The Arctic Treasure Hunt: A Game-Theoretic Approach to the Race for the North Pole Riches

ARNO REICHENVATER
Department of Economics and Business Administration, Faculty of Law,
University of Joensuu, P.O. Box 111, 80101 Joensuu, Finland
E-mail: arno.reichenvater@joensuu.fi

KOFI VONDOLIA
Department of Economics, School of Business, Economics and Law,
Gothenburg University, P. O. Box 640, 405 30 Gothenburg, Sweden
E-mail: kofi.vondolia@economics.gu.se

Abstract

The anthropogenic climate change presents a global challenge that threatens human survival. Paradoxically, this global catastrophe provides huge economic opportunities in the form of the Arctic Treasure Hunt with strategic interactions: an estimated quarter of Earth's oil reserves are to lie under the glaciers of the North Pole. In this paper, we use game-theoretic models to characterize the strategic interactions for different management regimes of the Arctic Treasure Hunt. Non-cooperative games are used to describe scenarios of sole management regime and characteristic function games those of cooperation in the form of joint management regime.

Keywords: characteristic function games, exclusive economic zones, game theory, oil reserves, management regimes, North Pole, supermodular games.

1. Introduction

In the 1980s, Young (1986) perceived the strategic importance of the Arctic Circle and declared the Age of the Arctic. Little has been seen of this prediction. However, the current global warming coupled with high and volatile international oil prices renews the quest for the Age of the Arctic with huge economic and strategic interests. For centuries, the North Pole has been considered an international common property resource of no significant economic value. With global warming, the Arctic region is expected to become more accessible. For instance, the Northern Sea Route (NSR) is estimated to reduce travelling distance from Europe to North America, Northeast Asia and Far East by 40% (Yenikeyeff and Krysiek, 2007). Paradoxically, the danger posed by the global warming at the Arctic is also creating one of the treasure hunts of this century: the so-called Arctic Treasure Hunt. This could eventually lead to easier access to natural resources that have, according to some estimates, turned out to be enormous. According to the US Geological Survey a quarter of Earth's oil reserves are under the glaciers of the North Pole. In addition, gas reserves, some bacteria and shipping routes are hoped to add economic benefits.

Hoping to take advantage of these benefits, Russia, Norway, Denmark, Canada and the United States have laid claims on the North Pole. These claims seek to extend the territories of each country beyond the 200 nautical miles defined by the United Nations Convention on Laws of the Sea. These claims are consistent with the United Nations Convention on Laws of the Sea which allows countries to claim more only if they can prove their continental shelf extends further into the sea. Although each of the countries has justifiable reasons for claiming parts of the region, some of the claims lack credibility. What is common among each country's aspirations is the expectation of economic gains. Some could even call these aspirations as imperialism.

The competition between the five littoral countries involves strategic interaction, which further complicates the analysis of the situation. The methodology used in game theory can offer an insight into the problem and greatly simplify the analysis. Game theory can be divided into two main categories: cooperative and non-cooperative. Both of these are applicable in the Arctic Treasure Hunt.

Russia claims that the 1240 mile underwater Lomonosov Ridge in the Arctic is connected to East Siberia and therefore is a part of its continental shelf. In addition to the Lomonosov Ridge, Canada has also laid claims to the North West Passage. Canada first claimed the passage in the early 1970s. The Russian and Canadian claims are being strongly contested by the others. The Danes argue that the Lomonosov Ridge is connected to Greenland. Since America has not ratified the 1982 Law of the Sea Convention (LOSC) they cannot stake a claim. Rather they assert that the North West Passage is international waters. Although the present claims are in the form of sole management regimes, there is likelihood of multilateral (or joint) management regime. According to the Law of the Sea Convention, in addition to a 200-mile exclusive economic zone (EEZ), signatories may also claim as additional territory any extensions to their continental shelves that they can scientifically substantiate. Russia, Denmark and Canada all claim that the Lomonosov Ridge is natural extension from their continental shelves, so the claims are overlapping. Also, financial resources and technology will impose significant constraints to enable joint claims to be credible. There might also be an agreement for joint exploration. Both these scenarios would depend on the desire for compromise and co-operation between the various parties.

One notable development of the Arctic Treasure Hunt is the formation of coalitions, which obviously implies cooperation. Norway has been reported to be willing to form a coalition with Russia (Yenikeyeff and Krysiek, 2007): this is because the Norwegian expertise and capital would be required by Russia in the exploration of the Arctic if Russia receives all the land it has claimed.

