_m \cdot PPMF + \beta_s \cdot PPSE RV} \overline{L}_s$$

$$KSERV^* = \frac{(1 - \beta_s) \cdot PPSE RV}{\beta_a \cdot PPAGR + \beta_m \cdot PPMF + \beta_s \cdot PPSE RV} \overline{K}_s$$

Optimal output levels therefore are (we replace the optimal factor demand functions into the production frontiers):

$$\begin{aligned}
PRODAGR^* &= CTAGR \cdot \left[ \beta_a \frac{PPAGR}{w} \right]^{\beta_a} \left[ (1 - \beta_a) \frac{PPAGR}{r} \right]^{1-\beta_a} \\
PRODMF^* &= CTMF \cdot \left[ \beta_m \frac{PPMF}{w} \right]^{\beta_m} \left[ (1 - \beta_m) \frac{PPMF}{r} \right]^{1-\beta_m} \\
PRODSERV^* &= CTSERV \cdot \left[ \beta_s \frac{PPSERV}{w} \right]^{\beta_s} \left[ (1 - \beta_s) \frac{PPSERV}{r} \right]^{1-\beta_s}
\end{aligned}$$

Export of products may be obtained such as to satisfy the balance of trade and the market clearing condition or, in order to simplify analytical results, we may define exports as a fixed proportion out of the total production. Market clearing conditions for tradeable goods and for services, are such that at optimum, the output value ( $PRODAGR^*$ ) equals demand (domestic demand plus exports, in the case of tradeables):

$$\begin{aligned}
PRODAGR^* &= XAGRDOM + EXPAGR \\
&= (1 - \psi_a) \alpha_a \frac{Inc}{PAGR} + EXPAGR
\end{aligned}$$

$$\begin{aligned}
PRODMF^* &= XMFDOM + EXPMF \\
&= (1 - \psi_m) \alpha_m \frac{Inc}{PMF} + EXPMF
\end{aligned}$$

$$\begin{aligned}
PRODSERV^* &= XSERV^* \\
&= (1 - \alpha_a - \alpha_m) \frac{Inc}{PSERV}
\end{aligned}$$

in which  $PRODAGR^*$ ,  $PRODMF^*$  are optimal levels of production and we used:

$$\begin{aligned}
XAGRIMP &= \psi_a \alpha_a \frac{Inc}{PAGR} \\
XAGRDOM &= (1 - \psi_a) \alpha_a \frac{Inc}{PAGR} \\
XMFIMP &= \psi_m \alpha_m \frac{Inc}{PMF} \\
XMFDOM &= (1 - \psi_m) \alpha_m \frac{Inc}{PMF} \\
XSERV^* &= (1 - \alpha_a - \alpha_m) \frac{Inc}{PSERV}
\end{aligned}$$

We choose to define exports as a fixed proportion out of the total production.

**Definition 9** *We consider that exports, like imports, represent a fixed proportion  $\gamma$  from the total production. This proportion  $\gamma$  may be calibrated from real social accounting matrix or it can be determined numerically from the trade clearing equations (see the next section). Therefore,*

$$\begin{aligned} EXPAGR &= \gamma_a \cdot PRODAGR^* \\ EXPMF &= \gamma_m \cdot PRODMF^* \end{aligned}$$

Under this assumption, export values for the agricultural and manufacturing aggregates are computed as:

$$\begin{aligned} PRODAGR^* &= \frac{1-\psi_a}{1-\gamma_a} \cdot \frac{\alpha_a Inc}{PAGR} \\ \implies EXPAGR &= \gamma_a \frac{1-\psi_a}{1-\gamma_a} \cdot \frac{\alpha_a Inc}{PAGR} \\ PRODMF^* &= \frac{1-\psi_m}{1-\gamma_m} \cdot \frac{\alpha_m Inc}{PMF} \\ \implies EXPMF &= \gamma_m \frac{1-\psi_m}{1-\gamma_m} \cdot \frac{\alpha_m Inc}{PMF} \end{aligned}$$

### 3 The balance of trade and trade theorems

The balance of trade value must be satisfied, therefore:

$$\begin{aligned} BT &= XAGRIMP \cdot PAGR + XMFIMP \cdot PMF \\ &\quad - EXPAGR \cdot PPAGR - EXPMF \cdot PPMF \\ &= \psi_a \alpha_a Inc + \psi_m \alpha_m Inc \\ &\quad - EXPAGR \cdot PPAGR - EXPMF \cdot PPMF. \end{aligned}$$