In addition, Anglo-American companies such as Exxon operated in Russia on the Sakhalin-1 oil and gas project in the Russian Far East. Therefore, the coalition between Russia and America is also feasible. Exxon has operated in Canadian territories before, so a US-Canada coalition is plausible. EU can also harmonise its Arctic strategy thus promoting a coalition among European contenders to the Arctic Treasure Hunt. The main battle is now among US, Russia, Denmark, Norway and Canada. It is thus imperative to study various coalitional structures of this game. This can be done with the help of characteristic function games (CFG), developed by von Neumann and Morgenstern (1944).

The formation of one or more coalitions is viable if the countries believe that cooperation benefits them. In the event that cooperation is not an option, there a few other courses of action that can take place. First, a two-stage non-cooperation game may be used. Here, the countries choose the optimal investment and land claim in the first stage, and compete in the second stage. Second, the non-cooperative game can be characterized in terms of supermodular games. The problem that we seek to address with supermodular games is similar to the arms race (e.g., Hendricks & Kovenock, 1989). The theoretical insight from arms race indicates that the perceived value of additional arms to a country depends on military capability of the adversary. Consider the following example: Russia goes to the Arctic Circle to gather evidence to support its claim to the Arctic Circle. The scientific expedition of Russia boosts similar sentiments among other contenders, creating incentives to pursue similar scientific expeditions. Typically, there are strategic complementarities. Supermodular optimization has been extended to analyse games with strategic complementarities (Topkis, 1979; Vives, 1990; and Milgrom and Roberts, 1990).

Uncertainties will also determine the profitability of the Arctic Treasure Hunt. According to Yenikeyeff and Krysiek (2007) the present estimates are made under current oil and gas price conditions. Therefore, the future of the Arctic shelf development will be determined by the dynamics of world oil prices in the next twenty years.

The purpose of this paper is to use game-theoretic models to offer insight into the strategic interaction among these Arctic countries that compete in the Arctic Treasure Hunt. However, the scope of this paper is extended only to an overall introduction of the three types of games mentioned above. Later work is required to estimate the possible outcomes for each type of games. Especially interesting results could be yielded from studying coalitions.

This paper is structured as follows. The initial stage concerning the policy a country decides to adopt is presented in the next section. There are several factors e.g. world oil prices, the distribution of oil at the Arctic, and the response of the international community that affect this decision. The third section contains the various game-theoretic formulations for different management regimes for the Arctic Treasure Hunt. The conclusions are presented in the final section.

2. First Stage

The initial set-up of the situation is one characterised by uncertainty about the oil deposits. There is no prior knowledge about how the oil is distributed at the Arctic, and no competing country is more knowledgeable than the others. For the sake of simplicity, let us assume two scenarios. In the first one, the oil is distributed across the whole area such that individual deposits are not connected to each other. In this case, the countries would want to conquer as much land as possible in order to maximize the oil reserves in their possession. Thus, rapid expansionism secures a greater amount of land.

In the second scenario, the oil is distributed evenly across the North Pole with all pockets of oil connected to each other by tunnels. Alternatively, the North Pole oil reserves can be thought to be merely one large deposit. This means that the competing countries would have to come up with a way to share the oil and benefit from it collectively. Thus, coalitions are possible. This simplification is done despite the fact that most likely neither of these extreme states of nature would occur. However, the extreme cases can help demonstrate behavior at both ends of the behavioral spectrum.

The actual state of nature was determined during the formation of the Earth. This constitutes the first stage of the game. However, there is uncertainties about the true state of nature since the players are not aware of the true state of the nature; this is built into the first stage of the game. . In this kind of a first stage, country *i* has to decide whether or not it is worth making additional territorial claims in an uncertain world. Uncertainty also arises from the fact that a country cannot know if investing in aggressive imperialism will be accepted by the international community, which might result in annulment.

A country has two policy options: first, settle with the amount of land designated to it according to the UN's Convention on the Law of the Sea that states that a state has sole exploitation rights over all natural resources within a 200-nautical mile zone that extends from its coastline. If the country can prove that its continental shelf extends another 150 nautical miles, it can claim the right to exploit the natural resources.

Although this approach of moderate diplomacy causes little friction in the political realm, it possesses its own weakness. There is a risk that another country can take advantage of the second option. That is, that despite the existing 200-nautical mile law, it chooses to aggressively claim more land, and if the international community agrees to that and oil is in small pockets, it will gain more land than with the moderate approach. There are risks, however. If the international community does not agree with the country's claims, its investment in establishing the claim, and costs associated with search and military forces will have been made in vain.

The main aim of each country is to maximize profit. It does so by choosing the optimal policy and the level of extraction. Factors that affect the decision-making process include the world's total oil reserves (Y), the size of the North Pole's oil reserves (z), the market price of crude oil ($p(x,Y)$, costs associated with search and extraction (c), and the strategies of a country and its competitors, s_i and s_{-i} for $i = 1,2,...,5$ and $M, A \in S$.

The price of oil is here simplified to be determined by the amount of oil in the market and expected oil reserves. The more oil there is in the market, the lower the price. Similarly, larger estimated oil reserves correspond with a lower oil price. Thus, $p = p(x,Y)$.

Initially, a country has to make a choice between a moderate and aggressive policy.

$$E(\Pi_i^M) = \delta[B^M(p(x,Y), z, Y, s_i, s_{-i}) - C^M(p(x,Y), z, c)]$$

and

$$E(\Pi_i^A) = (1-\delta)[B^A(p(x,Y), z, Y, s_i, s_{-i}) - C^A(p(x,Y), z, c)],$$

where δ expresses the perceived probability that the international community will condemn the hostile land claims. According to equation (1) when the country i chooses a moderate policy and the international community does not condemn hostile land claims the payoff of player i is zero. Simply put, the country will choose the policy that maximizes its expected profits:

$$\max E(\Pi_i) = \begin{cases} if \ \Pi_i^M > \Pi_i^A \ , i \quad chooses \quad M \\ if \ \Pi_i^M = \Pi_i^A \ , i \ indifferent \\ if \ \Pi_i^M < \Pi_i^A \ , i \quad chooses \quad A \end{cases}$$

The significance of this first-stage decision-making for the remainder of the game, regardless of which of the following three paths the game proceeds in, is that it affects the portion of the Arctic that each country will possess.

3. Second Stage

After the initial stage, several possible scenarios arise depending on the state of nature, propensity to cooperate and attitudes towards risk.

3.1. Scenario I: Non-cooperative Game

Non-cooperative games are probably the simplest forms of games. Despite of this, even the famous prisoner's dilemma can reveal a lot about strategic behaviour. However, at its most basic form, the prisoner's dilemma is a static game. We are more concerned about the dynamic games which offer an opportunity to study behaviour over time. Non-cooperative games are important in describing competition and how one country's decisions affect and are affected by other countries' decisions. Furthermore, the optimal strategy can be determined. This, of course, requires that rational choice is assumed.

In the game there are $n = 5$ players (i.e., the five competing countries). Each player maximizes its own economic gains from the resource by choosing its strategy. The outcome of the game – once again, assuming rationality – is a Nash equilibrium with all countries choosing optimal strategies.

Different approaches could be used to characterise the equilibrium with the choice among different approaches depending on the assumption about the players' commitment to future actions. The examples include modeling the players as choosing path strategies and search for the Nash equilibria (NE); and another being the use of the Subgame Perfect Nash equilibria (SPNE). However, strategic complementarities among the five players make the problem different; and Simaan and Cruz (1975) formulate the strategic complementarities version of this game. The choice of path strategies assumes that commitments extend over the entire future horizon whilst the decision rule strategies assume that no commitments are possible. Both approaches have been used to analyse non-renewable resources. Among the studies that use the path strategies for non-renewable natural resources are Crawford, et al. (1984) and Dasgupta and Heal (1995The solution for this game can be closed-loop or the open-loop controls.

Countries $i = 1, 2, ..., 5$ are engaged in non-cooperative extraction of Arctic oil pool. The discount rate is given as r. The objective for each country is to choose the optimal extraction path that will maximize the present value of its profits given the extraction path of other countries. Following Dasgupta and Heal (1995) coupled with symmetry assumption, the extraction path of each player will be given as:

$$y_{it} \frac{d}{dy_{it}} \left\{ p\left(\sum_{i=1}^{5} y_{it} \right) \right\} + p\left(\sum_{i=1}^{5} y_{it} \right) = \lambda_i e^{rt},$$

$\lambda_i > 0$ for $y_{it} > 0$, $i = 1, 2, ..., 5$. The Nash equilibrium is the solution to the above equation. Summing the above equation for the five players:

$$Y_t p' Y_t + 5p \ \ Y_t \ \ = \Omega e^{rt},$$

where $\Omega = \sum_{i=1}^{5} \lambda_i$ and $Y_t = \sum_{i=1}^{5} y_{it}$.