Under the assumption of a unique exchange rate ( $e = e_{pm} = e_a = e_m = e_{pa}$ ), we transform regional production and consumption prices, in functions of world prices by industry. Additionally, we consider that

transport rates are uniform across industries and that they vary across trading partners. In the case of bilateral trade with a unique trading partner, transport rate is a constant  $T$ :

$$\begin{array}{l}
PPAGR = \quad epa \cdot (PWAGR - T_a) = e \cdot (PWAGR - T_a) \\
PPMF = \quad epm \cdot (PVMF - T_m) = e \cdot (PVMF - T_m) \\
PAGR = \quad ea \cdot (PWAGR + T_a) = e \cdot (PWAGR + T_a) \\
PMF = \quad em \cdot (PVMF + T_m) = e \cdot (PVMF + T_m)
\end{array} \tag{1}$$

We must solve for the transport rate  $t$  the following system of equations:

$$\begin{array}{rcl}
& & BT = 0 \\
XAGRDOM - PRODAGR^* + EXPAGR & = & 0 \\
XMFDOM - PRODMF^* + EXPMF & = & 0
\end{array} \tag{2}$$

in which we know  $EXPAGR = \gamma_a \frac{1-\psi_a}{1-\gamma_a} \cdot \frac{\alpha_a Inc}{PAGR}$  and  $EXPMF = \gamma_m \frac{1-\psi_m}{1-\gamma_m} \cdot \frac{\alpha_m Inc}{PMF}$ . Replacing the prices along with the formulas for export values in the balance of trade, we obtain:

$$\begin{aligned}
BT = & \quad Inc \cdot (\psi_a \alpha_a + \psi_m \alpha_m) - EXPAGR (PWAGR - T_a) e \\
& \quad - EXPMF (PVMF - T_m) e
\end{aligned} \tag{3}$$

$$\begin{aligned}
\frac{BT}{Inc} = & \quad \psi_a \alpha_a + \psi_m \alpha_m - \gamma_a \alpha_a \frac{1-\psi_a}{1-\gamma_a} \frac{PWAGR - T_a}{PWAGR + T_a} - \\
& \quad - \gamma_m \alpha_m \frac{1-\psi_m}{1-\gamma_m} \frac{PVMF - T_m}{PVMF + T_m} = 0.
\end{aligned}$$

**Theorem 10** *Under the fixed-coefficient assumptions, under the framework of transport cost being paid by the producer, imports of goods INCREASE with the transport cost  $T_* \geq 0$  while exports DECREASE with  $T_*$ . Transports have a compensatory effect, on one hand when  $T$ 's are relevant they encourage imports in order to satisfy global demand, while*

they protect local producers from paying too much on exports. Vice-versa when  $T$ 's are small enough.

**Proof.** (we illustrate the result for the transport cost associated to agricultural goods,  $T_a$ ). The proof is straightforward. From  $XAGRIMP = \psi_a \alpha_a \frac{Inc}{PAGR} = \psi_a \alpha_a \frac{Inc}{e^{(PWAGR - T_a)}}$  we compute the derivative with respect to  $T_a$  :

$$\frac{dXAGRIMP}{dT_a} = \psi_a \alpha_a e \frac{Inc}{(PWAGR - T_a)^2} \geq 0$$

since  $\psi_a, \alpha_a, e \in ]0, 1[$ ,  $Inc \geq 0$ .

From  $EXPAGR = \gamma_a \frac{1 - \psi_a}{1 - \gamma_a} \cdot \frac{\alpha_a Inc}{PAGR} = \gamma_a \frac{1 - \psi_a}{1 - \gamma_a} \cdot \frac{\alpha_a Inc}{e^{(PWAGR + T_a)}}$  we compute derivatives:

$$\frac{dEXPAGR}{dT_a} = -\gamma_a \frac{1 - \psi_a}{1 - \gamma_a} \cdot e \frac{\alpha_a Inc}{(PWAGR + T_a)^2} \leq 0$$

since  $\gamma_a, \alpha_a \in ]0, 1[$ .

Q.E.D. ■

From the market closure conditions on tradeables, we obtain the optimal import demands:

$$\begin{aligned} XAGRIMP &= \psi_a \alpha_a \frac{Inc}{PAGR} \\ &= \psi_a \alpha_a \frac{Inc}{e^{(PWAGR + T_a)}} \end{aligned} \tag{4}$$

and in a similar way we obtain:

$$\begin{aligned} XMFIMP &= \psi_m \alpha_m \frac{Inc}{PMF} \\ &= \psi_m \alpha_m \frac{Inc}{e^{(PWMF + T_m)}} \end{aligned} \tag{5}$$

These formulas automatically lead us to the domestic local demands as functions of world prices:  $XAGRDOM = (1 - \psi_a) XAGRIMP$  and  $XMFDOM = (1 - \psi_m) XMFIMP$ .