The Nash solution is a reasonable simplification for the non-cooperative game. The Nash equilibrium of the problem in a differential game can take two forms depending on players' commitment to future extraction rates. In the open-loop form, the only argument of y_i^* is time: $y_i^*(t)$. This means each player determines its optimal extraction rate at $t = 0$. Simaan and Cruz (1975) distinguish between the closed-loop form and open-loop controls. However, in the optimal control problem both the open-loop and closed-loop forms coincide because of the Bellman's principle of optimality. The above strategic interaction between players can be formulated as a game.

A normal form game with n players is represented by a triple $\Gamma = (N, S, u_i)$. N is a finite or infinite set of n players. S_n is the strategy set available to the players $n \in N$ such that $S = S_1 \times ... \times S_n$. The typical element of each player's strategy set S_n is s_n while s_{-n} denotes the competitors' strategies. Thus, the complete strategy profile is $s = (s_n, s_{-n}) \in S$. u is a payoff function that can be represented by a von Neumann-Morgenstern utility correspondence $u_i : S \to \Re$.

Given the description of the game above, the pure strategy Nash equilibrium is defined as:

DEFINITION: *A strategy profile* $s = (s_1, s_2, ..., s_I)$ *constitutes a pure strategy Nash equilibrium of the game* $\Gamma_N = [N, \{S_i\}, \{u_i(.)\}]$ *if for every* $i = 1, 2, ..., N$, $u_i(s_i, s_{-i}) \geq u_i(s_i^{'}, s_{-i})$, $\forall s_i^{'} \in S_i$.

In Nash equilibrium, each player strategy choice is the best response to the strategies actually played by his rivals.

It is also possible to extend the concept of Nash equilibrium to situations where the players randomise over their pure strategies. The definition of mixed strategy equilibrium is given as:

DEFINITION: *A mixed strategy profile* $\sigma = (\sigma_1, ..., \sigma_N)$ *constitutes a Nash equilibrium of the game* $\Gamma_N = [N, \{\Delta(S_i)\}, \{u_i(.)\}]$ *if for every* $i = 1, 2, ..., N$, $u_i(\sigma_i, \sigma_{-i}) \geq u_i(\sigma_i^{'}, \sigma_{-i})$, $\forall \sigma_i^{'} \in \Delta(S_i)$.

The symmetry assumption is quite strong: players could differ in many important respects e.g. technology and efficiency in resource extraction. There are possibilities of dominant player in this instance and the Stackelberg solution concept is required.

Being naïve, a country could wait until the rest of the oil producing countries (including OPEC) have emptied their oil reserves, and begin to monopolize the oil markets. This would lead to an exorbitant oil price. A less naïve view is that a backstop technology will ultimately be available. In this case, it is less desirable to hoard oil until the end since by then a replacement could have been invented, thus making oil obsolete and plummeting the oil price.

3.2. Scenario II: Supermodular Games

In this type of scenario, the oil reserves at the North Pole are formed by a single deposit. Simply put, extracting oil from one part of the deposit will lower the oil level in all parts of the deposit. The reason for assuming a single deposit is merely for the sake of simplicity. This leads to a dilemma each country has to face: whether or not to form a coalition with other countries. Let us now assume that for some reason no coalitions are formed.

A country can choose to be selfish and drill oil at the highest possible rate, knowing very well that if the others do the same, then it will lose out on potential oil production. The end result will be that the oil is drilled and the stock is depleted rapidly. As Hendricks and Kovenock (1989) point out, if the players believe that the single pool of oil is large, then too much drilling will take place. Considering that the oil reserves of the North Pole constitute an estimated quarter of the world's total reserves, this rapid drilling could be a very reasonable simplification.

Here, supermodular games are appropriate to characterise the strategic interaction. Supermodular games are applicable when there are complementarities in strategy space. For example, consider the nuclear arms race between the United States and Soviet Union: both increased the number of their nuclear weapons as a response to the increase done by the other. In a sense, each additional weapon is more important than the previous. Similarly, in the Arctic Treasure Hunt if the competing countries -*i* increase their production, country *i*'s utility (payoff) will increase if it does the same.