**Theorem 11** *The demand for imports of tradeable goods, agriculture and manufacturing, are decreasing and convex functions of the transport cost  $T_*$ .*

**Proof.** We use results in equations (4) and (5). We define the vector of imports of tradeables,

$$XIMPDOM = (XAGRIMP, XMFIMP, XAGRDOM, XMFDOM)$$

The Jacobian of this vector is:

$$\begin{aligned} D_T XIMPDOM &= \begin{pmatrix} \frac{dXAGRIMP}{dT_a} & \frac{dXMFIMP}{dT_m} \\ \frac{dXAGRDOM}{dT_a} & \frac{dXMFDOM}{dT_m} \end{pmatrix} \\ &= \begin{pmatrix} -\frac{\psi_a \alpha_a Inc}{e[PWAGR+T_a]^2} & -\frac{\psi_m \alpha_m Inc}{e[PWMF+T_m]^2} \\ -\frac{\psi_a(1-\psi_a)\alpha_a Inc}{e[PWAGR+T_a]^2} & -\frac{\psi_m(1-\psi_m)\alpha_m Inc}{e[PWMF+T_m]^2} \end{pmatrix} \end{aligned}$$

which is negatively semidefined  $D_T XIMPDOM \leq 0$  since  $\psi_a, \alpha_a, \psi_m$  and  $\alpha_m \in ]0, 1[$ ,  $Inc > 0$  and  $e > 0$ . The Hessian of vector  $XIMPDOM$  is:

$$\begin{aligned} D_T^2 XIMPDOM &= \begin{pmatrix} \frac{d^2XAGRIMP}{dT_a^2} & \frac{d^2XMFIMP}{dT_m^2} \\ \frac{d^2XAGRDOM}{dT_a^2} & \frac{d^2XMFDOM}{dT_m^2} \end{pmatrix} \\ &= \begin{pmatrix} \frac{2e(\psi_a \alpha_a Inc)}{[PWAGR+T_a]^3} & \frac{2e(\psi_m \alpha_m Inc)}{[PWMF+T_m]^3} \\ \frac{2e(1-\psi_a)(\psi_a \alpha_a Inc)}{[PWAGR+T_a]^3} & \frac{2e(1-\psi_m)(\psi_m \alpha_m Inc)}{[PWMF+T_m]^3} \end{pmatrix} \end{aligned}$$

which is positively semidefined,  $D_T^2 XIMPDOM \geq 0$  since all the terms are non-negative. ■

Reassuming, Figure 2 in the "Appendix: Figures" reproduces the SAM presented in Figure 1 (see the previous section) by using the real model's notation.

## 4 Equilibrium values for $T_*$

**Lemma 12** *At equilibrium, the transport prices for agriculture  $T_a^*$  and manufacturing  $T_m^*$  are implicitly determined as:*

$$T_a^* = \left\{ PWAGR^2 - \frac{\alpha_a Inc}{e^2 CTAGR} \left[ \frac{w}{\beta_a} \right]^{\beta_a} \left[ \frac{r}{1-\beta_a} \right]^{1-\beta_a} \frac{1-\psi_a}{1-\gamma_a} \right\}^{\frac{1}{2}}$$

and respectively,

$$T_m^* = \left\{ PWMF^2 - \frac{\alpha_m Inc}{e^2 CTMF} \left[ \frac{w}{\beta_m} \right]^{\beta_m} \left[ \frac{r}{1-\beta_m} \right]^{1-\beta_m} \frac{1-\psi_m}{1-\gamma_m} \right\}^{\frac{1}{2}}$$

since optimal levels for  $w$  and  $r$  are functions of  $T$ .

**Proof.** is provided below. ■

We are able to determine equilibrium prices from the supply side equations referring to tradeable goods. Replacing the optimal output levels into the market clearing conditions and considering the price transformations in (??), we obtain:

$$\begin{aligned} \frac{1-\psi_a}{1-\gamma_a} \cdot \frac{\alpha_a Inc}{PAGR} &= CTAGR \left[ \beta_a \frac{PPAGR}{w} \right]^{\beta_a} \left[ (1-\beta_a) \frac{PPAGR}{r} \right]^{1-\beta_a} \\ e^2 (PWAGR^2 - T_a^2) &= \frac{\alpha_a Inc}{CTAGR} \left[ \frac{w}{\beta_a} \right]^{\beta_a} \left[ \frac{r}{1-\beta_a} \right]^{1-\beta_a} \frac{1-\psi_a}{1-\gamma_a} \end{aligned}$$

$$T_a^* = \left\{ PWAGR^2 - \frac{\alpha_a Inc}{e^2 CTAGR} \left[ \frac{w}{\beta_a} \right]^{\beta_a} \left[ \frac{r}{1-\beta_a} \right]^{1-\beta_a} \frac{1-\psi_a}{1-\gamma_a} \right\}^{\frac{1}{2}}$$

From the equations corresponding to the manufacturing market, we derive the transport rate functional at equilibrium:

$$\begin{aligned} PMF \cdot PPMF &= \frac{\alpha_m Inc}{CTMF} \left[ \frac{w}{\beta_m} \right]^{\beta_m} \left[ \frac{r}{1-\beta_m} \right]^{1-\beta_m} \frac{1-\psi_m}{1-\gamma_m} \\ e^2 (PWMF^2 - T_m^2) &= \frac{\alpha_m Inc}{CTMF} \left[ \frac{w}{\beta_m} \right]^{\beta_m} \left[ \frac{r}{1-\beta_m} \right]^{1-\beta_m} \frac{1-\psi_m}{1-\gamma_m} \end{aligned}$$

$$T_m^* = \left\{ PWMF^2 - \frac{\alpha_m Inc}{e^2 CTMF} \left[ \frac{w}{\beta_m} \right]^{\beta_m} \left[ \frac{r}{1-\beta_m} \right]^{1-\beta_m} \frac{1-\psi_m}{1-\gamma_m} \right\}^{\frac{1}{2}},$$

Remark that transport functionals are implicitly defined, since (with the price of services set up as numeraire  $PPSERV = PSERV = 1$ ):

$$\begin{aligned}
w^* &= \frac{\beta_a \cdot PPAGR + \beta_m \cdot PPMF + \beta_s \cdot PPSEEV}{LSUPTOT} \\
&= \frac{\beta_a \cdot e(PWAGR - T_a) + \beta_m \cdot e(PWMF - T_m) + \beta_s \cdot PPSEEV}{LSUPTOT} \\
&= \frac{\beta_a \cdot e(PWAGR - T_a) + \beta_m \cdot e(PWMF - T_m) + \beta_s}{L_s} \\
r^* &= \frac{\beta_a \cdot PPAGR + \beta_m \cdot PPMF + \beta_s \cdot PPSEEV}{KSUPTOT} \\
&= \frac{\beta_a \cdot e(PWAGR - T_a) + \beta_m \cdot e(PWMF - T_m) + \beta_s}{K_s}
\end{aligned}$$

Some simulations with different values of the parameters are presented in the appendix (see subsection "Graphics and numerical results" in the appendix), in order to provide better intuition regarding the transport price's determinants. For 28 replications of random meaningful prices PWMF and PWAGR, and some arbitrary values of the elasticities and import/export shares, we present results for the interactions:  $T = f(PWMF)$  and  $T = f(PWAGR)$ .

## 5 Comparative statics

In this section we analyse the situation in which small movements from the equilibrium point (we analyse partial derivatives for the agricultural industry; manufacturing will have identically similar conditions). All the derivatives taken on the model's decision variables are decreasing in transports, except for the global production function, which is increasing

in  $T_*$ , as demonstrated below:

$$\begin{aligned}
\frac{\partial PRODAGR}{\partial T_a} &= \frac{1-\psi_a}{1-\gamma_a} \frac{e \cdot \alpha_a Inc}{(PWAGR-T_a)^2} \geq 0 \\
\frac{\partial XAGRIMP}{\partial T_a T_m} &= -\psi_a \alpha_a \frac{e \cdot Inc}{(PWAGR+T_a)^2} \leq 0 \\
\frac{\partial XAGRDOM}{\partial T_a T_m} &= -(1-\psi_a) \alpha_a \frac{e \cdot Inc}{(PWAGR+T_a)^2} \leq 0 \\
\frac{\partial LAGR}{\partial T_a} &= -\frac{e\beta_a}{w} \leq 0 \\
\frac{\partial KAGR}{\partial T_a} &= -\frac{e(1-\beta_a)}{w} \leq 0 \\
\frac{\partial w}{\partial T_a} &= -\frac{e\beta_a}{L_s} \leq 0 \\
\frac{\partial r}{\partial T_a} &= -\frac{e\beta_a}{K_s} \leq 0
\end{aligned}$$

However, the negative effect of T on PRODAGR may be decomposed in two sub-effects related respectively: to the production oriented to satisfy the domestic demand and to the production for export. When the transport rate increases it affects only exports which diminish, while the overall production is re-oriented towards the local market (which has no transport cost).

## 6 Extension to a situation of trade among three regions

It worth noting that adding the third trade partner adds some complexity to the model. Now we deal with three regions (i.e. countries 1,2,3). Consider that fixed coefficients are bilaterally defined for each pair of countries (the first subscript for flow variables indicates the origin and the second subscript indicated the destination, i.e.  $XAGRIMP_{21}$  indicates imports of agricultural goods from country 2 into country 1 while  $\psi_{31}^a$  indicates the fixed proportion of agricultural imports from country 3 into country 1. Instead, the transport rate T is identical for all

destinations, therefore prices are uniform across countries.