A normal form game with n players is represented by a triple $\Gamma = (N, S, u_i)$. N is a finite or infinite set of n players. S_n is the strategy set available to the players $n \in N$ such that $S = S_1 \times ... \times S_n$. The typical element of each player's strategy set S_n is s_n while s_{-n} denotes competitors' strategies. Thus, the complete strategy profile is $s = (s_n, s_{-n}) \in S$. u is a payoff function that can be represented by a von Neumann-Morgenstern utility function $u_i : S \to \Re$. Following Milgrom and Roberts (1990), the game's solution is as follows.

The game is supermodular if the following conditions hold for all $n \in N$:

(A1) The strategy set S is a complete lattice.

(A2) u_n is supermodular in s_n (for fixed s_{-n}).

(A3) u_n has increasing differences in s_n and s_{-n}.

(A4) $u_n : S \to \Re \cup \{-\infty\}$ is order upper semi-continuous in s_n (for fixed s_{-n}) and order continuous in s_{-n} (for fixed s_n) and has a finite upper bound.

Often the above conditions can be checked by using the theorem below that closely resembles the above conditions.

THEOREM 1: *Suppose the number of players is finite, that the typical strategy for each player* n *is* $(s_{nj}; j = 1,..., k_n) \in \Re^{k_n}$ *and that the ordering is component-wise. Then,* Γ *is supermodular if assumptions (A1') – (A4') hold:*

(A1') S_n is an interval in $\Re^{k_n}$: $S_n = [\underline{y}_n, \overline{y}_n] = \{s \mid \underline{y}_n \le s \le \overline{y}_n\}$.

(A2') u_n is twice continuously differentiable on S_n.

(A3') $\partial^2 u_n / \partial s_{ni} \partial s_{nj} \ge 0$ for all n and for all $1 \le i < j \le k_n$.

(A4') $\partial^2 u_n / \partial s_{ni} \partial s_{mj} \ge 0$ for all $n \ne m$, $1 \le i \le k_n$ and $1 \le j \le k_m$.

The main characteristic of supermodular games follows directly from Topkis's monotonicity theorem. Each player's best-response correspondence has extremal selections that are increasing in each rival's strategy, s^{-i}. Therefore, the overall best response mapping has extremal selections that are increasing. The existence of a fixed point in either of these selections is a result from Tarski's fixed-point theorem. A fixed point implies a pure strategy equilibrium – a Nash equilibrium.

A pure Nash equilibrium is a tuple $s = (s_n; n \in N)$ such that each s_n maximizes $f(\hat{x}_n, x_{-n})$. By definition, any pure Nash equilibrium may be a mixed equilibrium as well as a correlated equilibrium. The sets of strategies $\bar{S} =_m \subset S_m$ (with $m = 1,..., N$) are rationalizable if for all n and $x_n \in \bar{S}_n$, x_n maximizes $E[f(\cdot, x_{-n})]$ for some probability distribution on x_{-n} with support in $\bar{S}_{-n}$. Furthermore, to be rationalizable, a strategy must belong to a rationalizable set.

A strategy x_n is *strongly dominated* by another pure strategy $\hat{x}_n$ if $f(x_n, x_{-n}) < f(\hat{x}_n, x_{-n})$ for all x_{-n}. Thus, it is rational to choose a dominating strategy over any dominated strategies. Given a product set $\hat{S}$ of strategy profiles, the set of n's *undominated responses* to $\hat{S}$ is defined by $U_n(\hat{S}) = \{x_n \in S_n \mid (\forall x_n' \in S_n)(\exists \hat{x} \in \hat{S}) f_n(x_n, \hat{x}_{-n}) \geq f_n(x_n', \hat{x}_{-n})\}$. Denote the list of undominated responses for each player by $U(\hat{S}) = (U_n(\hat{S}); n \in N)$. Furthermore, let $\bar{U}(\hat{S})$ denote the interval $[\inf(U(\hat{S})), \sup(U(\hat{S}))]$.