DEMAND (COUNTRY 1):

$$\begin{aligned}
 XAGR_1^* &= \alpha_a \frac{Inc_1}{PAGR} = \alpha_a \frac{Inc_1}{e^{(PWAGR+T_a)}} \\
 XMF_1^* &= \alpha_m \frac{Inc_1}{PMF} = \alpha_m \frac{Inc_1}{e^{(PVMF+T_m)}} \\
 XSERV_1^* &= (1 - \alpha_a - \alpha_m) \frac{Inc_1}{PSERV_1}
 \end{aligned}$$

and for tradeable goods:

$$\begin{aligned}
 XAGRIMPTOT_1 &= XAGRIMP_{21} + XAGRIMP_{31} \\
 &= \psi_{21}^a \alpha_a \frac{Inc}{PAGR} + \psi_{31}^a \alpha_a \frac{Inc}{PAGR} \\
 &= (\psi_{21}^a + \psi_{31}^a) \frac{\alpha_a Inc}{e^{(PWAGR+T_a)}} \\
 \\
 XAGRDOM &= XAGR_1^* - XAGRIMPTOT_1 \\
 &= (1 - \psi_{21}^a - \psi_{31}^a) \frac{\alpha_a Inc}{e^{(PWAGR+T_a)}} \\
 \\
 XMFIMPTOT_1 &= XMFIMP_{21} + XMFIMP_{31} \\
 &= (\psi_{21}^m + \psi_{31}^m) \frac{\alpha_m Inc}{e^{(PVMF+T_m)}} \\
 \\
 XMFDOM &= XMF_1^* - XMFIMPTOT_1 \\
 &= (1 - \psi_{21}^m - \psi_{31}^m) \frac{\alpha_m Inc}{e^{(PVMF+T_m)}}
 \end{aligned}$$

Concerning the supply side equations, they become:

$$\begin{aligned}
 \text{SUPPLY (COUNTRY 1):} \\
 LAGR_1^* &= \beta_a \frac{PPAGR}{w} = \beta_a \frac{e^{(PWAGR-T_a)}}{w} \\
 KAGR_1^* &= (1 - \beta_a) \frac{PPAGR}{r} = (1 - \beta_a) \frac{e^{(PWAGR-T_a)}}{w} \\
 LMF_1^* &= \beta_m \frac{PPMF}{w} = \beta_m \frac{e^{(PVMF-T_m)}}{w} \\
 KMF_1^* &= (1 - \beta_m) \frac{PPMF}{r} = (1 - \beta_m) \frac{e^{(PVMF-T_m)}}{r} \\
 LSERV_1^* &= \beta_s \frac{PPSERV}{w} \\
 KSERV_1^* &= (1 - \beta_s) \frac{PPSERV}{r}
 \end{aligned}$$

and similarly for the manufacturing sector:

$$\begin{aligned}
 PRODAGR^* &= \frac{1-\psi_{21}^a-\psi_{31}^a}{1-\gamma_{12}^a-\gamma_{13}^a} \cdot \frac{\alpha_a Inc}{PAGR} \\
 EXPAGRTOT_1 &= EXPAGR_{12} + EXPAGR_{13} \\
 &= (\gamma_{12}^a + \gamma_{13}^a) \frac{1-\psi_{21}^a-\psi_{31}^a}{1-\gamma_{12}^a-\gamma_{13}^a} \cdot \frac{\alpha_a Inc}{er(PWAGR+T_a)} \\
 \\
 PRODMF^* &= \frac{1-\psi_{21}^m-\psi_{31}^m}{1-\gamma_{12}^m-\gamma_{13}^m} \cdot \frac{\alpha_m Inc}{PMF} \\
 EXPMFTOT_1 &= EXPMF_{12} + EXPMF_{13} \\
 &= (\gamma_{12}^m + \gamma_{13}^m) \frac{1-\psi_{21}^m-\psi_{31}^m}{1-\gamma_{12}^m-\gamma_{13}^m} \cdot \frac{\alpha_m Inc}{er(PWMF+T_a)} \\
 PRODSERV^* &= (1 - \alpha_a - \alpha_m) \frac{Inc}{PSERV}
 \end{aligned}$$

The effects maintain their signs when derived with respect to the transport rate. The strategic elements now become the fixed trade parameters (import/ export coefficients,  $\gamma_*$  and  $\psi_*$ ). In particular, the coefficient of production dedicated to exports,  $\gamma_*$  has an ambiguous effect in the export value, while the coefficient associated to the demand of imports has a negative effect on exports (leverage effect).

## 7 Conclusion and future research

We build a simple model which is an alternative to the CES formulation traditionally used in foreign trade. Our specification represents better the economic situations like: non-homogeneous trading partners coexisting on the same geographical plan (i.e. under-developed countries and developed neighbours), second-best trade policies against market imperfections, the typical situation when a domestic market is supplied by a foreign monopoly firm (or market). The domestic market consists of many consumers who demand the product but has no domestic producers of the product. All supply of the product comes from a single foreign firm; some examples are: Palestinian major industrial sectors which depend on their major supplier, Israel; industrialized economies depending

on the oil resources exploited by foreign oligopolies, etcetera. In this case the market imperfection is that there are not a multitude of firms supplying the market, but rather by a monopoly. Further implementations will be added to the model, as: the introduction of an environmental tax on transports paid by the a representative transport firm. In this case, the tax becomes a part of the transport price and will be added to the trade distortionary effect. A government will collect this tax and will provide subsidies to a second type of households: the poor consumers.

## 8 Appendices

### 8.1 Nomenclature

XAGR,XMF,XSERV	=total agric/manufact/services qty's demanded
$\alpha_a, \alpha_m$	=elasticities in the utility function
$\psi_a$	=proportion of imports in total agricultural cons.
$\psi_m$	=proportion of imports in total manufacturing cons.
PAGR,PMF,PSERV	=consumption price indexes for agric/manufact/serv.
$\lambda$	=lagrange multiplier for utility maximisation
XAGR*,XMF*,XSERV*	=optimal indirect demand functions
PRODSERV,PRODAGR,PRODMF	=production functions for agric/manufct/services
PRODSERV*,PRODAGR*,PRODMF*	=optim. production functions for agric/manuf./serv
LAGR,KAGR/LMF,KMF/LSERV,KSERV	=factors demand (labor/capital) in agriculture/ manufact/services;
LAGR*,KAGR*/LMF*, KMF*/LSERV*,KSERV*	=optim. indirect factors demand functions
$\beta_a, \beta_m$	=factor demand elasticities in the prod. functions for agriculture/manufacturing;
CTAGR,CTMF,CTSERV	=constants in the production functions
r,w	=factor remunerations (returns to capital,wage)
PPAGR,PPMF,PPSERV	=production price indexes for agric/manufact/serv.
PWAGR,PWMF,PWSERV	=world prices for agric/manufact/services
EXPAGR,EXPMF	=exported quantities of agric/manuf.goods (supply)
XAGRIMP,XMFIMP	=imported quantities of agric/manuf. goods (demand)
$T_a, T_m$	=transport rates for agric/manufact.
e	=exchange rate

## 8.2 Equations summary (for the two regions model)

We reassume the final equations of the model with two regions in the following:

DEMAND:

$$\begin{aligned}
 XAGR^* &= \alpha_a \frac{Inc}{PAGR} \\
 XMF^* &= \alpha_m \frac{Inc}{PMF} \\
 XSERV^* &= (1 - \alpha_a - \alpha_m) \frac{Inc}{PSERV} \\
 XAGRIMP &= \psi_a \alpha_a \frac{Inc}{PAGR} \\
 XAGRDOM &= (1 - \psi_a) \alpha_a \frac{Inc}{PAGR} \\
 XMFIMP &= \psi_m \alpha_m \frac{Inc}{PMF} \\
 XMFDOM &= (1 - \psi_m) \alpha_m \frac{Inc}{PMF}
 \end{aligned}$$

SUPPLY:

$$\begin{aligned}
 LAGR^* &= \beta_a \frac{PPAGR}{w} \\
 KAGR^* &= (1 - \beta_a) \frac{PPAGR}{r} \\
 LMF^* &= \beta_m \frac{PPMF}{w} \\
 KMF^* &= (1 - \beta_m) \frac{PPMF}{r} \\
 LSERV^* &= \beta_s \frac{PPSERV}{w} \\
 KSERV^* &= (1 - \beta_s) \frac{PPSERV}{r} \\
 EXPAGR &= \gamma_a \frac{1 - \psi_a}{1 - \gamma_a} \cdot \frac{\alpha_a Inc}{PAGR} \\
 EXPMF &= \gamma_m \frac{1 - \psi_m}{1 - \gamma_m} \cdot \frac{\alpha_m Inc}{PMF} \\
 PRODAGR^* &= \frac{1 - \psi_a}{1 - \gamma_a} \cdot \frac{\alpha_a Inc}{PAGR} \\
 PRODMF^* &= \frac{1 - \psi_m}{1 - \gamma_m} \cdot \frac{\alpha_m Inc}{PMF} \\
 PRODSERV^* &= (1 - \alpha_a - \alpha_m) \frac{Inc}{PSERV}
 \end{aligned}$$

EQUILIBRIUM PRICES:

$$w^* = \frac{\beta_a \cdot PPAGR + \beta_m \cdot PPMF + \beta_s \cdot PPSE RV}{LSUPTOT}$$

$$r^* = \frac{\beta_a \cdot PPAGR + \beta_m \cdot PPMF + \beta_s \cdot PPSE RV}{KSUPTOT}$$