U may be used to represent the iterated elimination of strongly dominated strategies. Let us define $S^0 = S$ as the full set of strategy profiles, and $S^\tau = U(S^{\tau-1})$ for $\tau \geq 1$. For all τ, a strategy x_n is serially undominated if $x_n \in U_n(S^\tau)$. These are the strategies that survive the iteration of strongly dominated strategies. This has significant importance since only the surviving serially undominated strategies are rationalizable and can be played with a positive probability at both a pure and mixed Nash equilibrium as well as at a correlated equilibrium. The remaining strategies form the *dominance solution*. If, at the end, there exists only one strategy that has survived iteration, the game is called *dominance solvable*. All serially undominated strategies lie in an interval $[\underline{x}, \overline{x}]$ with supremum and infimum points being the largest and smallest Nash equilibria respectively.

3.3. Scenario III: Characteristic Function Games

The previous two scenarios dealt with non-cooperative games. Characteristic function games, on the other hand, are concerned with cooperation and specifically which coalitions should be formed. A coalition is a subset of players that has the right to make binding agreements with one another. Usually it is assumed that any subset of players can do this. In the Arctic Treasure Hunt game, characteristic function games allow us to study which combination of players would yield the greatest utility or profits. Characteristic function games have been applied to various industries with cooperation. Lindroos and Kaitala (2000) and Kronbak and Lindroos (2006) have studied coalitions in fisheries. Applications of characteristic function games in other areas include regional cooperation in investments in electric power among four states in India Gately (1974). Horvat and Bogataj (1999) apply the characteristic function to analyse interactions between business decision units. Mesterton-Gibbons (2001) provides interesting applications in car pool, log hauling, antique dealing and team long-jumping.

We assume that utility is transferable, i.e., that the payoffs attainable by any particular coalition (subset of N) consist of all individual payoffs that sum to no more than a particular number. When utility is transferable, it is possible to compare the utilities of different players, for instance, in monetary terms. In describing the characteristic function games we follow Friedman (1991)

Let $N = \{1,2,...,n\}$ denote the set of players in a characteristic function game. If coalitions are formed, they are denoted by *K*, *L*, *M* and so forth. The lower case letters denote the number of players: *K* has *k* players, *L* has *l* players and *M* has *m* players. For example, suppose $n = 20$ and $K = \{1,3,6,13\}$. Then $k = 4$.

DEFINITION: *A coalition is a subset of the set of players,* N*, that is able to make a binding agreement.*

In characteristic function games the actual strategies recede into the background. Instead, attention is given to what payoffs the players and coalitions are able to achieve for themselves. These can be characterized with the help of characteristic functions which can be defined as follows.

DEFINITION: *The transferable utility characteristic function of a game having the set of players* N *is a scalar valued function,* $v(K)$*, that associates* $v(K) \in R$ *with each* $K \subset N$*. The characteristic function value for the empty coalition is 0. That is,* $v(\Phi) = 0$.

We interpret $v(K)$ as the maximum payoff to members of the coalition *K* that the coalition can secure for itself. We adopt the so-called α-characteristic function approach, which describes what a player/coalition can guarantee himself/itself when the remaining players act to minimize his/its payoff. The α-characteristic function is defined as follows. Let $\Gamma = (N, S, u_i)$ be the game in strategic form. The joint strategy space of the players in a coalition *K* is $S^K = \times_{i \in K} S_i$. Elements of S^K are denoted s^K. Let $(s \setminus \mathrm{t_i})$ denote the strategy combination in which player i is using s_i if $i \notin K$ and t_i if $i \in K$.

Each player can be certain that his payoff does not exceed his maximin value in the game. This can be imagined to be a situation in which the remaining players form a coalition with the sole purpose of minimizing the payoff of player i. Hence, the largest payoff that player i can secure for himself is

$$y_{\alpha i} = \max_{t_i \in S_i} \min_{s^K \in S^K} P_i(s \setminus \mathrm{t_i})$$

Each coalition aims to maximize its payoff. Thus, the α-characteristic function is

$$v_\alpha(K) = \max_{t^K \in S^K} \min_{s^K \in S^K} \sum_{\mathrm{i \in K}} P_i(s \setminus \mathrm{t^K})$$

An important assumption of the characteristic function games is that they are superadditive. This means that coalitions can achieve at least as much as the sum of what their members can achieve. If K and L are subsets of N with $K \cap L = Ø$, then $v(K \cup L) \geq v(K) + v(L)$.

ASSUMPTION: *The characteristic function,* $v(K)$, *for a game* $\Gamma = (I, S, u_i)$ *is superadditive. That is, for any disjoint coalitions, K and L contained in n,* $v(K \cup L) \geq v(K) + v(L)$.