PRICES TRANSFORM:

$$PPAGR = epa \cdot (PWAGR - T) = e \cdot (PWAGR - T)$$

$$PPMF = epm \cdot (PVMF - T) = e \cdot (PVMF - T)$$

$$PAGR = ea \cdot (PWAGR + T) = e \cdot (PWAGR + T)$$

$$PMF = em \cdot (PVMF + T) = e \cdot (PVMF + T)$$

### 8.3 Figures

Figure 1: SAM reproducing the real local economy

	agriculture	manufacturing	services	labor	capital	residents	rest of the economy	TOTAL	
agriculture	inter-sector transactions						domestic consumption of local agricultural products	final and intermediate agroicultural exported goods	total sales of agriculture
manufacturing							domestic consumption of local manufactured products	final and intermediate manufactured exported goods	total sales of the manufacturing
services							expenditure of local residents on services	exported local services	total sales of services (other sectors)
labor	labor employed in agriculture	labor employed in manufacturing	labor employed in services				income of residents employed in non-local firms	total labor of residents and labor of non-residents employed in local firms	
capital	capital employed in agriculture	capital employed in manufacturing	capital employed in services				total employed capital in local firms		
residents							labor income	other income	residents households incomes
rest of the economy	imported intermediate goods	imported intermediate goods	imported intermediate goods	non-resident workers employed in local firms	capital supply	domestic consumption of imported goods and services and savings	balancing account		
TOTAL	total agriculture production	total manufacturing production	total services production	labor supply	capital supply	total residents' consumption	balancing account		

Figure 2: SAM values

	agriculture	manufacturing	services	labor	capital	residents	rest of the world	TOTAL
agriculture	0	0	0			XAGRDOM*PAGR	EXPAGR*PPAGR	PRODAGR*PPAGR
manufacturing	0	0	0			XMFDOM*PMF	EXPMF*PPMF	PRODMF*PPMF
services	0	0	0			XSERVOPT*PSERV	0	PRODSERV*PPSERV
labor	LAGR*w	LMF*w	LSERV*w					LDEMTOT*w
capital	KAGR*r	KMF*r	KSERV*r					KDEMTOT*r
residents				Inc				Inc
rest of the world	XAGRIMP*PAGR	XMFIMP*PMF	0					BA
TOTAL	total agriculture production	total manufacturing production	total services production	LSUPTOT	KSUPTOT	total residents' consumption	BA	

## 8.4 Graphics and numeric results

### SIMULATION 1

Industrialized country case

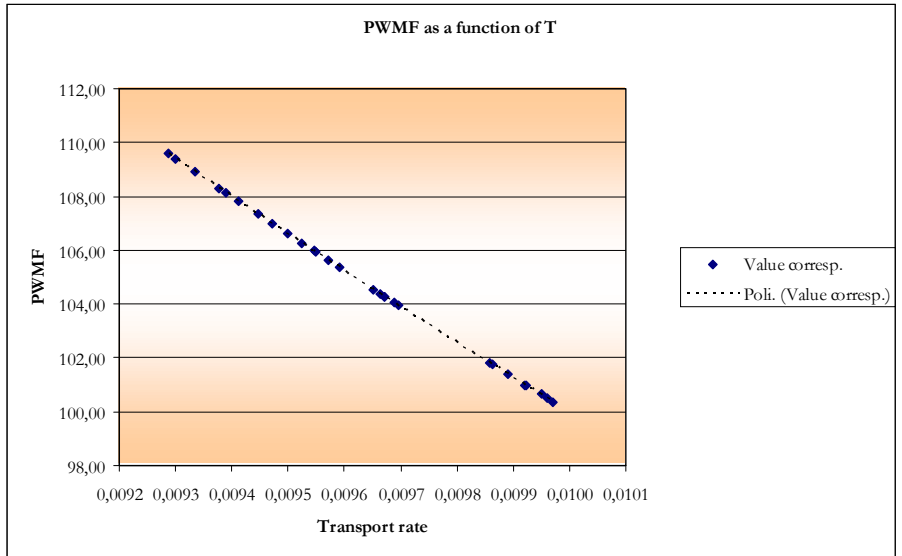
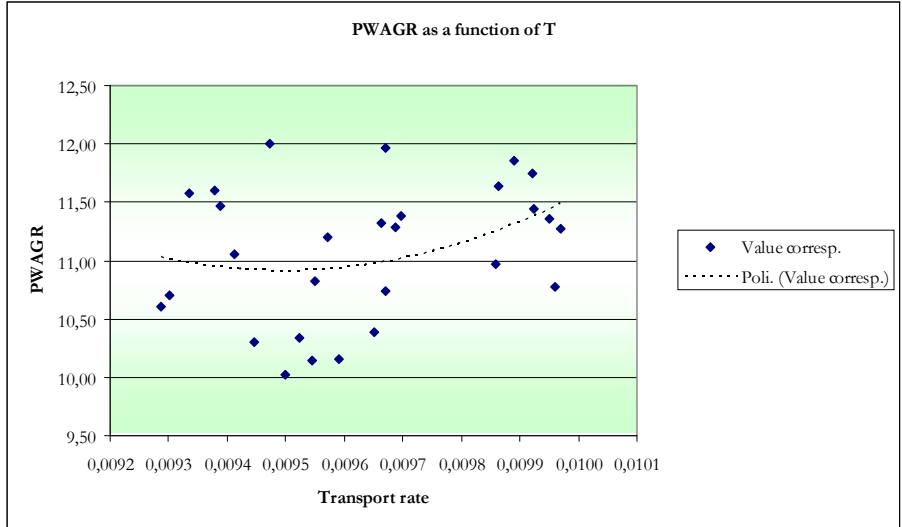
$$\alpha_a = 0.3; \alpha_m = 0.5; \alpha_s = 0.2; \gamma_a = 0.4; \gamma_m = 0.3;$$

$$\beta_a = 0.7; \beta_m = 0.4; \psi_a = 0.2; \psi_m = 0.3; e = 1.5;$$

$$CTAGR = 2; CTMF = 10; L_s = 1000; K_s = 10000;$$

Iteration	T	PWMF	PWAGR
1	0,0096	105,61	11,20
2	0,0100	100,51	10,77
3	0,0093	108,92	11,57
4	0,0097	104,28	10,74
5	0,0097	104,04	11,28
6	0,0093	109,40	10,70
7	0,0097	104,53	10,39
8	0,0095	105,92	10,83
9	0,0096	105,35	10,16
10	0,0099	101,74	11,64
11	0,0094	108,15	11,47
12	0,0100	100,38	11,27
13	0,0093	109,59	10,61
14	0,0097	104,28	11,97
15	0,0094	107,35	10,30
16	0,0099	101,41	11,85
17	0,0099	100,64	11,35
18	0,0094	107,82	11,05
19	0,0097	104,37	11,32
20	0,0095	106,63	10,02
21	0,0099	101,81	10,97
22	0,0097	103,93	11,38
23	0,0095	105,97	10,14
24	0,0099	100,97	11,44
25	0,0094	108,31	11,60
26	0,0099	101,00	11,75
27	0,0095	106,28	10,33
28	0,0095	106,99	12,00

We also provide a graphic representation of these dependencies:



## SIMULATION 2

Poor country case

$$\alpha_a = 0.7; \alpha_m = 0.2; \alpha_s = 0.1; \gamma_a = 0.2; \gamma_m = 0.1;$$

$$\beta_a = 0.5; \beta_m = 0.5; \psi_a = 0.4; \psi_m = 0.7; e = 1.5$$

$$CTAGR = 2; CTMF = 10; L_s = 1000; K_s = 10000;$$

Iteration	T	PWMF	PWAGR
1	0,0095	105,61	11,20
2	0,0100	100,51	10,77
3	0,0092	108,92	11,57
4	0,0096	104,28	10,74
5	0,0096	104,04	11,28
6	0,0092	109,40	10,70
7	0,0096	104,53	10,39
8	0,0095	105,92	10,83
9	0,0095	105,35	10,16
10	0,0098	101,74	11,64
11	0,0093	108,15	11,47
12	0,0100	100,38	11,27
13	0,0092	109,59	10,61
14	0,0096	104,28	11,97
15	0,0094	107,35	10,30
16	0,0099	101,41	11,85
17	0,0099	100,64	11,35
18	0,0093	107,82	11,05
19	0,0096	104,37	11,32
20	0,0094	106,63	10,02
21	0,0098	101,81	10,97
22	0,0096	103,93	11,38
23	0,0095	105,97	10,14
24	0,0099	100,97	11,44
25	0,0093	108,31	11,60
26	0,0099	101,00	11,75
27	0,0095	106,28	10,33
28	0,0094	106,99	12,00

## 8.5 Bibliography

1. Feenstra Robert C. Advanced International Trade. Theory and evidence. 2004
2. Fujita M. and Thisse J-F. Economics of Agglomeration. Citiesm industrial location and regional growth. Cambridge University

Press, 2002

3. Fujita M. and Thisse J-F. An introduction to geographical economics. 2002
4. J.R. Markusen, J. Melvin, W Kaempfer and K.E.Maskus. International Trade: Theory and Evidence. USA, McGraw Hill Irwin, 1995.
5. H. Yilmazkuday. Distribution of Consumption, Production and Trade within the U.S.. Letters in Spatial and Resource Sciences, 2008
6. H.Yilmazkuday. Is the Armington elasticity really constant across Importers?. Temple University - Dept. of Economics. wp October 2008.