It is worth mentioning that it is convenient to refer to the characteristic function and the set of players as *(N,v)* instead of using $\Gamma = (N, S, u_i)$ since actual strategies recede into the background. The reason for this is that $\Gamma = (N, v)$ contains all the required information.

DEFINITION: *For transferable utility games, the characteristic function form of a game, also called the coalitional form, is given by* $\Gamma = (N, v)$*. It is characterized by the set of players, N, and the characteristic function, v.*

An imputation is a payoff vector that gives each player at least as much as he could guarantee himself and gives all players together *v(N)*. The set of imputations is a set that contains all reasonable outcomes – certain payoff vectors – for a cooperative game.

DEFINITION: *A payoff vector,* $x \in R^n$*, is an imputation in the game* $\Gamma = (N, v)$ *if* $x_i \geq \underline{u}_i$ *for all* $i \in N$ *(i.e., x is individually rational) and* $\sum_{i \in N} x_i = v(N)$ *(i.e., x is group rational). The set of imputations is denoted I(N,v).*

With a basic understanding of some of the essentials of characteristic function games, it is time to introduce some solution concepts of them, namely the core, the nucleolus and the Shapley value. Characteristic function games are convenient because they can often produce results with relatively little technical rigour.

The core is at the heart of cooperative games, dating back over a century. Edgeworth (1881) was the first to discuss it in economics literature. The core is a set of trades that cannot be ruled out as final trades. Put in another way, the core is a set of plausible equilibria. In these equilibria, each trader has a level of utility that is at least as great as the utility gained from acting alone.

DEFINITION: *Let C(N,v) be the core of a game* $\Gamma = (N, v)$*. The core is a subset of the set of imputations consisting of the imputations that are not dominated.*

Therefore, the core cannot reveal any precise answer to a problem. The nucleolus, on the other hand, is generally unique and consists of a single imputation. A payoff vector is in the nucleolus if the excesses for all coalitions for that payoff vector are made as small as possible. If the core is not empty, the nucleolus belongs to the core. More formally,

DEFINITION: *The nucleolus of a game (N,v) is the imputation that minimizes the function θ(x) in the lexicographic order*. (*Vector α is lexicographically smaller than β if* $\alpha_l = \beta_l$ *for* $1 \leq I \leq k$ *and* $\alpha_k \leq \beta_k$ *.)*

The last solution concept introduced in this paper is the Shapley value (Shapley, 1953). It can be calculated for any superadditive game in characteristic function form that has a finite number of players. It has a wonderful quality of satisfying both individual and group rationality. The payoff of each player is the weighted average of the contribution that he makes to the coalition he belongs to. The weighted payoff depends on the number of players in the coalition and the total number of players in the game.

Let $\phi(v)$ denote the Shapley value. The Shapley value payoff for player i is

$$\varphi_i(v) = \sum_{K \subset N} \left[v)K) - v(K \backslash \ \mathrm{i} \ ) \right] \frac{(k-1)!(n-k)!}{n!}$$

There are four conditions that characterize the Shapley value, each included in the above equation. These conditions are (a) group rationality, $\sum_{i \in N} \phi_i(v) = v(N)$; (b) if player i no more than v({i}) to any coalition, then i receives only v({i}); (c) if two games are identical except for the order in which the players are listed, then the Shapley values for the players are the same; (d) if a game is formed by adding two games together, the Shapley value of the new game is the sum of the values of the two original games.

4. Concluding Remarks

Game theory is a useful tool in describing the strategic interaction of the Arctic Treasure Hunt. It can be applied to situations with and without cooperation, and can yield useful results for policy-makers. Although this paper makes several simplifying assumptions, including the distribution of oil and policy choices of countries, it highlights the fact that no country can plan ahead without considering the strategies of other countries.

While the scope of this paper is limited to the theoretic modeling of the problem, future research can offer important applicable results. For example, it would be interesting to determine which coalitions are desirable when using real data. As was mentioned in the introduction, there has already been speculation that some of the countries might combine their forces and work together in order to gain extra benefits.

It would also be interesting to consider how a grand coalition might contemplate on competing with OPEC if the Arctic oil reserves do end up being as enormous as estimated. This could have significant consequences on the price of crude oil. Also, it is worth noting that some of the competing countries (Russia, the United States and Norway) are already major players in the oil industry.

However, it would be foolhardy to rush into decisions just based on the few game-theoretic models presented in this paper. The time-frame for the Arctic Treasure Hunt is long, and in the next 50 or

100 years major technological breakthroughs may take place, dwarfing oil's role as the most important energy source. This is one of the many factors that the models in this paper do not account for, and that could bring additional headache for decision-makers. Is it rational for a country to, for instance, refrain from drilling oil in the hope of making a much greater profit later when the oil fields of other countries have dried up and the oil price has sky-rocketed? What if an alternative fuel is invented before that? When is such an alternative energy source expected to be invented and should funds be channelled to that rather than to the Artic Treasure Hunt?

Acknowledgements:

We are very grateful to Pedro Pintasilgo and Marko Lindroos for their comments. The second author is also grateful to Nordic Marine Academy and *Sida* for financial support.

References

Crawford, V. P., Sobel, J. & Takahashi, I. (1984). Bargaining, Strategic Reserves and International Trade in Exhaustible Resource. *American Journal of Agricultural Economics,* 66 (4), 472 -480.

Dasgupta P. S. & Heal, G. M. (1995). *Economic Theory and Exhaustible Resources*. (Cambridge: Cambridge University Press).

Edgeworth, F.Y. (1881). *Mathematical Physics*. (London: Kegan Paul).

Eswaran, M. & Lewis, T. (1985). Exhaustible Resources and Alternative Equilibrium Concepts. *Canadian Journal of Economics,* XVII (3), 459 – 473.

Friedman, J.W. (1991). *Game Theory with Applications to Economics*. (New York: Oxford University Press).

Gately, D. (1974). Sharing the gains from regional cooperation: A Game Theoretic Application to Planning Investment in Electric Market, *International Economic Review,* 15 (1), 195 – 208.

Guth, W. (1988). Game theory and the nuclear arms race - The strategic position of Western Europe, *European Journal of Political Economy*, 4, 245-261.

Hendricks, K. & Kovenock, D. 1(989). Asymmetric Information, Information Externalities, and Efficiency: The Case of Oil Exploration**.** *The RAND Journal of Economics*, 20, 164-182.

Hoffman, E., Marsden, J. & Siadi, R. (1991). Are joint bidding and competitive common value auctions markets compatible - Some Evidence from Offshore Oil Auctions, *Journal of Environmental Economics and Management*, 20, 99 – 112.

Horvat, L. & Bogataj, L. (1999). A market game with the characteristic function according to MRP and input-output analysis model. *International Journal of Production Economics*, 59 (1999), 281 – 288.

Kronbak, L. G. & Lindroos, M. (2006). An Enforcement-Coalition Model: Fishermen and Authorities Forming Coalitions. *Environmental & Resource Economics*, 35, 169 – 194.

Lindroos M. & Kaitala V. (2000). Nash Equilibria in a Coalition Game of the Norwegian Spring-spawning Herring Fishery. *Marine Resource Economics*, 15, 321 – 340.

Mesterton-Gibbons, M. (2001). *An Introduction to Game Theoretic Modelling*. 2nd edition, (Providence: American Mathematical Association).

Milgrom, P. & Roberts, J. (1990). Rationalizability, learning and equilibrium in games with strategic complementarities, *Economica,* 58, 1255 – 1278.

Shapley, L.S. (1953). A value for *n*-person games. (In H. W. Kuhn & A. W. Tucker (Eds.), *Annals of mathematics studies (pp. 307 – 317).* Princeton: Princeton University Press).

Simaan, M. & Cruz, J. B. (1975). Formulation of Richardson's Model of Arms Race from a Differential Game Viewpoint. *Review of Economic Studies,* 42 (1), 67 -77.

Topkis, D. (1979). Equilibrium points in nonzero-sum n-person submodular games. *Journal of Control and Optimization,* 17, 773 – 787.

Vives, X. (1990). Nash equilibrium with strategic complementarities. *Journal of Mathematical Economics,* 19, 305 -321.

von Neumann, J. & Morgenstern, O. (1944). *Theory of Games and Economic Behavior*. (Princeton, NJ: Princeton University Press).

Yenikeyeff, S. M. & Krysiek, T. F. (2007). The Battle for next Energy Frontier: The Russian Polar Expedition and the Future of Arctic Hydrocarbons, *Oxford Energy Comment,* August 2007.

Young, O. R. (1986). The Age of the Arctic. *Foreign Policy,* 61, 160 – 179.